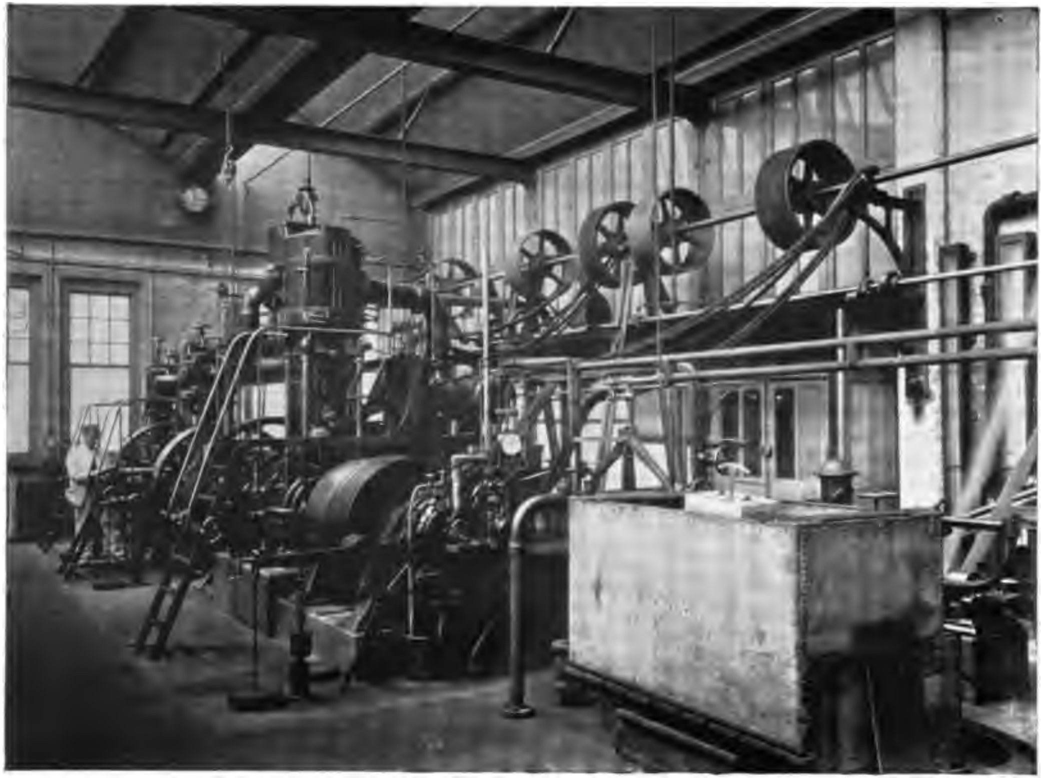

EXPERIMENTAL ENGINE.

Frontispiece. Man. St. Eng., Pt. I.

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A MANUAL

OF THE

STEAM-ENGINE.

FOR ENGINEERS AND TECHNICAL SCHOOLS;

ADVANCED COURSES.

PART I.

STRUCTURE AND THEORY.

BY

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SIXTH REVISED EDITION.

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PREFACE TO EDITION OF 1900.

In revising this volume for a new edition to be issued in the closing year of the nineteenth century, the author and publishers desire to renew their acknowledgments of the appreciation shown by readers of their efforts to keep their work fully up to date, and to state that the endeavor has here been made to exhibit the growth of the art and the science of steam-engine design, construction, and operation in that period which has seen its practical introduction as the motor of the civilized world, and its perfection in such degree as leaves little expectation of either rapid or extensive improvement, in structure or operation, hereafter.

The critical reader and the practitioner will find new illustrations, recent data, and some important, although not extensive, changes in the text and some notable matter in the appendix. No changes have been made for the mere purpose of introducing novelty; but no hesitation has been felt regarding any however troublesome and costly breaking up of plates and reconstruction where it has seemed possible to perfect the text or needed to bring the work fully up to its date.

Thanks are due many readers of earlier editions for kindly and appreciative comments and still more for equally kindly and still more helpful notice of detected misprints and for suggestions of possible improvement. It will be found by them that their suggestions have not been overlooked.

This "end of the century edition" is presented to its

prospective readers as a compendium of the work of the century in the evolution of the steam-engine as the motive power of the industrial world. The ninteenth century will forever be recognized as that period in which the steam-engine, the gas-engine, the factory system, the systems of all manufactures, the railroad, the steamship, the automobile common-road carriage, the telegraph, the telephone, the whole modern mechanical system of production and distribution of material and machines, of foods, and of all other products, and even the transmission and distribution of intelligence were practically introduced and substantially perfected. We cannot expect any coming century to leave such a record of complete evolution in these departments of progress.

SIBLEY COLLEGE, CORNELL UNIVERSITY,
ITHACA, N. Y., July, 1900.

PREFACE.

In the work of which this is the first volume, the endeavor has been to condense the essential facts and principles constituting the theory of the steam-engine, both in the ideal form usually assumed by older writers and in the actual form familiar to the practitioner, and also to give the more important facts and methods of its design, construction, maintenance, operation and trial. The first part contains the salient points of theory and an account of the gradual development of the engine from the crude forms of earlier times to the elegant and efficient types familiar to the engineer of to-day, and also a description of the general structure and the various special forms of the modern engine. The second volume gives the principles of general design, of the construction of the details of the machine, and the methods of operation and repair found satisfactory in recent practice.

In the construction of this work, it has been assumed that the reader is familiar with the higher mathematics and the principles of thermal physics, and generally well-read in those subjects which constitute the essential scientific basis of the professional training of the engineer. This assumption, which, a generation ago, would have been unjustifiable, is to-day perfectly reasonable. The profession of engineering has become one of the learned professions in a single generation, a consequence of the rapid development of the system of technical education now forming an essential and, often, the most extensive department of modern education in all civilized countries. The book is intended especially for the use of educated, practising engineers and of students, undergraduate and graduate,

in those technical schools which are sufficiently extensive in curriculum, and which have so large a student body as to justify specialization and the offering of advanced courses of instruction; institutions which include graduate schools of professional, specialized, work; for example, in the mechanical engineering of railways, of naval construction, of steam-engine building.

In the introduction of the reference to the use of this work in technical schools, in its title, it is not assumed that many such schools can find time or place for such a treatise. It is considered that possibly a few may find in it work for the senior year of their undergraduate course, and that still fewer among existing schools may find the two volumes and appropriate collateral reading suitable work for a year in graduate schools of steam-engineering. It is only in the highest class of such undergraduate schools and in a few special graduate schools that it would be justifiable to attempt such an extended course of instruction in this department. It is in part for such cases in Sibley College and elsewhere that it has been prepared.

Referring to the general plan and to the special and characteristic matter of the work, it will be observed that it differs greatly from other treatises on the subject, and that an attempt is here made to construct a theory of application for the *real* engine. In earlier works, no such attempt was made. The thermodynamic theory, that of the ideal engine, was long since completed; but the same statement could not be made in regard to the real engine. It has seemed to the Author that the subject has now reached such a stage, in its development, though still by no means complete or wholly satisfactory, that some advance might be made toward that end which only would be accepted by the practitioner as the true purpose of applied theory. In this belief, he has planned and worked out this scheme, in which he has endeavored to embody the most recent and useful results of the later researches of engineers and physicists looking toward this reduction of the theory of the steam-engine to a practically applicable form.

The work will, ere long, undoubtedly, seem, in view of further progress, crude and unsatisfactory; but we may at least hope that it cannot be long before some later writer will achieve full success.

In the construction of the theory of the engine—ideal and real—the purely thermodynamic theory is first given form, and in this the general methods of Rankine and Clausius, substantially identical, and, after a generation, entirely unchanged by their successors, are adhered to. In detail, the work of Clausius, and his methods, are mainly followed in the production of the principal equations of thermodynamics; then, in application, the course taken by Rankine is adopted. Rankine's initial processes are too obscure for the first part of the work, but those of application are admirably simple and convenient. Clausius, developing his equations with beautiful precision, and in simple, logical, and exact mathematical ways, is less satisfactory when we come to deal with the practical problems of the engineer. Combining the two, we obtain what has seemed to the Author a much more satisfactory system than either, as originally presented. The theory of the Real Engine, the "experimental theory" as Hirn called it, is necessarily still incomplete and imperfect. The facts and laws of internal wastes of heat in the engine are as yet too imperfectly understood to permit the framing of an exact theory of this part of the subject; but, fortunately, so much work has been done that we are now come to a point which permits us to formulate a provisional theory, and to adopt processes of computation sufficiently accurate, in many cases, to at least afford the engineer some assistance in his endeavor to anticipate what may be hoped for in the performance of the machine, the design of which he may have taken in hand.

The treatise of Professor Rankine, now ranked among the noblest of the engineer's classics, was published in 1859. The Author, then just out of college and engaged in steam-engine design as a special line of professional work, in the old firm of Thurston, Green & Co., probably like many other young engineers, read the work with avidity, anticipating that it might

give him an applied theory of the heat-engines, and a guide in their design and proportioning. But the results of thermodynamic computation were in such evident disaccord with the practice of the time that he threw it aside as disappointing and misleading. Later, during ten years and more of service in the U. S. N. Engineer Corps, a considerable part of the time in active service at sea, during the civil war and later, and during a half-dozen years of duty at the Naval Academy, detailed to give instruction in the departments of physics, chemistry, and applied mechanics, the works of Rankine, of Clausius, and of their numerous successors and imitators, were in constant use by the Author, and he still found that the same broad gulf between the pure and the applied theory, or rather the same deficiency of an applied science of the heat-engines, rendered it impossible for the engineer to make practical use of works on thermodynamics in his work of constructing engines for specified conditions. Practical experience was the only guide—a light only from the past. It was only when Professor Cotterill made the experimental work of Clark, of Hirn, of Isherwood, and of Emery a basis for his beautiful treatise on "The Steam-engine considered as a Heat-engine" that engineers began to find the thermodynamic theory, now supplemented by something approximating a satisfactory study of losses of heat and of work, really useful in office-work.

In the course of, now, twenty-five years of unintermitted employment as a specialist in technical college work, of thirty years of practical experience and work in the design, the construction, the management, and the scientific investigation of the principles of the steam-engine, the Author has been much interested in watching the gradual closing of this gap between the ideal and the real case, and the slow but steady growth of a philosophy of the real heat-engine competent to at least direct and aid, if not to form an exact science of the subject. In this development of an applied science, the honors are won by the engineers who have undertaken—however crudely, judged by the refined methods of modern science—to ascertain by experimental investigation precisely how heat-energy en-

tering the engine is distributed by transfer and transformation into useful and useless work, and to what extent it is subject to waste as heat. The mathematical physicists gave us the thermodynamic theory; but the engineers have been compelled to supply the essential complement, in order that we might make the science useful in engineering. When it became possible to write out a correct balance-sheet of itemized receipts and expenditures, it was possible for the engineer to make the science of the steam-engine the basis of the most refined operations of his art in the design and construction of the engine and its adjustment to its purposes with maximum economical result.

The long-established thermodynamic theory of the heat-engines, supplemented by what is rapidly coming to be a well-understood extra-thermodynamic theory of wastes, constitutes the complete theory of the machine, its operation, and its efficiency.

The attention of scientific men and engineers, throughout the world, has now become so earnestly drawn toward this matter, and researches are so generally in progress, under the direction of so many skilled investigators, that it cannot be long before this thermal division of the theory of the engine will be as well developed and as well understood as is now the thermodynamic. That it will ever be possible to secure as simple expression of the physical laws involved can perhaps hardly be hoped, still less expected. The simple expressions adopted by the Author seem to him likely to prove representative of a class which will always supply the engineer with his working equations. As far as accuracy is concerned, the best that can ever be said of them, probably, is that they enable us to predict, more closely than the pure thermodynamic theory, the probable performance of the engine. In other words, they give the engineer processes of application, where, formerly, theory was often useless and sometimes even misleading. The supplementing of the pure theory of the ideal engine by the physical theory of the real engine gives us a theory of application that enables us to ascertain, in a general way, the

effects of variation of the conditions of operation, to approximately compute the demand for steam and fuel, and to determine the most economical proportions and method of use of the machine, as affected by the commercial conditions of its environment. It is only now that the true problem of the engineer in this field can be solved, the problem: How may a given quantity of mechanical energy and power be obtained, by transformation from its potential form in fuel at minimum total cost? The fact that this is the first attempt to give some consistency and unity to the theory of the real engine will be possibly accepted as a justification of the perhaps somewhat over-liberal introduction of illustrative examples, and of the occasional repetition of statements of the more essential facts and principles.

The concluding chapter in the first part of the work represents an attempt to make the later facts and recent theory of the engine a basis for an investigation impossible of completion earlier. The beautiful method of Rankine, modified by the introduction of the theory of the thermal wastes of the real engine, becomes applicable to the solution of a great variety of problems which were, formerly, entirely beyond the reach of the designer or of the operator of the machine. They are problems, nevertheless, of extreme importance, and, in fact, constitute the first step in the logical series of processes which lead to the final perfection of the design of an engine precisely adapted to its place and purpose, mechanically and commercially. As now applicable to the case of the real engine, they permit the substitution of a more accurate and correct method for that unscientific "guesswork" of earlier practice which is responsible for so many and such unfortunate failures of the designing engineer in the adaptation of the machine to its work. The Author is convinced that here, as in the further investigation of the internal wastes of the engine, the highest talent of the skilled in research may for a long time find profitable employment in effecting closer approximations and in finding better and more exact systems of development.

The now familiar distinction between the ideal and the

real engine also makes it easy to bring into strong relief the principles controlling the reduction of the wastes which constitute the distinctive feature of the latter, and to show how the various familiar expedients for narrowing the range between the two cases operate. The theory of the compound engine, of jacketing, of superheating, can to-day be readily constructed and the influence of these and other expedients looking toward the same end may be clearly seen. It thus becomes now possible to intelligently employ them and to judge when and to what extent, their use is desirable and justifiable on the ground of ultimate economy. The designer is beginning to find use for his theory, as now made an applied theory, in every direction. This will be further illustrated in the second part of this work when the proportioning of the compound engine is taken in hand. The general principles are exhibited in Chapter VI of the first part; while the computation of dimensions comes properly into the second, which includes the designing of parts in detail. The computations of efficiencies for the single and multiple-cylinder engines, introduced into Chapter VI in Part I, as in all other cases, must be taken as illustrative only. The engineer must, in every case in his own practice, satisfy himself as to the exact conditions involved and determine for himself the precise values of the quantities to be employed in his own computations. No two cases are likely to involve the same conditions or give the same figures.

Part II deals with the designing, the construction, the operation and maintenance, and the determination of the power and efficiency of the engine. The principles of both parts of the work are summarized by a chapter on specifications and contracts. The discussion of the principles of regulation, of governor-construction, of the action of reciprocating parts, of the designing and proportioning of valve-motions, also fall into this division of the work.

In the preparation of the whole, every known available source of information has been resorted to, and, in many instances, in the absence of such records of fact, the Author has, as in the

preparation of his work on the Materials of Engineering, at an earlier date, been compelled to resort to experiment and to secure by direct investigation the facts considered by him essential to the completion of his task. Fortunately, the rapid progress of technical schools, and the general introduction of research as a feature of their higher work, are making this part of the work vastly easier and more satisfactory by constantly bringing into light new areas of the previously unexplored field. It has been the intention of the Author to give every essential reference to such authorities as he has consulted; their number and variety may give some idea of the magnitude of the task which has been here assumed, and justify, in some small measure, its imperfections.

PREFACE TO FOURTH, REVISED, EDITION.

THE favorable judgment of the profession and of readers, critical and other, has permitted the Author to print several editions of this work, and has secured him opportunities for more or less extensive revision at short intervals; thus keeping its contents well up to date and allowing, also, correction of such errors as have been discovered in the earlier issues. This kindly and appreciative reception by readers on both sides the Atlantic, and especially the translation of the work into French, by the talented and competent technicist, M. Demoulin, engaged for that work by MM. Baudry & Cie.,* have greatly encouraged the publishers and the Author in their endeavor to make the treatise as complete and perfect as possible, without regard to time or cost. The present revision has afforded opportunity to add a chapter in which are given the latest aspects of the applied theory of the steam-engine, the newer graphical methods of treatment of steam-engine efficiencies, and descriptions of the most modern advances in the science and the art of steam-engine design, construction, test-trial, and operation. Profuse illustration has given a combination of fulness with brevity, in discussion, that only thus could be secured.

The "standards of efficiency" are discussed, their characteristics noted, their relations to the various efficiencies of the Ideal and the Real Engine are shown, and the progress of energy from the potential store, in the fuel, through the furnace, the boiler, the engine, and the condenser, exhibited, and the

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* Traité de la Machine à Vapeur; par R. H. Thurston; Baudry & Cie., Paris, 1893.

methods of utilization and waste are described and illustrated, as exemplified in the action of the "real engine" of the latest and best types. The effects of variation of load, of speed, and of character of working fluid are traced to their influence upon the final and total efficiency, and on the economy of the engine, as measured in heat, steam, fuel, and money. The limit of the latest and best practice is brought into view as determined by results of trials of famous or peculiar engines. It is thought that it will be found that there is thus produced and illustrated, in application at least, an approximation to that "complete and perfect" theory of the real steam-engine, the need for which was discussed in the preface to the first edition of this work.

Minor errors have been, meantime, corrected and numerous improvements and changes of text have been ventured upon, in the hope that the presentation of the subject may be found more precise, explicit, and clear. The appendix will be found to contain a considerable amount of new and, it is hoped, valuable matter, reference and explanatory, which may add to the value of the book sufficiently to give full compensation for its increased bulk. No radical changes have been made; neither formal nor informal, kindly nor other and harsher criticism having developed any serious errors of fact or of philosophy. The authoritative critics have all dealt most kindly with this attempt to modernize the theory of the steam-engine, and the Author heartily appreciates the many kind and encouraging words which have been bestowed upon his work by distinguished members of the profession on both sides the Atlantic.

A MANUAL OF THE STEAM-ENGINE.

PLAN.

PART I. STRUCTURE AND THEORY.

PART II. DESIGN, CONSTRUCTION, OPERATION.

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The modern Savery Engine (pp. 8–10) with automatic action.

PART I.

HISTORY, STRUCTURE, AND THEORY

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MANUAL OF THE STEAM-ENGINE

PART I.

CHAPTER I.

THE DEVELOPMENT OF THE STEAM-ENGINE.

1. **The Purpose of any Heat-engine** is the useful and economical transformation, in the largest possible degree, of the heat-energy derived from combustion, or other source, and temporarily stored, in greater or less quantity, in a fluid capable of variation of pressure and volume with changes of heat and of temperature and pressure. In all familiar forms this heat is derived from the combustion of coal, or of some product of fuel-distillation, natural or artificial, and is transferred from the products of combustion to the working fluid, which may be gas, air, steam, or other vapor; or it may be that the storage medium and the vehicle of transfer, the working fluid, is the mixture composing those products of combustion themselves.

2. **The General Methods of Energy-transformation** are the same for any working substance. It is caused to undergo such changes of pressure, volume, and temperature as will effect the conversion of a portion of the stored heat-energy into mechanical energy, usually by driving a piston, but very rarely by the reaction of a jet passing out from under high pressure and at very high velocity. During these changes the fluid drives the piston forward by its expansion at comparatively high temperature and pressure, and is, later, compressed by the

piston on its return-stroke at a lower temperature and pressure; the net work done being thus a positive quantity and measured by the difference in the amount of work done, positively and negatively, in the complete revolution of the crank of the engine and a double-stroke of the piston.

3. **Heat-engines are classified** variously: as according to the physical state of their working fluids; according to the specific fluid used; or according to the method of their operation of that fluid. Thus we have gas-engines, vapor-engines, binary-vapor engines; or, we have steam-engines, ammonia or carbon-disulphide engines; petroleum-vapor engines; illuminating-gas engines; or, engines employing working fluids of constant or variable weight. All are, however, subject to the same general principles of heat-transformation and, ordinarily, to the same methods of thermal or thermo-dynamic, or of dynamic, waste.

In all cases their operation involves the thermo-dynamic science of the purely ideal engine, combined with the physical science of heat as applied to the phenomena of real engines. The steam-engine represents simply a single case among numerous heat-engines and motors; and its problem is merely a single application of principles involved in the philosophy of all.

4. **The Definition of a Steam-engine** may be enunciated thus:

The *steam-engine* is a machine designed and constructed especially for the purpose of converting the heat-energy stored in the vapor of water, in as large proportion as may be practicable, into dynamical, or mechanical, energy, and to apply that energy as directly and effectively as possible to the performance of useful work.

It may consist of a single element, or vessel, as in the oldest form of steam-engine—to be presently described; or it may, as in modern forms of engine, consist of a train of mechanism of considerable complexity. It may actuate a reciprocating system, as in pumping-engines of several forms; or it may turn a shaft; it may even impel a projectile, as in Perkins' steam-

gun; but, in all cases and in all forms, it is a thermo-dynamic machine, subject to thermo-dynamic and thermal losses and to wastes of dynamical energy.

5. **The Origin and Growth of the Steam-engine** are historically notable for great antiquity and long and, until within a century, slow progress. Precisely when the power of steam began to attract the attention of mankind is quite unknown; but it was certainly before history had begun to record any other than political events and before any industrial developments, any inventions, any useful art had become a matter of notice among historians. The people of some early prehistoric time deified their great mechanics and inventors, as they did their great warriors; but at the beginning of historic times this appreciation of those classes had largely ceased.

The first period of invention of the steam-engine was one of purely speculative knowledge, and it was known, at some time before the Christian era, as simply a toy, and the force of steam was only thought of as possibly applicable to the purposes of the priestly prestidigitators of that time. This period of speculation continued until the middle of the seventeenth century, when the Marquis of Worcester and his contemporaries and predecessors sought to make useful application of the latent powers of steam. A second period of application was thus inaugurated which continued up to the end of the first quarter of the nineteenth century; when, the inventions of Watt and others having revealed the value, the power, and the wide adaptability of the machine, in all its principal forms, a third period of refinement and of improvement in all details and all applications brought the engine into substantially its existing form.*

6. **Hero's Engine** is described by Hero the Younger of Alexandria and dated about 120 B.C., and here we find the first record of the early history of the steam-engine.

In the home of Euclid, the great geometrician, and possibly contemporary with that talented engineer and mathematician

* History of the Steam-engine; R. H. Thurston. New York: D. Appleton & Co. International Series.

Archimedes, Hero produced a manuscript which he entitled "Spiritalia seu Pneumatica." The work is still extant, and has been several times republished. In it are described a number of interesting though primitive forms of water and heat engines, and, among the latter, that shown in Fig. 1,* an apparatus moved by the force of steam.

This earliest of steam-engines consisted of a globe suspended between trunnions, through one of which steam enters through pipes from the boiler below. The hollow bent arms cause the vapor to issue in such a direction that the reaction produces a rotary movement of the globe, just as the rotation of reaction water-wheels is produced by outflowing water.

It is quite uncertain whether this machine was ever more than a toy, although it has been supposed by some authorities that it was actually used by the Greek priests for the purpose of producing motion of other apparatus in their temples.

FIG. 1.—HERO'S ENGINE, B.C. 200.

It seems sufficiently remarkable that, while the power of steam had been, during all the many centuries that man has existed upon the globe, so universally displayed in so many of the phenomena of natural change, mankind lived almost up to the Christian era without making it useful in giving motion even to a toy; but it must excite still greater surprise that, from the time of Hero, we meet with no good evidence of its application to any practical use for many hundreds of years. Here and there, in the pages of history and in special treatises, we find a hint that the knowledge of the force of steam is not forgotten; but biographers and his-

* *Vide* Woodcroft's "Translation of Hero." The cut is from Thurston's History of the Steam-engine

torians have devoted little time to the task of seeking and recording information relating to the progress of this and other important inventions and improvements in the mechanic arts.

7. **Early Knowledge of Steam** and of its power was confined to the understanding that the vapor of water was capable of exerting some force in its exit from closed vessels, and that it might be given application to a few simple and unimportant operations. Hero shows a variety of such applications, some of them very ingenious but all of no importance. For example, he sketches and describes methods of applying the expansive force of steam to the opening and closing of temple doors, to the working of various automata, and to the production of sounds. Nothing indicates that any ancient writer or mechanic had the slightest idea or expectation of the future use of this, to them, concealed power in the operations of the arts.

8. **Steam-power in the Middle Ages** was but little better understood and appreciated than in earlier times. "Æolipiles," such as Hero's machine for use as a turnspit, and the various forms of apparatus in which steam was produced and from which it was allowed to issue in a jet for the purpose of "blowing the fire," seem to have been the earliest and latest productions of this period; although predictions of a later application to important purposes were sometimes made by the speculative philosophers and inventors of those centuries succeeding the tenth and up to about the beginning of the seventeenth. At this latter date a number of crude schemes and rude forms of apparatus, as those of Porta (1601), of Da Caus (1615), and of Branca (1629), were suggested by various ingenious philosophers and writers; but none seems to have been actually constructed and used, even experimentally, until later.

9. **The Marquis of Worcester, and Papin** the distinguished contemporary physicist and philosopher, were the first of these schemers who seem to have actually constructed their apparatus.

In 1663 Edward Somerset, second Marquis of Worcester, published a curious collection of descriptions of his inventions, couched in obscure and singular language, and called a "Century of the Names and Scantlings of Inventions by me already practised." One of these inventions is an apparatus for raising water by steam. The description was not accompanied by a drawing, but the sketch here given probably resembles his contrivance very closely. Steam is generated in the boiler *D*, and thence is led into the vessel *A*, already nearly filled with water. It drives the water in a jet out through a pipe, *F* or *F'*. The vessel *A* is then shut off from the boiler and again filled "by suction," after the steam has condensed, through the pipe *G*, and the operation is repeated, the vessel *B* being used alternately with *A*.

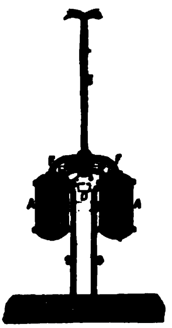

FIG. 2. WORCESTER'S ENGINE, A.D. 1650.

This apparatus was used for the purpose of elevating water for practical purposes at Vauxhall, near London. It was still earlier used at the home of Worcester, Raglan Castle, where the openings cut in the wall for its reception are still to be seen. The *separate boiler*, as here used, constitutes a very important improvement upon the preceding forms of apparatus, although the idea was original with Porta.

The "water-commanding engine," as its inventor called it, was, therefore, the first instance in the history of the steam-engine in which the inventor is known to have "reduced his invention to practice."

It is evident, however, that the invention, important as it was, does not entitle the marquis to the honor claimed for him by many authorities of being *the inventor* of the steam-engine. Somerset was simply *one* of those whose works collectively make the steam-engine.

The invention of the Marquis of Worcester was revived twenty years later by Sir Samuel Morland, but in what form is not now known. In a memoir which he wrote upon the subject in 1683, he exhibited a degree of familiarity with the properties of steam that could hardly have been expected of

any one at that early date. In his manuscript, now preserved at the Haarlem Collection of the British Museum, he states the size of the cylinders required in his machine to raise given quantities of water per hour, and gives very exactly the relative volumes of equal weights of water and of steam under atmospheric pressure. He tells us that one of his engines, with a cylinder six feet in diameter and twelve feet long, was capable of raising 3240 pounds of water through a height of six inches, 1800 times an hour.

From this time forward the minds of many mechanicians were earnestly at work on this problem—the raising of water by aid of steam. Hitherto, although many ingenious toys, embodying the principles of the steam-engine separately, and sometimes, to a certain extent, collectively, had been proposed and even occasionally constructed, the world was only just ready to profit by the labors of inventors in this direction. But, at the end of the seventeenth century, English miners were beginning to find the greatest difficulty in clearing their shafts of the vast quantities of water which they were meeting at the considerable depths to which they had penetrated, and it had become a matter of vital importance to them to find a more powerful aid in that work than was then available. They were, therefore, by their necessities, stimulated to watch for, and to be prepared promptly to take advantage of, such an invention when it should be offered them. The experiments of Papin, and the practical application of known principles by Savery, placed the needed apparatus in their hands.

When Louis XIV. revoked the Edict of Nantes, the persecutions at once commenced drove from the kingdom some of its greatest men. Among these was Denys Papin, a native of Blois and a distinguished philosopher. He studied medicine at Paris, and, when expatriated, went to England, where he met the celebrated philosopher Boyle, who introduced him into the Royal Society, of which Papin became a member and to whose "Transactions" he contributed several valuable papers. He invented, in 1680, the "Digester," in which substances, unaffected by water boiling under atmospheric pressure, can be

subjected to the action of water boiling under high pressure, and thus thoroughly "digested" or cooked. The danger of bursting these vessels caused him, in 1681, to invent and apply the *lever safety-valve*,* now an indispensable appurtenance to every steam-boiler.

In 1690 he constructed a working model of an engine, consisting of a steam-cylinder with a piston which was raised by steam pressure, and which descended again when the condensation of the steam produced a vacuum beneath it. This apparatus the inventor proposed to use as a motor for working pumps and for driving paddle-wheels; but he never built a successful working machine on this plan, so far as we can ascertain.†

Papin, in 1707, proposed to avoid the loss due to condensation of steam in the vessel to some extent at least by the use of his piston, which he interposed between the steam and the water.‡ This engine is in principle a Marquis of Worcester engine, in which the piston is introduced to separate the steam from the water which it impels, and thus to reduce the amount of loss by condensation. This engine was never constructed except experimentally, however, and is principally of interest in a history of the steam-engine from the fact that it was a useful suggestion to succeeding inventors.

10. **Savery's "Fire-engine"** was the first among all the earlier devices which came into actual use in the application of the energy stored in steam to the purposes of industry.

The constant and embarrassing expense and the engineering difficulties presented by the necessity of keeping the British mines, and particularly the deep pits of Cornwall, free from water, and the failure of every attempt previously made to provide effective and economical pumping machinery, were

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* Other forms of safety-valve had been previously used.

† "Recueil des diverses Pièces touchant quelques nouvelles Machines et autres Sujets philosophiques," M. D. Papin, Cassel, 1695.

‡ "Nouvelle Manière de lever d'Eau par la Force de Feu, mise en Lumière." Par M. D. Papin, Docteur en Médecine, Professeur en Mathematique à Cassel, 1707.

noted by Savery, who, July 25, 1698, patented the design of the first engine which ever was actually employed in this work.

A working model was submitted to the Royal Society of London in 1699,* and successful experiments were made with it. This engine is shown in Fig. 3, as described by Savery himself in 1702 in the "Miners' Friend." *L L* is the boiler, in which steam is raised, and through the pipes *O O* it is alternately let into the vessels *P P*.

Suppose it to pass into the left-hand vessel first. The valve *M* being closed and *r* being opened, the water contained in *P* is driven out and up the pipe *S* to the desired height, where it is discharged. The valve *r* is then closed, and also the valve in the pipe *O*. The valve *M* is next opened, and condensing water is turned upon the exterior of *P* by the cock *Y*, leading water from the cistern *X*. As the steam contained in *P* is condensed, forming a vacuum, a fresh charge of water is driven by atmospheric pressure up the pipe *T*. Meantime, steam from the boiler has been let into the right-hand vessel *P*, the cock *W* having been first closed and *R* opened. The charge of water is driven out through the lower pipe and the cock *R*, and up the pipe *S* as before, while the other vessel is refilling preparatory to acting in its turn. The two vessels thus are alternately charged and discharged as long as is necessary. Savery's method of supplying his boiler with water was at once simple and ingenious.

The small boiler *D* is filled with water from any convenient source, as from the stand-pipe *S*. A fire is then built under it, and when the pressure of steam in *D* becomes greater than in the main boiler *L*, a communication is opened between their lower ends and the water passes under pressure from the smaller to the larger boiler, which is thus "fed" without interrupting the work. *G* and *N* are *gauge-cocks* by which the height of water in the boilers is determined, and these attachments were first adopted by Savery.

Here we find, therefore, the first really practicable and

* "Transactions of the Royal Society," 1699.

commercially valuable steam-engine. Thomas Savery is entitled to the credit of having been the first to introduce into general use a machine in which the power of heat, acting through the medium of steam, was rendered useful. It will be noticed that Savery, like the Marquis of Worcester and like Porta, used a boiler separate from the water-reservoir. He added to the "water-commanding engine" of the Marquis the system of *surface-condensation*, by which he was enabled to change his vessels when it became necessary to refill them; and the secondary boiler, which enabled him to supply the working boiler with water without interrupting its action. The machine was capable of working uninterruptedly for a period of time only limited by its own endurance. Savery never fitted his boilers with the safety-valve, although it was subsequently used on Savery engines by Desaguliers; and in deep mines he was compelled to make use of higher pressures than his rudely-constructed boilers could safely bear. The introduction of his machines was therefore greatly retarded by the fear, among miners, of the explosion of his boilers. In fact, such explosion did occur on more than one occasion.

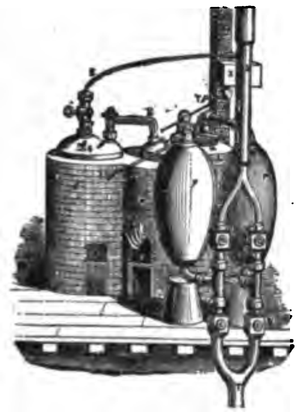
FIG. 3.—SAVERY'S ENGINE, A.D. 1699.

The Savery engine was improved, about 1716 or 1718, by Dr. Desaguliers, who attached to it Papin's safety-valve, and substituted a jet-injection from the stand-pipe into the "forcing-vessels" for the surface-condensation of Savery's original arrangement. The Savery engine, however, after all improvement in design and construction, though a working and a useful machine, was still a very wasteful one. The steam from the boiler, passing into the cold, wet water-reservoir or forcing-vessel, was condensed in large quantity, and also to a very

serious extent, by coming into actual contact with the water itself.

11. **The Performance of the Savery Engine** was thus evidently unsatisfactory, as judged from the modern standpoint; yet, as the first machine applying natural forces to a great task, and for the first time accomplishing it, it was a grand success. The operation of deep mines had become impracticable where water was met with in any considerable quantity, and, in some cases, hundreds of horses had been kept employed, at enormous and even fatal expense, to keep the lower levels in working. These were displaced by steam and the Savery engine, and mines which must otherwise have been abandoned were once more made profitable.

The defects of this class of engines were nevertheless great. Their enormous consumption of fuel was one serious difficulty everywhere except in the coal districts; their heavy pressures needed at deep shafts and for high lifts gave rise to dangers which threatened constantly both life and property when, as was very usual, the workmanship of the "forcing-vessel" was defective. In fact, the invention of the Savery engine was introductory of the steam-boiler explosion; several of the boilers exploding while at work and doing some damage. This new and intimidating experience, and the evident wastefulness of the machine, led mechanics, very soon, to study the problem anew with a view to improvement in these respects; its extravagant consumption of fuel, the inconvenient necessity of placing it near the bottom of the mine to be drained, and of putting in several for successive lifts where the depth was considerable, and, especially, the risk which its use with high pressures involved even in its best form, had considerably retarded its introduction, and it therefore came into use very slowly, notwithstanding its superiority in economic efficiency over horse-power.

Many years after Savery's death, in 1774, Smeaton made the first duty-trials of engines of this kind. He found that an engine having a cylindrical receiver 16 inches in diameter and 22 feet high, discharging the water raised 14 feet above the

surface of the water in the well, making 12 strokes, and raising 100 cubic feet per minute, developed $2\frac{5}{8}$ horse-power, and consumed 3 hundredweight of coals in four hours Its duty was, therefore, 5,250,000 pounds raised one foot per bushel of 84 pounds of coals, or 62,500 "foot-pounds" of work per pound of fuel. An engine of slightly greater size gave a duty about 5 per cent greater.*

12. **Newcomen's Engine.**—The first important step taken towards remedying the defects of Savery's machine was taken by Thomas Newcomen and John Cawley, or Calley, two mechanics of the town of Dartmouth, Devonshire, England, who produced what has been known as the Atmospheric or Newcomen Engine. Newcomen was a blacksmith, and Cawley a glazier and plumber. It has been stated that a visit to Cornwall, where they witnessed the working of a Savery engine, first turned their attention to the subject; but a friend of Savery has stated that Newcomen was as early with his general plans as Savery. After some discussion with Cawley, Newcomen entered into correspondence with Dr. Hooke, proposing a steam-engine, to consist of a steam-cylinder containing a piston similar to those of Huyghens's and Papin's engines, and driving a separate pump, similar to those generally in use where water was raised by horse or wind power. Dr. Hooke advised and argued strongly against their plan; but, fortunately, the obstinate belief of the unlearned mechanics was not overpowered by the disquisitions of their distinguished correspondent, and Newcomen and Cawley attempted an engine on their peculiar plan.

This succeeded so well as to induce them to continue their labors, and in 1705 to patent †—in combination with Savery, who held the right of surface-condensation, and who induced them to allow him an interest with them—an engine combining a steam-cylinder and piston, surface-condensation, and a separate boiler and separate pumps. In the atmospheric en-

* History of the Steam-engine, R. H. Thurston, p. 45; Farey on the Steam-engine, p. 125.

† It has been denied that a patent was issued; but there is no doubt that Savery claimed and received an interest in the new engine.

gine as first designed, the slow process of condensation by the application of the condensing water to the exterior of the cylinder to produce the vacuum caused the strokes of the engine to take place at very long intervals. An improvement was, however, soon effected which immensely increased this rapidity of condensation. A jet of water was thrown directly *into* the cylinder, thus effecting for the Newcomen engine what Desaguliers had previously done for the Savery engine. As thus improved, the Newcomen engine is shown in Fig. 4.

FIG. 4.—NEWCOMEN'S ENGINE, A.D. 1705.

Here *d* is the boiler. Steam passes from it through the cock *d*, and up into the cylinder *a*, equilibrating the pressure of the atmosphere, and allowing the heavy pump-rod *k* to fall, and, by its greater weight, acting through the beam *i i*, to raise the piston *s* to the position shown. The cock *d* being shut, *f* is then opened, and a jet of water from the reservoir *s* enters the cylinder, producing a vacuum by the condensation of the steam. The pressure of the air above the piston now forces it down, again raising the pump-rods, and thus the engine works on indefinitely. The pipe *h* is used for the purpose of keeping the upper side of the piston covered with water, to prevent air-leaks—a device of Newcomen. Two gauge-cocks, *c*, *c*, and a safety-valve, *N*, are represented in the figure, but it will be noticed that the latter is quite different from the now usual form. Here, the pressure used was hardly greater than that of the atmosphere, and the weight of the valve itself was ordinarily sufficient to keep it down. The rod *m* was intended to carry a counter-weight when needed. The condensing water, together with the water of condensation, flows off through the open pipe *p*.

Newcomen's first engine made six or eight strokes a minute; the later and improved engines made ten or twelve.

The steam-engine had now assumed a form that somewhat

resembled the modern machine. An important defect still existed in the necessity of keeping an attendant by the engine to open and shut the cocks. A bright boy, however, Humphrey Potter, to whom was assigned this duty on a Newcomen engine, in 1713 contrived what he called a *scoggan*—a catch rigged with a cord from the beam overhead—which performed the work for him. The boy, thus making the operation of the valve-gear automatic, increased the speed of the engine to fifteen or sixteen strokes a minute, and gave it a regularity and certainty of action that could only be obtained by such an adjustment of its valves.

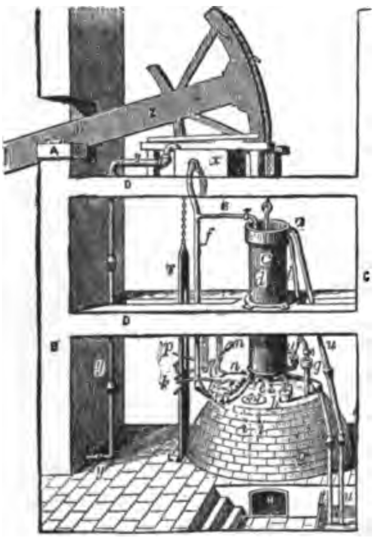
FIG. 5.—BEIGHTON'S VALVE-GEAR, A.D. 1718.

This ingenious young mechanic afterward became a skilful workman and an excellent engineer, and went abroad on the Continent, where he erected several fine engines. Potter's rude valve-gear was soon improved by Henry Beighton, and the new device was applied to an engine which that talented engineer erected at Newcastle-on-Tyne in 1718, in which engine he substituted substantial materials for Potter's unmechanical arrangement of cords, as seen in Fig. 5.

In this sketch *r* is a plug-tree, plug-rod, or plug-frame, as it is variously called, suspended from the great beam with which it rises and falls, bringing the pins *p* and *k*, at the proper moment, in contact with the handles *k k* and *n n* of the valves, moving them in the proper direction and to the proper extent. A lever safety-valve is here used, at the suggestion, it is said, of Desaguliers. The piston was packed with leather or with rope, and lubricated with tallow.

In illustration of the application of the Newcomen engine

to the drainage of mines, Farey describes a small machine, of which the pump is 8 inches in diameter, and the lift 162 feet. The column of water to be raised weighed 3535 pounds. The steam-piston was made 2 feet in diameter, giving an area of 452 square inches. The net working-pressure was assumed at $10\frac{3}{4}$ pounds per square inch; the temperature of the water of condensation and of uncondensed vapor after the entrance of the injection-water being usually about 150° Fahr. This gave an excess of pressure on the steam-side of 1324 pounds, the total pressure on the piston being 4859 pounds. One half of this excess is counterweighted by the pump-rods, and by weight on that end of the beam; and the weight, 662 pounds, acting on each side alternately as a surplus, produced the requisite rapidity of movement of the machine. This engine was said to make 15 strokes per minute, giving a speed of piston of 75 feet per minute, and the power exerted usefully was equivalent to 265,125 pounds raised one foot high per minute. As the horse-power is equivalent to 33,000 "foot-pounds" per minute, the engine was of $\frac{265125}{33000} = 8.034$—almost exactly 8 horse-power.

It is instructive to contrast this estimate with that made for a Savery engine doing the same work. The latter would have raised the water about 26 feet in its "suction-pipe," and would then have forced it, by the direct pressure of steam, the remaining distance of 136 feet; and the steam-pressure required would have been nearly 60 pounds per square inch. With this high temperature and pressure, the waste of steam by condensation in the forcing-vessels would have been so great that it would have compelled the adoption of two engines of considerable size, each lifting the water one half the height, and using steam of about 25 pounds pressure.

Further improvements were effected in the Newcomen engine by several engineers, and particularly by Smeaton, and it soon came into quite extensive use in all of the mining districts of Great Britain, and it also became generally known upon the Continent of Europe. Its greater economy of fuel as compared with the Savery engine in its best form, its greater safety

—a consequence of the low steam-pressure adopted,—and its greater working capacity, gave it such manifest superiority that its adoption took place quite rapidly, and it continued in general use in some districts where fuel was cheap up to a very recent date. Some of these engines are even now in existence. From about 1758 to the time of the introduction of the Watt engine, this was the machine in almost universal use for raising large quantities of water.

13. **The Merits and Demerits** of the Newcomen engine were those characterizing a novel and radically altered form of machine, which was the first of a new type: that which may be called the modern type of steam-engine. A complete revolution had been thus effected, and the genius of the great inventors had produced a more complete and thorough change of type than had been previously seen, or even than has ever been since effected by even Watt and his contemporaries and successors. It may then be said that, defining the steam-engine as a train of mechanism, Newcomen and Cawley were its inventors, and that their machine was the first steam-engine. The invention of the modern type of steam-engine is to be credited to them, and not to any of those later inventors who simply improved upon it in matters of detail. In this respect Newcomen antedates Watt.

Comparing the engine with those preceding it, we see that at first we find a single vessel performing the functions of all the parts of a modern pumping-engine; it was at once boiler, steam-cylinder, and condenser, as well as both a lifting and a forcing pump. The Marquis of Worcester, and, still earlier, Da Porta, divided the engine into two parts; using one part as a steam-boiler, and the other as a separate water-vessel. Savery duplicated those parts of the earlier engine which acted the several parts of pump, steam-cylinder, and condenser, and added the use of the jet of water to effect rapid condensation. Newcomen and Cawley next introduced the modern type of engine, and separated the pump from the steam-engine proper. In their engine, as in Savery's, we will observe the use of surface-

condensation first; and subsequently that of a jet of water thrown into the midst of the steam to be condensed.

Thus an engine was produced which, by the separation of the boiler from the engine, made it practicable to secure the economical production of steam by correct design and giving ample areas of heating surface. By the liberty thus gained, also, of proportioning the pumps independently, it was practicable to obtain the needed power with steam of low pressure; it became practicable to apply simply atmospheric pressure to the work, using steam simply to remove the atmosphere from the opposite side of the piston, thus at once and entirely evading all dangers coming of the employment of high pressures. Finally, by the separation of the engine from the other elements of the machine, it became possible to appreciably reduce the wastes by initial condensation of steam while doing its work of impulsion. It was by these several ways that an enormous advance was made, economically, in the application of steam to raising water.

The defects of the engine, as judged from a modern standpoint, were the great size and weight of the machine, relatively to its power; its still enormous consumption of steam and fuel; and its rude construction. It was still far from perfect in either design or construction, or satisfactory as to economical performance, even as finally built by Smeaton, the great engineer of that time who made its very best examples. The latter raised the best duty of the engine from about ten per cent to more nearly twelve per cent of that of the better class of modern pumping-engines.

Smeaton made a number of test-trials of Newcomen engines to determine their "duty"—i.e., to ascertain the expenditure of fuel required to raise a definite quantity of water to a stated height. He found an engine 10 inches in diameter of cylinder, and of 3 feet stroke, could do work equal to raising 2,919,017 pounds of water one foot high, with a bushel of coals weighing 84 pounds.

Thus, by the end of the third quarter of the eighteenth century, the steam-engine had become generally introduced, and

had been applied to nearly all of the purposes for which a single-acting engine could be used. The path which had been opened by Worcester had been fairly laid out by Savery and his contemporaries, and the builders of the Newcomen engine, with such improvements as they had been able to effect, had followed it as far as they were able. The real and practical introduction of the steam-engine is as fairly attributable to Smeaton as to any one of the inventors whose names are more generally known in connection with it. As a mechanic he was unrivalled; as an engineer he was head and shoulders above any constructor of his time engaged in general practice. There were very few important public works built in Great Britain at that time in relation to which he was not consulted; and he was often visited by foreign engineers, who desired his advice with regard to works in progress on the Continent.*

14. **James Watt** and his engine now come into view. The success of the Newcomen engine naturally attracted the attention of mechanics, and of scientific men as well, to the possibility of making other applications of steam-power. The greatest men of the time gave much attention to the subject; but until Watt began the work that has made him famous, nothing more was done than to improve the proportions and to slightly alter the details of the Newcomen and Cawley engine, even by such skilful engineers as Brindley and Smeaton.

This great man was born at Greenock, January 19, 1736. He was a bright boy, but exceedingly delicate in health, and quite unable to attend school regularly, or to apply himself closely to either study or play. At the age of eighteen Watt was sent to Glasgow, there to reside with his mother's relatives, and to learn the trade of a mathematical-instrument maker. The mechanic with whom he was placed was incapable of giving much aid in the project; and Dr. Dick, of the University of Glasgow, with whom Watt became acquainted, advised him to go to London. Accordingly, he set out in June, 1755, for the metropolis, where, on his arrival, he arranged with Mr. John

* History of the Steam-engine.

Morgan, in Cornhill, to work for a year at his chosen business, receiving as compensation twenty guineas. At the end of the year he was compelled by serious ill-health to return home. Having become restored to health, he went again to Glasgow, in 1756, with the intention of pursuing his calling there. Dr. Dick employed him to repair some apparatus which had been bequeathed to the college. He remained here until 1760, when he took a shop in the city, and in 1761 moved again into a shop on the north side of the Trongate, where he earned a scanty living, still keeping up his connection with the college. He spent much of his leisure time in making philosophical experiments. The introduction of the Newcomen engine in the neighborhood of Glasgow, and the presence of a model in the college collections, which model was placed in his hands in 1763 for repairs, led him to study the history of the steam-engine, and to conduct for himself an experimental research into the properties of steam, using a set of improvised apparatus.

15. **The Newcomen Model**, as it happened, had a boiler, which, although made to a scale from engines in actual use, was quite incapable of furnishing steam enough to work the engine. It was about nine inches in diameter, and the steam-cylinder was two inches in diameter, and of six inches stroke of piston. Watt at once noticed the defect referred to, and immediately sought, first the cause and then the remedy.

He soon concluded that the sources of loss of heat in the Newcomen engine—which loss would be greatly exaggerated in a small model—were: first, the dissipation of heat by the cylinder itself, which was of brass, and was both a good conductor and a good radiator; secondly, the loss of heat consequent upon the necessity of cooling down the cylinder at every stroke in producing the vacuum; and, finally, a loss of power was due to the existence of vapor beneath the piston, the presence of which vapor was a consequence of the imperfect method of condensation which characterizes the Newcomen engine.

He first made a cylinder of non-conducting material—wood soaked in oil and then baked—and found a decided advantgae

in the economy of steam thus secured. He then conducted a series of experiments upon the temperature and pressure of steam at such points in the scale as he could readily reach, and, constructing a curve with his results, the abscissas representing temperatures, and the pressures being represented by the ordinates, he ran the curve backward until he had obtained approximate measures of temperatures less than 212°, and of pressures less than atmospheric. He thus discovered that, with the amount of injection-water used in the Newcomen engine, bringing the temperature of the interior, as he found, down to from 140° to 175° Fahr., a very considerable back-pressure would be met with.

Continuing his research still further, he measured the amount of steam used at each stroke; and, comparing it with the quantity that would just fill the cylinder, he found that at least three fourths was wasted. The quantity of cold water necessary to produce condensation of a given weight of steam was next determined, and he found that one pound of steam contained enough heat to raise about six pounds of cold water, as used for condensation, from the temperature of 52° Fahr. to the boiling-point; and, going still further, he found that he was compelled to use, at each stroke of the Newcomen engine, four times as much injection-water as should suffice to condense a cylinder full of steam. Thus was confirmed his previous conclusion that three fourths of the heat supplied to the engine was wasted.

His experiments having revealed to him the now well-known fact of the existence of latent heat, he went to his friend Dr. Black, of the university, with this intelligence; and the latter then informed him of the Theory of Latent Heat which had but a short time earlier been discovered by Dr. Black himself.

Watt had now, therefore, determined by his own researches, as he himself enumerates them,* the following facts:

(1) The capacities for heat of iron, copper, and of some sorts of wood, as compared with water.

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* Robinson's "Mechanical Philosophy," edited by Brewster.

(2) The bulk of steam compared with that of water.

(3) The quantity of water evaporated in a certain boiler by a pound of coal.

(4) The elasticities of steam, at various temperatures greater than that of boiling water, and an approximation to the law which it follows at other temperatures.

(5) How much water, in the form of steam, was required, at every stroke, by a small Newcomen engine, with a wooden cylinder six inches in diameter and twelve inches stroke.

(6) The quantity of cold water required, at every stroke, to condense the steam in that cylinder, so as to give it a working power of about seven pounds on the square inch.

After these well-devised and truly scientific investigations, Watt was enabled to enter upon his work of improving the steam-engine with an intelligent understanding of its existing defects, and with a knowledge of their cause. It was on a Sunday afternoon, in the spring of 1765, that he devised his first and his greatest invention—the separate condenser. His object in using it was, as he says himself, *to keep the cylinder as hot as the steam that entered it.* He was therefore the first to apprehend and to state a problem which the modern engineer is still vainly endeavoring completely to solve.

Watt was, at this time, twenty-nine years of age. Having taken this first step and made such a radical improvement, the success of the invention was no sooner determined than others followed in rapid succession as consequences of the exigencies arising from the first radical change in the old Newcomen engine. But in the working out of the forms and proportions of details in the new engine, even Watt's powerful mind, with its stores of happily-combined scientific and practical information, was occupied for years.

In attaching the separate condenser, he first tried surface condensation; but this not succeeding well, he substituted the jet. Some provision became at once necessary for preventing the filling of the condenser with water.

Watt at first intended adopting the same expedient which worked satisfactorily with the less effective condensation of

Newcomen's engine, i.e., leading a pipe from the condenser to a depth greater than the height of the column of water which could be counterbalanced by the pressure of the atmosphere; but he subsequently employed the air-pump, which relieves the condenser, not only of the water, but of the air which also usually collects in considerable volume, and vitiates the vacuum.

He next substituted oil and tallow for the water previously used in lubrication of the piston and keeping it steam-tight, in order to avoid the cooling of the cylinder incident to the use of water. Still another cause of refrigeration of the cylinder, and consequent waste of power in its operation, was seen to be the entrance of the atmosphere, which came in at the top and followed the piston down the cylinder at each stroke. This the inventor concluded to prevent by covering the top of the cylinder, and allowing the piston-rod to play through a "stuffing-box," which device had long been known to mechanics. He accordingly not only covered the top, but surrounded the whole cylinder with an external casing or "steam-jacket," and allowed the steam from the boiler to pass around the steam-cylinder and to press upon the upper surface of the piston, where its pressure was readily variable and therefore more manageable than that of the atmosphere. It also, besides keeping the cylinder hot, could do comparatively little harm should it leak by the piston, as it might be condensed and readily disposed of.

16. **The Single-acting Engine** of Watt was now fully developed from the "atmospheric engine" of Newcomen. As improved it is shown in Fig. 6, which represents the engine as patented in April, 1769. Watt's first engine was erected with the pecuniary aid of Dr. Roebuck, the lessor of a coal mine on the estate of the Duke of Hamilton, at Kinneil, near Borrowstounness. This engine, which was put up at the mine, had a steam-cylinder eighteen inches in diameter.

In the figure, the steam passes from the boiler through the pipe *d* and the valve *c* to the cylinder casing, or steam-jacket, *Y Y*, and above the piston *b*, which it follows in its

descent in the cylinder *a*, the valve *f* being at this time open to allow the exhaust to pass into the condenser *h*.

The piston now being at the lower end of the cylinder, and the pump-rods at the opposite end of the beam *y* thus raised, and the pumps filled with water, the valves *c* and *f* close, while *e* opens, allowing the steam which remains above the piston to flow beneath it, until, the pressure becoming equal above and below by the weight of the pump, it is rapidly drawn to the top of the cylinder, while the steam is displaced above, passing to the underside of the piston.

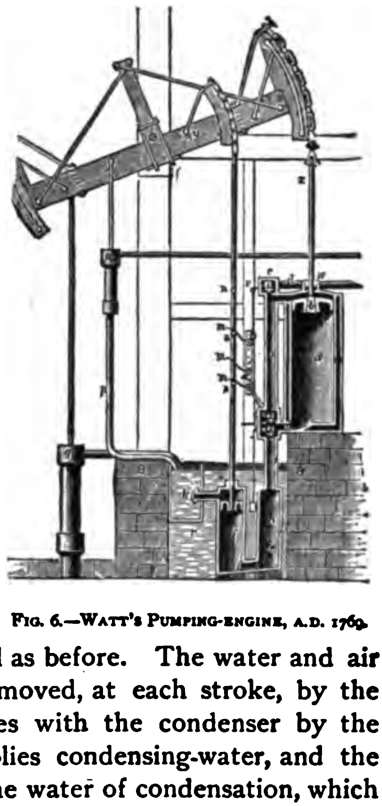


FIG. 6.—WATT'S PUMPING-ENGINE, A.D. 1769.

Now the valve *e* is closed, and *c* and *f* are again opened, and the down-stroke is repeated as before. The water and air entering the condenser are removed, at each stroke, by the air-pump *i*, which communicates with the condenser by the passage *s*. The pump *q* supplies condensing-water, and the pump *A* takes away a part of the water of condensation, which is thrown by the air-pump into the "hot-well" *k*, and with it supplies the boiler. The valves are moved by valve-gear very similar to Beighton's, by the pins *m m* in the "plug-frame" or "tappet-rod" *n n*.

The engine is mounted upon a substantial foundation, *B B*. *F* is an opening, out of which, before starting the engine, the air is driven from the cylinder and condenser.

17. Watt's Double-acting Engine was the next of his great inventions; and his scheme of the expansion of steam was quite as important.

Watt conceived the idea of economizing some of that power,

the loss of which was so plainly indicated by the violent rush of the exhaust steam into the condenser, and described the advantages that would follow the use of steam expansively. by means of a "cut-off," in a letter to Dr. Small, of Birmingham, dated Glasgow, May, 1769. He also planned a "compound engine." This invention of the expansion of steam, which, in importance, was hardly exceeded by any other improvement of the steam-engine, was adopted at Soho in 1776, but the patent was not obtained until 1782.

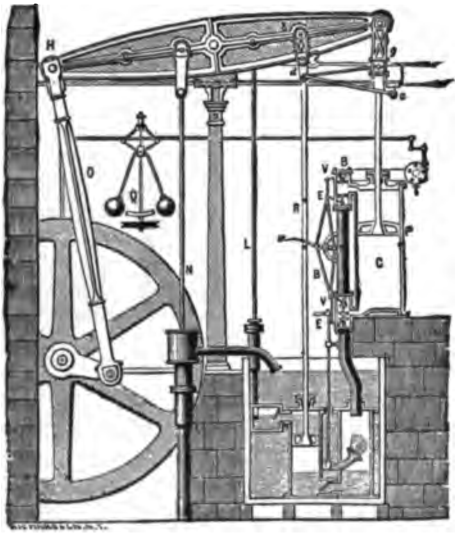


FIG. 7.—WATT'S ENGINE, A.D. 1780.

During this interval, Watt invented the crank and fly-wheel, but, as the former had been first patented by Wasborough, who is supposed to have obtained a knowledge of it from workmen employed by Watt, the latter patented several other methods of producing rotary motions, and temporarily adopted that known as the "sun-and-planet wheels," subsequently using the crank. The adaptation of the steam-engine to the production of rotary motion was soon succeeded by the introduction of the Double-acting Engine, the Fly-ball Governor, the Counter, the Steam-engine Indicator, and other minor but

valuable improvements, which were the final steps by which the Watt steam-engine became applicable to driving mills, to use on railroads, to steam-navigation, and to the countless purposes by which it has become, as it has already been denominated, the great material agent of civilization.

Fig. 7 represents the Watt Double-acting Engine. It will be noticed that it differs from the Single-acting Engine in having steam-valves, *B B*, and exhaust-valves, *E E*, at each end of the cylinder, thus enabling the steam to act on each side of the piston alternately, and practically doubling the power of the engine.

The end of the beam opposite to the cylinder is usually connected with a crank-shaft.

18. The Later Pumping-engine, Cornish type, is shown in the succeeding figure, exhibiting the principal form of pumping-engine now rarely constructed.

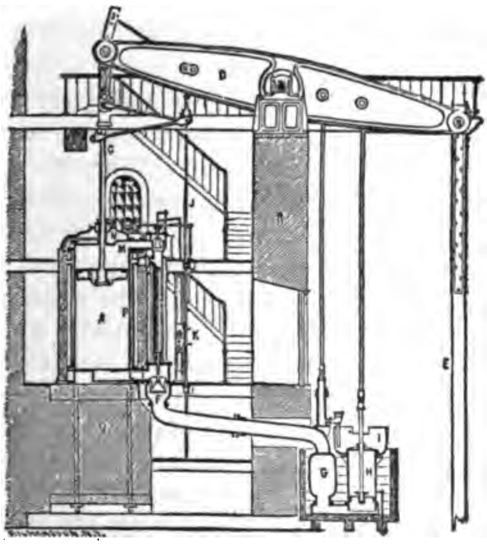


FIG. 8.—THE CORNISH PUMPING-ENGINE, 1877.

Fig. 8 represents the Cornish pumping-engine, which, in spite of its great weight and high cost, is still in use.

It will be seen that it is the engine of James Watt in all its

general features, with the addition, in its operation, of the application of Watt's idea of expansion of steam to something approximating the extent customary at the present time.

It is single-acting, and has a steam-jacket and a plug-rod valve-gear, *J K*. The improvements are principally in the form and proportions of its parts, and in its adaptation to high steam and "short 'cut-off.'" *A* is the steam-cylinder, *B C* the piston and rod, *D* the beam, and the pump-rod. The condenser is seen at *G*, and the air-pump at *H*. The steam-cylinder is "steam-jacketed," and is surrounded by a casing, *O*, composed of brickwork or other non-conducting material. Steam is first admitted above the piston, driving it rapidly downward and raising the pump-rod. At an early point in the stroke the admission of steam is checked by the sudden closing of the induction-valve, and the stroke is completed under the action of expanding steam assisted by the inertia of the heavy parts already in motion. The necessary weight and inertia are afforded in many cases, where the engine is applied to the pumping of deep mines, by the immensely long and heavy pump-rods. Where this weight is too great, it is counterbalanced; and where, as when used for the water-supply of cities, too small, weights are added. When the stroke is completed, the "equilibrium-valve" is opened, and the steam passes from above to the space below the piston, and, an equilibrium of pressure being thus produced, the pump-rods descend, forcing the water from the pumps and raising the steam-piston.

The absence of the crank or other device which might determine absolutely the length of stroke compels a very careful adjustment of steam admission to the amount of load. Should the stroke be allowed to exceed the proper length, and should danger thus arise of the piston striking the cylinder-heads, the movement is checked by buffer-beams. The regulation is effected by a "cataract," a kind of hydraulic governor, consisting of a plunger-pump with a reservoir attached. The plunger is raised by the engine, and then automatically detached. It falls with greater or less rapidity, its velocity being determined by the size of the eduction orifice, which is adjustable by hand.

When the plunger reaches the bottom of the pump-barrel, it disengages a catch, a weight is allowed to act upon the steam-valve, opening it, and the engine is caused to make a stroke. When the outlet of the cataract is nearly closed, the engine stands still a considerable time while the plunger is descending, and the strokes succeed each other at long intervals. When the opening is greater, the cataract acts more rapidly, and the engine works faster. This has been, from about the middle of the nineteenth century, steadily displaced by crank and fly-wheel engines, usually compounded.

19. **The Compound Engine** originated in Watt's time. Fig. 9 represents the first "compound" or "two-cylinder" engine. This class of engines, in which the steam exhausted from one cylinder is further expanded in the second, was first introduced by Hornblower, in 1781, and was patented, in combination with the Watt condenser, by Woolf, at a later date (1804), with a view to adopting high steam and considerable expansion. The Woolf engine was to some extent adopted, but was not successful in competing with Watt engines where the latter were well built, and, like Hornblower's engine, was soon given up.

The compound engine has come up again within a few years, and with what is *now* considered high steam and considerable expansion, and designed with more intelligent reference to the requirements of economy of working steam in this manner, it is gradually displacing other forms of engine.

The engine patented by Hornblower in 1781 was first described by the inventor in the "Encyclopædia Britannica." It consists, as is seen by reference to the engraving, of two steam-cylinders, *A* and *B*—*A* being the low- and *B* the high-pressure cylinder—the steam leaving the latter being exhausted into the former, and, after doing its work there, passing into the condenser, as already described. The piston-rods, *C* and *D*, are both connected to the same part of the beam by chains, as in the other early engines. These rods pass through stuffing-boxes in the cylinder-heads, which are fitted up like those seen on the Watt engine. Steam is led to the engine through the pipe, *G Y*, and cocks, *a*, *b*, *c*, and *d*, are adjustable, as required,

to lead steam into and from the cylinders, and are moved by the plug-rod, *W*, which actuates handles not shown. *K* is the exhaust-pipe leading to the condenser. *V* is the engine feed-pump, and *X* the pump-rod carrying the pump-buckets at the bottom of the shaft.

The cocks *c* and *a* being open and *b* and *d* shut, the steam passes from the boiler into the upper part of the steam-cylinder, *B*; and the communication between the lower part of *B* and the top of *A* is also open. Before starting, steam being shut off from the engine, the great weight of the pump-rod, *X*,

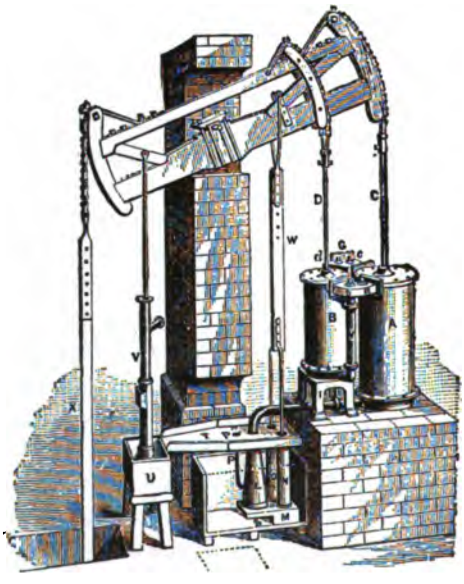


FIG. 9.—HORNBLOWER'S COMPOUND ENGINE, 1781.

causes that end of the beam to preponderate, the pistons standing, as shown, at the top of their respective steam-cylinders.

The engine being freed from all air by opening all the valves and permitting the steam to drive it through the engine and out of the condenser through the "snifting-valve," *O*, the valves *b* and *d* are closed, and the cock in the exhaust-pipe opened.

The steam beneath the piston of the large cylinder is immediately condensed, and the pressure on the upper side of that piston causes it to descend, carrying that end of the beam with it, and raising the opposite end with the pump-rods and their attachments. At the same time, the steam from the lower end of the small high-pressure cylinder being let into the upper end of the larger cylinder, the completion of the stroke finds a cylinder full of steam transferred from the one to the other with corresponding increase of volume and decrease of pressure. While expanding and diminishing in pressure as it passes from the smaller into the larger cylinder, this charge of steam gradually resists less and less the pressure of the steam from the boiler on the upper side of the piston of the small cylinder, *B*, and the net result is the movement of the engine by pressures exerted on the upper sides of both pistons and against pressures of less intensity on the under sides of both. The pressures in the lower part of the small cylinder, in the upper part of the large cylinder, and in the communicating passage, are evidently all equal at any given time. When the pistons have reached the bottoms of their respective cylinders, the valves at the top of the small cylinder, *B*, and at the bottom of the large cylinder, *A*, are closed, and the valves *c* and *d* are opened. Steam from the boiler now enters beneath the piston of the small cylinder; the steam in the larger cylinder is exhaused into the condenser, and the steam already in the small cylinder passes over into the large cylinder, following up the piston as it rises.

Thus, at each stroke a small cylinder full of steam is taken from the boiler, and the same weight, occupying the volume of the larger cylinder, is exhausted into the condenser from the latter cylinder.

Referring to the method of operation of this engine, Prof. Robison demonstrated that the effect produced was the same as in Watt's single-cylinder engine—a fact which is comprehended in the law enunciated many years later by Rankine, that, "so far as the theoretical action of the steam on the pis-

ton is concerned, it is immaterial whether the expansion takes place in one cylinder, or in two or more cylinders." It was found, in practice, that the Hornblower engine was no more economical than the Watt engine; and that erected at the Tin Croft Mine, Cornwall, in 1792, did even less work with the same fuel than the Watt engines.

The plan unsuccessfully introduced by Hornblower was subsequently modified and adopted by others among the contemporaries of Watt; and, with higher steam and the use of the Watt condenser, the "compound" gradually became a standard type of steam-engine.

Arthur Woolf, in 1804, re-introduced the Hornblower or Falck engine, with its two steam-cylinders, using steam of higher tension. His first engine was built for a brewery in London, and a considerable number were subsequently made. Woolf expanded his steam from six to nine times, and the pumping-engines built from his plans were said to have raised about 40,000,000 pounds one foot high per bushel of coals, when the Watt engine was raising but little more than 30,000,000. In one case a duty of 57,000,000 was claimed.

The accompanying engraving exhibits a modern and successful type of compound engine, which may be taken for comparison in style, general design, proportions, and performance with the earlier forms of pumping-engine. It was designed by Mr. E. Reynolds and is in operation in the city of Milwaukee, where it was constructed.

Here the pumps are in line with the steam-cylinders, bringing the working-strain direct to the plungers. The valve-gear has a cut-off on both cylinders, which allows the steam to be worked from boiler-pressure down to 8 or 9 pounds. The cylinders are steam-jacketed. The pump, condenser, boiler feed-pumps, and air-chambers are placed below the floor. The contract required a delivery of 12,000,000 gallons of water, 150 feet high, every 24 hours, and a duty of 97,000,000 foot-pounds for every 100 lbs. of coal consumed.

Still more remarkable types and higher duties are discussed later. See § 37, pages 175, 180.

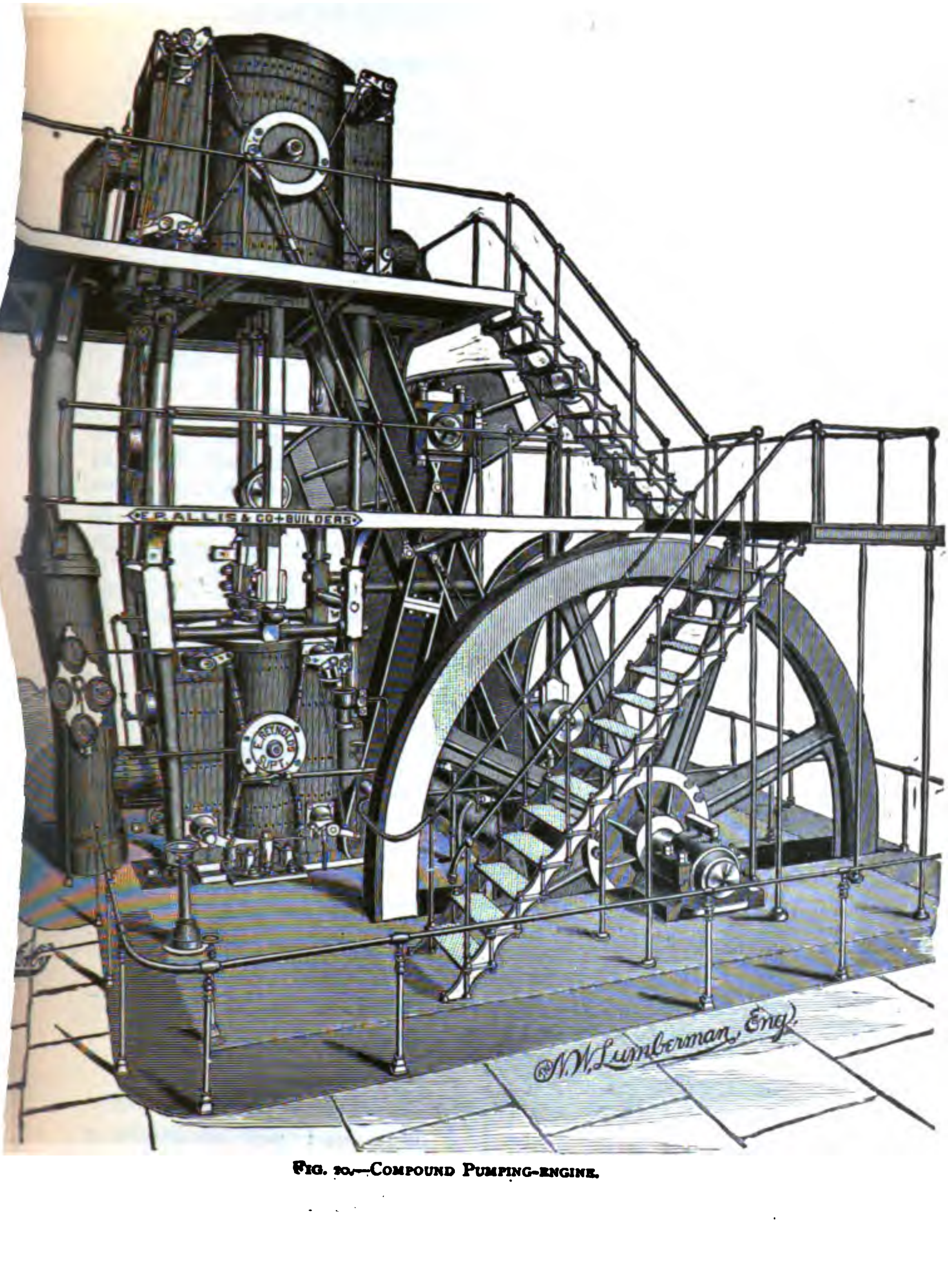


FIG. 10.—COMPOUND PUMPING-ENGINE.

The principal dimensions of the engines are:

Diameter high-pressure cylinder	inches,	34
Diameter low-pressure cylinder	"	66
Diameter of pump	"	41.78
Diameter of pump-plunger	"	30
Length of stroke	"	60

The performance of this engine may be compared with those reported for Savery's, Newcomen's, and Watt's machines to obtain some idea of the progress of modern times in the economical use of steam.

The following are the results of the trial:

Duration of trials	hours,	48
Steam-pressure in engine-room	pounds,	74.81
Vacuum by gauge	inches,	26.25
Water-pressure gauge	pounds,	62.02
Total head, including suction-lift	"	67.29
Revolutions of engine per minute		25.51
Piston speed per minute	feet,	255.10
Coal consumed	pounds,	32.395
Duty in foot-pounds, per 100 pounds of coal consumed		104,820,431

Exceeding the duty and the capacity guaranteed under the ordinary, every-day conditions, and the actual weight of coal consumed being charged up without deductions of any kind.

The progress of steam-pumping engine efficiency, from the time of Newcomen and of Watt to date, is seen in the following figures (see p. 81):

Date.	Engine.	Duty; ft.-lbs. per 100 lbs.
1769	Newcomen (by Smeaton)	7,000,000
1772	" " "	12,000,000
1776	Watt	21,600,000
1778	" expansive	26,600,000
1830	Cornish	86,585,000
1860	Compound	100,000,000
1880	Triple-expansion	120,000,000
1900	Quadruple-expansion	160,000,000

The duties given are those either guaranteed or actually resulting from trials. The fuel demanded per horse-power per hour thus has decreased from about 35 pounds in Smeaton's Newcomen engines, and 8 in Watt's best work, to 2¼

pounds in the Cornish, and to less than 1.75 in later engines of the compound type; the minimum given above being 1.5. Even this figure has been reduced with later engines of the three- and four-cylinder types to not far from unity.

20. **The Stationary Engine** is, as has been already seen, an evolution from the earlier types of pumping-engine, and is a product of the fertile and fruitful brain of James Watt. The Watt double-acting engine, turning a shaft, regulated by a "fly-wheel" and controlled by the Watt governor, represents the type of the modern stationary engine as well as that of Watt's own time. The changes which have occurred since that period have been mainly in matters of detail.

The old "parallel motion" guiding the head of the piston-rod has now become generally superseded by the guides and sliding cross-head. The valve-gear has been simplified and better adapted to efficient action as a "cut-off" gear. The governor has been so attached as to adjust the steam supply to work momentarily performed, by variation of the point of cut-off, and, revolution by revolution, fixing the ratio of expansion. The general design and construction of the engine have been modified in the direction of simplicity, cheapness, and lightness, combined with strength. The use of the direct-acting engine, rather than the beam-engine, is now general, and, for all but "high-speed" engines which make 150 to 300 revolutions or more per minute, some form of "detachable valve-gear" is employed.

The first successful "drop cut-off" engine was that of F. E. Sickels, of 1841, which employed "puppet-valves" on the steam side, which could be detached and allowed to fall into their seats at any desired point in the stroke, by a detaching mechanism operated either by hand or by the governor. To prevent injury by the impact of the valve on the seat, a "dash-pot" was used, consisting of a vessel, containing either water or air, into which a loosely-fitted piston was fitted. This piston, attached to the valve-stem, directly or indirectly, rose and fell with the latter, and when the valve was about to strike the seat at the end of its descent, the fall was checked and the

valve "eased" down to the seat by the resistance of the fluid in the dash-pot, on which the piston fell, and through which, for a very short distance, it then forced its way.

Modifications of these devices were devised by G. H. Corliss in 1849, and constitute the so-called Corliss engine of the present time, which will be described later. Many other inventors have since constructed still other engines of the same general character.

The latest improvements of the stationary engine relate to what are distinctively known as the "high-speed" engine, and have led to the production of engines especially adapted to driving machinery at very high speeds of revolution. In the most successful engines of this type it is usual to make the engine itself of the simplest possible design; to adopt a simple valve-motion, and to secure regulation by means of a governor placed on the main shaft and adjusting the point of cut-off by shifting the eccentric. A single valve is often used. These engines will be fully described in the next chapter.

Where the cost of securing the needed condensing water is not too great, and where the steam-pressure is moderate, a condenser may be economically added to the non-condensing engine, thus obtaining a gain in power of considerable amount and an increase in economy of steam and of fuel, if the engine is well proportioned to its work when thus altered, of often one third—three pounds and two pounds of good coal per horse-power and per hour being common figures for such engines working non-condensing and condensing. The gain in power is often one fourth or one third. But with increasing pressure of steam this gain becomes lessened.

21. The Locomotive was one of the fruits of the inventive genius of Watt and his contemporaries.

When the steam-engine had so far been perfected that the possibility of its application to other purposes than the elevation of water had become generally recognized, the problem of its adaptation to the propulsion of carriages was attacked by many engineers and inventors.

As early as 1759 Dr. Robison called the attention of Watt

to the possibility of constructing a carriage to be driven by a steam-engine. Watt, at a very early period, proposed to apply his engine to locomotion, and contemplated using either a non-condensing engine or an air surface-condenser. He included the locomotive-engine in his patent of 1784, and his assistant, Murdoch, in the same year made a working-model locomotive which was capable of running at a rapid rate.

The first actual experiment was made, as is supposed, by a French army officer, Nicolas Joseph Cugnot, who in 1769 built a steam-carriage, which was set at work in presence of the French Minister of War, the Duc de Choiseul. The funds required were furnished by the Comte de Saxe. Encouraged by the partial success of the first locomotive, Cugnot, in 1770, constructed a second, which is still preserved in the Conservatoire des Arts et Métiers Paris. This more powerful carriage was fitted with two non-condensing single-acting cylinders thirteen inches in diameter. Although the experiment seems to have been successful, there appears to have been nothing more done with it.

An American of considerable distinction, Nathan Read, patented a steam-carriage, 1790.*

In 1804 Oliver Evans completed a flat-bottomed boat to be used at the Philadelphia docks, and, mounting it upon wheels, drew it by its own steam-engine to the river-bank. Launching the craft, he propelled it down the river, using its steam-engine to drive its paddle-wheels. Evans's "*oructor amphibolis*," as he named the machine, was the first road-locomotive that we find described after Cugnot's time. Evans asserted that carriages propelled by steam would soon be in common use; and offered a wager of three hundred dollars that he could build a "steam-wagon" that should excel in speed the swiftest horse that could be matched against it.

Trevithick and Vivian built a locomotive-engine in 1804 (Fig. 11) for the railway at Merthyr-Tydvil, in South Wales, which was quite successful, although sometimes giving trouble

* "Nathan Read and his Steam-engine." New York: Hurd & Houghton, 1870.

by slipping its wheels. This engine had one steam-cylinder 4¾ inches diameter, and carried forty pounds steam.

Colonel John Stevens, of Hoboken, was undoubtedly the greatest engineer and naval architect living at the beginning of the present century. Without having made any one superlatively great improvement in the mechanism of the steam-engine, like that which gave Watt his fame; without having the

FIG. 11.—TREVITHICK'S LOCOMOTIVE, 1804.

honor of being the first to propose navigation by steam, or steam-transportation on land, he exhibited a far better knowledge of the science and of the art of engineering than any man of his time, and he entertained and urged more advanced opinions and more statesmanlike views, in relation to the economical importance of the improvement of the steam-engine, both on land and water, than seem to have been attributable to any other leading engineer of that time.

In 1812 he published a pamphlet embodying "Documents tending to prove the Superior Advantages of Railways and

Steam-carriages over Canal Navigation." * At this time the only working locomotive in the world was that of Trevithick and Vivian, at Merthyr-Tydvil, and the railroad itself had not grown beyond the old wooden tram-roads of the collieries. Yet Colonel Stevens says in this paper: "I can see nothing to hinder a steam-carriage moving on its ways with a velocity of one hundred miles an hour"—adding in a footnote: "This astonishing velocity is considered here merely possible. It is probable that it may not, in practice, be convenient to exceed twenty or thirty miles per hour. Actual experiments can only determine this matter, and I should not be surprised at seeing steam-carriages propelled at the rate of forty or fifty miles an hour."

He proposed rails of timber, protected when necessary by iron plates, or to be made wholly of iron. The car-wheels were to be of cast iron, with inside flanges to keep them on the track. The steam-engine was to be driven by steam of fifty pounds pressure and to be non-condensing.

He gives 500 to 1000 pounds as the maximum weight to be placed on each wheel, shows that the trains—or "suites of carriages," as he calls them—will make their journeys "with as much certainty and celerity in the darkest night as in the light of day," shows that the grades of proposed roads would offer but little resistance, and places the whole subject before the public with accuracy of statement and evident appreciation of its true value.

In 1814 George Stephenson, to whom is generally accorded the honor of having first made the locomotive-engine a success, built his first engine at Killingworth, England.

In 1815 he applied the blast-pipe in the chimney, by which the puff of the exhaust steam is made useful in intensifying the draught, and applied it successfully to his second locomotive, here seen in section (Fig. 12). This is the essential characteristic of the locomotive-engine. In 1815, therefore, the modern locomotive steam-engine came into existence, for it is this

* Printed by T. & J. Swords, 1160 Pearl Street, New York, 1812.

invention of the blast-pipe that gives it its life, and it is the mechanical adaptation of this and of the other organs of the steam-engine to locomotion that gives George Stephenson his greatest claim to distinction.

In 1825 the Stockton and Darlington Railroad was opened, and one of Stevenson's locomotives, in which he employed his "steam-blast," was successfully used, drawing passenger as well

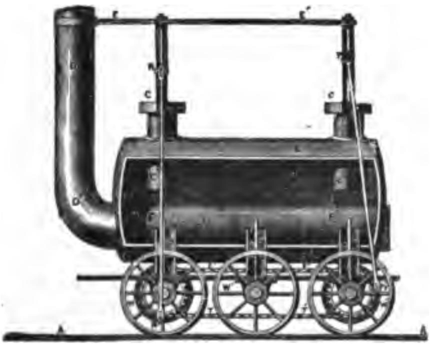


FIG. 12.—STEPHENSON'S LOCOMOTIVE, 1815.

as coal trains. Stephenson had at this time become engineer of the road. The time required to travel the distance of twelve miles was two hours.

One of the most important and interesting occasions in the history of the application of the non-condensing steam-engine to railroads, as well as in the life of Stephenson, was the opening of the Liverpool and Manchester Railroad in the year 1829. When this road was built, it was determined, after long and earnest discussion, to try whether locomotive-engines might not be used to the exclusion of horses, and a prize of £500 was offered for the best that should be presented at a date which was finally settled at the 6th of October, 1829. Four engines competed, and the "Rocket," built by Stephenson, received the prize.

This engine (Fig. 13) weighed four and one fourth tons, with its supply of water. Its boiler was of the fire-tubular type, a form that had grown into shape in the hands of several

inventors,* and was three feet in diameter, six feet long, with twenty-five three-inch tubes, extending from end to end of the boiler. The steam-blast was carefully adjusted by experiment, to give the best effect. Steam pressure was carried at fifty pounds per square inch.

The average speed of the Rocket on its trial was fifteen miles per hour, and its maximum was nearly double that, twenty-nine miles an hour; and afterward, running alone, it reached a speed of thirty-five miles.

FIG. 13.—THE ROCKET, 1829.

In America the locomotive was set at regular work on railroads, for the first time, on the 8th of August, 1829. This first locomotive was built by Foster, Rastrick & Co., at Stourbridge, England, and was purchased by Mr. Horatio Allen for the Delaware and Hudson Canal Company's road from Carbondale to Honesdale, Pennsylvania.

It was at about this time (1831) that Mr. Horatio Allen introduced the first eight-wheeled locomotives ever built, and gave them a form which was the prototype of a recently-built locomotive which has been brought out in Great Britain. In this year, also, an engine, the De Witt Clinton, was built for John B. Jervis of the Mohawk and Hudson Railroad. At about the time of the opening of the early railroads, the introduction of steam-carriages on the common highway had become a favorite idea with engineers.†

In December, 1833, about twenty steam-carriages and traction road-engines were running or were in course of construction in and near London.

In our own country the roughness of roads discouraged inventors, and in Great Britain, even, the successful introduc-

* Barlow and Fulton, 1795; Nathan Read, Salem, United States, 1796; Booth, of England, and Séguin, of France, about 1827 or 1828.

† "History of the First Locomotive in America," W. H. Brown. D. Appleton & Co., New York, 1872.

tion of road-locomotives, which seemed at one time almost an accomplished fact, finally met with so many obstacles that even Hancock and Gurney, the most ingenious, persistent, and successful of constructors, gave up in despair. Hostile legislation procured by opposing interests, and possibly also the rapid progress of steam-locomotion on railroads, caused this result. It has now reappeared as the "automobile."

The steam-blast of Hackworth, the tubular boiler of Séguin, and the link-motion of Stephenson constitute the essential features of the modern locomotive-engine. Locomotives have gradually and steadily increased in size and power from the date of their introduction. The Rocket, which first proved conclusively, in 1829, the value of steam-locomotion, weighed 4½ tons. In 1835 Robert Stephenson, who had constructed it with his father, writing to Robert L. Stevens, said that he was making his engines heavier and heavier, and that the engine of which he enclosed a sketch weighed nine tons, and could draw "100 tons at the rate of sixteen miles an hour, on a level." Locomotives are now built weighing seventy tons, and even one hundred, and powerful enough to draw more than 2000 tons at a speed of twenty miles an hour. The modern locomotive consists of a boiler, mounted upon a strong light frame of forged iron, by which it is connected with the wheels. The largest engine yet constructed in the United States is said to have a weight of about 200,000 pounds, which is carried on twelve driving-wheels. A locomotive has two steam-cylinders, either side by side within the frame, and immediately beneath the forward end of the boiler, or on each side and exterior to the frame. The engines are non-condensing and of the simplest possible construction. The whole machine is carried upon strong but flexible steel springs. The steam-pressure is usually more than a hundred pounds. The pulling-power is generally about one fifth the weight under most favorable conditions, but becomes as low as one tenth on wet rails. The fuel employed is wood in new countries, coke in bituminous-coal districts, and anthracite coal in the eastern part of the United States. The general arrangement and the proportions of loco-

motives differ somewhat in different localities, as will be seen later.

The common three-ported slide-valve was invented by Murdoch, while with Watt, about 1799. This valve, driven by a system of single loose eccentrics and stops, for either forward or backward gear, was adopted by Stephenson and others, and probably by some of the first builders of the marine engine, as well as on the locomotive, as early as or earlier than 1820. At about this latter date the heart-shaped cam and its frame came into temporary use, to be superseded in 1840 or 1842 by the so-called Stephenson link. The two eccentrics, for forward and backward motion, with their hooks and the wedge-motion, were also in use during this period, the hooks being the favorite arrangement, towards its close, on locomotives. The link

FIG. 14.—MODERN LOCOMOTIVE.

continues in use as, on the whole, the most satisfactory gear, although, since 1855–60, many modifications and the later class of "radial" gears have been brought into competition with it.

After their introduction, the growth of railroads and the use of locomotives extended in the United States and in Europe with great rapidity. The first railroad in the United States was built near Quincy, Massachusetts, in 1826. In 1850 there were about 700 miles in operation; in 1860 there were over 30,000, and in 1890 about 160,000 miles of completed road in the United States; and the rate of increase had risen in 1873 to above 7000 miles per year, as a maximum, and the consumption of rails for renewal alone amounts to probably a million tons yer year.

The now standard engine for any given class of traffic has assumed such exact proportions and such generally accepted form that the engines of any two well-known builders, though readily distinguishable by the expert engineer, appear to the inexperienced observer to be duplicates. Thus the two engines here shown, the one by the Baldwin Works, the other by the Brooks Company, have every essential feature common; and all are more or less obviously related and modernized forms of the older types of engine.

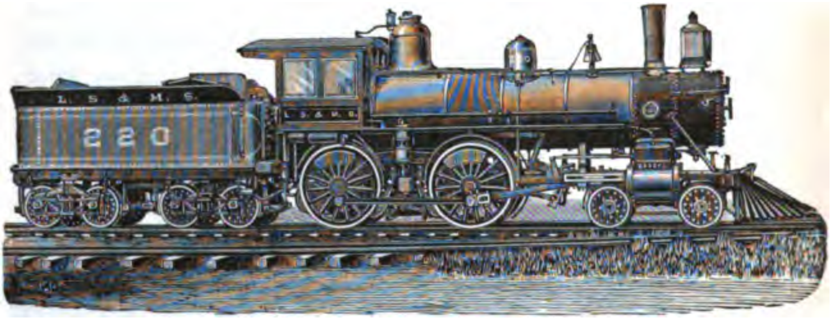


FIG. 15.—BROOKS ENGINE.

The tubular boiler has been given better proportions and has greatly gained in size; the steam-blast and smoke-pipe are as used in Stephenson's day; the whole system of "running gear" is that of Stephenson; the bell, sand-box, and whistle are characteristic of American practice, but are substantially the same with all American builders. The frame and the general external arrangements differ from those of the British engine, presently to be shown; but in even this comparison, the main characteristics of the locomotive-engine remain equally distinguishable and equally striking in both forms.

The Gooch and Allan forms of link were brought out about 1855, both giving nearly equal lead at both ends, and simple kinematic chains. Engelmann, in 1859, substituted pins and links for the sliding-block, while Stewart and Fink had already adopted (1857) a single eccentric.* The Von Waldegg-Walschaerts gear came out in 1861.

* Trans. Engrs. of Scotland; Nov., 1890.

Hackworth's, the first radial gear, came out in 1859, and many years later (1878–88) those of Brown, Marshall, Joy, and Strong.

The "drop," the "trip," or the "detachable" gears came in in 1840 with the Hogg, the Sickels (1841), the Corliss (1849), the Greene (1855), and numerous others, both in Europe and the United States.

The Steam Fire-engine is still another form of transportable engine, and is peculiarly an American production.

As early as 1830, Braithwaite and Ericsson, of London, England, built an engine with steam and pump cylinders of 7 and 6½ inches diameter, respectively, with 16 inches stroke of piston. This machine weighed 2¼ tons, and is said to have thrown 150 gallons of water per minute to a height of between 80 and 100 feet. It was ready for work in about 20 minutes after lighting the fire. The first attempt made in the United States to construct a steam fire-engine was probably that of Hodge; who built one in New York in 1841. It was a strong and very effective machine, but was too heavy for rapid transportation. The late J. K. Fisher, who throughout his life persistently urged the use of steam-carriages and traction-engines, designing and building several, also planned a steam fire-engine. Two were built from his designs by the Novelty Works, New York, about 1860, for Messrs. Lee & Larned. They were "self-propellers," and one of them, built for the city of Philadelphia, was sent to that city over the highway, driven by its own engines. The other was built for and used by the New York Fire Department, and did good service for several years. These engines were heavy but powerful, and moved at good speed under steam. The Messrs. Latta, of Cincinnati, soon after succeeded in constructing comparatively light and very effective engines, and the fire department of that city was the first to adopt steam fire-engines definitely as their principal reliance.

The steam fire-engine has now entirely displaced the old hand-engine. It does its work at a fraction of the cost of the latter. It can force its water to a height of 225 feet, and to a

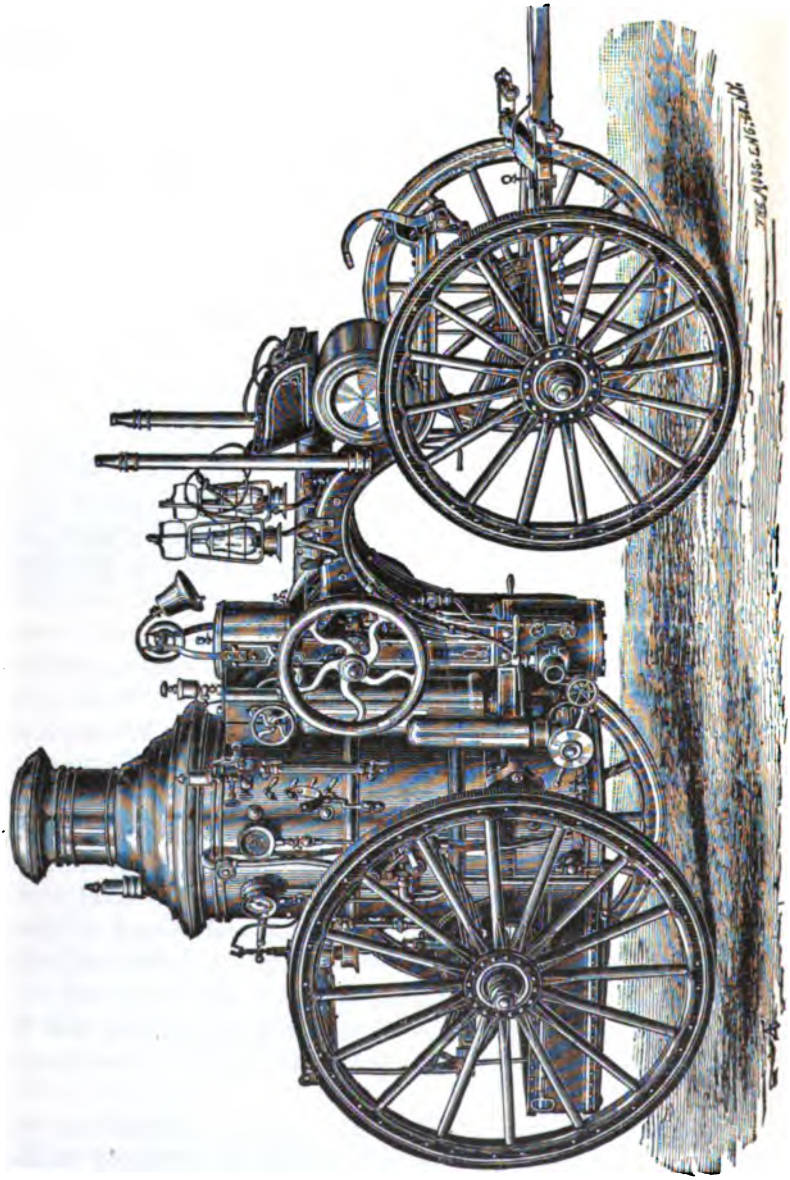


FIG. 16.—CLAPP & JONES FIRE-ENGINE.

distance of more than 300 feet horizontally, while the hand-engine can seldom throw it one third these distances; and the "steamer" may be relied upon to work at full power many hours if necessary, while the men at the hand-engine soon become fatigued, and require frequent relief.

In the modern standard steam fire-engine, Fig. 16, reciprocating engines and pumps are adopted. There are pairs of engines and companion-pumps, working on cranks, set at right angles, and turning a balance-wheel set behind them.

Such machines illustrate the most remarkable concentration of power in small compass, with lightness and strength of parts. As constructed by the best builders, they are composed of choice materials, are exceedingly carefully and well proportioned, and are beautifully finished. Their boilers contain little water, and are crowded with heating surface; they therefore make steam with great rapidity; their pumps have large passages and valves of small lift, and deliver large volumes of water easily; and they are arranged on a carriage permitting rapid and easy haulage. The heaviest of these engines rarely weigh much over three tons, and they are made as light as two tons.

22. **The Early Marine Engine** was an early outgrowth of the work on the steam-engine in the latter part of the eighteenth and early portion of the nineteenth century.

In 1690 Papin proposed to use his piston-engine to drive paddle-wheels to propel vessels; and in 1707 he applied the steam-engine which he had proposed as a pumping-engine to driving a model boat on the Fulda, at Cassel. His pumping-engine forced up water to turn a water-wheel, which, in turn, was made to drive the paddles. An account of his experiment is to be found in manuscript in the correspondence between Leibnitz and Papin, preserved in the Royal Library at Hanover.

December 21, 1736, Jonathan Hulls took out an English patent for the use of a steam-engine for ship-propulsion, proposing to employ his steamboat in towing. He proposed using the Newcomen engine, fitted with a counterpoise weight, and a

system of ropes and grooved wheels, which, by a peculiar ratchet-like action, gave a a continuous rotary motion. There is no positive evidence that Hulls ever put his scheme to the test of experiment, although tradition does say that he made a model, which he tried with such ill success as to prevent his further prosecution of the experiment.

In 1774 the Comte d'Auxiron, a French nobleman and a gentleman of some scientific attainments, constructed a steam-boat, and tried it on the Seine, with the aid of M. Perier. This experiment proving unsuccessful, M. Perier built another boat, which he tried independently in 1775, but was again unsuccessful, owing principally to the small power of his engine. In 1778, and again 1781 or 1782, the French Marquis de Jouffroy, who, in his later experiments, used quite a large vessel, succeeded in obtaining such good results as to encourage him to persevere, but, political disturbances driving him from his country, his labors terminated abruptly.

About 1785, John Fitch and James Rumsey, two ingenious American mechanics, were engaged in experiments having in view the application of steam to navigation. Rumsey's experiments began in 1774, and in 1786 he succeeded in driving a boat at the rate of four miles an hour against the current of the Potomac, at Shepardstown, Maryland. Rumsey employed his engine to drive a great pump, which forced a stream of water aft, thus propelling the boat forward. This same method has been tried by the British Admiralty in the Water-witch, a gunboat of moderate size, using a centrifugal pump to set in motion the propelling stream, and with some other modifications which are decided improvements upon Rumsey's rude arrangements, but which have not done much more than did his toward the introduction of "hydraulic propulsion," as it is now called. John Fitch was an ingenious Connecticut mechanic. After roaming about until forty years of age, he finally settled on the banks of the Delaware, where he built his first steam-boat. In 1788 he obtained a patent for the application of steam to navigation. His boat was sixty feet long and twenty feet wide. The propelling apparatus was a system of paddles,

which were suspended by the upper ends of their shafts, and moved by a series of cranks, one to each, taking hold at the middle, and giving them almost exactly the motion which is imparted to his paddle by the Indian in his canoe. Fitch's boat, when tried at Philadelphia, was found capable of making eight miles an hour. It was laid up in 1792.

In 1788 Patrick Miller, James Taylor, and William Symmington attached a steam-engine to a boat with paddle-wheels, which had been built by the first-named, and tried it for the first time on Dalswinton Lake, in Dumfriesshire, Scotland. This boat having attained a speed of five miles an hour, another was constructed and was tried in 1789. This vessel was driven by an engine of twelve horse-power, and made seven miles an hour. This result, encouraging as it was, led to no further immediate action, the funds of the experimenters having failed.

In 1801, however, Symmington was employed by Lord Dundas to construct a steamboat, with a design of substituting steam for horse-power on canals. The Charlotte Dundas, as this boat was named, was so evidently a success that the Duke of Bridgewater ordered *eight* similar vessels for his canal; but his death, soon afterward, prevented the order being filled.

At this time, several American mechanics were also still working at this attractive problem. In 1802–'3, Robert Fulton, with Mr. Joel Barlow, in whose family he resided, and Chancellor Livingston, who had also then taken up a temporary residence in Paris, commenced a small steamboat eighty six feet long and of eight feet beam. The hull was altogether too slight to bear the weight of the machinery, and, when almost completed, the little craft literally broke in two, and sank at her moorings.

The wreck was promptly recovered and rebuilt, and in August, 1803, the trial-trip was made in presence of a large party of invited guests. The experiment was sufficiently successful to induce Fulton and Livingston to order an engine of Messrs. Boulton and Watt, directing it to be sent to America, where Livingston soon returned. In 1806 Fulton followed,

reaching New York in December, and at once going to work on the vessel for which the English firm sent the engine, without being informed of its intended use. In the spring of 1807 the Clermont (Fig. 17), as the new boat was christened, was launched from the ship-yard of Charles Brown, on the East River, New York. In August the machinery was on board,

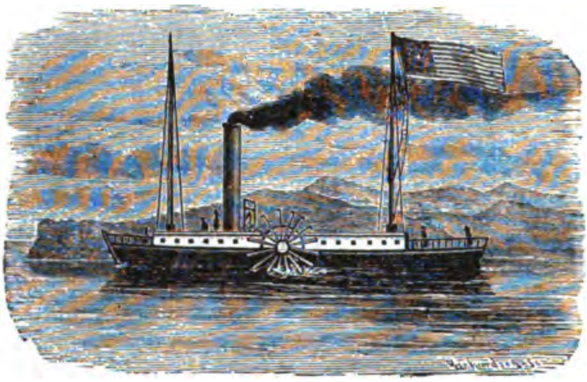


FIG. 17.—THE CLERMONT, 1807.

and in successful operation. The hull of this boat was one hundred and thirty-three feet long, eighteen feet beam, and seven feet in depth. The boat soon afterwards made a trip to Albany, making the distance of one hundred and fifty miles in thirty-two hours running time, and returning in thirty hours. The sails were not used on either occasion. This was the first voyage of considerable length ever made by a steam-vessel, and the Clermont was soon after regularly employed as a passenger-boat between the two cities.

Fulton, though not to be classed with James Watt as an inventor, is entitled to the great honor of having been the first to make steam-navigation an every-day commercial success, and of having thus made the first application of the steam-engine to ship-propulsion which was not followed by the retirement of the experimenter from the field of his labors before success was permanently insured.

The engine of the Clermont (Fig. 18) was of rather peculiar

form, the engine being coupled to the crank-shaft by a bell-crank, and the paddle-wheel shaft being separated from the crank-shaft, but connected with the latter by gearing. The cylinders were twenty-four inches in diameter and of four feet stroke. The paddle-wheels had buckets four feet long, with a dip of two feet.

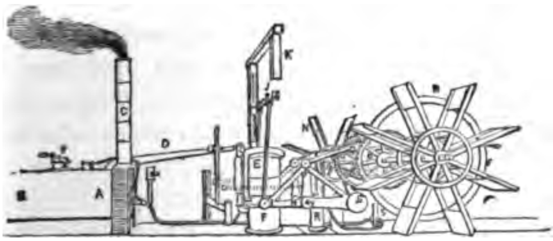


FIG. 18.—ENGINE OF THE CLERMONT, 1807.

Subsequently, Fulton built several steamers and ferry-boats, to ply about the waters of the States of New York and Connecticut. The Clermont was a boat of but 160 tons burden; the Car of Neptune, built in 1807, was 295 tons; the Paragon, in 1811, measured 331; the Richmond, 1813, 370 tons; and the Fulton the First, built in 1814–'15, measured 2475 tons. The latter vessel, whose size was simply enormous for that time, was what was then considered an exceedingly formidable steam-battery, and was built for the United States Navy. Before the completion of this vessel, Fulton died of disease resulting from exposure, February 24, 1815, and his death was mourned as a national calamity.

The prize gained by Fulton was, however, most closely contested by Colonel John Stevens, of Hoboken, who has been already mentioned in connection with the early history of railroads, and who had been, since 1791, engaged in similar experiments. In 1789 he had petitioned the Legislature of the State of New York for an act similar to that granted Livingston, and stated that his plans were complete and on paper.

In 1804, while Fulton was in Europe, Stevens had completed a steamboat sixty-eight feet long and fourteen feet beam, which combined novelties and merits of design in a

manner that was the best possible evidence of remarkable inventive talent, as well as of the most perfect appreciation of the nature of the problem which he had proposed to himself to solve.

The steamboat boiler of 1804 (Fig. 19) was built to bear a working pressure of over fifty pounds to the square inch, at a time when the usual pressures were from four to seven pounds. It consists of two sets of tubes, closed at one end by solid plugs, and at their opposite extremities screwed into a stayed water and steam reservoir, which was strengthened by hoops. The whole of the lower portion was inclosed in a jacket of iron lined with non-conducting material. The fire was built at one end, in a furnace inclosed in this jacket. The furnace-gases passed among the tubes, down under the body of the boiler, up among the opposite set of tubes, and thence to the smoke-pipe.

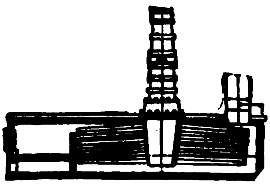


FIG. 19.—STEVENS'S "SECTIONAL" BOILER, 1804.

The engine (Fig 20) was a direct-acting, high-pressure condensing engine of ten inches diameter of cylinder, two feet stroke of piston, and drove a *screw* of four blades, and of a form which, even to-day, appears quite good.

FIG. 20.—MACHINERY OF TWIN-SCREW STEAMER OF 1804.

The first of Stevens's boats performed so well that he immediately built another one, using the same engine as before, but employing a larger boiler, and propelling the vessel by *twin-screws* (Fig. 21), the latter being another instance of his use of a device brought forward long afterward as new, and since frequently adopted. This boat was sufficiently successful to indicate the probability of making steam-navigation a commercial success, and Stevens, assisted by his sons, built a

boat which he named the Phœnix, and made the first trial in 1807, just too late to anticipate Fulton. This boat was driven by paddle-wheels. The Phœnix, shut out of the waters of the State of New York by the monopoly held by Fulton and Livingston, was placed for a time on a route between Hoboken and New Brunswick; and then, anticipating a better pecuniary

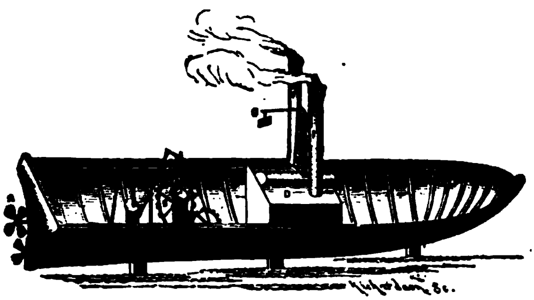


FIG. 21.—STEVENS'S TWIN-SCREWS, 1805.

return, it was concluded to send her to Philadelphia to ply on the Delaware.

At that time no canal offered the opportunity to make an inland passage, and in June, 1808, Robert L. Stevens, a son of John, started with Captain Bunker to make the passage by sea. Although meeting a gale of wind, he arrived at Philadelphia safely, having been the first to trust himself on the open sea in a vessel relying entirely upon steam-power. From this time forward the Messrs. Stevens, father and sons, continued to construct steam-vessels.

The steam-engine in most general use for sea-going ships when the introduction of the screw compelled its withdrawal, with the paddle-wheel which it drove, was that shown in Fig. 22, which represents the side-lever engine of the steamer Pacific, as designed by Charles W. Copeland.

In the sketch, *A* is the steam-cylinder; *B C* the side-rods, or links, connecting the cross-head in the piston-rod with the end-centre, *D*, of the side-lever *D E F*, which vibrates about the main centre *E*, like the overhead beams. A cross-tail at *G* is connected with the side-lever and with the connecting-rod *G H*;

which latter communicates motion to the crank *IJ*, turning the main shaft *J*. The air-pump and condenser are seen at *O M*. This engine was one of the earliest and best examples of the type, and perhaps the first ever fitted with a framing of wrought-iron.

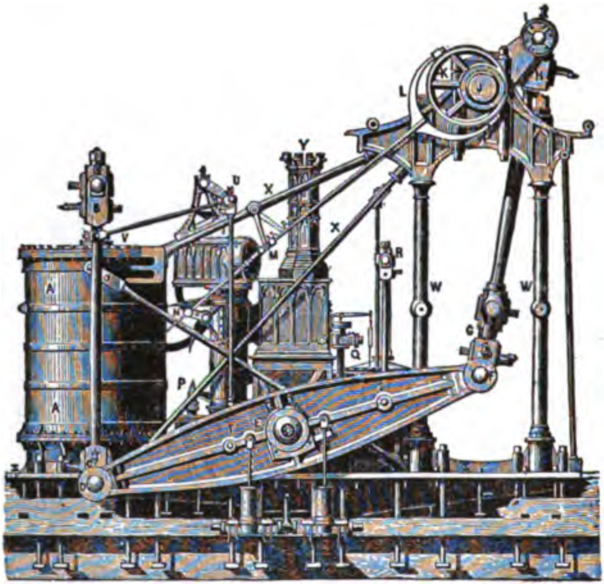


FIG. 22.—COPELAND'S SIDE-LEVER ENGINE, 1849.

After the experiments of Stevens, we find no evidence of the use of the screw, although schemes were proposed and various forms were even patented, until about 1836.

In 1836 Francis P. Smith, an English farmer who had become interested in the subject, experimented with a screw made of wood and fitted in a boat built with funds furnished by a Mr. Wright, a London banker. He exhibited it on the Thames and on the Paddington Canal for several months. In February, 1837, by an accident, a part of the screw-blade was broken off, and the improved performance of the boat called attention to the advisability of determining its best proportions. In 1837 Smith exhibited his courage and his faith in the

reliability of his little steamer by making a coasting-voyage in quite heavy weather, and the performance of his vessel was such as to fully justify the confidence felt in it by its designer. The British Admiralty soon had its attention called to the performance of this vessel, and to the very excellent results attained by the Archimedes, a vessel of 237 tons burden, which was built by Smith and his coadjutors in 1838 and tried in 1839, attaining a speed of eight knots an hour. By the performance of the Archimedes, the advantages of screw-propulsion, especially for naval purposes, were rendered so evident that the British Government built its first screw-vessel, the Rattler, and Brunel adopted the screw in the iron steamer Great Britain, which had been designed originally as a paddle-steamer.

Simultaneously with Smith, Captain John Ericsson was engaged in the same project. He patented, July, 1836, a propeller which was found at the first trial to be of such good form and proportions as to give excellent results. His first vessel was the Francis B. Ogden, named after the United States Consul at Liverpool, who had lent the inventor valuable aid in his work. The boat was forty-five feet long, eight feet beam, and drew three feet of water. It attained a speed of ten miles an hour, and towed an American packet-ship, the Toronto, four and a half miles an hour on the Thames. This was a splendid success.

Ericsson built several screw-boats, and finally, meeting Captain Robert F. Stockton, of the United States Navy, that gentleman was so fully convinced of the merits of Ericsson's plans that he ordered an iron vessel of seventy feet length and ten feet beam, with engines of fifty horse-power. The trial of the Stockton, in 1839, was eminently satisfactory. The vessel was sent to America under sail, and the designer was soon induced to follow her to this country, where his later achievements are well known. The engines of the Stockton were direct-acting, the first examples of engines coupled directly to the crank-shaft, without intermediate gearing, that we meet with after that of John Stevens. Soon after Ericsson arrived in the United States he obtained an opportunity to design a screw-

steamer for the United States Navy, the Princeton, and, at about the same time, the English and French governments had screw-steamers built from his plans, or from those of his agent in England, the Count de Posen. In these ships—the Amphion and the Pomona—the first horizontal, direct-acting engines ever built were used. They were fitted with double-acting air-pumps, having canvas valves and other novel features.

John Ericsson continued designing screw-ships for the merchant and the naval services, and his greatest triumph, the iron-clad "Monitor," embodied the essential features of this early revolution in marine-engine construction.

From 1840 the screw gained favor rapidly, and finally began to displace the paddle for deep-water navigation. Progress in this direction was at first somewhat slow. In 1840, and during the following ten years, many experiments were instituted between the performances of screw and paddle steamers without definitely settling engineering practice. The reason was, probably, that the introduction of the rapidly-revolving screw, in place of the slow-moving paddle-wheel, necessitated a complete revolution in the design of their steam-engines. And the unavoidable change from the heavy, long-stroked, low-speed engines, previously in use, to the light engines, with small cylinders and high piston-speed, called for by the new system of propulsion, was one that necessarily occurred slowly, and was accompanied by its share of those engineering blunders and accidents that invariably take place during such periods of transition.

The earliest days of screw propulsion witnessed the use of steam of ten or fifteen pounds' pressure, in a geared engine using jet-condensation, and giving a horse-power at an expense of perhaps seven or eight pounds of coal per hour. A little later came direct-acting engines with jet-condensation, and steam at twenty pounds pressure, costing about five or six pounds per horse-power per hour. The steam-pressure rose a little higher with the use of greater expansion, and the economy of fuel was further increased. The introduction of the surface-

condenser, which began to be generally adopted some ten or fifteen years ago, brought down the cost of power to between three and four pounds in the better class of engines.

At about the same time, this change to surface-condensation helping greatly to overcome the troubles arising from boiler-incrustation, which had checked the rise in steam-pressure above about twenty-five pounds, and it being at the same time learned by engineers that the deposit of the scale and sulphate of lime in the marine boiler was determined by temperature rather than by the degree of concentration, and that all the lime entering the boiler was deposited at the pressure just mentioned, a sudden advance took place. Careful design, good workmanship, and skilful management made the surface-condenser an efficient apparatus, and, the dangers of incrustation being thus lessened, the movement toward higher pressures recommenced and progressed so rapidly that, now, over one hundred pounds per square inch is very usual, and three hundred and fifty pounds has been attained in marine engines built by the Messrs. Perkins, who are said to have reached the remarkable economy of a horse-power for each pound of combustible in the fuel consumed in the boiler.

These high pressures, and the greater expansion of the steam, in turn, produced another revolution in engine-construction. It at last became generally known that one of the most serious losses of heat, and consequently of power, in the steam-engine, when expansion is carried to a considerable extent, occurs in consequence of condensation and the deposition of moisture upon the interior of the cylinder, which moisture, when the exhaust takes place, carries, by its re-evaporation, large quantities of heat into the condenser, without deriving any power from it. This loss is also, in some degree, prevented by dividing the expansive working of the steam among two or more cylinders, as in the compound system. Here the heat wasted in either cylinder is less, in consequence of the lessened range of temperature; and that lost by one cylinder is carried into the second, and there, to some extent, utilized.

The amount of saving effected by this means is considera-

ble—so great, in fact, as to have produced a complete revolution in engineering practice in the construction of marine engines by the best-known builders. They, under the lead of John Elder, adopted the Woolf engine, which had, in earlier times, with lower steam, less expansion, and less intelligent engineering, proved apparently a failure.

To-day all sea-going steamers are fitted with multi-cylinder engines having surface-condensers, and with tubular boilers, which are fitted, frequently, with superheaters.

The latest and largest of the paddle steamers of the Cunard line, the Scotia, built in 1862, was 379 feet long, and of 3871 tons burden; crossing the Atlantic in less than nine days. The engines were side-lever, and 100 inches diameter of cylinder, 12 feet stroke, making 18 revolutions per minute, and producing 4500 horse-power.

DIMENSIONS OF THE LARGEST OCEAN STEAMERS.

Name of Ship.	Date.	Length over all.	Beam.	Depth.	Draught.	Displacement.	Speed.
		Feet.	Feet.	Feet.	Feet.	Tons.	Knots.
Great Eastern	1858	692	83	57½	25½	27,000	14½
Paris	1888	560	63	42	26½	15,000	20
Teutonic	1890	585	57½	42	26	13,800	20
St. Paul	1895	554	63	42	27	16,000	21
Campania	1893	625	65	41½	28	19,000	22
Kaiser Wilhelm d. Grosse	1897	649	66	43	29	20,000	22.62
Oceanic	1899	704	68	49	32½	28,500	21½
Deutschland	1900	686	67	40	....	23,500	23.0
Celtic	1901	700	75	49	....	37,700	16.0

The later development on the ocean included the steamers Campania and Lucania, built in 1890–92. The former crossed the Atlantic, from Queenstown to New York, in 5 days, 9 hours, 27 minutes, and made 545 miles in twenty-four hours. These vessels are of 13,000 tons burden, 30,000 horse-power, and 622 feet long, 65 feet beam, and 43 feet depth. They have twin-screws, with four sets of triple-expansion engines. They carry 2000 people, of whom 1500 are passengers and 168 in the engineer's crew. The Deutschland has made 24.5 knots.

The steam-cylinders are of 37, 79, and 98 inches diameter, and 5 feet stroke, making, at speed, 90 revolutions per minute. The surface-condensers each contain 20 miles of brass tubes, $\frac{5}{8}$ inch diameter. The propellers are 19 feet diameter and $28\frac{1}{2}$ feet pitch; twin-screws, with four blades. Twelve boilers, containing 84 furnaces, with steam at 180 pounds, supply the engines. The feed-water amounts to 120 tons, the condensing water to 8000 tons, per hour, and the coal burned to 320 tons per day. The thrust on the two propellers is about 100 tons, total.* (See p. 222.)

The advances made in steam-navigation since the days of Stevens and Fulton may perhaps be best realized on comparing a modern steam-yacht of similar dimensions with the little screw boat of 1804. That here shown, as built by the Douglas Co., at Waukegan, Illinois, has very nearly the same measurement—26 feet length, 6 feet beam—but it weighs only

FIG. 23.—SMALL STEAM-YACHT.

one ton, carries an engine of 3 effective horse-power, and has a speed of about six miles an hour, a higher speed than that of Fulton's Clermont, a boat of five times its length.

23. **The Later Phases** of construction are given in more detail in § 24. By the year 1880, the standard form of marine engine, for large powers and for long voyages, had become the "compound," or double-cylinder type, expanding steam from a pressure of 75 to 90 pounds (5 to 6 atmospheres), by gauge, through two cylinders, "in series," into a condenser,

* London Engineer, Dec. 19, 1890.

the expansion terminating at 7 to 10 pounds per square inch ($\frac{1}{2}$ to $\frac{2}{3}$ atmosphere) above vacuum. The largest engines were constructed with a pair of low-pressure cylinders, to reduce the difficulties experienced in the attempt to make so large a single low-pressure cylinder; and these were called "three-cylinder compound engines."

In 1890, "triple-expansion engines" had become common, employing three cylinders "in series," and using steam of 10 to 12 atmospheres pressure (150 to 180 pounds per square inch by gauge), and the largest of these were given twin low-pressure cylinders.

Speeds of piston of 600 to nearly 1000 feet, and 70 to 90 revolutions per minute, were usual, with engines of 5 feet stroke and more, producing 10,000 to 20,000 I. H. P. in the propulsion of the largest and fastest steamships. Meantime, the weight of machinery fell from about 1000 to 300 or 350 pounds per horse-power.

Ratios of expansion were restricted, usually, to 3 or 5 in simple, 7 to 8 in compound, and 12 to 15 in triple-expansion engines, and the cost in fuel consumed dropped from $2\frac{1}{2}$ or 3 pounds per I. H. P. per hour to $2\frac{1}{2}$ and 2 and to $1\frac{1}{2}$ or even one, under favorable conditions.

The steady rise in steam-pressures during the century is best illustrated by naval steam-engineering. In the time of Watt and up to about 1840, the usual pressure in the low-pressure side-wheel engines of that period was from 4 to 7 pounds ($\frac{1}{4}$ to $\frac{1}{2}$ atmosphere) by gauge, and the rude flue-boilers then in use were of the simplest and weakest forms. By the middle of the century the fire-tubular boiler had come into quite common use, and pressures had risen to double those above stated. Between 1850 and 1860, the customary pressures in new engines and boilers had become 20 to 25 pounds ($1\frac{1}{3}$ to $1\frac{2}{3}$ atmospheres) and, the introduction of the surface-condenser removing the principal difficulty, the later rise in pressure was rapid and has never ceased.

At the pressure then reached, the deposition of the calcium sulphate contained in sea-water was complete and the conse-

quent loss of economy was very serious. The use of the surface-condenser, by reducing this loss, produced a gain of 15 or 20 per cent.

The type of boiler was next made the cylindrical, Scotch, form, with large flues serving as furnaces and the gases returned through tubes, both flues and tubes enclosed in one cylindrical shell, and, the compound engine introduced, the pressures rising rapidly to 60 or 75 pounds (4 or 5 atmospheres), by gauge, these changes resulting in a further economy of 30 or even 40 per cent in engines designed during the decade 1860–70. The next ten years carried pressures for compound engines up to 90 and 120 pounds (6 and 8 atmospheres) and the triple-expansion engine, coming into use, 1875–80, the pressure has risen one third or one half more, this type giving a gain of 15 or 20 per cent over the earlier compound engines. Lately water-tube boilers are common.

The following have been considered fair average figures, as representing what was good and standard practice at the dates given, and as illustrating the progress effected in marine engineering in the period 1870–90:

Type.	Date.	Pressure of Steam.	Coal per I. H. P. per hour.	Piston Speed.	Weight per I. H. P.
Simple	1870	50	2.1	375	500
Compound	1880	75	1.8	480	480
Triple	1900	180	1.3	800	300

In exceptional cases, as in torpedo-boats, the progress in lightening machinery, but not in efficiency, has been still greater, piston speeds having risen to above 1000 feet per minute. The weights of the two- and of the three-cylinder compound engine, as now customarily built, are not very different. For example, the following, as given by Mr. Hall in 1887, gives the weights of two selected cases:

	Two Cyl.	Three Cyl.
Boilers and accessories	88 tons	90 tons
Water in boilers	47 "	38 "
Engines and accessories	121 "	107 "
Water in condenser	2 "	2 "
Total	258 tons	237 tons
Power—I. H. P	1150	1160
Weight, lbs. per I. H. P.	502	457

The difference is here rather less than ten per cent, in favor of the later type.

The gradual reduction of weights of steam machinery during the period succeeding the middle of the nineteenth century is best illustrated by reference to the changes effected in naval work. The minimum weight in 1850 was about 200 pounds each, engines and boilers, per I. H. P., 400 pounds total; while these figures were reduced by 1860 to about 175 and 350; in 1870 to 150 and 300; in 1880 to 125 or 140, and 275 or 280; in 1885 to 80 or 90 for engines, and 100 for boiler, less than 200 total; and in 1890 to 40 or 50, 70 or 75, and 100 to 125 total, and even less in exceptional cases, as in fast yachts and torpedo-boats. The lightest examples are as low as 50 or 60 pounds, total, per horse-power. The adoption of simple types, of high engine-speed, of water-tube boilers, and of forced draught, is the secret of the rapid gain at the later dates.

According to Sennett, the reduction in weight of the machinery of naval vessels has steadily progressed since the early part of the nineteenth century, and since the advent of steam navigation. In 1832, with side-lever paddle-wheel engines, flue-boilers carrying but 4 lbs. of steam, and jet-condensers, there was but 1.45 I. H. P. obtained per ton of weight. Tubular boilers and 9 lbs. pressure increased the power to 3.14 I. H. P. per ton in 1845; oscillating engines and 14 lbs. of steam to 4.72 I. H. P. per ton in 1850; screw engines and 20 lbs. of steam to 5.52 I. H. P. in 1857; and the surface-condenser and 30 lbs. of steam to 7.5 I. H. P. per ton in 1870. The compound engine with 60 lbs. of steam only gave 6.4 I. H. P. per ton of machinery in 1876, but greatly reduced the total weight carried on account of reduced coal consumption. Triple compound engines produce a saving in fuel, rather than of weight, to be carried. The increase of weight due to compound and triple compound engines is chiefly caused by the heavier boilers required for the higher pressures, though the engines are also generally somewhat heavier. The introduction of forced blast has enabled the weight of the boiler to be reduced, and this, with high speed, reduces the weight of the engine so that tor-

;edo-boat machinery in 1880 gave 37.66 I. H. P. per ton of weight, and in a fast steamer built in 1882, 12.56 I. H. P. was obtained per ton of weight.

The later progress and current practice in the application of steam-power in small boats is well shown by the facts in the department of naval construction; and especially in the recent

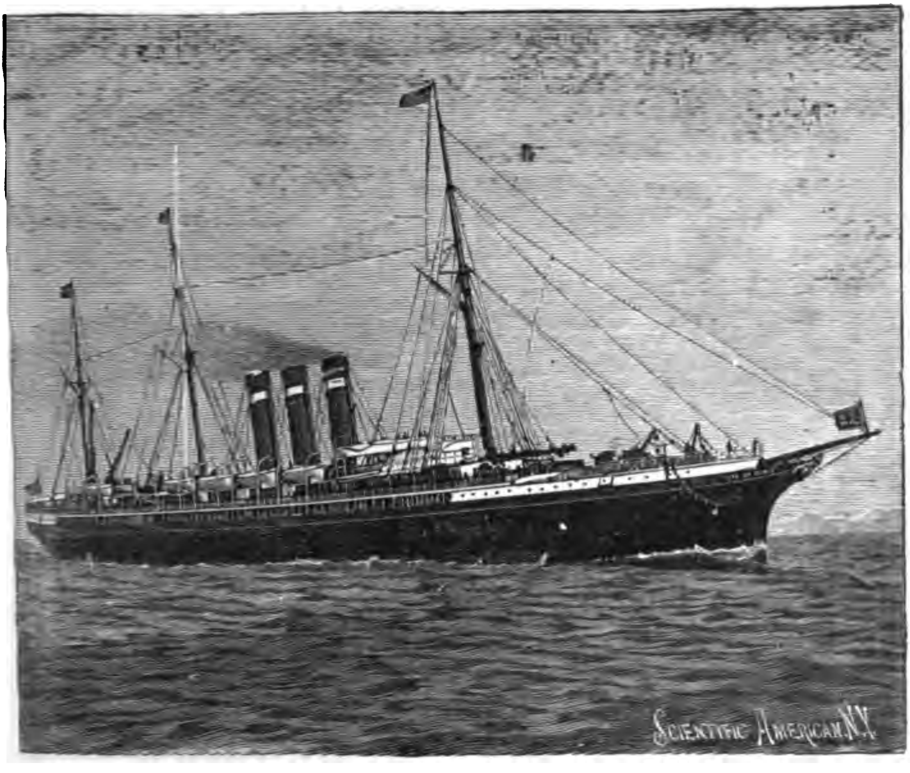


FIG. 24.—THE CITY OF PARIS.

introduction of surface-condensation and compounding and of a forced draught. About 1863–5, the naval steam-launch was about 40 feet long, was fitted with a high-pressure engine of 25 H. P., and had a speed of 6 knots. In 1880 the speed and power had risen to 20 knots and 500 I. H. P.; in 1890 to about 30 knots with over 1000 power, in consequence of im-

proved lines and higher efficiency of machinery and reduced weights. Recently (1897) a launch 100 feet long, with engines of 1500 H. P., and weighing 45 tons, attained a speed of 32¾ knots (37½ miles, nearly). Recent trials of simple and compound engines, in competition, as reported to the British Admiralty, gave 7½ and 4 pounds of fuel as respectively required. Their weights were nearly the same; 180 and 150 pounds, nearly, per I. H. P.* The last boat, above, is driven by a steam-turbine.†

By the introduction of forced combustion in the boiler-room, of steam steering, and of anchor- and cargo-hoisting machin-

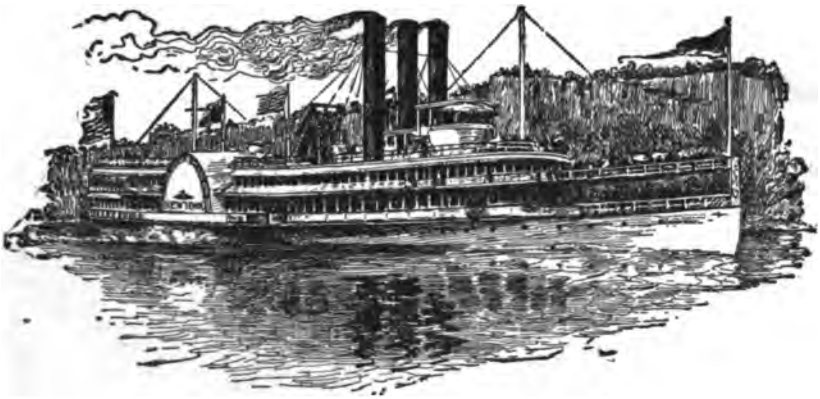


FIG. 25.—THE NEW YORK.

ery, and various other changes, the number of tons transported per person employed on shipboard has been increased from 2⅓ to 4⅓ between 1860 and 1900, or about doubled in the present century. The speeds of passenger-steamers now often exceed 20 knots (about 23 miles) an hour for an average, crossing the Atlantic. The mean of sixteen voyages of the City of New York, the City of Paris,‡ and the Teutonic was about six days and a quarter, while the Lucania has crossed in 5 days, 7 hours, 23 minutes.

In contrast with the Clermont, we may note the principal

* Machinery of Small Boats; A. Spyer; Trans. Brit. Inst. N. Archts., XXVIIth Session.

† Parsons on The "Turbinia," 1897.

‡ From paper by Mr. C. E. Emery in the Scientific American Supplement, 1890.

features of the steamer New York, built eighty years later, for the same route, by the Harlan & Hollingsworth Company, and "engined" by the W. & A. Fletcher Company. (Fig. 25.)

The dimensions of hull are as follows:

Length on the water-line	301	feet.
Length over all	311	"
Breadth of beam, moulded	40	"
Breadth of beam, over guards	74	"
Depth, moulded	12	" 3 ins.
Draught of water	6	"
Tonnage (net, 1091.89)	1552.52	

The wheels are aft of the centre of length, instead of forward—a great improvement in the appearance of the boat.

The engine is a beam-engine, with a cylinder 75 inches diameter and 12 feet stroke of piston, provided with Stevens' cut-off. The use of a surface-condenser, instead of a jet-condenser, in this river steamer, is a change made to overcome the evil of using mixed salt and fresh water in the boilers, as the tides extend to Albany and the water changes from salt to fresh *en route*.

Another change is the return to the use of Stevens' feathering wheels. These are 30 feet 2 inches diameter outside of buckets. There are twelve curved steel buckets to each wheel. Each bucket is 3 feet 9 inches wide and 12 feet 6 inches long. The wheels are overhung, and they have a bearing on the hull only. The feathering is effected in the usual manner by driving and radius bars, operated by a centre placed eccentric to the shaft and held by the "A-frame" on the guard. These wheels were introduced in the New York for the purpose of gaining speed, and the trial-trip shows that the builders' expectations were completely fulfilled. Absence of jar is another gain obtained by the use of these wheels, and the comparatively thin buckets enter the water so clean and smooth that one notices, not the shake so common on boats with the ordinary wheels, but an almost entire absence of it.

Steam is supplied to the engine by three return-flue boilers,

each $9\frac{1}{2}$ feet diameter of shell, 11 feet width of front, and 33 feet long, constructed for a working pressure of 50 pounds per square inch. Each boiler has a grate-surface of 76 square feet or 228 square feet in all, and with the forced draught produce 3850 horse-power.

The exterior is of pine, painted white relieved with tints and gold. The interior is finished in cabinet work, and is all hard wood, ash being used forward of the shaft on the main deck, and mahogany aft and in the dining-cabin. Ash is also used in the "grand saloons" on the promenade deck. The saloon-sides are almost entirely of glass, and the windows so low that persons seated inside have an opportunity to view the scenery.

The Puritan, Fig. 26, illustrates the adaptation of this type of steamer, so nearly perfected by Robert L. Stevens, to that kind of navigation, intermediate between river, or still-water, and oceanic, which permits the retention of some features of the former, while modifying the shape of hull and type of engine to meet the demands of "outside" navigation.

The plans of this steamer are by Mr. Pierce, the details of hull-construction by Mr. Faron, and the machinery by the W. & A. Fletcher Co. The principal dimensions are as follows: Length over all, 420 feet; length on the water-line, 404 feet; width of hull, 52 feet; extreme breadth over guards, 91 feet; depth of hull amidships, 21 feet 6 inches; height of dome from base-line, 63 feet; whole depth, from base-line to top of house over the engine, 70 feet. Her total displacement, ready for a trip, is 4150 tons, and her gross tonnage 4650 tons.

The ship is fire-proof and unsinkable, having a double hull, divided into 59 water-tight compartments, 52 between the hulls and 7 made by athwartship bulkheads. In the fastenings of hull and compartments there were used 700,000 rivets, and upwards of thirty miles of steel angle-bar. Her decks are of steel, wood-covered. Her masts are of steel, and hollow, to serve as ventilators, and are 22 inches in diameter. Her paddle-wheels are encased in steel.

The hull is of "mild steel," twenty per cent stronger than

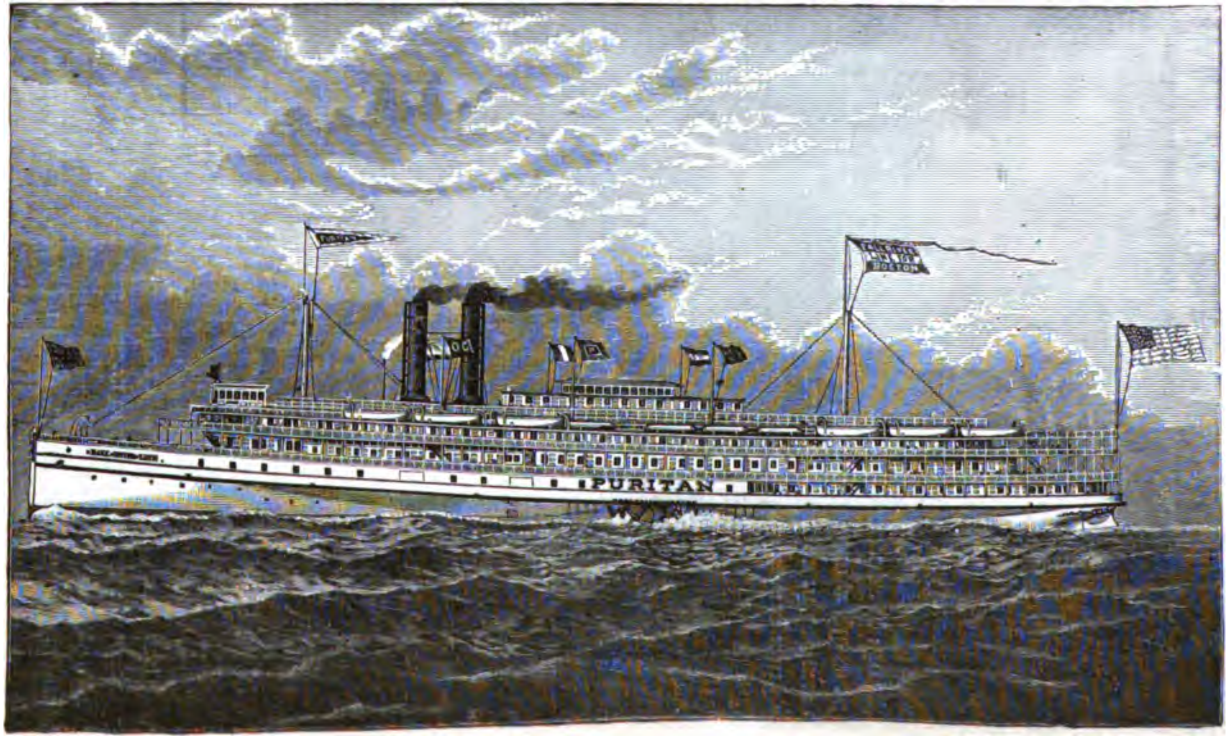


FIG. 26.—A "SOUND STEAMER."

iron. The wheels are of steel, and are 35 feet in diameter outside the buckets. The buckets are 14 feet long and 5 feet wide, each bucket of steel ⅞ inch thick, and weighing 2800 pounds without rocking arms and brackets attached. The total weight of each wheel is 100 tons. The wheels are "feathering," and turn at the rate of 24 revolutions a minute.

The boat has a compound, vertical, beam, surface-condensing engine of 7500 horse-power. The high-pressure cylinder is 75 inches in diameter, and 9 feet stroke of piston. The low-pressure cylinder is 110 inches in diameter, and 14 feet stroke of piston. The surface-condenser has 15,000 square feet of cooling surface and weighs 53 tons. Of condenser-tubes of brass there are 14½ miles. Her working-beam is 34 feet in length from centre to centre, 17 feet wide, and weighs 42 tons. The section of beam-strap measures 9½ × 11¼ inches. The main centre of the beam is 19 inches in diameter in its bearings. The shafts are 27 inches in diameter in main bearings, and 30 inches in gunwale bearing. They weigh 40 tons each. The cranks weigh 9 tons each. The crank-pin is 19 inches in diameter and 22 inches long.

The boilers contain 850 square feet of grate-surface and 26,000 square feet of heating surface. The products of combustion pass through two super-heaters, 8 feet 10 inches inside diameter, and 12 feet 4 inches outside diameter, by 12 feet high; thence into two smoke-stacks, the top of each being 101 feet 1 inch from the keel.

The dining-saloon is 108 feet 4 inches in length, by 30 feet in width, and 12 feet in height. There are 12 miles of electric-lighting wire, and, including annunciators, fire-alarm, etc., there are twenty miles of wire and twelve thousand feet of steam-pipe. There are capacious gangways and staircases, lofty cornices, and ceilings supported by tasteful pilasters, the tapering columns of which, in relief, flank exquisitely-tinted panelling throughout the length of her saloons. Every convenience known to civilization, and which can contribute to the ease and comfort of the traveller on land or when afloat, is included in the internal arrangements of this floating caravansary.

The electric-light currents are generated by four dynamos, each designed with a capacity of 400 lights, or a total of 1600 lights, but capable of maintaining 1850 lights if required.

These great steamers have all the essential features of the earlier river-boats of Stevens: the same long, flat, shallow hull, the widely-extended guards and main deck; the "hog-frames" stiffening the whole structure; the same type of "beam-engine," as a rule; and the high deck-houses; but the progress of the century is seen in their enormous size, great power and speed, and their innumerable conveniences and luxuries.

The fleets of vessels employed on the great lakes between the United States and Canada have become mainly steam-fleets; the principal part of the lake transportation of ores, timber, and grain being now carried on in craft like that seen in

FIG. 27.—A "LAKE STEAMER."

the accompanying illustration, a type of vessel peculiarly American. The figure represents the Tuscarora, built at Cleveland, by the Globe Iron Works, for the Lehigh Valley fleet, at a cost of about $250,000. Vessels of this class are built of steel and fitted with multiple-cylinder engines, and are both fast and economical.

The Tuscarora is 312 feet over all, 40 feet beam, and 25½ feet deep. The weight of hull exceeds 1600 tons. She has

two flush steel decks, the top covered with 3-inch pine, and an additional tier of deck-beams below, or a third deck. The water-bottom runs clear aft, and there are three longitudinal keelsons on either side of the main keelson. The triple-expansion engines have 24-, 38-, and 61-inch cylinders of 42 inches stroke. There are three boilers, 12 × 12½ feet, carrying 160 pounds of steam.

The growth of tonnage on these lakes now exceeds 100,000 tons per annum; or about the same as the total of the Atlantic and Pacific coasts. The steamers employed are usually very similar in general construction to that here illustrated, the high deck-houses and cabins of the river steamer being necessarily omitted as a matter of safety, and the comparatively smooth and low house of the ocean steamer substituted. The deeper water also permits the use of the screw on the largest vessels.

24. **Recent Applications** of the multiple-cylinder engine have become usual in every department of steam-engineering. The efforts of Hornblower and of Wolff and their contemporaries failed, partly because of the active business competition of Boulton and Watt, who possessed at the time enormous advantages and immense power, but mainly because the steam-pressures and speeds of piston then adopted were too low, and the practicable range of expansion was too small, to permit the advantages of the more complex type of engine to become obvious and important. But when the steam-pressure carried on other engines began to rise toward three and four atmospheres, the ratios of expansion to exceed three or five, the serious wastes arising from initial cylinder-condensation began to be seen, and were found to place an early limit to economically increased expansion. This limit, as well as the economical operation of the engine at the earlier limit, was promptly modified when the new construction was adopted; and it was found that not only was the efficiency of the engine at ratios of expansion then considered maxima greatly increased, but that it was possible to economically extend expansion very much farther than was practicable in a single cylinder. As steam-pressures continued

to rise, and as expansion was correspondingly increased, the gain by compounding became more observable and important, and the new engine found more general application; until now it is employed almost exclusively in marine engineering, and very extensively in other departments. The increase of steam-pressure above one hundred pounds per square inch, above six or seven atmospheres, has led to the introduction of the triple-compound, or "triple-expansion," engine, and pressures exceeding ten atmospheres are already making the "quadruple-expansion" engine a desirable type where great economy of fuel is essential. In all cases, in marine engines, it is found advisable, in good types of engine, to expand steam down to from ten to eight pounds per square inch above perfect vacuum, to about a half atmosphere pressure, to secure best results. The better the design the lower this limit.

The advantages of the multicylinder engines have become so evident that, since about 1870, they have been adopted as standard by the navies of the world, in spite of the obvious objections to high steam and their inflexibility of power adjustment in modern warfare.

Multiple-cylinder marine engines are used to the almost entire exclusion of the older forms of simple engine. Although invented by Hornblower in 1781, and, in the more common types, by Wolff in 1804, it was only when, a half-century later still, Messrs. Randolph and Elder in the screw-steamer Brandon (1854) and the paddle-steamers Valparaiso and Nica and others, still later, of the Pacific Steam Navigation Co., made this type practically a success, that it attracted the general attention of engineers. From that time it has steadily and rapidly displaced the simple engine. The gain of the two-cylinder compound engine, when compared with the standard simple marine engine, was found to be from 20 to 40 per cent, averaging in those early days probably 33 per cent. This was enough to secure their general introduction with great rapidity, once the fact was established.

The most common form given the two-cylinder compound engine, of the best construction, is that shown in a succeeding

illustration (Chap. II, Fig. 112), and is that almost universally adopted for vessels of the merchant marine. Many designs, differing greatly among themselves and from the above, have been introduced into the ships of the fighting classes in the navy, having mainly in view the reduction of their vertical dimensions and getting them well below the water-line and out of reach of shot. It is also sometimes attempted in naval engines to so make their steam-connections that either or both cylinders may be supplied with steam directly from the boilers, should any exigency or an emergency make it desirable. The principles of designing, of proportioning, and of construction are precisely the same, however, whatever the method of grouping the engine-cylinders or their details and accessories.

In the cases, becoming common in the United States, but comparatively rare in Europe, in which the engine is proposed to be made a beam-engine and is to drive paddle-wheels, the usual method of compounding is to place the two cylinders at the same end of the beam and as closely together as possible. In the Buckeye State, designed by Mr. Erastus Smith about the middle of the century, the low-pressure piston was an annulus working between the exterior surface of the high-pressure and the internal surface of the low-pressure cylinder; both pistons being connected to a common cross-head and, by the same pair of links, to the extremity of the beam. The compound engines of the City of Fall River were found to give higher efficiency, by one fourth or one third, than the simple engines customarily employed on Long Island Sound in the same work.*

Perhaps as near an approach to ideal efficiency as has yet been recorded, all things considered, is that of M. Normand's torpedo-boat in the French navy, No. 128; the engines of which are reported to have demanded but 0.462 kilogs of fuel per horse-power and per hour (1.16 lbs. per British H. P.).

These engines were "receiver-compounds," with steam

* Report on the City of Fall River, by Messrs. Sague and Adger; with introduction by R. H. Thurston; Jour. Frank. Inst., July, 1884.

entering at 4.3 atmospheres (70 lbs.), with clearances of 10.6 and 6.4 per cent.* The power attained was 940 I. H. P., the displacement of the hull being about 35 tons; and speed not far from 10 knots, the maximum, when driven, being 21 knots. The principal source of this exceptional economy is presumed to be a remarkably effective system of feed-water heating by intermediate steam to 212° F.; full compression in the small cylinder; and a slight degree of superheating by "wire-drawing" the steam. The boiler-steam had a pressure nearly three times as great as that in the steam-chest. M. Normand has since, nevertheless, substituted the triple-expansion engine for the compound.†

The Triple-expansion Engine has succeeded the ordinary two-cylinder compound machine in regular work of the merchant navy for long routes, and is also occasionally adopted for stationary engines where the cost of fuel is such as will justify the somewhat increased cost of construction. By its use, it is found practicable to raise the steam-pressure to above ten atmospheres (150 lbs. and upward) and to increase the ratio of expansion to 15 or more, with good results. The great cost of fuel and the value of tonnage-space on shipboard have hastened this advance in marine-engine design. Mr. A. E. Seaton, comparing sister-ships fitted with the two types of engine, found this change to produce a saving of about 20 per cent over the two-cylinder compound engine, a difference substantially that predicted by computations assuming the usual differences of pressure and ratios of expansion and a reduction by one-third of the cylinder-wastes.

"Triple-expansion" engines were introduced as early as 1874 by Mr. A. C. Kirk in designing the machinery of the S.S. Propontis of Liverpool, the steam being supplied at 160 pounds pressure by water-tube boilers of the Rowan type; Mr. Kirk observing that a ratio of expansion exceeding 2½ was not practically more advantageous than this value; as higher ratios so exaggerate internal wastes as not to be economical in a single

* Official Report: Mem. de la Soc. des Ing. Civils; Dec. '90; p. 854.

† Ibid.

cylinder. The result was a considerable gain in economy of steam and fuel.

This type of engine in the long voyage between London and Australia (1880) has given similar economy, saving 500 tons in the voyage and permitting the carrying of 500 tons additional freight.

The triple expansion in engines carrying 175 to 200 pounds steam has been introduced (1885), and many vessels are supplied with water-tube boilers with steam as high as 300 pounds pressure, reduced to 250 at the engines.

The quadruple-expansion engine (1890) with steam at 200 to 250 pounds pressure has still further promoted this advance, yielding the horse-power on a consumption of as little as 12 pounds of steam and 1½ pounds good fuel, including the demand of its accessory and usually independent air-pump and feed-pump systems.

In 1840 a pound of coal propelled a displacement weight of 0.578 ton 8 knots: but the earning weight was only one tenth of this, 90 per cent of the displacement representing the hull, machinery, and fuel. In 1850, with iron vessels and the screw propeller, a displacement weight of 0.6 ton was propelled 9 knots by one pound of coal; but the proportion of cargo had risen to 27 per cent, or 0.16 ton. In 1860, with higher boiler-pressure and the surface condenser, 0.82 ton displacement was propelled 10 knots, and the cargo was 33 per cent, or 0.27 ton. In 1870, after the compound engine had come into use, 1.8 tons displacement was propelled 10 knots, and here the cargo formed 50 per cent of the whole, being 0.9 ton. In 1885 there were two classes of freight boats; the "tramp" propelled 3.4 tons displacement 8½ knots, with 60 per cent, or 2 tons of cargo; at the same time the enormous cargo steamers of the North Atlantic were driving a displacement of 3.14 tons 12 knots, with 55 per cent, or 1.7 tons of cargo. On the modern express passenger steamers the cargo weight is down to 0.09 ton per pound of coal.*

* President A. J. McGinnis, Liverpool Engineering Society.

25. **The Process of Development** of the steam-engine is, *en résumé*, as follows: *

A century ago, James Watt had just begun to introduce the first engines belonging to a, then, new type.† A century before (1698), the ingenuity and practical skill of Captain Savery had conferred an enormous benefit upon the mining industries, and through them upon the world, by applying the "fire-engine" of the Marquis of Worcester to raising water from the then rapidly deepening mines. Savery used steam of 8 to 10 atmospheres (120 to 150 pounds) total pressure, in some cases; and he is entitled to fame as the first to introduce that now familiar concomitant of civilization, the steam-boiler explosion. The usual pressure was 3 atmospheres. These engines demanded about 30 pounds of coal, per horse-power per hour, as a minimum. The apparatus of Savery was not what would to-day be called a steam-engine, at all. It was not a train of mechanism, involving moving parts, cylinder, piston, crank, and fly-wheel. Huyghens (1680) and Papin (1690) proposed true engines with steam-pistons traversing their cylinders, and forming, on the whole, much such a train of mechanism as is now so well known;‡ but the Newcomen engine was the first of this type to come into practical use. A writer of that time states § that "Mr. Newcomen's invention of the fire-engine enabled us to sink our mines to twice the depth we could formerly do, by any other machinery;" but "every fire-engine of magnitude consumes £3000 worth of coal per annum." The coal-consumption was, at best, about 20 pounds per hour and per horse-power. It was this engine that Watt found in operation, when he entered upon the stage.

Watt was not simply a mechanic; he was a real philosopher, and a truly scientific investigator. He found that the sources of loss in engines were the conductivity and radiating power

* Stationary Steam-engines; R. H. Thurston; N. Y., J. Wiley & Sons.

† History of the Growth of the Steam-engine. International Series. N. Y., D. Appleton & Co.

‡ Mem. Acad. Sci.; Paris, 1680. *Acta Eruditorum;* Leipsic, 1690.

§ *Mineralogia Cornubiensis;* Price; 1778. Appendix.

of the steam-cylinder, the alternate heating and cooling of the metal at each stroke, the imperfect vacuum, and the wastes from boiler and steam-pipes. To correct these defects, he clothed his boilers and steam-pipes with non-conductors, sometimes even making boiler-shells of wood. Smeaton had already covered the pistons and cylinder-heads with wood. Watt made a more practicable improvement, however, when he devised the steam-jacket. He attached a separate condenser, closed the cylinder at the top, made the engine double-acting, and finally adapted the engine to drive machinery, fitting it with shaft and fly-wheel, throttle-valve and governor, and thus making the steam-engine such as we see it to-day, in all essential particulars. His engine was substantially complete by the year 1784.*

Later changes have been a succession of refinements, and of developments in application. Stephenson, and his contemporaries, applied steam on railroads; Stevens, Fitch, and Evans, and, finally, Fulton, in the United States, and Bell and others, in Europe, introduced steam navigation; Sickels invented the "detachable" cut-off valve-gear; Corliss introduced the peculiar type of engine that has given him fame, and so attached its governor as to determine the point of cut-off automatically, and thus to regulate the engine. Robert L. and Francis B. Stevens designed the American river steamboat, and its beam-engine, with so simple and effective a valve-gear that it remains, to-day, still standard. The compound engine, even, was brought out by contemporaries of Watt, and thus every prominent feature and essential detail of the modern steam-engine was introduced at, or before, the middle of the nineteenth century.

Yet practice has been steadily changing since his time; and the form and proportions of the steam-engine, and the methods of steam distribution, have been undergoing constant changes. In the days of Watt, steam was worked at about 7 pounds pressure, per square inch, in stationary engines; they were always fitted with condenser and air-pump, were slow in move-

* History of the Growth of the Steam-engine, p. 119. Farey on the Steam-engine.

ment, and were, consequently, of small power in proportion to their size; they wasted heat and fuel to such an extent as to demand 6 or 8 pounds of coal per horse-power and per hour. It is true that Wolff, in 1804, expanded 6 or 8 times, using higher steam, and obtained the horse-power with 4 pounds of fuel per hour, and that John Stevens and Oliver Evans, in the United States, and Trevithick, in Great Britain, had already used still higher steam in non-condensing engines; but these examples simply illustrated the fact that isolated examples which lead standard practice by a half-century, or more, are to be observed during the growth of every art.

Although the principles of steam-engine economy were, in the main, well understood by Watt and his competitors, and have become well settled in later years, we are still far from a completely satisfactory solution of the problem, which, as stated by the Author elsewhere, may be enunciated thus: To construct a machine which shall, in the most perfect manner possible, convert the energy of heat into mechanical power; the heat being derived from the combustion of fuel, and steam being the receiver and conveyer of that heat.

Watt's first condenser has been seen to have been a surface-condenser. He immediately afterward adopted a jet-condenser, however, to obtain "a surface sufficiently extensive to condense the steam of a large engine," and to avoid the difficulties that might arise should the condensing water "crust over the thin plates" of the surface-condenser.

The surface-condenser was used by Mr. S. Hall, in 1838, on the steamship Wilberforce. This condenser had 2374 copper tubes, 8 feet long and one-half inch in diameter, placed vertically in a box, cooling surface about 2486 square feet, and 8.72 square feet of condensing surface per horse-power. The tubes became coated with mud, and were removed; the surface-being changed to a jet-condenser. In 1859 the P. & O. Steamship Co. adopted surface-condensation on the Moulton. The condenser had 1178 tubes, 5 feet 10 inches long, $\frac{5}{8}$ inch in diameter, 0.05 inch thick, or a surface of 4200 square feet, 2.42 square feet of condensing surface per I. H. P. The tubes were

packed with linen tape and screwed glands. The circulating water was controlled by a centrifugal pump, probably the first independent circulating-pump ever used. The tubes were vertical and the refrigerating water ascended them on the outside.

Since that date their use has become general; the pioneers in the United States having been Lighthall and Sewall.

The general introduction of electric-lighting systems, which ordinarily employ "dynamos" driven at very high velocities of rotation, brought about a remarkable and radical change of practice in steam-engine design and construction. The demand became imperative for a motor system which should provide power with decreased weight and volume of engine and machinery, and this concentration of power required to be accompanied by a corresponding increase in speed of engine-piston and of rotation, and a much better regulation. Experience has generally led to the adoption, where practicable, of independent engines to each dynamo, and only the high speed of the modern engine is, ordinarily, considered suitable to this work.

Steam-pressures have risen, since the improvement of the steam-engine by Watt was begun, somewhat as follows, at sea and in condensing engines (see p. 81):

Year. A.D.	Steam Pressures. lbs.	atmos.
1800	0 to 5	0 to $\frac{1}{3}$
1810	5 " 7	$\frac{1}{3}$ " $\frac{1}{2}$
1820	5 " 10	$\frac{1}{3}$ " $\frac{2}{3}$
1830	10 " 20	$\frac{2}{3}$ " $1\frac{1}{3}$
1840	15 " 20	1 " $1\frac{1}{3}$
1850	15 " 25	1 " $1\frac{2}{3}$
1860	20 " 30	$1\frac{1}{3}$ " 2
1870	30 " 60	2 " 4
1880	60 " 90	4 " 6
1890	100 " 150	7 " 10
1900	200 " 300	20 " 30

In many cases, considerable variations from these figures have been observed; but they may be taken as representative

of what was generally thought good practice at the several dates.

The history of progress in marine engineering in the latter half of the nineteenth century is exceedingly instructive. As the power of the engine is, if properly proportioned, in the ratio of its speed of piston or, with any one engine, to its revolutions in the unit of time, these speeds have risen from 500 or 600 feet to 1000, and from 40 or 50 to 80 and 100 revolutions, with even large engines. Simple engines at 25 pounds pressure have been superseded by compound engines at 60 to 80 and these by triple and quadruple expansion from 150 and 250 pounds; while gaining 30 per cent or more in the first step and 20 or more in the second, all costs considered. Forced draught at 6 inches water pressure has been used, and the speed of similar ships raised from 10 or 12 knots to 15 and then to 18 and 20, each square foot of heating surface giving, in some cases, 20 horse-power.

In 1890 the combined power of all the prime movers in the world using steam as the working fluid was not far from 100,000,000 horse-power, of which the United States had about 15,000,000, Great Britain the same, France and Germany, collectively, a similar amount, and the balance was distributed among other nations. Taking the horse-power as the equivalent of the work of five men, as an average, including overtime, the work of steam is the equivalent to that of a population of working men amounting to 500,000,000, to a total population of 2,500,000,000, or to about quadrupling the work of the globe.

26. **The Philosophical Study** of this development will be seen to give rise to the following:

We may rapidly note the prominent points of improvement, and the most striking changes of form; and may thus obtain some idea of the general direction in which we are to look for further advance.

Beginning with the earlier machines, we there found a single vessel performing the functions of all the parts of a modern pumping-engine; it was at once boiler, steam-cylinder, and

condenser, as well as both a lifting and a forcing pump. The Marquis of Worcester, and, still earlier, Da Porta, divided the engine into two parts; using one part as a steam-boiler, and the other as a separate water-vessel. Savery duplicated those parts of the earlier engine which acted the several parts of pump, steam-cylinder, and condenser, and added the use of the jet of water to effect rapid condensation. Newcomen and Cawley next introduced the modern type of engine, and separated the pump from the steam-engine proper: in their engine, as in Savery's, we notice the use of surface-condensation first; and, subsequently, that of a jet of water thrown into the midst of the steam to be condensed. Watt finally affected the crowning improvement of the single-cylinder engine, and completed this movement of differentiation by separating the condenser from the steam-cylinder, thus perfecting the general structure of the engine. Here this movement ceased, the several important processes of the steam-engine now being conducted each in a separate vessel. The boiler furnished the steam; the cylinder derived from it mechanical power; the vapor was finally condensed in a separate vessel; while the power, which had been obtained from it in the steam-cylinder, was transmitted through still other parts to the pumps, or wherever work was to be done.

Watt and his contemporaries also commenced that movement toward higher pressures of steam, used with greater expansion, which has been the most striking feature noticed in the progress of the steam-engine since his time. Newcomen used steam of barely more than atmospheric pressure, and raised 105,000 pounds of water one foot high, with a pound of coal consumed. Smeaton raised the steam-pressure to eight pounds, and increased the duty to 120,000. Watt started with a duty of double that of Newcomen, and raised it 320,000 foot-pounds per pound of coal, with steam at ten pounds. To-day, Cornish engines of the same general plan as those of Watt, but worked with forty to sixty pounds of steam, and expanding three to six times, do a duty that will probably average, with good ordinary engines, above 600,000 foot-pounds per pound of coal.

The increase of steam-pressure and expansion which has been seen since Watt's time has been accompanied by a very great improvement in workmanship, a consequence of rapid increase in the perfection and the wide range of adaptation of machine-tools, of higher skill and intelligence in designing engines and boilers, increased piston-speed, greater care in obtaining dry steam, and in keeping it dry until thrown out of the cylinder—either by superheating, or by steam-jacketing, or by both means combined; and it has been further accompanied by greater attention to the important matter of providing carefully against losses by conduction and radiation, and by internal wasteful transfer of heat. The use, finally, of the "compound," or the multicylinder, engine for the purpose of reducing friction, as well as of saving some of that heat which is usually lost in consequence of internal condensation and re-evaporation due to great expansion, has still further aided in this progress and giving a duty of 1,500,000 or more.

An important consequence of the still unchecked rise of piston-speed in the modern-steam-engine is the approach to a limit beyond which the now standard form of "drop cut-off," or "detachable" valve-gear, cannot be used. For the piston would, at that limit of speed, reach the end of its stroke before the dropped valve could reach its seat, and the point of cut-off and degree of expansion could no longer be determined accurately and invariably by the governor. This limit has probably already been attained in some engines; and the engineer is driven back to the use of the older types of "positive-motion" valve-gearing, and is compelled to devise special forms of governor which shall have sensitiveness, and yet power sufficient to control these less tractable kinds of mechanism, and to invent reliable and durable forms of balanced valves, and to practise every available expedient for making the movement of the valve, and its adjustment by the regulator, perfectly easy. Positive motion and ease of adjustment by the governor are, therefore, evidently the requisites of a successful valve-gear for the high-speed engine which will succeed the standard engine of to-day for many purposes.

We may now summarize the results of our examination of the development of the steam-engine thus:

(1) The process of improvement has been one, primarily, of "differentiation;" the number of parts has been continually increased, while the work of each part has been simplified, a separate organ being appropriated to each process in the cycle of operations.

(2) A kind of secondary process of "differentiation" has, to some extent, followed the completion of the primary one, in which secondary process one operation is conducted partly in one and partly in another part of the machine. This is illustrated by the cylinders in series of the multicylinder engine.

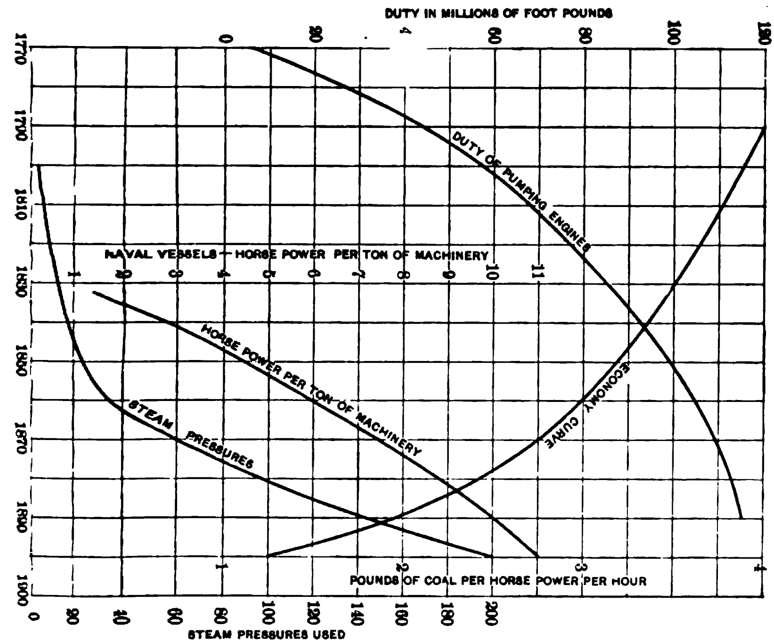
(3) The direction of improvement has been marked by a continual increase of steam-pressure, greater expansion, special provision for obtaining dry steam, higher piston-speed, careful protection against loss of heat by conduction or radiation internally, as well as externally, and, in marine engines, by surface-condensation.

The direction of further improvement, as indicated by science as well as by our review of the actual steps already taken, would seem to be: *En résumé*, working between the widest attainable limits of temperature, and the saving of heat previously wasted in the apparatus or rejected from it. Steam must enter the machine at the highest possible temperature, must be protected from waste or loss of heat, and must retain, at the moment before exhaust, the least possible proportion of originally available heat. He whose inventive genius, or mechanical skill, contributes to effect either of these objects—to secure either the use of higher steam with safety, or the more effective conversion of heat into mechanical power without waste, or the reduction, by transformation into work, of the temperature of the rejected working-fluid—confers an inestimable boon upon mankind.

In detail, in the engine proper the tendency is, and may be expected to continue, in the near future at least, toward higher steam, greater expansion in more than one cylinder, steam-jacketing, superheating, a careful use of non-conducting pro-

tectors against waste, and higher piston-speed with rapid rotation, and to the adoption of special proportions and of forms of valve-gear adapted to such high-speed engines. In the boiler, more complete combustion, without excess of air passing through the furnace, is sought, and a more thorough absorption of heat from the furnace-gases. The latter may be ultimately found most satisfactorily attainable by the use of a mechanically-produced draught, in place of the far more wasteful method of obtaining it by the expenditure of heat in the chimney. In construction, we may anticipate the use of better materials, as already seen in the substitution of "mild steels" for the cruder material, iron, and more careful workmanship, especially in the boiler, and still further improvement in forms and proportions of details.

Progress may be well summarized by "curves of progress," as the author has called them. He is indebted to Mr. Koon for those here given.*



PROGRESS IN STEAM-ENGINE EFFICIENCY.

* Sibley Journal of Mechanical Engineering, 1902.

CHAPTER II.

STRUCTURE OF THE STEAM-ENGINE.

27. **The Structure and Uses of the Steam-engine** have been well defined and mutually adapted, each to the other, since the middle of the nineteenth century, and in such manner as to have led to the production of certain fairly definite forms of engine; which are each employed very generally, sometimes exclusively, for equally specific purposes.

Thus: the modern mill-engine, simple or compound, is commonly a direct-acting, horizontal engine—at least for moderate and large powers—with effective provision for adjusting the point of cut-off by the action of the governor; the engine employed especially to drive fast machinery is commonly a machine having a "positive-motion" valve-gear and as simple of construction, as compact, and as well balanced as the art of the builder can make it; while the locomotive and the marine engines are each of a type which has been the product of years of change and of evolution which have resulted in their very perfect adaptation to their peculiar work. It has thus happened that engines are divided into classes; each class having its characteristic form and structure, and its own special nomenclature.

28. **The Classification into Types** has been usually followed in substantially the manner indicated in the scheme given in the next article. It is not invariably the fact, however, that the classification with reference to use is adhered to in the actual use of engines; and it is often the fact that one type is applied to the purposes ordinarily considered specially appropriate to another class. For example: we find the portable engine, and sometimes a retired locomotive, doing duty as

a stationary, mill, engine; as may also be the case sometimes with an engine constructed on what are recognized generally as the characteristic plans of the marine engine.

Nevertheless, as a rule, each kind of work is best performed by a form of engine which has been found, by the experience of years, to be the best for that place. The engineer is therefore inclined to be somewhat cautious in accepting any suggestion looking to interchange of duties in this manner.

According to Weisbach's system, the various piston-engines may be grouped under the following classes: *

I. According to the number of cylinders:
  (1) Single cylinder.
  (2) Multiple-cylinder engines.

II. With reference to the construction of cylinders:
  (1) Fixed cylinder.
  (2) Movable cylinder.

  In the first case, the engines are—
    (*a*) Vertical.
    (*b*) Horizontal.
    (*c*) Inclined.

  In the second case, they are—
    (*a*) Oscillating.
    (*b*) Rotary.

III. With reference to the action of the steam:
  (1) Single acting.
  (2) Double-acting.

IV. With reference to the transmission of the steam-power:
  (1) Direct-acting.
  (2) Indirect-acting.

  And in the latter case either—
    (*a*) With balance lever, or beam.
    (*b*) Without lever or working beam.

29. **Steam-engines Classed** according to their purpose and use, as in the following scheme, may be taken as practically including all existing standard and approved types.

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* Weisbach's Mechanics, vol. ii. part 2, § 452, p. 285.

STANDARD TYPES OF ENGINES.

GENERAL CLASS.

Stationary, or Mill, Engines:
Moderate Speed or
High Speed.
Agricultural Engines.
Pumping engines.
Crank and fly-wheel.
Direct-acting.
Portable Engines and
Semi-portable Engines.
Steam Fire-engines.
Road Locomotives.
Railway "
Marine Engines.
Paddle-engines.
Screw-engines.
Special Types.

Engines may also be classed according to structure: as simple or compound; as direct-acting, beam, vertical, inverted, horizontal, or inclined; or as condensing or non-condensing; high-pressure or low-pressure; or as reciprocating, vibrating, as steam-turbines, or as rotary engines; or as directly connected or geared; as jet-condensing or surface-condensing. They are very frequently designated by the name of the inventor, designer, or constructor: as the Watt, the Corliss, or the Porter engine.

In the first classification—that by reference to proposed use—the title is sufficiently indicative of its own reason and meaning; and this is commonly the case with the nomenclature based on structural characteristics. A simple engine does its work in a single cylinder; while a "compound or multicylinder engine has two or more cylinders," so connected "in series" that the steam exhausted from one shall be successively worked, under decreasing pressures, in the others.

Direct-acting engines are directly connected from head of piston-rod and the cross-head to the crank; beam-engines have a "working-beam" interposed; and the geared engine drives its load—as the screw-shaft in marine engines—by means of pinions on the crank-shaft and gears on the screw-shaft; thus enabling the latter to be driven at higher speed than the former, or, in very rare instances, the reverse. Vertical, inverted, horizontal, or inclined engines are so named to indicate the direction of their "centre-lines" and their position. Condensing and non-condensing engines are distinguished by the fact that the latter possesses a condenser. The condenser, however, is not always made to produce a vacuum, when high steam-pressures are adopted; it is occasionally worked at atmospheric pressure, and is then simply either a heater or an expedient for securing pure feed-water for the boilers.

Reciprocating engines are those—the usual type—in which the piston moves backward and forward in a true cylinder; vibrating engines constitute a rare type in which the piston swings in an arc inside a cylinder of appropriate form; while rotary engines are those in which the piston continuously revolves on an axis, usually parallel to its own plane.

The classification adopted by the Author as that which will be followed in the arrangement of this work is the first, as presented in the table above; but separate articles or chapters will be devoted to such modifications as are comprehended in the other methods of classing engines falling under those heads.

30. **The Principles and Aim in Designing** any engine, as guiding the selection of type and details, are such as will insure the most exact adaptation of the machine to the specified work. The ultimate purpose is always to secure the best possible combination of minimum first cost with minimum running expenses. That is the best engine which, at the end of a life terminated either by its own wear and tear and natural decay, or by the substitution of a later and better form, gives the best total effect, as measured on the books of the treasurer, and as including interest on first cost, regular operating ex-

penses, compensation of attendant labor, rents, insurance, oil, fuel, and incidentals, making the sum of all such charges a minimum.

Hence the stationary engine may be chosen without much regard to weight or space occupied; locomotive and marine engines must be light, compact, and powerful; and the latter must be chosen and constructed, especially for long voyages, with primary regard to high economy in use of fuel. In all cases, other things equal, a direct application of the engine to its intended work is desirable; and it thus happens that we may prefer an engine of moderate speed for mill-work and a "high-speed engine" for driving dynamo-electric machines. In districts remote from coal-fields every known method is applied to insure maximum economic efficiency; while among coal-mines steam-jacketing, superheating, or "compounding" are expedients which have no interest for either the engineer or his client. It is such considerations as these which sometimes lead to the use of one standard form of engine where another type would ordinarily be employed—as a portable engine to drive a factory, where to be used temporarily, or as when cramped for space; as in the application of locomotive boilers in the torpedo fleet.

31. **The Principles of Construction** of the selected type of engine are determined by precisely the same considerations. The engine must be so built that the costs of maintenance shall be made a minimum for the life of the machine. It must be as light as possible, yet the strength of every part must be sufficient to make it safe against all ordinary contingencies; bearings must not only be designed in proper number, location, and dimensions, but they must be made of good material for their purpose; good material and good workmanship will invariably, "in the long-run," afford full compensation for their cost. It is the consideration of these principles and the deductions from a now long period of extensive and continuous experience which have led to the production and use, for their prescribed purposes, of the several standard types of engine to be presently described.

32. **The Exigencies of Operation** determine many matters of detail in every type of engine; and the designing engineer, or the purchaser or user, of an engine can never be secure of a satisfactory result unless the conditions of operation and possible accidents and exigencies are considered. Thus: lubrication must be absolutely continuous and certain on engines working at high speed of rotation; provision must be made, especially with marine engines, and elsewhere where "priming" or "foaming" may endanger the engine, for the safe expulsion of water from the steam-cylinder; reversing-gears must be fitted to rolling-mill engines; an adjustable distribution of steam is essential in the case of the locomotive.

33. **The Stationary Engine** has a variety of forms, differing with the special nature or with the location of the machinery to be driven. It is usually a simple engine; but is getting to be more and more frequently "compound," or even "triple-expansion;" it is usually driven at moderate speed, and has a "detachable valve-gear" or "drop cut-off;" but it is often of the high-speed type, with a positive-motion valve-gear and a shaft-governor. Among the most common forms are:

(1) The Mill-engine;

(2) The Pumping-engine, and others of the moderate low-speed class;

(3) The High-speed Engine, of various kinds, but mainly used for mills or electric-lighting establishments; and a few peculiar forms that need not be here considered.

Each of these types or forms is built both simple and compound; the latter will be specially considered in a distinct chapter.

The best known and most generally used class of stationary engines at the present time, as has been stated, is that which has the so-called "drop cut-off," or "detachable valve-gear." The oldest well-known form of valve-motion of this description is the Sickels cut-off, previously mentioned, patented by Frederick E. Sickels about the year 1841. It was introduced by the inventor in a form which especially adapted it to the beam-engine used on the Eastern waters of the

United States, and was adapted to stationary engines by Messrs. Thurston, Greene & Co., of Providence, R. I., who employed it for some years before any other form of "drop cut-off" came into general use.

The Sickels cut-off consisted of a set of steam-valves, made independent of the exhaust-valves, and each raised by a catch, which could be thrown out, at the proper moment, by a wedge with which it came in contact as it rose with the opening valve. This wedge, or other equivalent device, was so adjusted that the valve should be detached and fall to its seat when the piston reached that point in its movement, after taking steam, at which expansion was to commence. From this point, no steam entering the cylinder, the piston was impelled by the expanding vapor. The valve was usually the double-poppet. Sickels subsequently invented what was called the "beam-motion," to detach the valve at any point in the stroke. As at first arranged, the valve could only be detached during the earlier half-stroke, since at mid-stroke the direction of motion of the eccentric-rod was reversed and the valve began to descend. By introducing a "wiper" having a motion transverse to that of the valve and its catch, and by giving this wiper a motion coincident with that of the piston by connecting it with the beam or other part of the engine moving with the piston, he obtained a kinematic combination which permitted the valve to be detached at any point in the stroke, adding a very simple contrivance which enabled the attendant to set the wiper so that it should strike the catch at any time during the forward movement of the "beam-motion."

On stationary engines, the point of cut-off was afterward determined by the governor, which was made to operate the detaching mechanism, the combination forming what is sometimes called an "automatic" cut-off. The attachment of the governor so as to determine the degree of expansion had been proposed before Sickels's time. One of the earliest of these contrivances was that of Zachariah Allen, in 1834, using a cut-off valve independent of the steam-valve. The first to so attach the governor to a *drop cut-off* valve-motion was George

H. Corliss, who made it a feature of the Corliss valve-gear, already referred to, in 1849. In the year 1855, N. T. Greene introduced a form of expansion-gear, in which he combined the range of the Sickels beam-motion device with the expansion-adjustment gained by the attachment of the governor, and with the advantage of flat slide-valves at all ports—both steam and exhaust.

Many other ingenious forms of expansion valve-gear have been invented, and several have been introduced, which, properly designed and proportioned to well-planned engines, and with good construction and management, should give economical results little if at all inferior to those just named. Among the most ingenious of these devices is that of Babcock & Wilcox, in which a very small auxiliary steam-cylinder and piston is employed to throw the cut-off valve over its port at the instant at which the steam is to be cut off. A very beautiful form of isochronous governor was used on this engine, to regulate the speed of the engine by determining the point of cut-off.

In some forms of Wright's engine the expansion is adjusted by the movement, by the regulator, of cams which operate the steam-valves so that they shall hold the valve open a longer or shorter time, as required.

The Older Forms of stationary engines were usually simple in design, of plain construction, durable, economical in first cost and in maintenance; but, as compared with more recent engines, wasteful of steam and fuel. But little space need be here given to their description. They were either beam-engines or direct-acting, and their valves and gear, from the first quarter of the century, consisted often of a single three-ported slide-valve like that of the modern locomotive, driven by a single eccentric and effecting the desired expansion and compression of steam by the lap and lead of the valve, in a manner to be described in a succeeding chapter. The beam-engine gradually fell into disfavor, on account of its size and cost, and was displaced very generally, by the middle of the century, by the horizontal direct-acting engine; and the increased steam-pressures and improved economy of the non

condensing engine also resulted in the increasing employment of that form of machine, to the exclusion of the condensing engine, which is, however, still much used, especially for large powers.

Where economy was particularly sought, the engine was often fitted with a separate cut-off valve, often mounted on the back of the main valve; sometimes, however, as a distinct organ in its own valve-chest. In the most common system—that of Mayer—this cut-off valve consisted of two blocks sliding on the back of the main valve, actuated by an independent eccentric, and capable of being separated or brought together, as desired, by a right and left screw, in such manner as to vary the point of cut-off to any required extent. The eccentric is set with or 180° from the crank, accordingly as the cut-off is effected by the inside or the outside edges of the cut-off blocks.

Where much power is required, the stationary engine is now usually a horizontal direct-acting engine, having a more or less effective cut-off valve-gear, according to the size of engine and the cost of fuel. A good example of the simpler form of this kind of engine is the small horizontal slide-valve engine, with the Meyer system of valve-gear. This form is a very effective machine, and does excellent work when properly proportioned to yield the required amount of power. It is well adapted to a ratio of expansion of from four to five. Its disadvantages are the difficulty which it presents in the attachment of the regulator, to determine the point of cut-off, by the heavy work which it throws upon the governor when attached, and the rather inflexible character of the device as an expansion valve-gear. The best examples of this class of engine have heavy bed-plates, well-designed cylinders and details, smooth-working valve-gear, the expansion-valve adjusted by a right-and left-hand screw, and regulation secured by the attachment of the governor to the throttle-valve.

The engine shown in the accompanying illustration (Fig. 28) is an example of an excellent stationary engine, and is simple, strong, and efficient. The frame, front cylinder-head, cross-head guides, and crank-shaft "plumber-block," are cast

in one piece. The cylinder is secured against the end of the bed-plate, as was first done by Corliss. The crank-pin is set in a counterbalanced disk. The valve-gear is simple, and the governor effective and provided with a safety-device to pre-

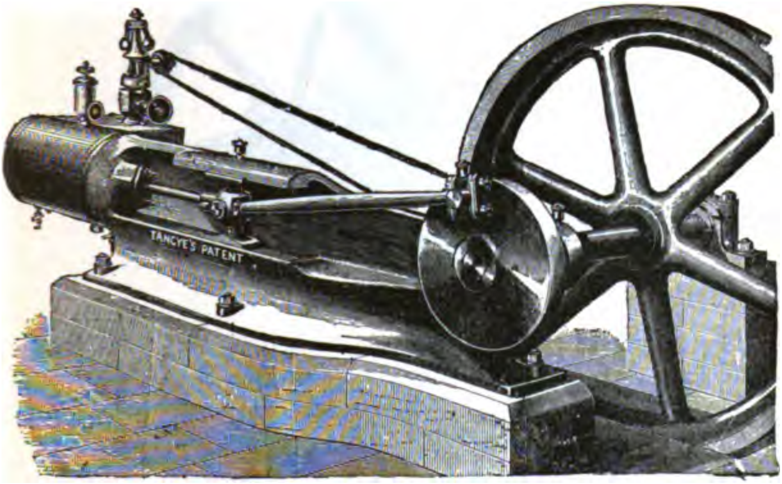


FIG. 28.—STATIONARY ENGINE.

vent injury by the breaking of the governor-belt. In this example all parts are made to exact size by gauges standardized to Whitworth's sizes.

With many engines (as is seen in Fig. 29) two supports are placed—the one under the main bearing, and the other under the cylinder—to take the weight of the engine; and through them it is secured tc the foundation. A valve is sometimes used consisting of two pistons connected by a rod and worked by an ordinary eccentric. By a simple arrangement these pistons have always the same pressure inside as out, which prevents any leakage; and they are said always to work equally as well and free from friction under high as under low pressure.

Engines of the class just described are especially well fitted, by their simplicity, compactness, and solidity, to work at the high piston-speeds which are gradually becoming generally adopted in the effort to attain increased economy of fuel by

the reduction of the immense losses of heat which occur in the expansion of steam in the metallic cylinders through which we are now compelled to work it.

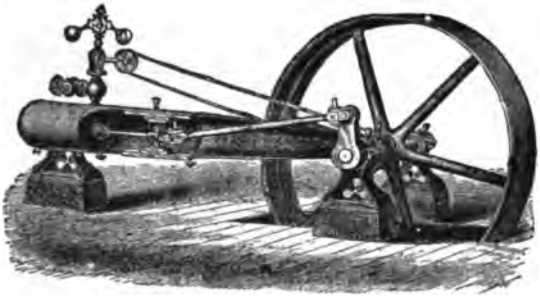


FIG. 29.—HORIZONTAL STATIONARY ENGINE.

The technical expressions "right-hand" and "left-hand" engines are thus defined as applied to engines of this class:

Stand by the end of the cylinder, face the shaft and observe the position and direction of the main driving-pulley, and class the engine as follows:

Right-hand engines have the main driving-pulley on the right of the observer. Left-hand engines have the main driving-wheel on the left of the observer.

Forward-running engines move the top of the main driving-pulley away from the observer.

Backward-running engines move the top of the main driving-pulley towards the observer.

In deciding on the direction in which an engine is to run, it is well to remember that forward-running engines are preferable, on account of the thrust of the connecting-rod being received on the lower guides, which are always stiffer and better lubricated than the upper.

One of the neatest and best modern designs of stationary engine for small powers is seen in Fig. 30. which represents a "vertical direct-acting engine," with base-plate—a form which is a favorite with many engineers.

The engine shown in the engraving consists of two principal parts, the cylinder and the frame, which is a tapering column

having openings in the sides, to allow free access to all the working parts within. The slides and pillow-blocks are cast with the column, so that they cannot become loose or out of

FIG. 30.—VERTICAL STATIONARY ENGINE.

line; the rubbing surfaces are large and easily lubricated. Owing to the vertical position, there is no tendency to side wear of cylinder or piston. The packing-rings are self-adjusting; the crank is counterbalanced; the crank-pin, cross-head pin, piston-

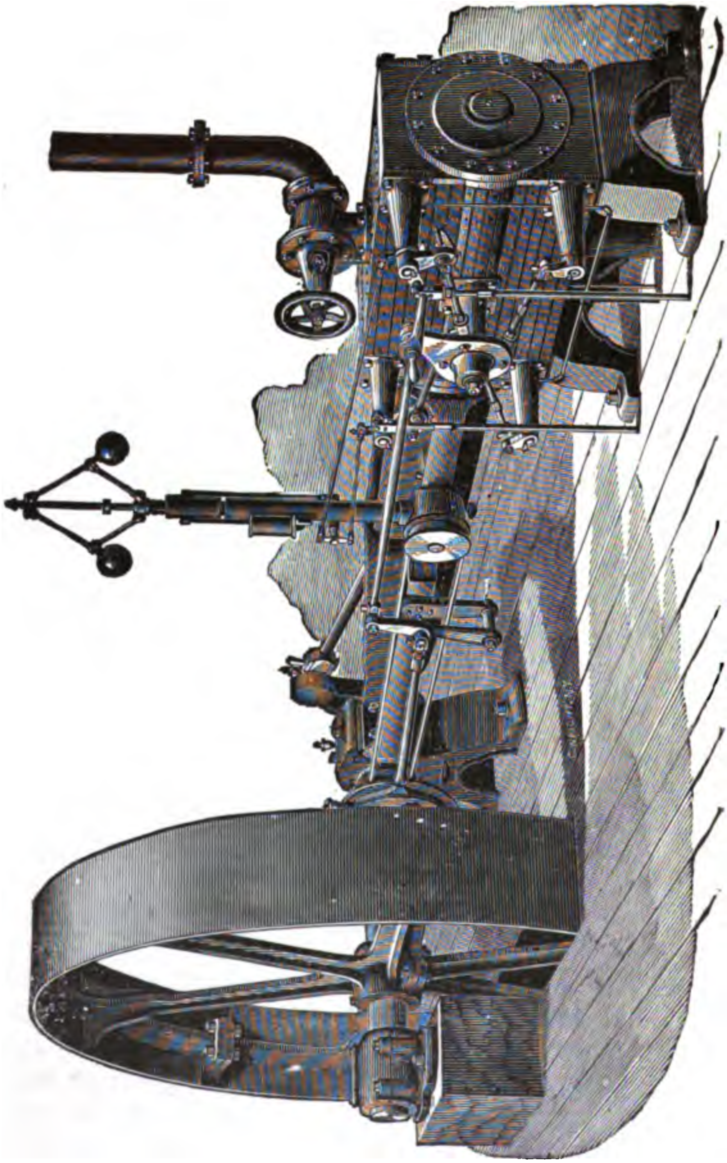


FIG. 32.—THE CORLISS ENGINE.

the valve continuously rotates, closing without reciprocation.* When of small size, the stationary is made non-condensing; when of large power, it is very frequently a condensing engine. When large and where economy is very essential, it is frequently a "compound," and often a "triple-expansion," engine; the steam-pressure being carried higher as a higher ratio of expansion is adopted. In many cases, as in cotton-mills making fine grades of product, or for electric-lighting, precise regulation of speed is required, and this may determine the choice of type of engine, and the selection of a "releasing gear."

The best-known engine of this class is the Corliss engine. It is very extensively used in the United States, and has been copied very generally by European builders. Fig. 32 represents the Corliss engine. The horizontal steam-cylinder is bolted firmly to the end of the frame, which is so formed as to transmit the strain to the main journal with the greatest directness. The frame carries the guides for the cross-head, which are both in the same vertical plane. The valves are four in number, a steam- and an exhaust-valve being placed at each end of the steam-cylinder. Short steam-passages are thus secured, and this diminution of clearance is a source of some economy. Both sets of valves are driven by an eccentric operating a disk or wrist-plate, *E* (Fig. 33), which vibrates on a pin projecting from the cylinder. Short links reaching from this wrist-plate to the several valves, *DD*, *FF*, move them with a peculiarly varying motion, opening and closing them rapidly, and moving them quite slowly when the port is either nearly open or almost closed. This effect is ingeniously secured by so placing the pins on the wrist-

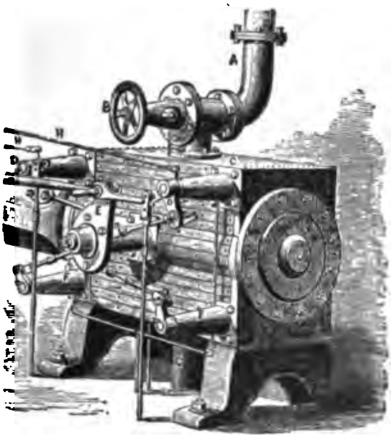


FIG. 33.—CORLISS ENGINE VALVE-MOTION.

* Report on Machinery and Manufactures at Vienna in 1873; R. H. Thurston; Washington, Gov't Printing Office; 1875.

plate that their line of motion becomes nearly transverse to the direction of the valve-links when the limit of movement is approached. The links connecting the wrist-plate with the arms moving the steam-valves have catches at their extremities, which are disengaged by coming in contact, as the arm swings around with the valve-stem, with a cam adjusted by the governor. This adjustment permits very perfect regulation by automatic variation of the ratio of expansion by the governor.

The standard form of Corliss valve is very well exhibited

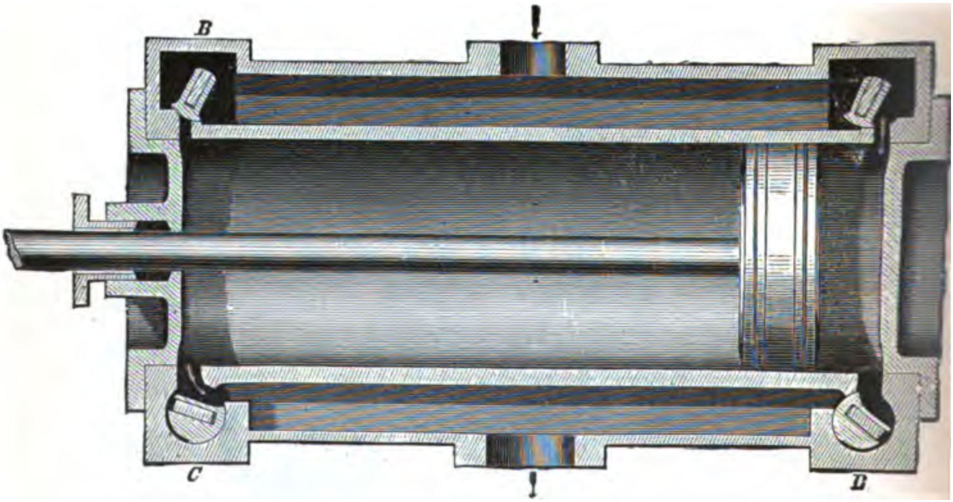


FIG. 34.—THE CORLISS ENGINE-CYLINDER.

by the illustrations here given, which are taken from the drawings of Mr. Harris.

Those marked A are the steam-, and those marked B are the exhaust-valves. Both consist, as is seen, of cylinders, parts of which have been cut away, leaving the working and bearing surfaces of no greater extent than is necessary to subserve the purposes of the valve. These surfaces are of the simplest possible form and are easily fitted up in the lathe. In order that they may come to a bearing with certainty, and without regard to the position of the spindle relatively to the valve,

they are made with a longitudinal slit into which fits, without jamming, the blade of the rock-shaft. The valves are thus allowed to come to a bearing, and even to wear down in their seats without causing leakage.

The next figure shows the arrangement of this valve as seen in longitudinal section of the chest. As this maker constructs it, the stem goes through a fitted opening, without stuffing-box, and the slight drip is carried off from the closed

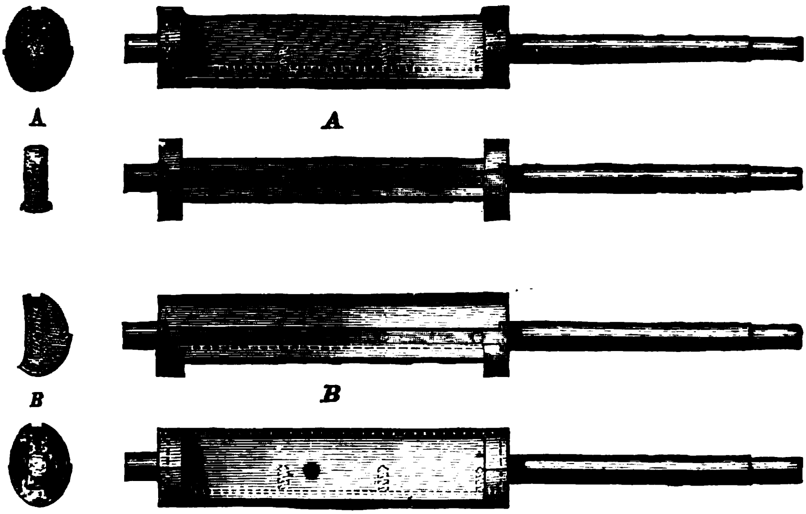


FIG. 35.—HARRIS-CORLISS VALVES.

space at *D*; thus none escapes into the engine-room. The steel collar at *F*, which is shrunk on the stem, fits into the recess at *a* and serves as a packing. As the tendency of the stem to shift outward always causes the collar to wear to a fit, it is not likely often to wear leaky.

Another detail of interest in the Corliss engine is the "dash-pot." When the valve is suddenly closed, some device is necessary to prevent jar at the instant of its coming to rest. This device is the dash-pot. The form adopted by Corliss consists of a shallow cup into which a piston on the valve-stem fits,

cushioning the enclosed air, and thus checking the motion of the valve without shock. This dash-pot, made by Watts, Campbell & Co., who have successfully introduced Corliss engines into electric-light establishments in New York City and elsewhere, is that seen in the figures.

The annular piston, *E*, *E*, fits the cylinder, *D*, *D*, *E*, *E*, and a space, seen above *B*, forms a vacuum-chamber which assists the spring or weight, closing the valve by the formation of a more or less complete vacuum, as the piston is raised while the valve is opening. A small cock, not seen, is arranged to adjust the degree of exhaustion of this chamber. When the valve has nearly reached its seat, the piston, *D*, passes the opening

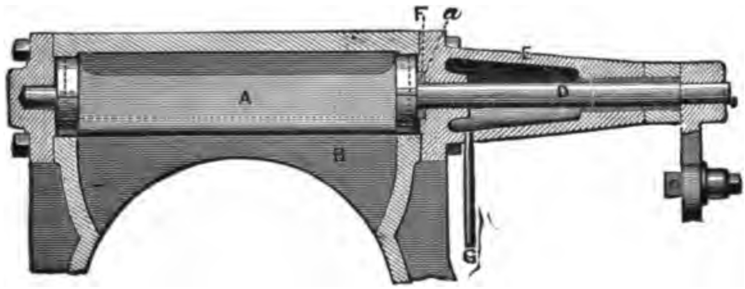


FIG. 36.—HARRIS-CORLISS VALVE.

from *F* into the outer space and the enclosed air then acts as a cushion, checking the movement of the valve.

The "dash-pot" was invented originally by F. E. Sickels. In the original water dash-pot of Sickels, the cylinder is vertical, and the plunger or piston descends upon a small body of water confined in the base of the dash-pot. Corliss's air dash-pot is now often set horizontally.

The Corliss engine is the prototype of a large number of engines constructed in Europe and America, having the same or very similar structure and methods of operation.

The leading features of this machine are thus:

(1) The use of four valves—two steam and two exhaust—so placed as to reduce "clearance" to a minimum.

(2) The use of a rotating valve, capable of being cheaply and readily fitted up, of being easily moved, and of being conveniently worked by connections outside the steam-spaces.

(3) The use of a "wrist-plate," caused to oscillate by a single eccentric, and directly so connected with all four valves that each may be given a rapid opening and closing movement, and be held open and nearly still, at either end of its range, by swinging the line of connection nearly into the line between centres, thus permitting nearly a full opening of port to be

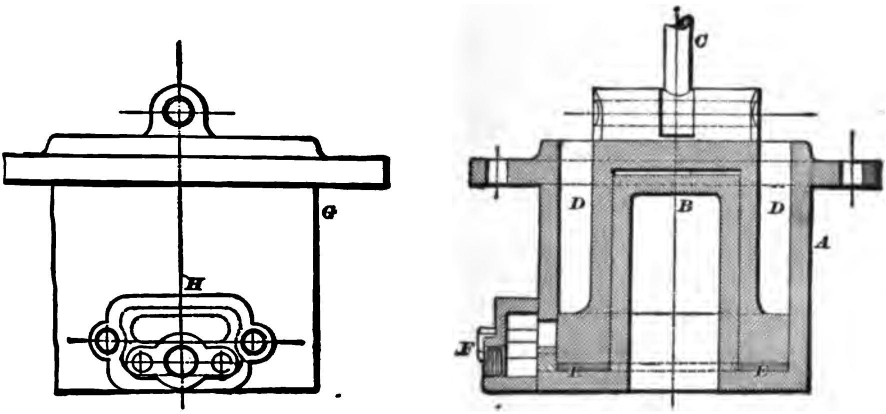


FIG. 37.—THE DASH-POT.

maintained during an appreciable interval, and a free and complete steam supply and exhaust.

(4) A beautifully simple and effective method of detaching the steam-valve from the driving mechanism, and of insuring its rapid and certain closure at the proper moment, to produce any desired expansion of steam.

(5) A direct connection of the governor, so as to determine the ratio of expansion, while so adjusting the power of the engine to the work to be done that the variation of speed with changing loads becomes a minimum.

(6) Making this latter adjustment in such a way as to throw the least possible work on the regulating mechanism, and thus

to give the governor the greatest possible sensitiveness and accuracy of action.

(7) A form of frame and general design of engine which gives maximum strength and stiffness, with least cost and weight.

All these features are combined to form a steam-engine essentially different, in general and in detail, from all earlier engines. In operation, the engine was found to exhibit a remarkable economy of fuel, and a singularly perfect regula-

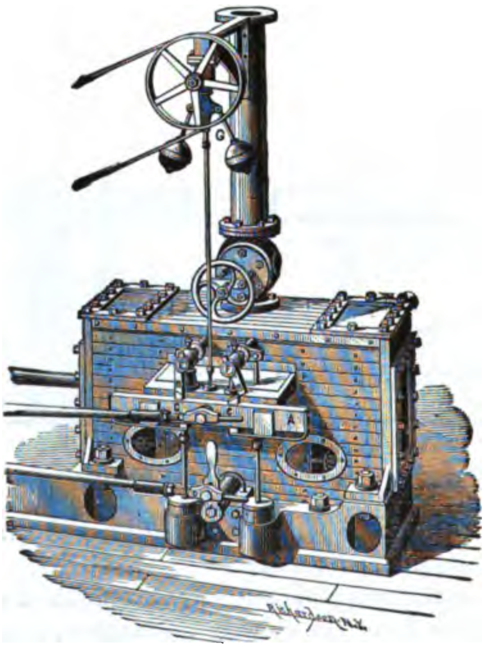


FIG. 38.—GREENE ENGINE. (Scale $\frac{1}{32}$.)

tion, and to be far more durable and more economical in cost of repairs, on the average, than was generally supposed possible.

The Greene Steam-engine (Fig. 38) has four valves, as in the Corliss. The cut-off gear consists of a bar A, moved by the steam-eccentric in a direction parallel with the centre-line of the cylinder and nearly coincident as to time with the piston. On this bar are tappets, C C, supported by springs and adjustable

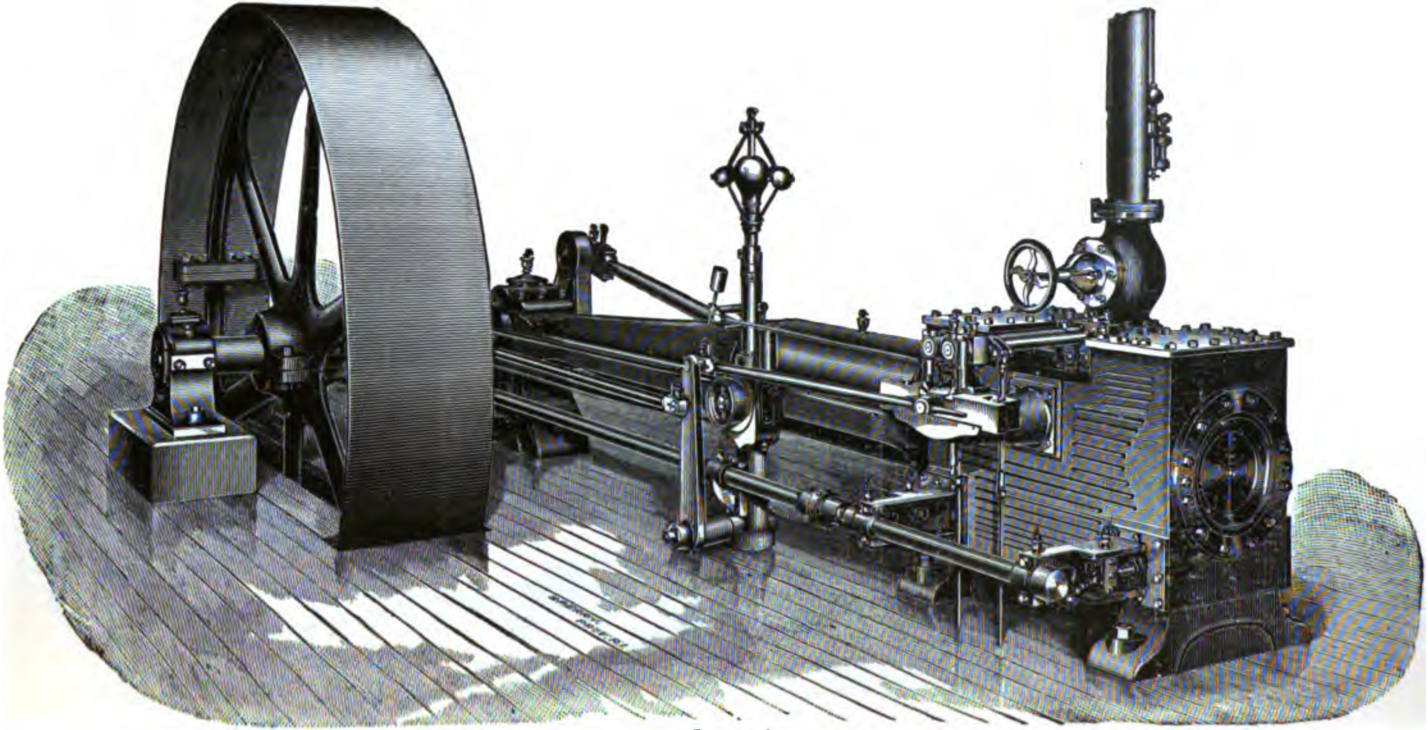


FIG. 39.—THE GREENE ENGINE.

in height by the governor. These tappets engage the arms, *B B*, on the ends of rock-shafts, *E E*, which move the steam-valves and remain in contact with them a longer or shorter time, and holding the valve open during a greater or less part of the piston-stroke, as the governor permits the tappets to rise with diminishing engine-speed, or forces them down as speed increases. The exhaust-valves are moved by an independent eccentric-rod, which is itself moved by an eccentric-set, as is usual with the Corliss and with other engines generally, at right angles with the crank. This engine, in consequence of the independence of the steam-eccentric, and of the contemporary movement of steam valve-motion and steam-piston, is capable of cutting off at any point from beginning to nearly the end of the stroke. The usual arrangement, by which steam and exhaust valves are moved by the same eccentric, only permits expansion with the range from the beginning to half-stroke. In the Corliss engine the latter construction is retained, with the object, in part, of securing a means of closing the valve by a "positive motion," should, by any accident, the closing not be effected by the weight or spring usually relied upon.

There are other engines belonging to the class here considered—engines having a detachable cut-off valve closed independently of the motion of the valve-gear,—of which space will not permit description. Among these are the Wright engine, constructed by one of the oldest and best known designers in the United States; the Brown engine, a machine which has been extensively adopted for driving mills in New England, and is famous for the excellence of its workmanship and finish, as well as for its durability and efficiency; the Fitchburg engine, and others.

An ingeniously arranged engine of the class considered in this division of the subject, the Wheelock engine, is seen in the accompanying engraving.

The steam-chest is placed below the cylinder, and the steam- and exhaust-valves are set side by side, the latter serving both as induction and eduction valve, and having the same action, nearly, as the common three-ported slide-valve; while the func-

FIG. 40.—THE WHEELOCK ENGINE.

tion of the former is principally that of a cut-off valve. The latter, or main valve, is set nearest the end of the cylinder, and the exhaust steam is thus permitted to escape directly, and promptly from the engine. The valve and seat are independent, and coned slightly, and may be adjusted to take up wear, or to relieve pressure on the seats. These valves are carried on steel trunnions, and with hardened surfaces of contact are but little subject to wear. The steam or cut-off valve is set farther away from the cylinder than in the standard arrangements of Corliss and other builders of that class of engines, and this enables the maker of this engine to secure a single port with reduced clearance and less liability to leakage, should the expansion-valve leak. In the later engines of this class a gridiron valve is used in a shell of the same general form, as illustrated in Volume II. In this engine—and it should be the case in every engine in which the regulator is driven by belt—the connection from shaft to governor is so made that the breaking of the belt permits an automatic closing of the valve and the stopping of the engine. The regularity of motion of the class of engines described in this section may be inferred from the fact stated in regard to the engine here studied, that it has been known to vary but a half-revolution per minute when five sixths of the load was thrown off.

Simple and Compound Stationary Engines are both in common use as mill-engines; and all the familiar classes of engines are constructed in both forms. Until recently the mill-engine has been very generally a single-cylinder engine, or a pair of simple engines coupled with cranks at right-angles where great power was demanded; but the Corliss and other mill-engines are now often "compounded," and it is not unusual to compound comparatively small high-speed engines. In such cases the elements of the combination are commonly similar in design to the simple form of the same engine. The combination is often made by constructing a "tandem" engine, in which the cylinders are placed in the same line, end to end, and often with their pistons on the same rod. In other cases, the engines are set side by side, actually constituting each a complete engine,

with their cranks set at right-angles for a two-cylinder compound, or at angles of 120° for a "triple-expansion," engine, and with a common frame. In such cases, as will be seen later, an intermediate "receiver" is introduced into which the high-pressure cylinder exhausts and from which the low-pressure cylinder takes its supply without seriously affecting the working of the fluid.

Nearly all the engines to be described are thus built "compound," and some are "triple expansion."

The Stationary Multiple-cylinder Engine is rarely given the ..ne form as the marine engine. The necessity of having a ..ir of cranks, and the objection to the employment of the fly-wheel, do not here exist; nor does either the volume or the weight of the machine become so vitally important a matter as at sea. The design adopted is, for these reasons, one which will be of minimum first cost, irrespective of these considerations.

The "Tandem" Engine is perhaps the most common form of stationary compound engine. In this type, as shown in the accompanying illustration, the two cylinders are set in line, have a common piston-rod, and drive the same crank. The high-pressure cylinder is commonly placed behind the low-pressure, and the latter is directly attached to the frame of the engine. The exhaust of the smaller cylinder is carried in any convenient manner to the large engine; but the more direct and the larger the conduits employed, the better. In some cases, the two cylinders are set directly in contact. This plan involves a difficulty, usually, in packing the rod between them, but it has the advantage of great compactness.

The Compound Corliss Engine was first introduced by other builders; but no one was more successful in the economical working of the machine than was its great originator, the late George H. Corliss. The usual method of compounding this engine for stationary purposes is that known as the "tandem" system, in which the high-pressure cylinder is set behind the low-pressure, both pistons having a common rod and driving a common set of reciprocating parts and having valve-gearing

actuated by the same eccentric and rod. The plan is simple, inexpensive, convenient, and compact, and is found to be very satisfactory in operation, the economy attained by it being about as high as that of any other arrangement yet devised.

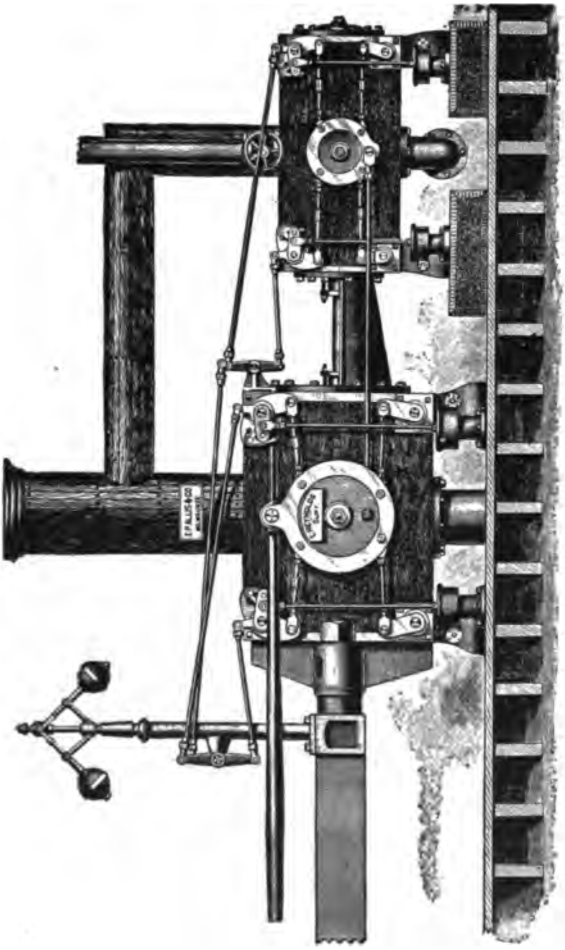


FIG. 41.—"TANDEM" COMPOUND ENGINE. (Scale $\frac{1}{16}$.)

This method is illustrated by Fig. 41, which exhibits a form of the engine designed by Mr. Edwin Reynolds. It is readily seen that it would probably be impossible to find a better method of combining maximum efficiency with minimum cost of con-

struction than this, or to make a more compact disposition of parts. It is necessarily of considerable length; but in other directions has no greater dimensions than the single engine of the simple type.

The performance of this type of engine has been most excellent. For example, the engines of the Nourse steam-mill, as constructed by Mr. Corliss, were found to demand no more than 1.62 pounds of good fuel per horse-power and per hour. The same engine as a simple engine, the high-pressure cylinder disconnected, if equal to the best of its class, under similar conditions of operation, would probably not require less than two pounds; which may be taken as about the limit of economical working for that type of engine, with a good condenser and dry steam.

One disadvantage of this type of engine—the "tandem"—is the length of passage between the exhaust-port of the high-pressure and the induction-passage of the low-pressure cylinder when the former is taking steam in the backward stroke; but this is partly compensated, at least, by the very short passage obtainable for the opposite movement. The valve-gearing is commonly the same on both cylinders; but it is often so arranged that the governor operates on the one cylinder only, leaving the ratio of expansion of the other to be determined by the measure of expansion in the first.

Another not uncommon system of compounding this engine, especially for large powers, is oftener practised in Europe than in the United States. This is the coupling of two engines, side by side, as in common marine practice; while another method sometimes adopted is the adaptation of two independent engines of properly-adjusted sizes to act, the one as the high-, the other as the low-pressure engine of a compound system. These engines are occasionally set at some distance apart, when the local conditions make that a more convenient disposition. The efficiencies of these several types of compound Corliss engines are substantially the same. They are all subject to about one half the internal wastes of the simple engine of similar dimensions, to about double the external

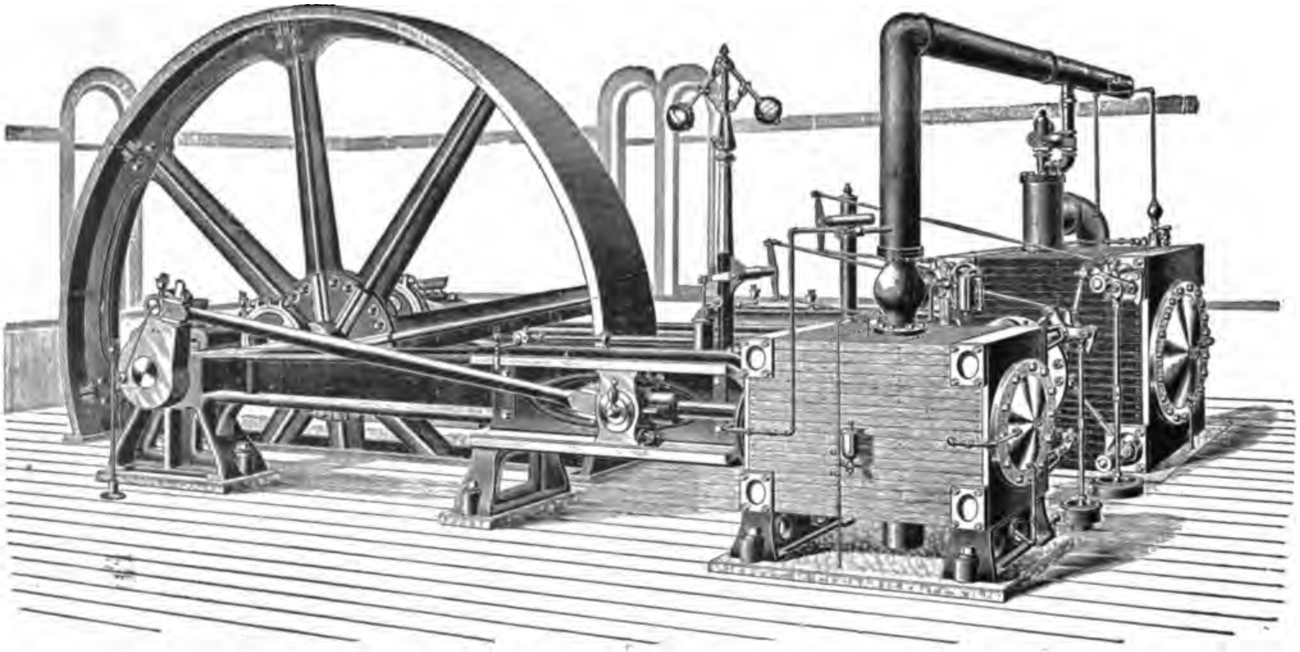


FIG. 42.—"CROSS-COMPOUND" ENGINE.

wastes of heat, and have a trifle more friction. On the whole, they will ordinarily give an increased economy amounting to about twenty per cent of the heat and fuel consumption of the simple engine.

In some cases the arrangement of a pair of complete engines, of properly selected sizes, in such manner that either the exhaust of one may be used in the other, or steam may be taken direct from the boiler to either, is found advantageous. When less power is demanded, or when one is disabled, the available engine may then be used alone. Economy has been attained by this plan, even when the two engines are placed at considerable distances apart, the precaution being taken to carefully guard against loss of heat between them.

The "Cross-compound" type of Corliss engine is illustrated by the accompanying sketch of a pair designed by Mr. Reynolds and built by Allis & Co. for the Namquit Mills. The cranks are set at right-angles, and the receiver is placed beneath the floor. This is a less common variety than the "tandem" form; but is still often adopted.

The general arrangement and disposition of the parts of a triple-expansion engine, as built by the Corliss Co., is seen in Fig. 43. Here the low-pressure cylinder is divided, one of its two elements being coupled with the high-pressure cylinder on the right, and the twin with the intermediate cylinder on the left. The cranks are set at 90°. These engines have cylinders 20, 34, 36, and 36 inches diameter and 5 feet stroke of piston. All cylinders are completely steam-jacketed, heads included, and the steam is somewhat superheated. Jet-condensers are used. The capacity of the engine is 1000 I. H. P. or more, and its "duty" is about 135,000,000 pounds; the fuel used, when of good quality, amounting, on test, to 1.44 pounds per horse-power per hour.

"Compounding" simple engines is often a very economical and profitable plan. The method depends mainly upon the design of the engine to be so altered. The common forms of stationary beam-engine are commonly improved by what is called "McNaughting," placing a new high-pressure cylinder

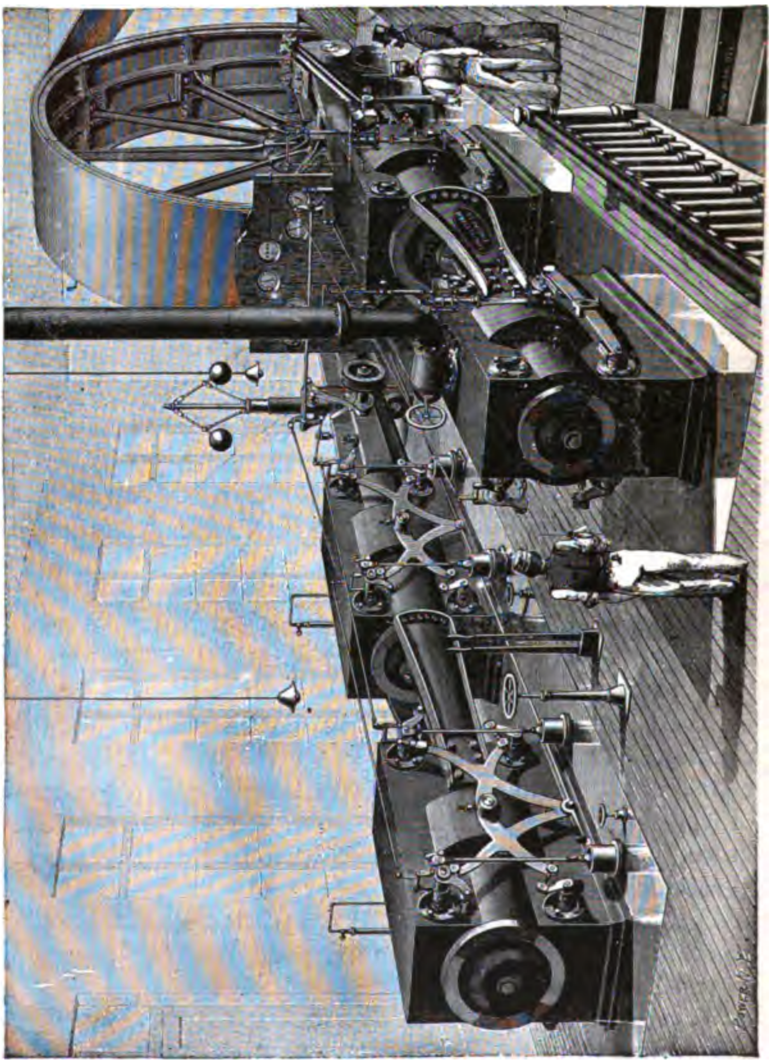


FIG. 43.—TRIPLE-EXPANSION CORLISS ENGINE.

beside the old cylinder and connecting it to the beam either at the old air-pump centre, if condensing, or to the point at which the air-pump would have been attached, if the engine

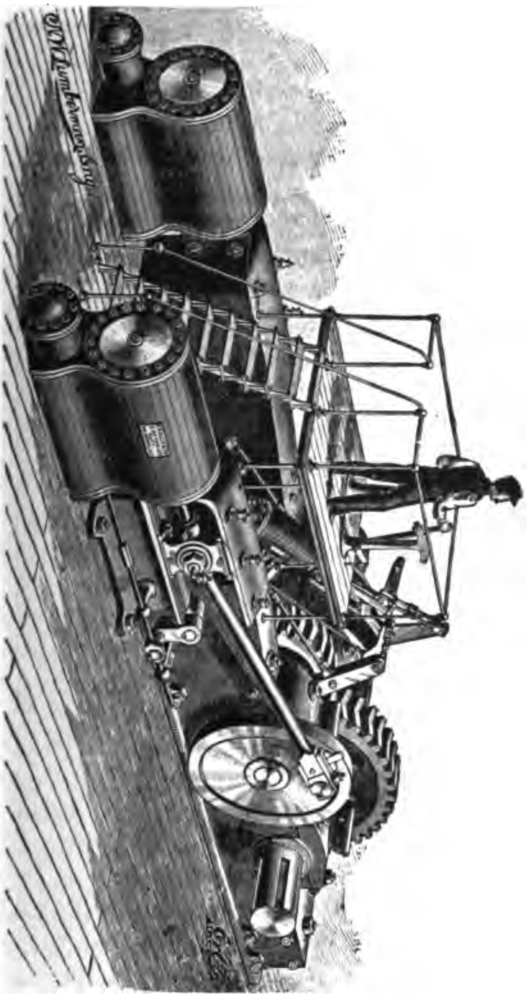


FIG. 44.—REVERSING-ENGINE.

be non-condensing. The vertical marine engine may sometimes be altered into the compound form by placing the new cylinder above the old and the two pistons on a common rod.

Many engines cannot be satisfactorily compounded, and others only by the establishment of a separate complete high-pressure engine in close proximity to the old and arranging the latter to take its steam from the former.

The gain to be anticipated by such improvement and alteration of type will depend upon the character of the altered machine. Should it be a very wasteful engine, enormous gains may be anticipated if, while adding the new construction, the old is put in good order. For cases in which the old engine is reasonably economical, the gain is simply that due to reduction of cylinder-condensation, and this is at least partly compensated by the friction of the added parts. Savings as great as one half are not unusual in such cases as the first, and as little as ten per cent, in cases like the second, are common. Whether such a gain is, on the whole, financially advantageous is still another question to be settled for each case.

Rolling-mill Engines are often constructed especially for their work. For heavy mills they are often made to reverse.

The last figure illustrates a common form of reversing-engine. The engine frames are heavy cast-iron girders having a bearing the entire length on the foundation. On the top side of the frames are the main journals. These journals are provided with means for taking up wear and adjusting the helical gears which transmit motion from one shaft to the other.

The main valves are placed *under* the cylinders, the valve-chambers forming a part of the cylinder casting, thus bringing the steam-ports on the lower side of the cylinder, to allow water of condensation to pass out through the exhaust-ports without danger to either cylinder or head. As an additional means of safety the builders often use "snifting-valves" on each end of the cylinder.

Where very heavy rolls are employed, as in making armor-plate, for example, an engine is often demanded which may be instantly reversed, driving with equal facility in either direction. Fig. 45 exhibits such an engine as built by the Allis Co., from Mr. Reynolds's plans, for Messrs. Carnegie, Phipps & Co. of Pittsburgh. The fly-wheel is here, also, dispensed

with, and the engines are designed for high speeds of rotation and very heavy work.

The steam-cylinders are forty inches diameter by fifty-four

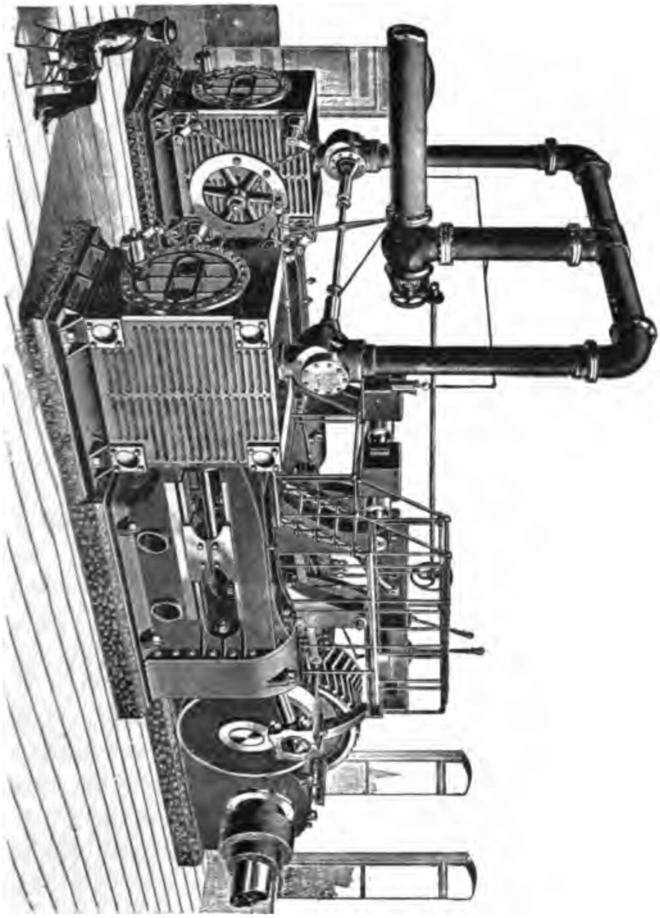


FIG. 45.—REVERSING ROLLING-MILL ENGINE.

inches stroke, with Reynolds' Corliss valve-gear without the drop cut-off mechanism; the speed of the engines is controlled by the operator, and is varied in every-day practice from 5

revolutions to 120 revolutions per minute. Power from the crank-shaft is transmitted to the roll-shaft by means of a pair of shrouded helical-tooth steel gears.

The reversing mechanism, operated by steam, is controlled by a lever on the engineer's platform; from this position he has unobstructed view of all parts of the engine and roll-train.

35. **High and Low Speed** distinguish a more modern type from those engines already described. Classified with reference to their method of driving machinery, we may thus designate the two classes:

(1) Engines which may be used in driving by belt, and which are not adapted for direct connection.

(2) Engines especially designed and constructed to be coupled directly to the "dynamo."

The first class of engines is, by many of the more conservative engineers, still preferred to the second. The latter constitute the so-called modern "high-speed" type of engine, and are gradually coming into use; some engineers adopting them both for direct and for indirect connection. The most experienced engineers are not yet fully in accord in regard to the question whether they have passed the experimental stage in such general application.

One of the methods of securing economy in the working of steam has been stated to be the driving of the engine up to the highest safe velocity of piston, and giving it maximum speed of rotation. The time allowed for "initial" condensation of each charge, and for the necessary change of temperature preceding such condensation, is thus reduced, and the amount of steam condensed within the cylinder being thus made a minimum, in any given time, the percentage of loss of the increased quantity of steam worked off by the engine becomes correspondingly less.

Engines of this class have a number of advantages, consequent upon their high speed: they are, *other things being equal*, more economical in the use of steam; they can be given a very much smaller fly-wheel; they have, in consequence of the enormously reduced weight of wheel, less friction; they are more easily

held to their speed by the governor; they are less subject to variation of speed between beginning and end of any one stroke; and they are often less troublesome and expensive to connect to the load than slow-running engines. These advantages are common to all classes of engines, if they can be driven up to high speeds. The class here considered is better fitted to realize these advantages than the older forms of engines, because they are especially designed for high speed.

The objection to this type of engine is the increased risk of wear, and of accident, due to their rapid motion, and especially the danger that when accidents do occur they may be more serious than with engines working at ordinary speeds. The precautions taken by builders of fast engines are all directed toward meeting this contingency, making their machines safe against accident. These precautions are seen to be the strengthening, and especially the stiffening, of all the parts exposed to the stresses due to the action of inertia in the reciprocating pieces; the adjustment of all parts to each other in such a manner as to avoid spring; the use of the best material, and of an effective system of lubrication; and the securing of the most perfect workmanship.

As actually constructed, they are of proportionally shorter stroke than the preceding types, and are consequently more subject to internal waste by cylinder-condensation and have large clearance and "dead" spaces, and thus, also, both exaggerate internal heat-waste, and become liable to greater loss of cushion-steam. As a rule, in actual work, this class of engine is not usually distinguished by peculiarly high economical results, in competition with the "low-speed" engines.

The latter, on the other hand, usually are at a disadvantage for fast running, both through complication of parts and the use of a detachable valve.

The Porter-Allen Engine was the first of the class known as "high-speed" engines. Its designers were Mr. C. T. Porter and Mr. J. F. Allen, the latter being the inventor of its valve-gear; while the former was the pioneer in the introduction of engines of this class.

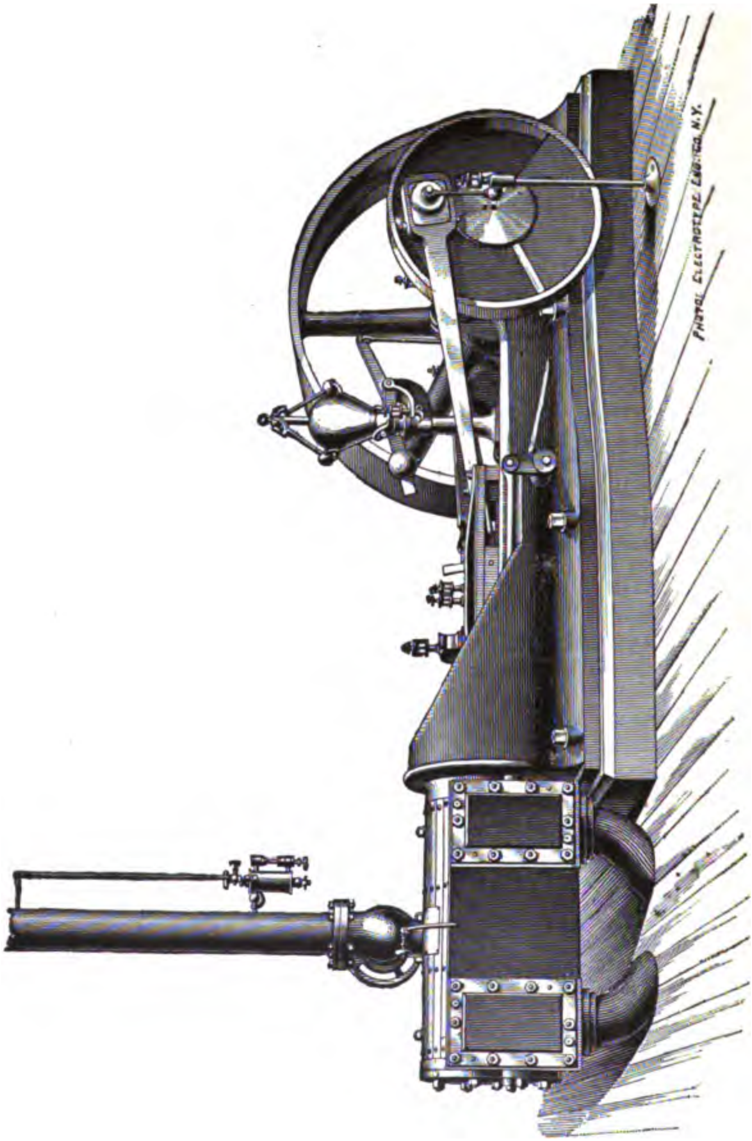


FIG. 46.—PORTER-ALLEN ENGINE.

In the Allen engine (Fig. 46), the cylinder and frame are connected as in the engine seen in Fig. 28, and the crank-disk, shaft-bearings, and other principal details are not essentially different. The valve-gear differs in having four valves, one at each end on the steam as well as on the exhaust side, all of which are balanced and worked with very little resistance. These valves are not detachable, but are driven by a link attached to and moved by an eccentric on the main shaft; the position of the valve-rod attachment to which link is determined by the governor, and the degree of expansion is thus adjusted to the work of the engine. The engine has usually a short stroke, not exceeding twice the diameter of cylinder, and is driven at very high speed, generally averaging from 600 to 800 feet per minute.* This high piston-speed and short stroke give high velocity of rotation. The effect is, therefore, to produce an exceptional smoothness of motion, while permitting the use of small fly-wheels. Its short stroke enables solidity to be attained in a bed of rigid form, making it a self-contained engine, adapted to heavy work, and requiring but a small foundation.

The journals of the shaft, and all cylindrical wearing-surfaces of such engines, are finished by grinding, and are thus made perfectly cylindrical. The crank-pin and cross-head pin are hardened before being ground. The joints of the valve-gear consist of pins turning in solid ferrules in the rod-ends, both hardened and ground. After years of constant use thus, no wear occasioning appreciable lost time in the valve-movements occurs.

Where great steadiness of motion is desired, the expense of coupled engines is often incurred. Quick-running engines do not often require to be coupled; a single engine may give greater uniformity of motion than is usually obtained with coupled engines at ordinary speeds.

The governor used on this engine is known as the Porter governor. It is given power and delicacy by weighting it

* Or not far from 600 times the cube root of the length of stroke, measured in feet.

down, and thus obtaining a high velocity of rotation, and by suspending the balls from forked arms, which are given each two bearing-pins separated laterally so far as to permit considerable force to be exerted in changing speeds without cramping those bearings sufficiently to seriously impair the sensitiveness of the governor.

In "high-speed" engines, the possibilities in the direction of increasing speeds are sought to be made the most of. Their market is not only to be found in the domain of the electrical generation of light, and electrical transmission of power, but in older fields of work as well. The loss of power in the "jack-shafts," or "first-motion shafts," of mills and workshops driven by the low-speed engines is an item of no inconsiderable amount in many cases. The tendency is now observable toward the adoption of the higher speed of engine, in direct connection with the main line of shafting, even where not quite as economical in the use of steam, through the intermediary of a single belt or pair of gears, or even by directly attaching the crank-shaft of the engine to the main line by a coupling.

Mr. Allen's invention of a valve-gear placed in the hands of Mr. Porter, who was endeavoring to design a "high-speed" engine, the device needed to carry out the idea.

This arrangement consists of a single eccentric driving a link-motion to operate the steam-valve and to work the exhaust at the same time. The link is controlled by a Porter governor, and is so connected and driven that the gear may be readily and quickly adjusted by the governor to any desired point of cut-off. The eccentric and link are shown in the next illustration. The eccentric is set on the shaft in such a position that its motion is coincident with that of the crank. The link is a slotted curved arm, forming one piece with the eccentric-strap, pivoted at the middle on trunnions sustained by an arm rocking about a pin set in the bed of the engine. The upper end of the link carries a pin, from which a rod leads off to the exhaust, which is driven without variable connections. The link-block is fitted to work in the slot of the link, from the end nearest the exhaust-rod pin, down to the point opposite the

pivotal point at which the trunnions are set. When it is at the upper end, the throw of the valve is a maximum; when at the lower point, it is a minimum. As the link-block is moved up and down in the slot, the motion of the valve is varied, and the ratio of expansion correspondingly altered. By an ingenious adjustment of a still more ingenious form of valve-motion, it is thus possible to obtain a valve movement of perfect precision at all speeds, and on both the forward and the back-

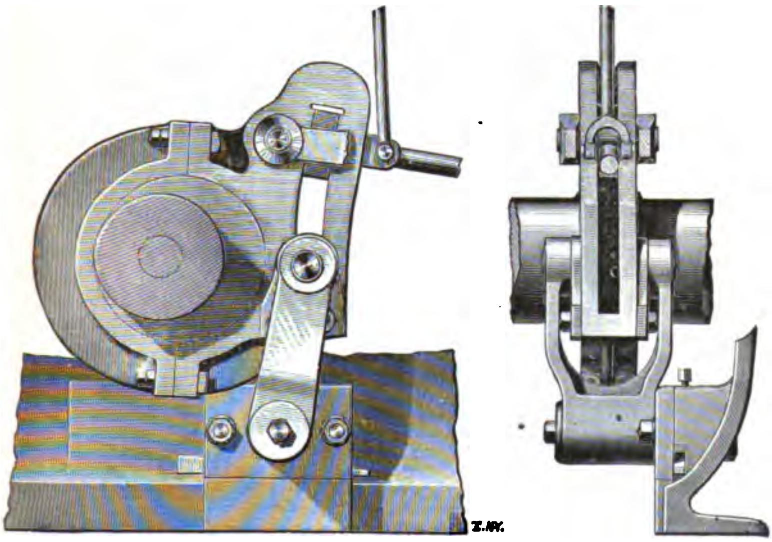


FIG. 47.—THE ALLEN LINK. (Scale $\frac{1}{16}$.)

ward stroke, with a quicker closing action, as the cut-off is later. The steam is allowed to enter the cylinder, at nearly boiler pressure, almost up to the point of cut-off, and the expansion line is a smooth curve very nearly from the junction with the steam line.

The four valves are shown in the next figure, which is a section through the steam-cylinder showing valve, ports, and general construction. The two valves at the upper side of the cylinder are the steam-valves; the lower are the exhaust-valves. This section is, however, horizontal, the valves being set on their edges at either side of the cylinder. The exhaust-

valves are so placed as to drain the cylinder of any water that may have entered with the steam, or may have been produced by internal condensation. Both sets of valves are so made,

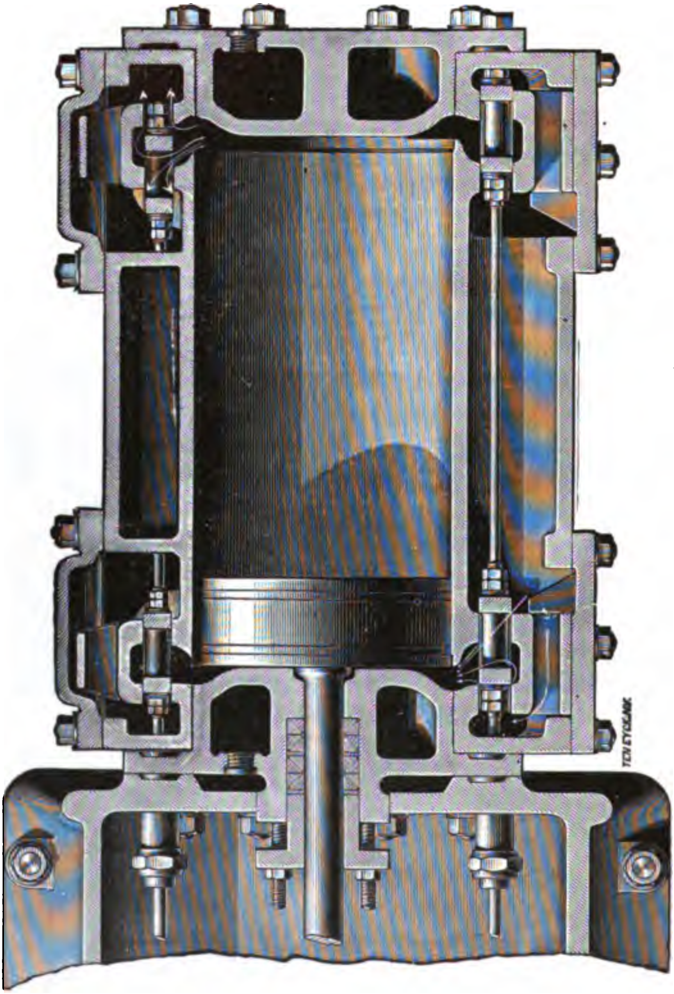


FIG. 48.—STEAM-CYLINDER (SECTION). (Scale $\frac{1}{4}$).

and set, as to be well balanced, and so as to be capable of having the wear taken up when it occurs. The steam-valves are provided with packing-plates, which are adjustable by hand, to

make them steam-tight, as well as to secure a perfect balance. Each valve is placed in a separate valve-chest, and can be independently adjusted. Each valve opens four ports; each is so set that it is actuated by a rod in the line of its own centre; and all are thus rendered but little liable to either wear or leakage. The rock-shaft arm on the intermediate rock-shaft, between the eccentric and the steam-valve stem, assists in securing the quick opening and closing motion essential to a satisfactory distribution of the steam.

The features which have been described are not necessarily distinctive of a "high-speed engine." A positive-motion valve-gear, and a good steam-distribution, are desirable in such engines, and the first point is, in fast-running machines, an essential requisite; but the engine, so far as it has been described, may be as well considered a slow as a fast engine. There are some details which are essentially and peculiarly characteristic of the class to which this machine is assigned. Among these points are the strength and rigidity of parts which distinguish such engines; the great nicety of fitting; the excellence of all material in every part exposed to the straining action of inertia, and the minor modifications of details to adapt them to service in a machine in which play in joints or bearings will make trouble.

The bed is stiff and solid, especially in those parts which take the stresses of the reciprocating pieces. It is broad and deep, with the line of thrust of piston-rod carried close to its surface between the guides, and with a box form which gives great resistance to forces tending to twist it. The steam-cylinder is secured to the bed by the end, a construction adopted by Corliss many years ago, and one which gives all desirable strength, with freedom from those strains which come of connection of two large masses at different and constantly varying temperatures. The main journal-boxes are made in four pieces, and are set up by adjustable wedges, so set as to avoid the springing of the shaft that is sometimes found to occur with a less effective arrangement. The main-shaft journals, and the journals of the crank-pins, are made with especial care, skil-

fully ground to size and form, and nicely finished before the engine is assembled. The pin is of "mild" steel, carefully case-hardened to give it a surface that will wear well and will not "cut." The provisions for lubrication in such engines are among the most important of its details.

The action of inertia in the moving parts is made by Mr. Porter the means of securing smoothness in working and evenness of crank-pin pressures. At the beginning of the stroke the inertia of the piston, its rod, the cross-head, and to a certain extent the connecting-rod, of all reciprocating parts, causes them to offer a certain resistance to the accelerated motion which they are compelled to take up. This resistance becomes less and less up to zero at half-stroke, the point at which their velocity is a maximum. Passing this point, they are rapidly retarded, and this same property of inertia causes them to offer a resistance to retardation, which resistance now is felt as an impelling force at the crank-pin. Thus, the effect of the presence of these heavy masses in the line of connection produces a reduction of pressure upon the pin at the commencement, and an increase of pressure at the end, of stroke. But in consequence of the varying action of the steam, producing an excess of pressure at the beginning and a deficiency of pressure at the end of stroke, we may combine these two effects, and the result is a comparatively uniform load upon the crank-pin throughout the stroke. This compensation is capable of being, in many cases, very nicely adjusted by properly proportioning the weight of the reciprocating parts. It is evident, however, that at some higher speed, the weight of these parts, as proportioned for strength simply, would be sufficient to give this desirable adjustment of the load on the crank-pin. There is no reason to suppose that this, which would seem to be a natural speed of the steam-engine, may not, at any time, be attained.

The Porter-Allen engine, the earliest of the "high-speed" engines, was also one of the first of its class to be constructed as a compound engine. Since the best engines of this type have about the efficiency of good Corliss engines, it is evident

that the opportunity offered for economical improvement is here equal, and the result of the experiment has been as satisfactory. The usual methods of compounding are substantially the same as those familiar in the case of the Corliss engine, and they may be expected to exhibit a similar ratio of improvement when compared with the corresponding simple machine. In some cases this gain is not sufficient to compensate the increased cost and complication, added expense of maintenance, and greater weight and volume; but at pressures exceeding sixty or seventy-five pounds it is found that they give real advantage, and the more as the pressures and ratios of expansion are increased. At still higher pressures, as for those exceeding 125 or 150 pounds, it is probable that still further subdivision of the total expansion-ratio, and the construction of the triple-expansion engine, would prove to be an improvement; while at pressures exceeding 200 or 225 pounds the quadruple-expansion machine would be as profitable, comparatively, as in those departments of application in which they have been already set at work. A maximum ratio of expansion of about three in each cylinder is probably advisable.

Another engine of this class is that first designed by Mr. J. W. Thompson, and known as the "Buckeye engine." This engine was not a radical competitor of the pioneer engine; but was, from the beginning, a moderately-high-speed engine. It was fitted with a positive motion, "automatic" or self-adjusting valve-gear, and a balanced valve, and had sufficient stability and excellence of workmanship to make it safe at high speeds; while the peculiarities of its construction were such as gave it a very high place as an economical machine. In this case the cylinder is carried on a pedestal, as is that of the Corliss engine, usually; the frame consists of a girder uniting the cylinder and the main pillow-block and carrying the guides; the crank-shaft end is carried by another pillow-block. The main frame is, however, supported by a strut which is now usually seen in other engines, and which takes the load tending to spring the girder under the guides.

The valves are so constructed that the steam enters balance-

engine is doing all that it can do. In this governor (Fig. 50) two levers are set on either side the crank-shaft, in a frame or a pulley to which they are pivoted at *b*, *b*. These rods carry weights, *A*, *A*, which may be adjusted to any desired position by means of the bolts seen in the cut. The outer end of each rod is linked to the loose eccentric, *C*, *C*, by the rods *B*, *B*, and is controlled by the springs *F*, *F*, which resist the effort of centrifugal force tending to throw the weights outward. As the weights swing outward or inward, as the one or the other of the two opposing forces predominates, the eccentric is turned on the shaft in such a manner as to give the valves that motion which is necessary to produce the proper distri-

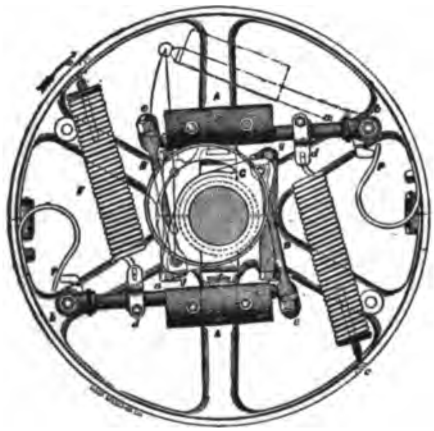


FIG. 50.—THOMPSON'S GOVERNOR.

bution of steam to bring the engine to its speed. The adjustment of this regulator to its work is easily obtained by the shifting of the weights along the levers, or by increasing or diminishing their amount, as is found necessary.

The general arrangement of this system and the appearance of an engine of this class are illustrated in the accompanying engraving.

A dash-pot has sometimes been used on the governor to correct the tendency to violent fluctuation when nearly isochro-

nous, and this was probably the first case of its use on this class of engines.

The independence of the cut-off and main valves, in consequence of the use of two eccentrics, permits any ratio of expansion to be adopted that may be desired, and the fact that the cut-off eccentric is set, at starting, nearly "with the crank," gives a wide range determinable by the governor, nearly from full-stroke to complete suppression. As the governor shifts the eccentric about the shaft, it gives increased angular advance and a shorter and shorter cut-off.

Here the main valve is actuated as in the common forms of valve; but its eccentric, instead of being set ahead of the

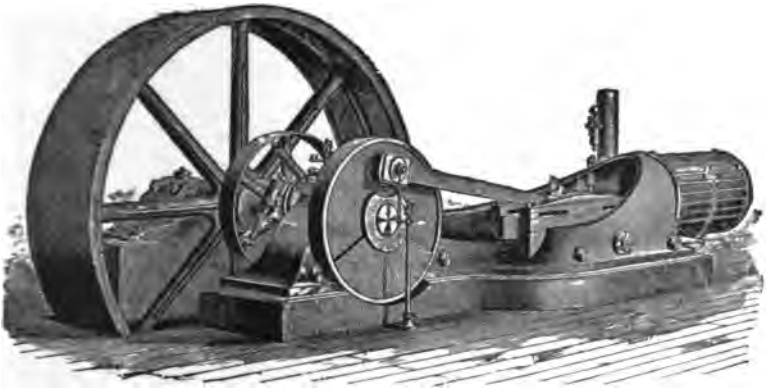
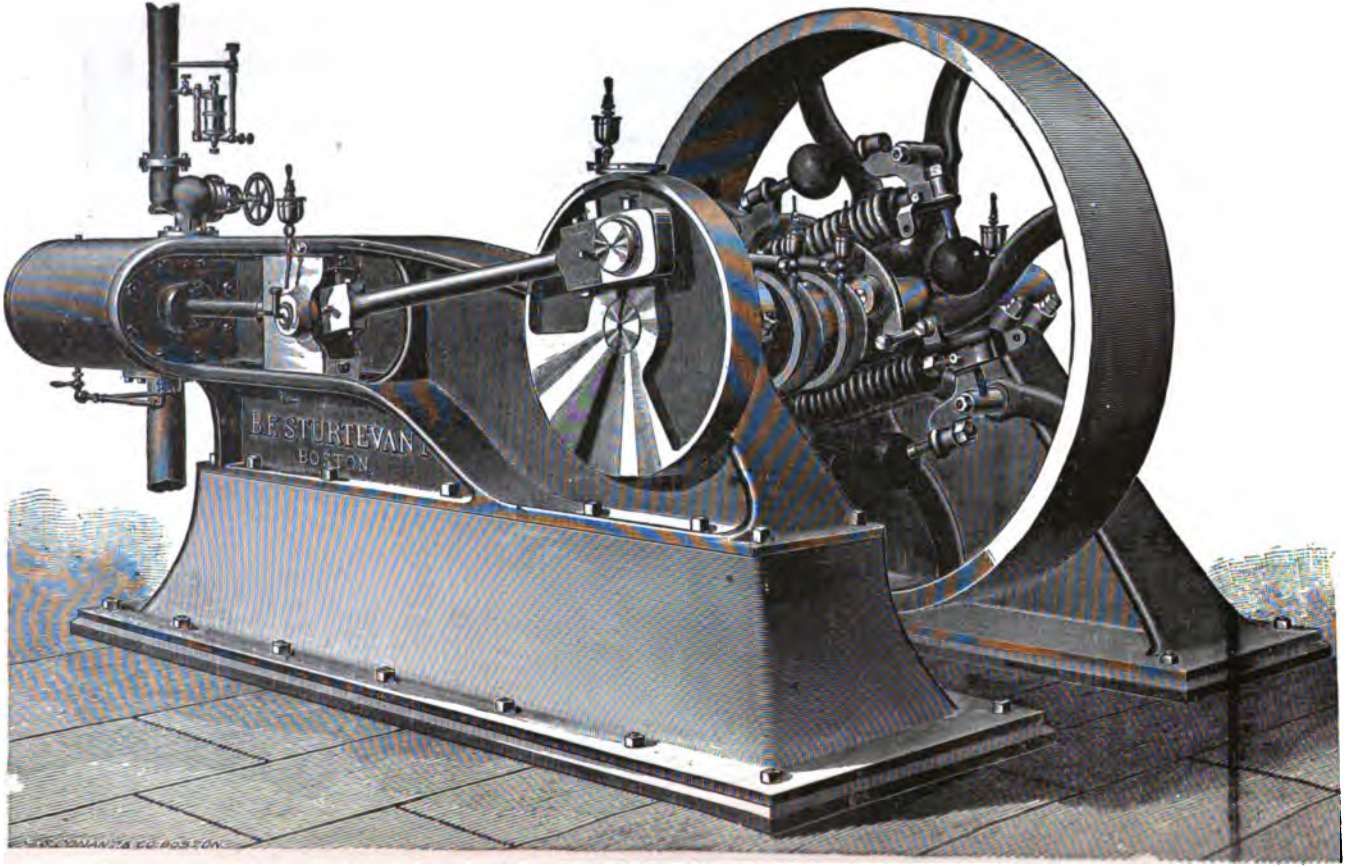


FIG. 51.—THOMPSON'S SYSTEM.

crank, follows, the exhaust- and steam-openings being, by the structure of the valve, reversed, and their acting edges transposed.

By carrying the pivot of the cut-off rock-shaft on the main rock shaft arm, uniform travel of the cut-off valve on the back of the main valve is secured, whatever the variation of cut-off. This insures uniform wear. In this, as in all engines similarly regulated, any mishap to governor or its connections stops the engine, a "run-away engine" being thus impossible.

In some cases, the use of an independent cut-off valve actuated by an "automatic" regulation system is adopted with the simpler forms of valve. The following figure illustrates


B.F. STURTEVANT
BOSTON.

such a plan, as constructed by Sturtevant, for all powers up to 150 H. P. Here the passages in the main valve, for the admission of steam, do not extend through the entire thickness of the valve. Within the main valve is a cylindrical seat in which runs a piston-valve, which receives from its eccentric a differential movement relatively to that of the main valve, just before the beginning of the stroke, opening the passage into the cylinder. The valve returns to cut off the steam at a time determined by the governor. As, at this time, the two valves are moving in opposite directions, this action is very prompt.

This form of cut-off valve has very little motion in its seat, and is subject to no lateral pressure. The main valve is set to cut off at three-quarters stroke. The main valve is balanced by pressure-plates upon its back.

The Straight-line Engine differs as radically from the two preceding as do they from each other. In this engine we find but a single valve, which does duty both as a distributing and as a cut-off valve.

This engine is the invention of, and is designed by, Prof. J. E. Sweet, and has some interesting points, which will bear much more extended study than they can be given in the space which can here be allowed.

The engine takes its name from its peculiar form of frame, which is seen to consist of two perfectly straight diverging struts extending from the end of the cylinder directly to the two main bearings, thus carrying the line of resistance to the pull and push of the connections exactly along its own central line. The engine is carried on three points as is the practice with "surface-plates," which must have an absolutely invariable system of supports, to avoid danger of "spring." These are under the main bearings, and beneath the steam-cylinder. The two journals receive equal loads; the crank-pin is not subject to the deflecting forces met with where a crank is overhung; danger of unequal wear of journals, and of springing the pin, is thus avoided. The fly-wheel is placed in twin form between the main bearings, and also serves as a crank as

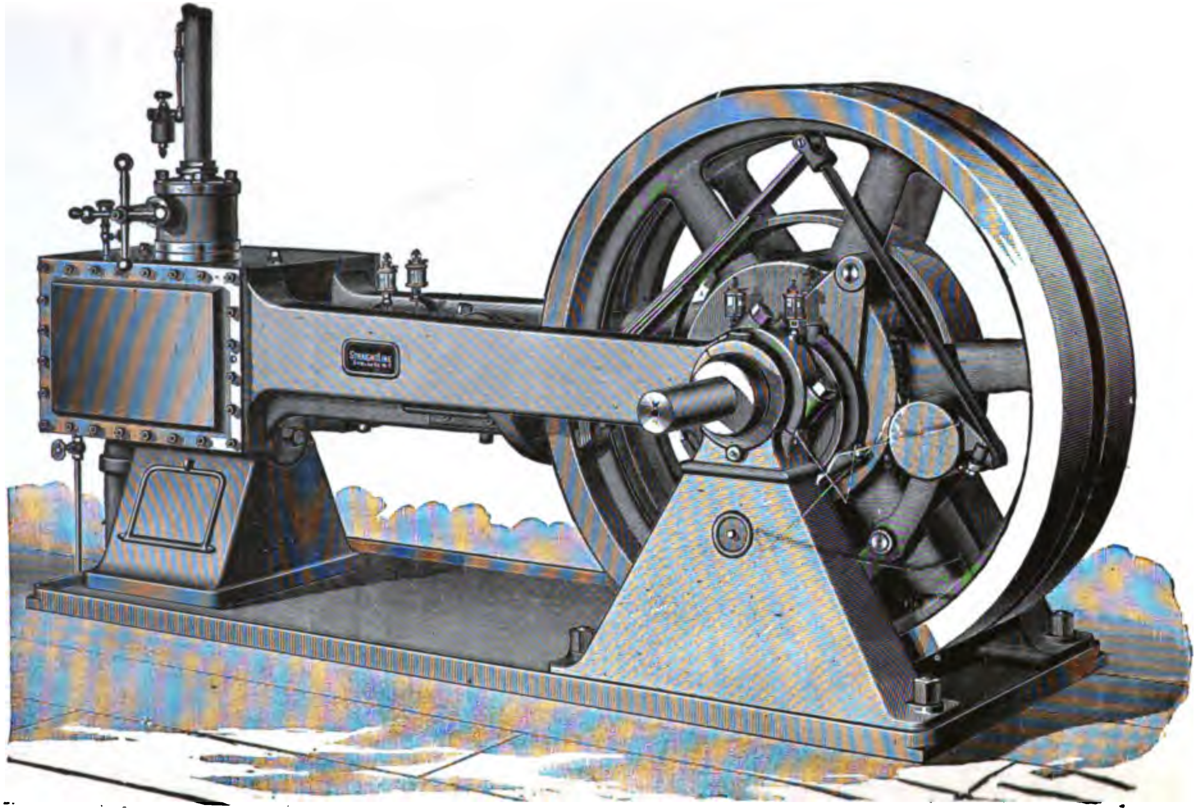


FIG. 53.—SWEET'S STRAIGHT-LINE ENGINE.

well as balance-wheel. By its action at this point it intercepts heavy and objectionable stresses, which, otherwise, might be transmitted to the main shaft; and the reciprocating action of counterweights and equilibrating parts is thus only felt within a mass of metal which can resist them with safety and without affecting the main journal; which is also less liable to spring under the loads transmitted through it. To secure better distribution of wear, the crank-shaft is allowed some end-play.

The steam-cylinder has the valve-chest placed at the end nearest the crank, and the ports and passages are carried as in those engines. The valve-stems have no stuffing-boxes, but pass into the chest through unusually long and carefully fitted holes in a hub, made about five one-thousandths of an inch larger than the rod inside the Babbitt-metal bushing, for a length of six diameters, or more. The hub is loose in the hole in the end of the valve-chest, and is packed at the ends by a washer fitted on a flat seat on the inside. The piston-rod is similarly fitted.

In this engine, wear is avoided at the cross-head pin by cutting away the surfaces which do little or no work, and thus securing overrunning surfaces, which are not subject to this distorted wear to so great an extent.

The valve is what may be called a "piston-valve" of rectangular section, the space in which it slides having, therefore, also a rectangular section.

The compound form of the Sweet engine is one of the best of illustrations of the compactness which may be given the "tandem" type of the machine. The engine is built, as to its high-pressure cylinder and working parts, precisely like the standard type of the simple engine of the same design. It has exactly the same characteristic form of frame and methods of connection and of steam-distribution and governor. Directly behind the high-pressure cylinder, however, is placed the larger, low-pressure, cylinder, the whole forming, practically, one structure. The whole machine can be taken apart and reassembled without disturbing the cylinders or the frame. Both pistons, which are mounted on one rod, can be removed and replaced;

the intermediate head coming away with its stuffing-box through the larger cylinder. The packing of the rod between

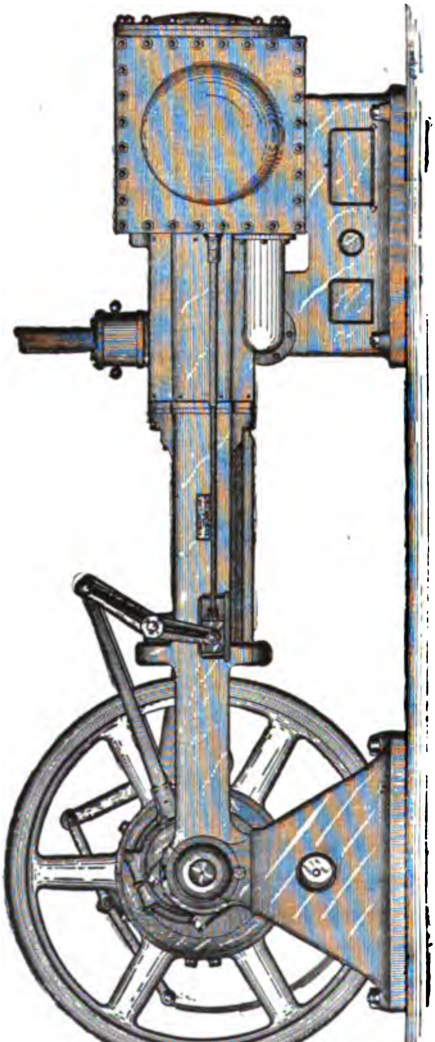


FIG. 54.—THE SWEET COMPOUND STRAIGHT-LINE ENGINE.

the two cylinders is a metallic sleeve, solid and free from liability to produce trouble or to require readjustment, once in place

and properly fitted. It is free from liability to wear or to bear upon the rod in such a manner as to produce undue friction and heating, while it is loose enough to work smoothly and yet tight enough to prevent leakage of steam past its shell. The valve of the low-pressure cylinder is worked by an independent, fixed, eccentric, and the expansion is adjusted by the action of the governor, affecting the point of cut-off on the high-pressure cylinder, precisely as in the simple engine. Where the load is fairly steady this arrangement is perfectly satisfactory. The inventor has also planned a triple-expansion vertical engine of equal simplicity.

The Armington and Sims Engine is of the same general class with the last described forms of engine, but differs from them in its details and in its proportions, somewhat, and especially in the form of its valve, and in the devices intermediate between governor and valve. In this engine the "piston" valve is used, combined with a double port. The following engraving, Fig. 55, presents a view of this engine. The bed, or frame, is seen to be similar to that of the Porter-Allen engine, heavy, solid, stiff, taking the bending stresses of the guides at its upper surface, and insured against twisting strains by the box form of its section. Two main pillow-blocks carry its steel crank-shaft, and support the two wheels, one of which is a balance-wheel, and the other of which is the pulley, from which the engine is belted to its work; or, perhaps oftener, both being used in driving, thus equalizing the load on the shaft and preventing tendency to wear out of line. The steam-cylinder is overhung, and the exhaust-pipe is carried down below the floor, clear of the foundation, which latter has a minimum extent and cost, while sufficiently heavy and strong enough to carry the engine steadily. In some cases the frame is made with but one pillow-block, and the crank is overhung.

The journals are calculated for the speeds and pressures adopted. The lubrication is a matter of vital importance in all engines of this class. In this engine the "sight feed" is used, in which each drop of oil falls through a clear space, on its way

to the point to be oiled, in full view of the man in charge, and any failure of the oil to "feed" is thus promptly detected. The crank-pin is supplied by a "wiper," which takes its supply of the lubricant from the oil-cup at every revolution of the crank. This device has been used, in very similar form, by the Author, on fast marine engines, with perfect satisfaction.

A governor, of the same type as that exhibited in the articles describing the "Buckeye" and the "Straight Line" engines, is secured to the arms of the pulley on the frame, and

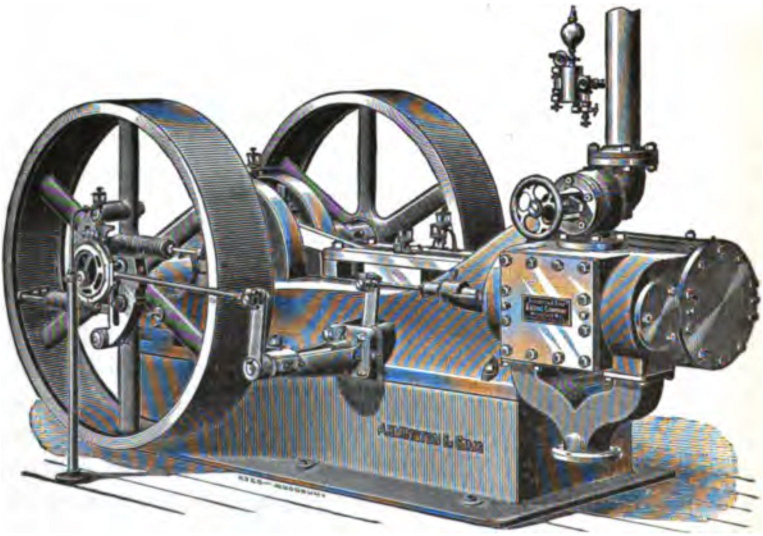


FIG. 55.—ARMINGTON & SIMS ENGINE.

adjusts the position of the eccentrics which give motion to the valve through a rod and valve-stem, the connection between which two parts is made at a point at which they can be conveniently supported by a rock-shaft and arm carried at the middle of the length of the frame. The cranks are two disks in which the balancing mass can be secured at any desired point.

The cylinder, steam-chest, and valve-seat are all in one casting.

The steam-chest is in direct communication with the boiler, and the valve, which is of the piston form with a double steam-

port, is surrounded by the "live steam," thus taking steam at the middle and exhausting it at the ends of the chest. The valve moves precisely as does the ordinary locomotive slide-valve, and the steam is introduced, at the beginning of the stroke, through a double length of port, and hence with unusual promptness when the engine is running at high speed.

The total "dead space" in these engines, including piston-clearance, is sometimes as low as 5 per cent on large sizes. In all cases, compression should fill this space at every stroke. This piston-valve possesses a novelty in the double port. Its advantages are the ease and cheapness with which it can be made and fitted, and with which it can be replaced when worn; its perfect balance and ease of working under any practicable steam-pressure, its permanence, tightness, and remarkable durability when properly cared for and used with boilers supplied with good water. Its disadvantages are the rapidity with which it sometimes wears, when it is not kept well lubricated, or when it is exposed to the action of steam carrying over from the boiler acidulated or dirty water, the danger of injury to the cylinder or its heads when priming occurs, and the proneness of the attendant to neglect its repair.

The governor is the same, in principle, as those already described as adapted to the adjustment of the eccentric on the main or the governor shaft. It has the two weights carried on, and forming a part of arms pivoted to the governor pulley, and revolving in the vertical plane as usual in that class of governors. The position of these weights, as determined by the speed and the action of the springs, determines the position of the eccentrics, and thus the position and motion of the valve, and the point of cut-off, flying out and giving a higher ratio of expansion as the load on the engine is diminished, or as steam-pressure rises in the slightest degree, and a lower ratio as these conditions are reversed. In the device here adopted, however, the valve is driven by an eccentric which is "duplex." One eccentric is set inside another, and connected to the governor arms in such a way that, as the weights separate with increasing speed of engine, both eccentrics are turned on the shaft so

as to cause their "throws" to coincide, or to separate as may be necessary. When they coincide, the travel of the valve is due to a greater total throw, and is a maximum; when they are separated as far as possible the travel is reduced to a minimum. The action is almost precisely the same as that of a "Stephenson link," worked between full and mid-gear. When the two eccentrics give maximum travel, the action is that of the link-motion in full gear; when they are at opposite sides of the shaft, the action is that of a link in mid-gear. By setting them at intermediate points, the throw is made that is required to give an intermediate action of the valve, and thus the distribution of steam is made to accord with the demands of the work by such a variation of the ratios of expansion and of compression as is obtained by the link-motion, and, in this case, with the advantage in promptness of opening and of closure obtainable with a double-ported valve. The range of action given in this engine is sufficient to permit a range of cut-off from 0 to about three-quarters stroke. The lead remains unchanged, and the compression increases as the ratio of expansion is increased. The springs of the governor are used in compression.

Among the first of the "single-valve automatic" engines to find a place in electric lighting was the Armington & Sims engine, which was also one of the earliest to be built as a compound engine. An experimental engine was built about 1880; but the engine was not constructed as a multiple-cylinder engine regularly and as a standard type until some years later. The form given this engine is seen in the accompanying illustration, which represents the machine as constructed to give 100 horse-power at high speed. The regulation and the general construction of each of the two elements of the compound engine are similar to those already described in the simple engine. The two cranks are placed opposite, and this gives that perfection of balance which cannot be secured by any other device. It is also the best method of obtaining transfer of steam from the one engine to the other with minimum loss of pressure. The attainment of a speed of 800

revolutions a minute is not unusual. Both cylinders are steam-jacketed. Such engines are usually made up to about 200 horse-power. In the type here shown, the cranks being opposite, the engine balanced, it can safely be run at a high speed; the peculiar form of the valve provides for quick admis-

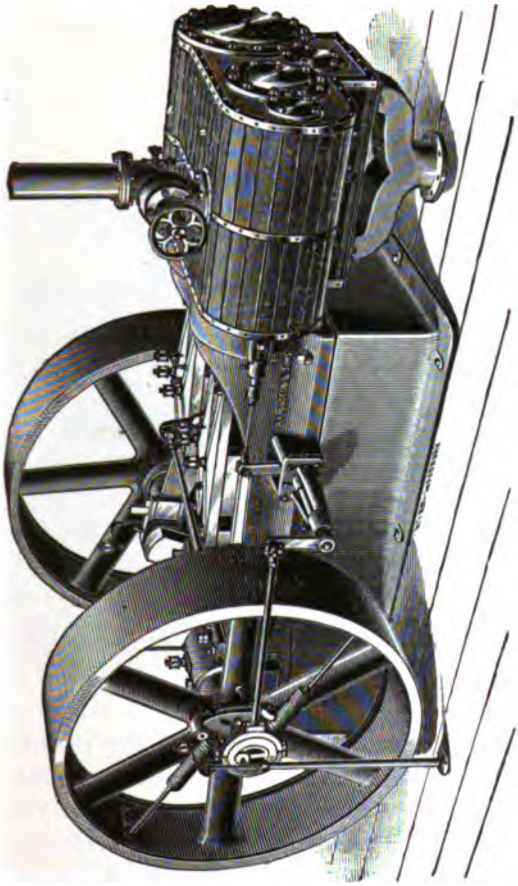


FIG. 56.—ARMINGTON & SIMS COMPOUND ENGINE.

sion of steam, and the large wearing surfaces insure it more or less fully against leakage; the pistons and stuffing-boxes used are more easily got at than ordinarily with engines of the "tandem" type.

In the Ide engine, of this class, shown herewith, a similarly compact form of "automatic" engine is illustrated; with its shaft-governor, and peculiarly solid frame.

The top of the frame extends from cylinder to main bearing, the full width of bearing. The caps are put on at an angle, which gives an adjustment in line with the wear of the

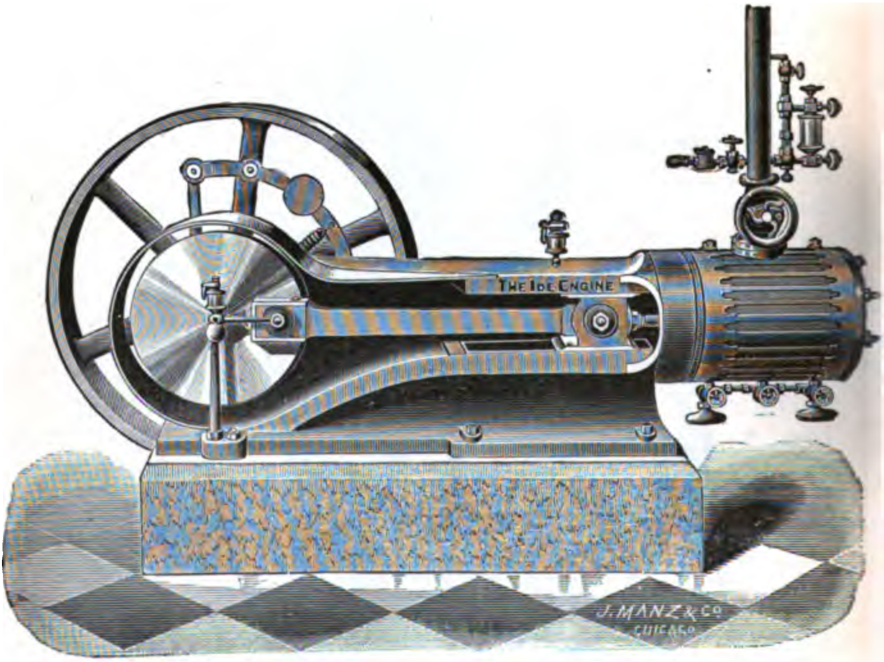


FIG. 57.—THE IDE ENGINE.

parts. The adjustment is given by reducing the thickness of the liner plates, and the cap is always drawn up solid.

A straight vertical web of metal connects the upper and lower portions of the frame, and forms a stiff girder. This web extends from the cylinder to the front side of bearing, close to the crank-disk.

The fly-wheel is set as close as possible to the crank, in order to reduce the strain on the shaft. The base of the frame is rectangular, and forms a box girder, the top of which forms

the bearing for the lower guide, which receives the pressure of the connecting-rod.

Piston-valves are used, and, in this engine, the steam-chest is bored out and fitted with bushings which have supporting bars to prevent the valve catching upon the ports. When worn they can be withdrawn and new ones inserted, and a new valve introduced, without delay.

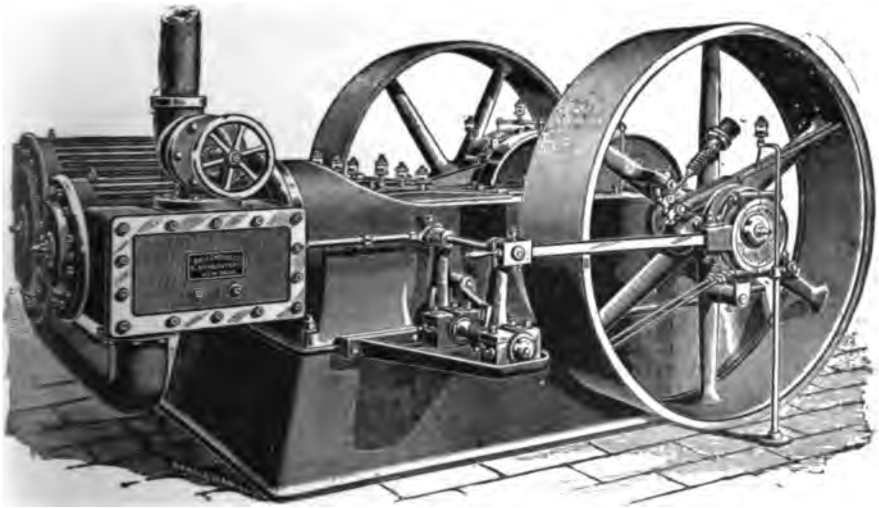


FIG. 58.—"CROSS" COMPOUND ENGINE.

Fig. 58 represents an automatic compound engine designed by Mr. F. H. Ball, especially for use in driving dynamo electric machinery.

The illustration represents engines using steam at 125 pounds pressure, and of 250 horse-power each.

It was thought best to build these engines in the form of a double engine rather than the "tandem" type of compound, because it was believed that higher rotative speed could be successfully used where the work was distributed over two sets of crank-pins and journals of smaller sizes, rather than with the use of a single set of bearings of larger size, as in the case of a tandem engine developing the combined power of the double compound.

The cylinder-dimensions selected after working up a large number of provisional diagrams were as follows:

High-pressure cylinder: diameter 13″; stroke 16″. Low-pressure cylinder: diameter 25″; stroke 16″.

The maximum power attained on trial was 325 I. H. P.

The next figure illustrates the same make of engine compounded in the more usual way, a "tandem," compound, high-speed engine, for electric-lighting or other purposes, which is found to be one of the best combinations of efficiency with simplicity and small cost.

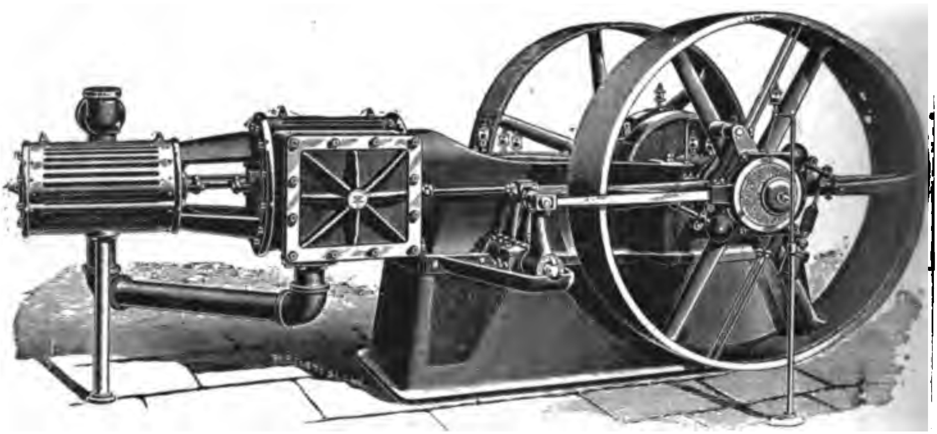


FIG. 59.—TANDEM COMPOUND HIGH-SPEED ENGINE.

Nearly all makers now use this method of compounding for all cases except where, as in marine engines, a double engine with cranks at right-angles is considered desirable on other grounds. They are nearly as simple in form, as cheap of construction, and as inexpensive in repairs as the simple engine.

An engine designed by Mr. Ide, Fig. 60, illustrates both the "tandem" form of compound high-speed engine, and some features of design of peculiar interest. This engine has its running parts covered in to insure that the oil, which is freely supplied, may not be wasted or spattered about, to the injury of surrounding objects, while thus also obtaining thoroughness

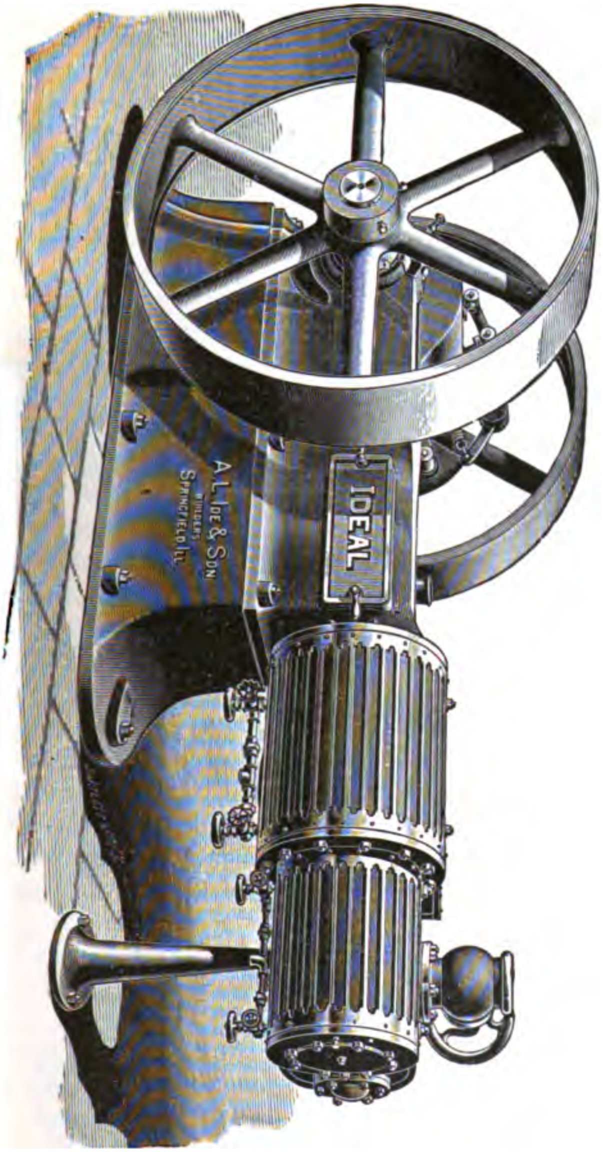


FIG. 60.—IDE'S "TANDEM" COMPOUND ENGINE.

of lubrication approximating that of the "oil-bath." This gives, when fully effected, very great decrease in the wasted energy of internal friction of engine and corresponding increase of efficiency. The design is simple, inexpensive of construction, and embodies details of construction coming to be generally recognized as essential to high efficiency. The engine has a shaft-governor, controlled by a dash-pot, and thus enabled to regulate more closely. Its running parts are usually of steel.

The low-pressure cylinder is bolted direct to the engine-bed, and to the head of this cylinder is cast the high-pressure cylinder. By this arrangement steam from the high-pressure cylinder has a short, direct passage into the low-pressure cylinder, and four stuffing-boxes are dispensed with on the rods between cylinders, reducing friction and dispensing with considerable external radiating surface.

The cylinders and steam-chests are encased with a finished iron jacket, with two-inch air-space, between cylinder and jacket, filled with non-conducting material. Both cylinder-heads are protected in the same manner.

The head between the cylinders is cored out leaving a space, which is filled with non-conducting materials.

The next figure exhibits the same type of engine as arranged for a "cross-compound" by the Harrisburg Co. The "tandem" engine has an advantage in small cost, in compactness, and small friction; but the cross-compound, with cranks at 90°, has no "dead-centres," is somewhat steadier in its revolution, and has lighter stresses on its running parts. A receiver is here needed, and is seen between the two engines. It is made an expansion-piece to avoid temperature-strains.

In designing the twin form, or cross-compound engine, it is advisable to secure compactness without sacrificing accessibility; independence of parts exposed to independently varying temperatures, and a nice adjustment of steam-distribution with respect to both the cylinders and the intermediate receiver. The next figure illustrates the arrangement of the Harrisburg engine as seen from behind the cylinders.

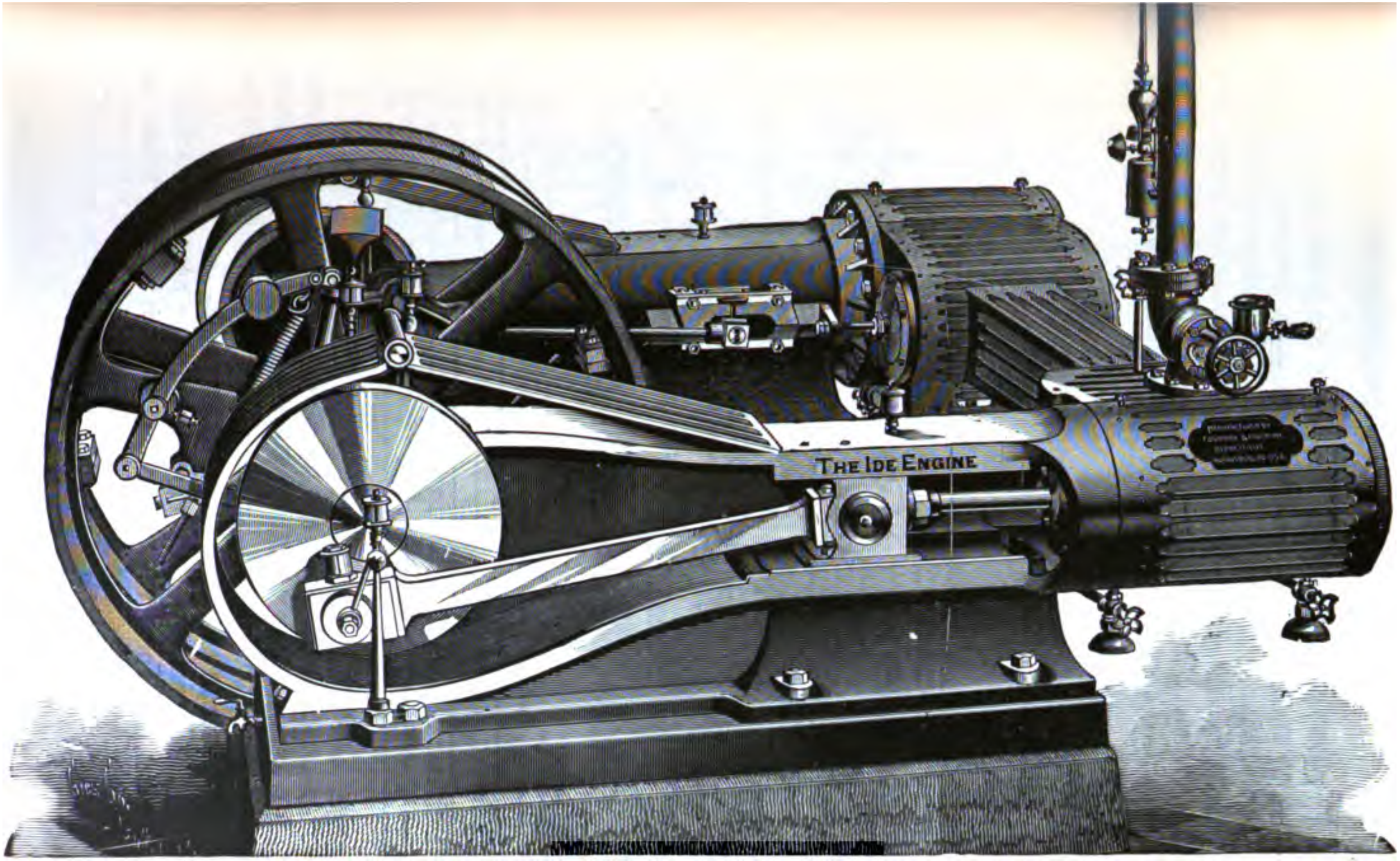


FIG. 61.—"CROSS-COMPOUND" ENGINE.

In the plans it is to be noted that the power to be given off by the engines is transferred through the intermediately situated pulley fly-wheel, which is the only element separating the two machines. The shaft is made of minimum length; the space afforded by the mounting of the wheel in this manner also serves to admit the two valve-chests and a very short

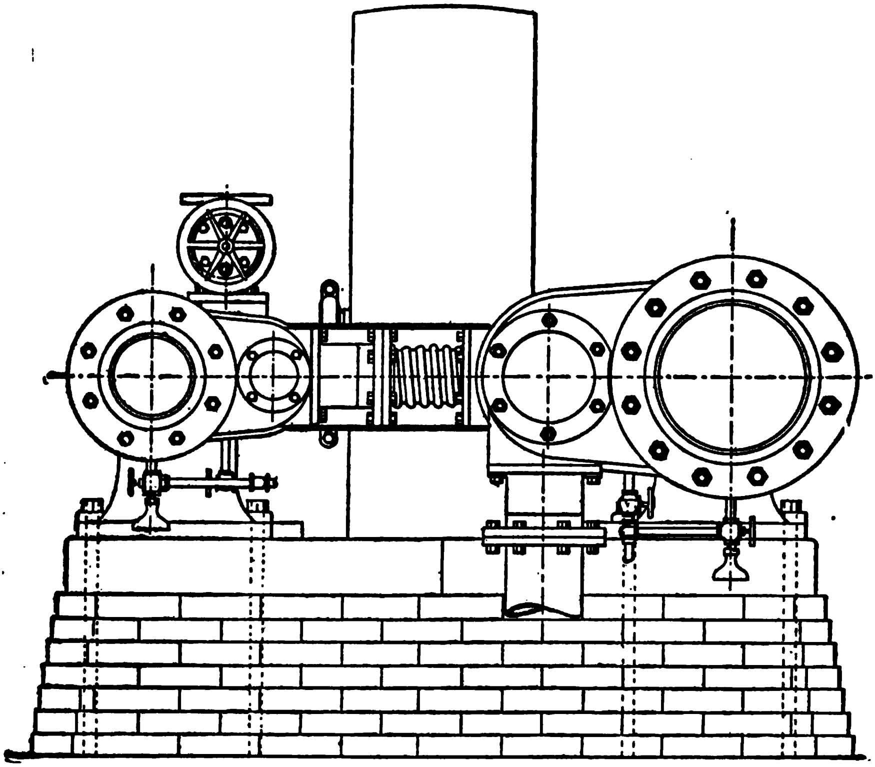


FIG. 62.—SECTION: CROSS-COMPOUND ENGINE.

connection serving as receiver and constructed with an expansion-piece, to avoid introduction of strains. The whole design, which is now a not uncommon one, illustrates well the most compact possible form of this engine.

These points are also observable in the next illustration, in which a plan of the Porter compound is given. Where, as

in this case, the valve is on a level with the centre-line of the engine, care must be taken to secure immunity from danger

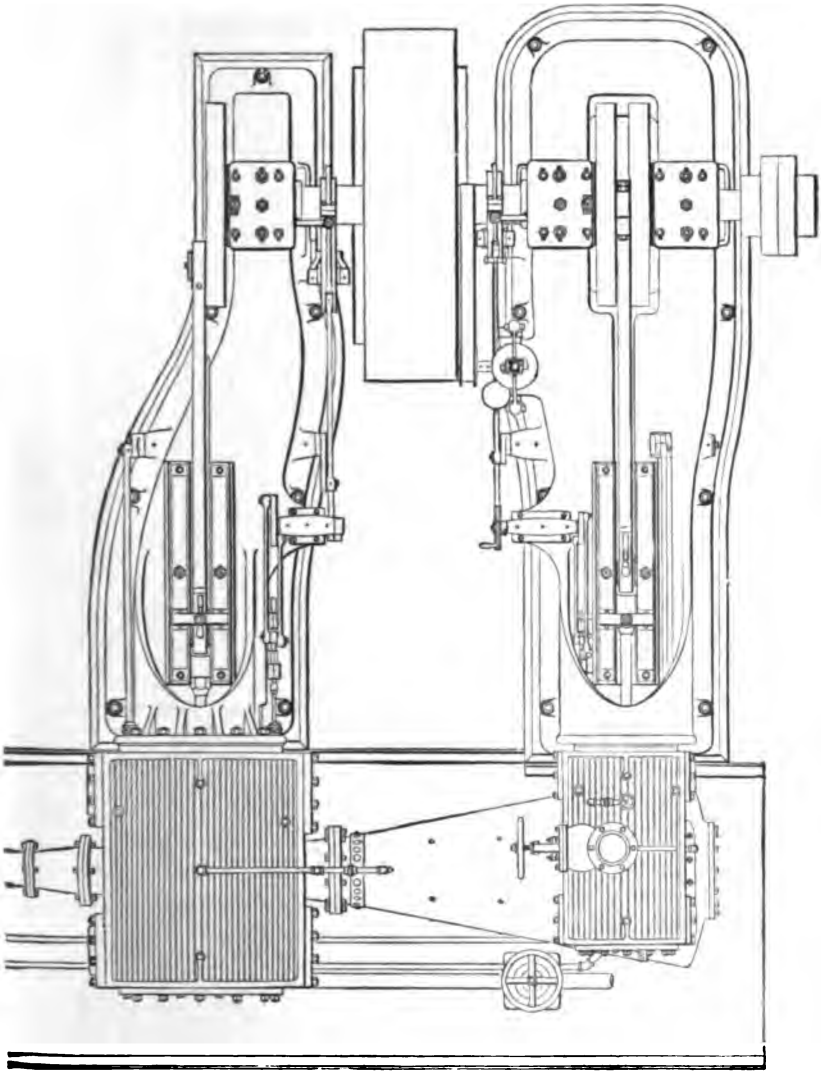


FIG. 63.—PLAN OF ROLLING-MILL ENGINE.

from water entering the cylinders, by the use of an automatic relief-valve, or a "breaking-cap."

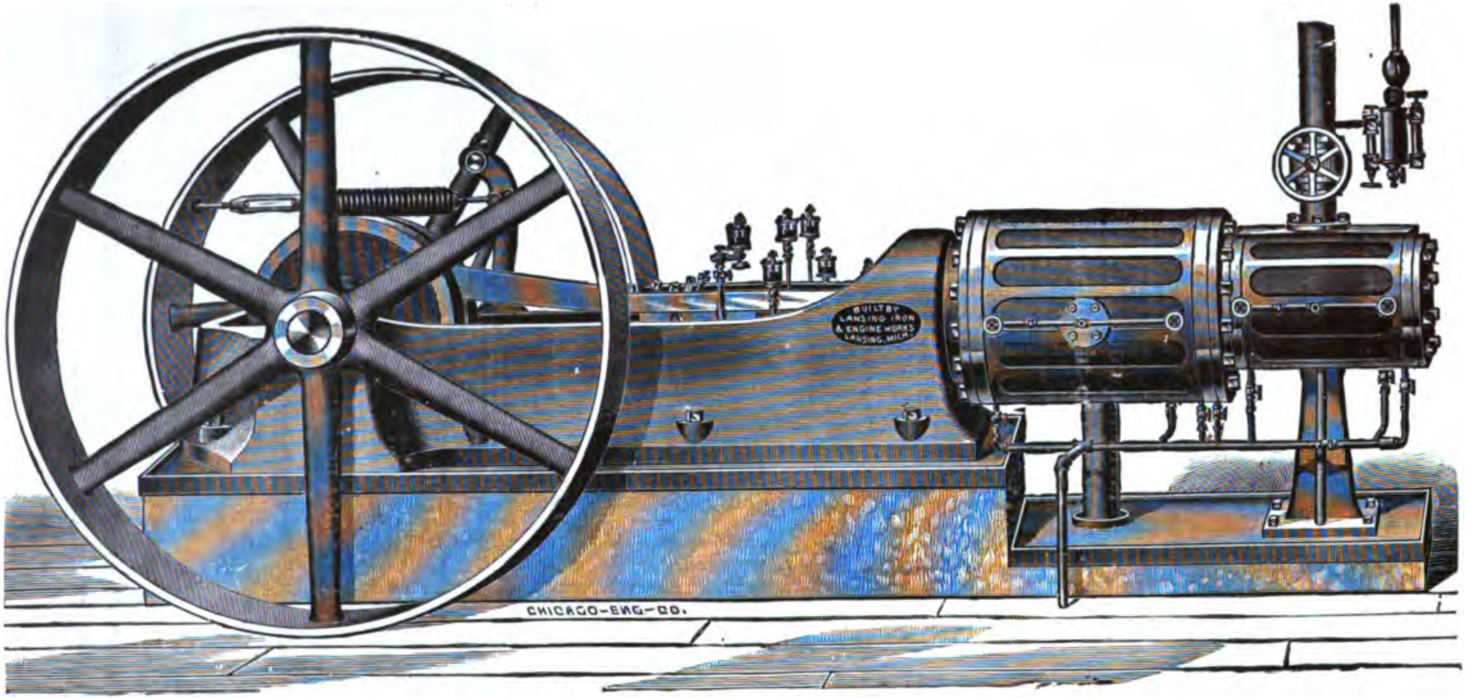


FIG. 64.—THE LANSING COMPOUND ENGINE.

The engraving on page 146 shows the usual construction of foundation, which may be either brick or stone, but is commonly preferred of brick with, often, stone blocks on which the engine is immediately supported.

Where, as often in rolling-mills, the power of the engine must be transmitted along the shaft, a fly-wheel of the simpler kind may be placed between the cylinders, and still greater compactness thus sometimes attained. Thus, in Fig. 63, the plan of a Porter-Allen rolling-mill engine, this arrangement is made, the shaft being extended to the right, toward the roll-train, to which it is coupled as shown. The arrangement of the machine, in detail, illustrates the special methods of combining two engines of this type, as dictated by its special construction.

The Lansing engine, planned by Mr. Jarvis, illustrates still another design of the "tandem" compound variety. In this case both steam-chests are on the same side, giving short connection between the two chests, and diminishing the surface exposed to steam, which exposure is detrimental to economy. It is seen in Fig. 64.

The shaft-governor, keyed to the shaft, obviates danger due to the breaking of belts or gears. This governor is of the class in which the eccentric is hung on an arm, which allows it to swing across the shaft by levers pivoted to the spider of the wheel. In its details it is the design of Prof. R. C. Carpenter.

To obtain the astatic or isochronous property, the governor must be so arranged that with a slight variation in speed it may move through its entire range. This end is attained by setting up the springs with an initial tension, so that the weights with their arms remain against the inner stops until the speed has nearly reached its governing range. A slight additional increase would then cause the weight and arms to move, if the increase were not checked, through the entire range of action.

This action is restrained by the air dash-pot, seen in Fig. 65. Inertia is made to act usefully by so pivoting the arms that, when the governor is in operation, the weights are in

such a position that a line drawn through their centre of gravity perpendicular to the radius will pass to one side of the arm-pivot. The force due to inertia, when the speed changes, acts nearly along this line, and tends to turn the arm about the pivot, and thus move the eccentric in the same manner

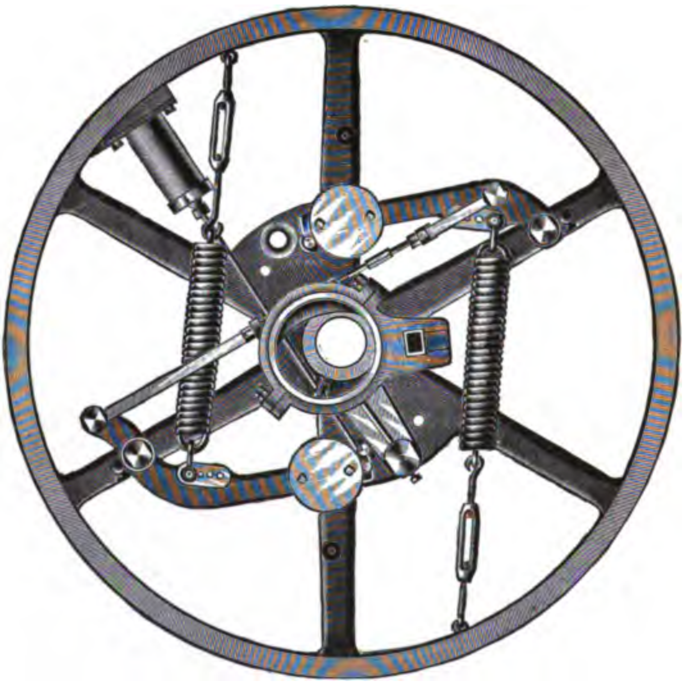


FIG. 65.—CARPENTER'S GOVERNOR

as the centrifugal force, and acting most quickly, it gives the governor a greater sensitiveness.

36. **The Single-acting High-speed Engine** is a peculiar but now familiar type. In the "single-acting engine," the steam drives the piston in but one direction, and the return-stroke must be made without the production of useful work. In the "double-acting engine," the steam acts upon the piston in both directions, and with practically equal effect. Thus, a more regular action is secured with a given weight of balance-

wheel, or the same regularity with a wheel of less weight than is required for the other form of engine. This smoothness of motion is one of the most essential features of steam-engine economy. At the speeds which have been lately attained, however, the inertia of moving parts becomes so great that moderate variations in the impelling power become comparatively insignificant, and have no perceptible effect upon the smoothness of revolution of the crank-shaft.

The double-acting engine evidently possessed greater power than its predecessor, when of the same size, and the "efficiency of the machine" was correspondingly increased.

But the very conditions which have been thus made to aid in securing regularity have introduced a new difficulty: At every revolution of the engine, the crank "turns the centre" twice; and, at every passage of the centre, the direction of pressure upon the crank-pin is reversed, thus producing a shock which varies with the difference of pressure, the suddenness with which it is felt at the pin, and the extent of the "lost motion" between the pin and its bearings. Some lost motion must always be permitted, to avoid danger of heating the journal and injury to the machine. The counteracting adjustments are found to be, usually, the utilization of the inertia of the reciprocating parts; the adoption of heavy compression, and very careful adjustment of the fit of the brasses on the pin. With skilful use of these expedients, and with the introduction of perfection of workmanship, and of qualities of material, such as have only been attained in late years, the "high-speed engine" has been made successful at as high as 300 and even, in some cases, 600 or more revolutions per minute.

But much higher speeds than these are sometimes demanded; and engines must, in the future, be built to run, regularly, steadily, and safely, at, probably, very much higher velocities. This may, ultimately, lead to radical changes in the design of the now standard forms of fast engines. Nevertheless, the limit of speed has by no means been reached, even at the higher of the above speeds, with the common type

of engine. The speed of even 450 times the cube root of the length of stroke, now a common figure, and over three times that given by Watt's rule, is occasionally greatly exceeded. Ericsson designed an engine, some years ago, for electric lighting, which ran, for years, at 1250 revolutions per minute, without accident. The piston-speed was about twice that of

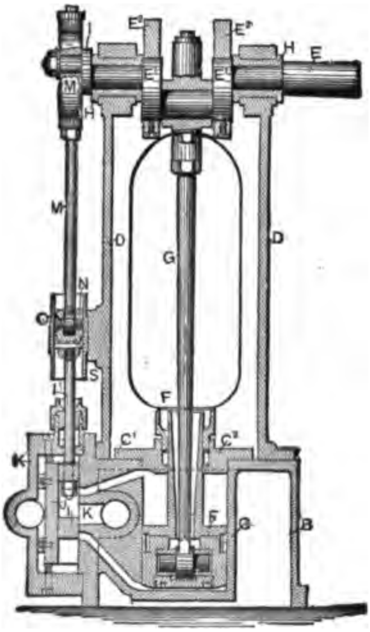


FIG. 66.—ERICSSON'S ENGINE. (Scale $\frac{1}{16}$.)

the average "high-speed" engine, and nearly ten times that adopted by Watt.

The object of the inventor was to design a steam-engine for the special work of driving small dynamo-electric machines, and hence to secure great stability and strength, a minimum number of parts requiring lubrication, and absolute certainty that the parts retained should be, at all times, thoroughly supplied with the lubricant. The engine is therefore made a "half-trunk" engine, the trunk, *F*, *F*, Fig. 66, serving as an

oil-reservoir. The joint in the eccentric-rod is provided with a piston moving in a cylindrical guide, *N*, which is also an oil-reservoir. The cylinder, *C*, and base-plate, *B*, are in one casting, upon which is set the hollow frame supporting the crank-shaft, *H E*, and balance-wheel. Every journal and rubbing part has an oil-reservoir and special provision for effective lubrication. This engine is at Sibley College.

There comes a time, in the attempt to secure smooth working, and as speeds are increased, when the weight of running parts, as calculated for strength only, becomes as great as is desirable to effect this object by their inertia; there comes a time, also, as compression is increased, when the "cushioned" steam is carried up to boiler-pressure, and this would seem the natural limit. The next device adopted by the engineer, in chronological order, is that of preventing the lift of the brasses of the crank-pin and of the cross-head pin when turning the centres, while still leaving the freedom of fit required to give safety from heating. This last expedient is that which has led to the construction of a class of engines which are as peculiar and as typical as either of the classes which have been already described.

Westinghouse's Engine belongs to this class, and is here taken as its representative. The change of construction characteristic of this type of engine is a return to the original "single-acting" plan of engine. The simple form of this engine, Figs. 67, 68, has two cylinders, *A A*, fitted with single-acting pistons, *D D*, forming trunks filling the bore of the cylinder, giving a long steam-tight bearing, and taking the connecting-rod pin, *A B*, at a point at which no tendency to rock the piston can be produced. The top of the piston is cored out to prevent transfer of heat from the working to the non-working end. The rods, *F F*, take hold of the crank-pins within an inclosed chamber, *C*, forming part of the engine-frame, *E C*. This frame and bed-plate also acts as a reservoir for oil lubricating the journals and pistons, which oil floats on water and is dashed up over the moving parts so enclosed, at every revolution of the engine. No other attention is required

than to keep a supply of oil in the chamber, by filling as loss occurs by leakage. In fact, the whole engine is thus shut in by its frame, and its working parts are invisible while working —an arrangement at once a means of security and convenience.

The valve adopted in this engine is a piston-valve of the class already described, but having some peculiarities specially adapting it to its use in this engine. Its guide, *J*, Fig. 67, is a

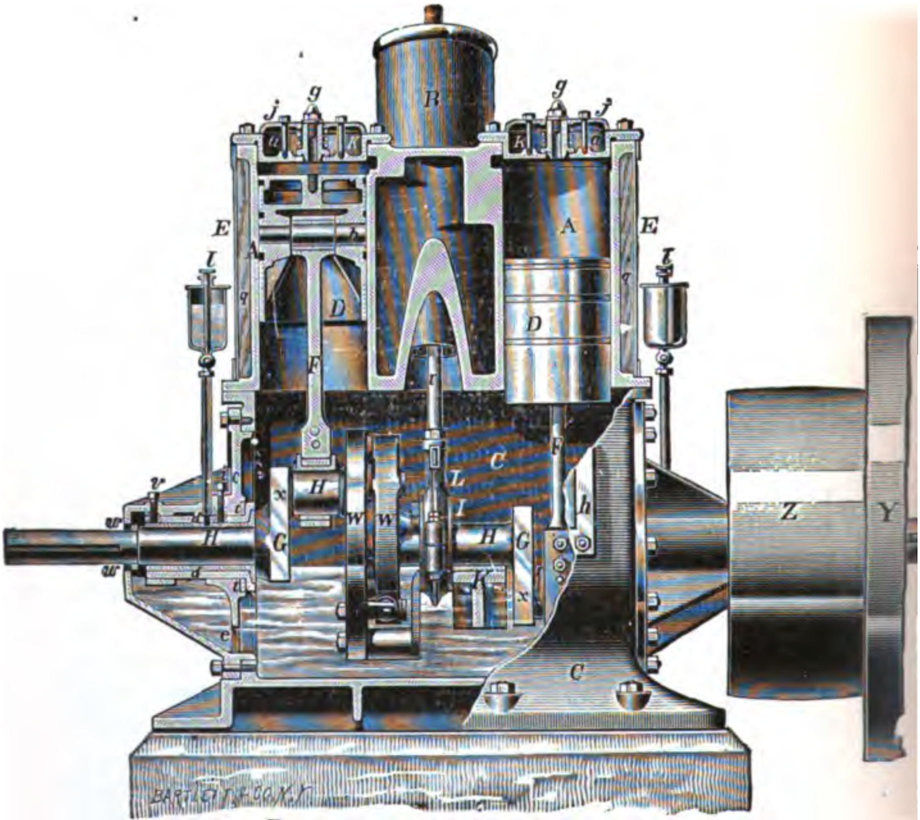


FIG. 67.—WESTINGHOUSE ENGINE. (Scale 1/12.)

piston traversing a cylinder separating the exhaust space from the chamber below. This one valve, *V*, distributes steam to both cylinders, the two cranks being set directly opposite each other. This adjustment of the cranks also gives a perfect

balance of reciprocating parts, and secures smoothness of movement of the whole machine, whatever speed may be adopted; and exceptional speeds of 1000 revolutions, or more, per minute are reached without observable vibration.

The governor, *I*, and its action, are precisely like the same

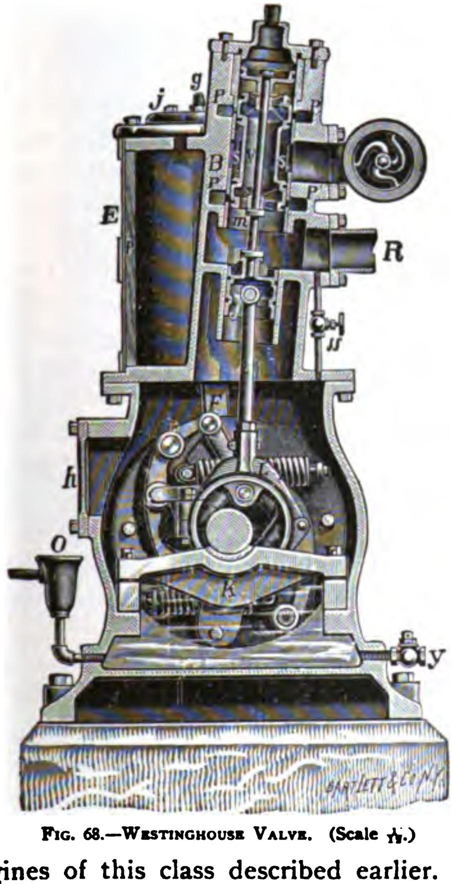


FIG. 68.—WESTINGHOUSE VALVE. (Scale $\frac{1}{10}$.)

parts in engines of this class described earlier. It actuates the eccentric, and determines the point of cut-off by varying the throw of the valve, while retaining constant lead. The governor is usually so adjusted that it will not come into play until the engine falls one per cent below, or rises one per cent above, the normal speed; its full traverse is effected, also,

within this range, the intention being that the speed shall never vary more than one per cent from that fixed as its proper velocity. The range of expansion is from 0 to about $\frac{5}{8}$ stroke.

One of the dangers to which fast-running engines are peculiarly exposed is that of injury by the entrapping of water in the cylinder, and the plunging of the piston against the mass of incompressible fluid which then fills the clearance-spaces. In this engine, in addition to the relief-cocks, or valves, which are always fitted to such engines, a safeguard is introduced in the form of what engineers are accustomed to call the "breaking-piece," a part which is made purposely weaker than other portions of the machine, exposed to a common danger, so that this piece may go when danger arises. This piece is always one the replacement of which will give little trouble, and make but little expense. Such a breaking-piece is made to form a part of the cylinder-head. This may be knocked out without injury to any important, or costly, part of the structure.*

The Single-acting Multicylinder Engine is often adopted for work in which high speed of rotation is an advantage. The Westinghouse compound engine, illustrated in the engraving, is a good typical representative of this class, and is one of the simplest devices of its kind. A single piston-valve, set horizontally above the two cylinders, distributes the steam and is regulated by a shaft-governor which properly varies its throw. The cranks are set opposite each other; the motions of the pistons are synchronous in opposite directions, and no receiver is needed. Both engines are single-acting, and high compression does away, largely, with the wastes due to considerable clearance. The cut-off in the high-pressure cylinder is effected by the lap of the valve. It has been found possible by this arrangement to bring down the consumption of steam to less than 20 pounds (9 kilos) per horse-power per hour when con-

* The Author planned an engine, about the year 1860, in which the whole cylinder-head was made a safety-valve which could lift and discharge the water into the chamber behind it, the cover of the latter being bolted on, while the cylinder-head was only held in place, against a faced joint, by steam-pressure.

densing, and below 25 pounds (11 kilos) when working non-condensing.

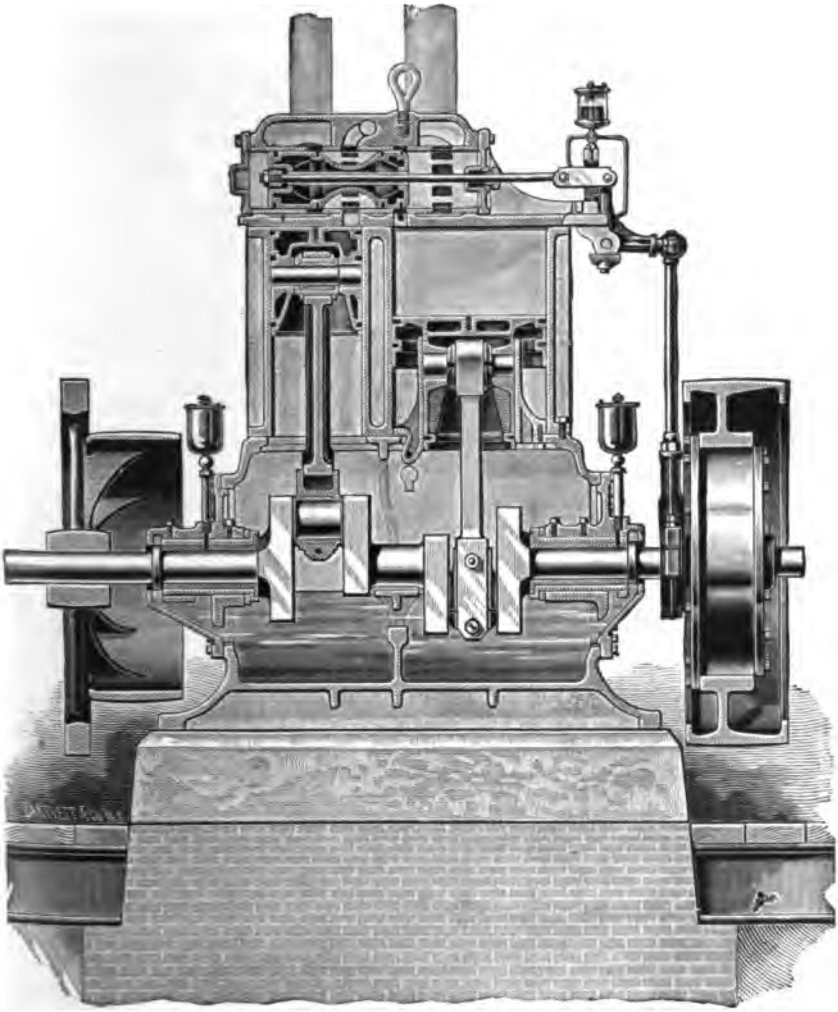


FIG. 69.—WESTINGHOUSE COMPOUND ENGINE. (Scale $\frac{1}{12}$.)

In such single-acting engines, it is usually intended that the rod shall never leave the crank-pin, in order that pounding may not occur. It is therefore evidently necessary that they

should be so proportioned and speeded that the action of the inertia of their reciprocating parts shall not produce stresses, on turning the centre, in excess of the sum of weights and steam-pressure.

An ingenious modification of the enclosed single-acting compound type of engine, the "central-valve engine" of Mr. Willans—which is also interesting as having been the subject of exceptionally complete scientific investigation—is seen in Fig. 70.* It was studied as a simple, a compound, and a triple-expansion engine; being easily adapted to either system.

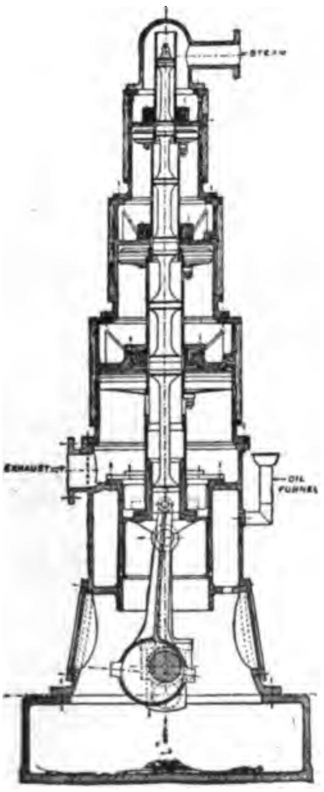


FIG. 70.—WILLANS' ENGINE. (Scale $\frac{1}{16}$.)

As here shown, its three cylinders are placed in series and "tandem." The valves are on one rod, driven by a single eccentric on the crank-pin; the rod being in the axis of the engine and the valves within the hollow piston-rod. Cut-off is effected by the passage of the ports into metallic rings in the ends of the cylinders, and is adjustable by hand or by the governor. Compression is effected in the separate cushion-chamber. †

These engines are usually grouped in pairs, with cranks at right-angles.

As the valve-faces move with the pistons, the valve-motion must here be taken from the pins to secure the desired movement relatively to the pistons.

* The discussion of this paper is remarkably interesting.—Trans. Brit. Inst. C. E.; March, 1888; 1887–1889; No. 2306; vol. xciii.

† Ibid., vol. lxxxi. p. 166.

The work on the main journals and pins is substantially all on the upper "brass" of the latter and the lower of the former,

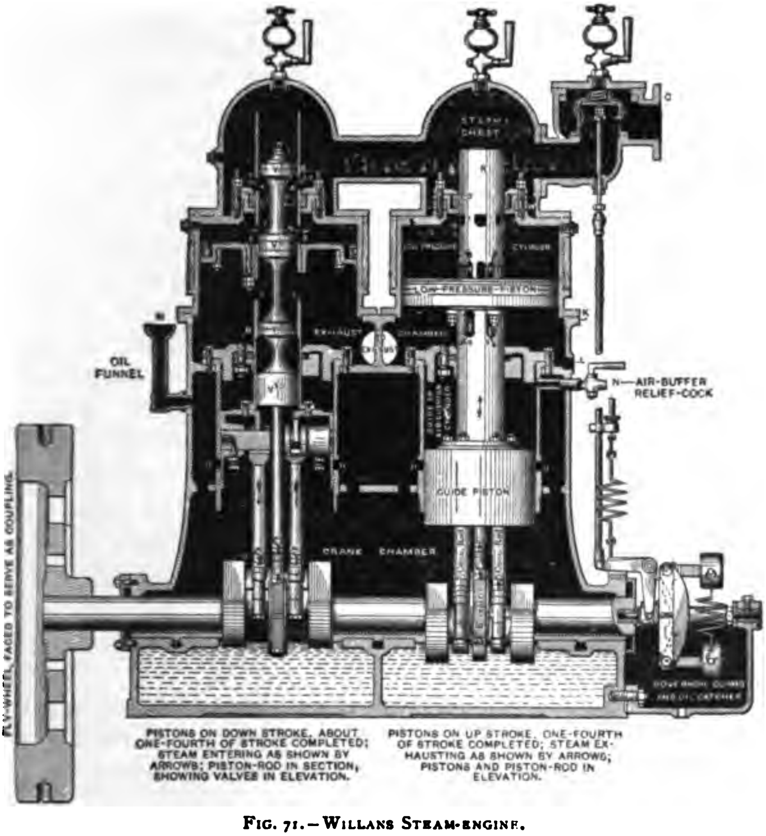


FIG. 71.—WILLANS STEAM-ENGINE.

and the crank-pin working-side is never expected to leave the pin. The eccentric-rod, like the connecting-rod, is always

in compression, and the main bearings also are always under constant downward thrust. Lubrication is secured, by the Westinghouse method, by the dipping of the crank into a pool of oil and water in the crank-case. The guide-pistons are arranged to produce the needed cushion by compressing the air in the compression-chambers and this is adjustable as may prove to be advisable. The governor is of the now familiar Hartnell type.

Another recent and peculiar example of this class of enclosed engines is the so-called ' Triumph " engine of Mr. Eickershoff, a " valveless " engine, in which the piston of one of its elements serves to distribute the steam to the others. It consists of three engines, side by side, each having the general construction shown in Fig. 71, coupled to cranks set at angles of 120°. Its simplicity is its striking feature, having neither valves, eccentrics, piston- or valve-rods, cross-heads nor stuffing-boxes. The distribution is remarkably good. Regulation is effected by a throttling-governor on the steam-chest.

With the exception of the cut-off, each piston controls the steam in the cylinder next preceding in the order of rotation, and when acting as a valve is at or near its maximum speed, while at the same moment the pistons in the preceding cylinder are at their slowest speed. This simple expedient controls the steam in this engine in a manner remarkable for its very great efficiency.

The indicator-diagram here given was taken from a $7 \times 14 \times 8$ inch engine, non-condensing. *XX* is the atmospheric line; *AB* is the admission-line in the high-pressure cylinder; *BC*, the steam-line; *C*, the point of cut-off; *CD*, expansion-line for high-pressure cylinder only; *D*, point of release to low-pressure cylinder; *DEF*, expansion-line, showing expansion in both high- and low-pressure cylinders, represented also by expansion-line *LM* of the low-pressure card; *F*, point of compression in the high-pressure cylinder connection with the low-pressure closing; *FA* is the compression-line.

In the low-pressure card *KL* is the admission-line; *LM*, the expansion-line, corresponding to line *DEF* of the high-pressure

card; *M* is the point of cut-off of the high-pressure cylinder, corresponding with compression-point *F* of the high-pressure card; *MN* is the expansion-line for the low-pressure cylinder only; *N* is the point of release to the exhaust; *NP* is the exhaust-line; *PQ*, line of back-pressure; *QK*, compression-line. It must be remembered that the piston in the cylinder from which the high-pressure card is taken is 120° in advance of the piston in the cylinder from which the low-pressure card is taken. The

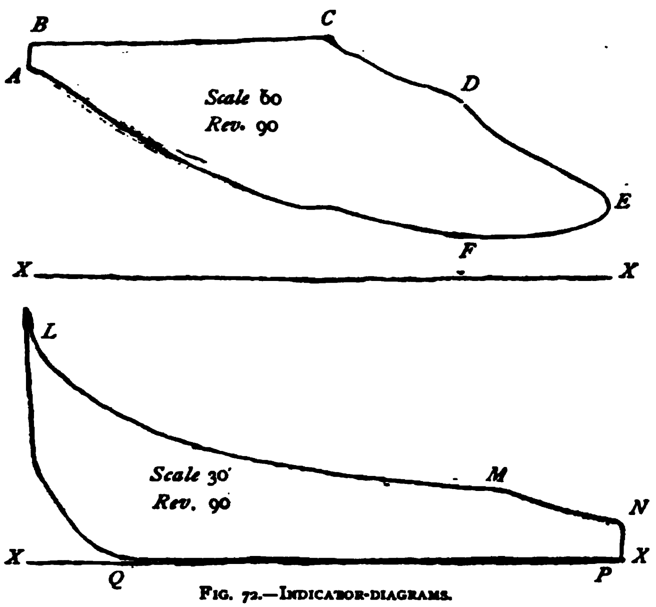


FIG. 72.—INDICATOR-DIAGRAMS.

ratio of the clearance to the volume of the high-pressure cylinder is such that the compression is always brought to initial pressure, irrespective of change in load. By this means the cylinder-walls are brought to the temperature of the entering steam and condensation prevented, and shock in passing the centres is avoided.

The plan of enclosing the "running parts" of the engine to insure freedom from dust, flooded journals, and exemption from expense in finishing small parts, is illustrated, in a

special case, as here shown, an upright single-valve automatic engine designed by Sturtevant. In this case, a pair of engines

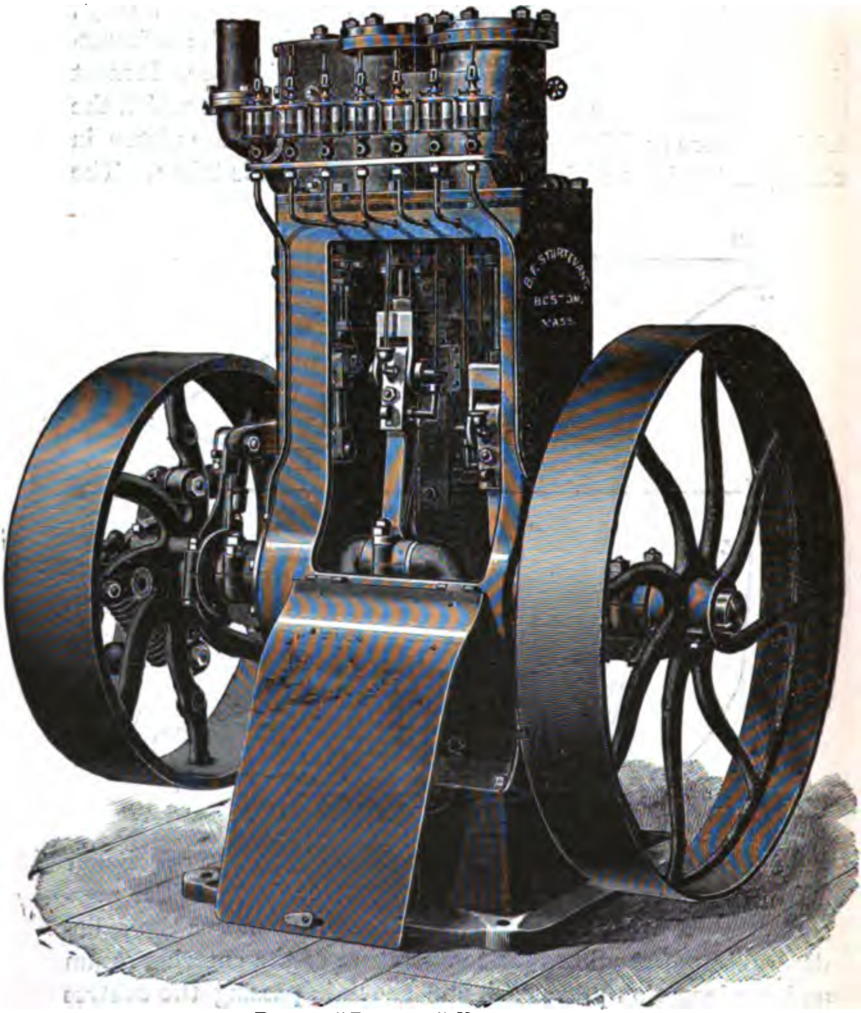


FIG. 73.—"ENCLOSED" UPRIGHT ENGINE.

are set with cranks opposite to secure a balance, and a single valve answers for both. An excellent and often practised arrangement of oil-cups is here shown; all being of the

sight-feed" class, all set in view and together, and where readily accessible. This general plan is adopted for engines of 10 to 35 horse-power.

37. **Pumping-engines** are built, as a rule, compound, and will be considered as such in the chapter relating to that class of constructions. Their principal types may, however, be properly described here.

A simple form of pumping-engine without fly-wheel is the now common "direct-acting steam-pump." This engine is generally made use of as a forcing- and fire-pump, and wherever

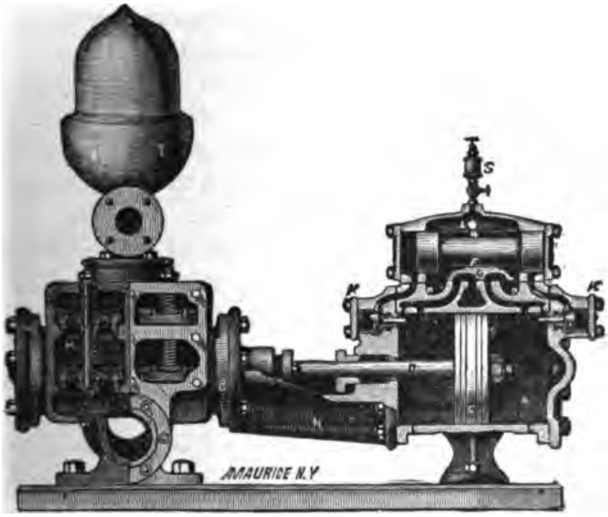


FIG. 74.—STEAM-PUMP. (Scale $\frac{1}{16}$.)

the amount of water to be moved is not large, and where the pressure is comparatively great. The steam-cylinder, *AR*, and feed-pump, Fig. 74, are in line, and the two pistons have usually one rod in common. The two cylinders are connected by a strong frame, and two standards fitted with lugs carry the whole, and serve as a means of bolting the pump to the floor or to its foundation.

The method of working the steam-valve of the modern steam-pump is very ingenious and peculiar. As shown, the pistons are moving toward the left; when they reach the end

of their stroke, the face of the piston strikes a pin or other contrivance, and thus moves a small auxiliary valve, *I*, which opens a port, *E*, and causes steam to be admitted behind a piston, or permit steam to be exhausted, as in the figure, from before the auxiliary piston, *F*, and the pressure within the main steam-chest then forces that piston over, moving the main steam-valve, *G*, to which it is attached, admitting steam to the left-hand side of the main piston, and exhausting on the right-hand side, *A*. Thus the motion of the engine operates its own valves in such a manner that it is never liable to stop working at the end of the stroke, notwithstanding the absence of the crank and fly-wheel, or of independent mechanism, like the cataract of the Cornish engine. There is a very considerable variety of pumps of this class, all differing in detail, but all presenting the distinguishing feature of auxiliary valve and piston, and a connection by which it and the main engine each works the valve of the other combination.

In some cases these pumps are made of considerable size, and are applied to the elevation of water in situations to which the Cornish engine, described in the preceding chapter, was formerly considered exclusively applicable. Fig. 75 illustrates such a pumping-engine, as built for supplying cities with water. This is a Worthington "compound" direct-acting pumping-engine. The cylinders, *A B*, are placed in line, working one pump, *F*, and operating their own air-pumps, *D D*, by a bell-crank lever, connected to the pump-buckets by links. Steam exhausted from the small cylinder, *A*, is further expanded in the large cylinder, *B*, and thence goes to the condenser, *C*. The valves are moved by valve-gear which is actuated by the piston-rod of a similar pair of cylinders placed by the side of the first. These valves are constructed substantially on the plan of the Corliss and are thus very fairly balanced, are easily and promptly moved, and give little clearance. By connecting the valves of each engine with the piston-rod of the other, it is seen that the two engines must work alternately, the one making a stroke while the other is still, and then itself stopping a moment while the latter makes its stroke.

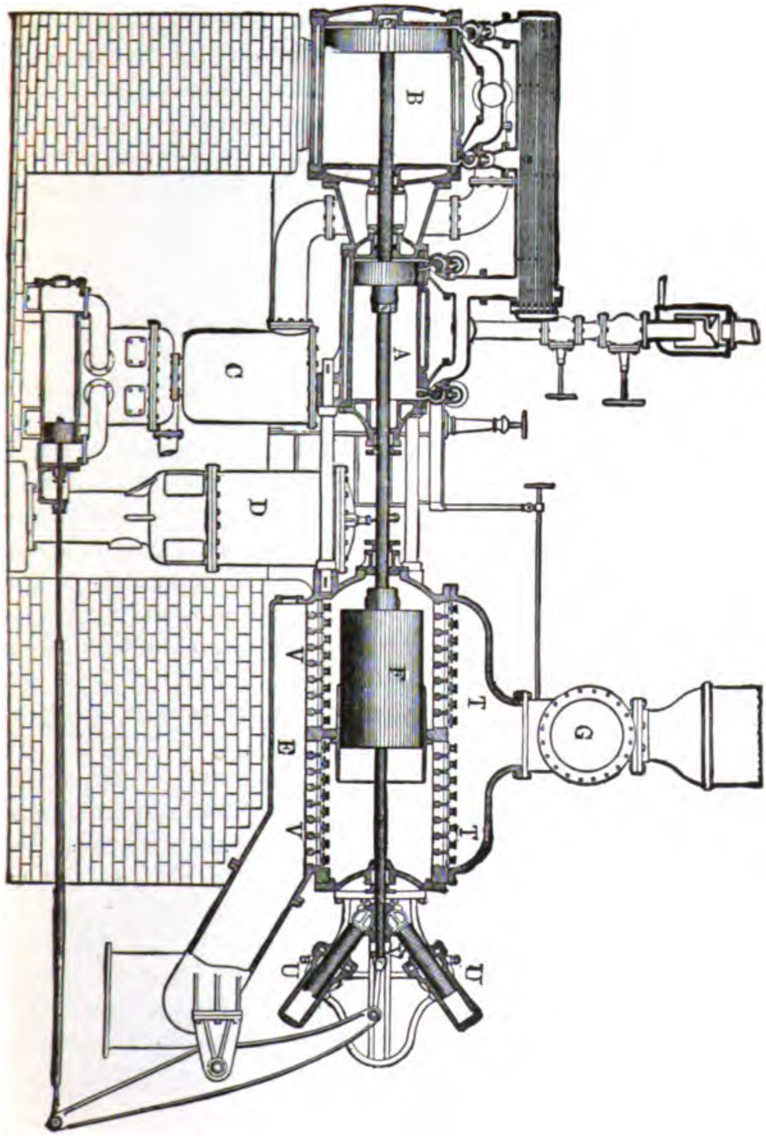


FIG. 75.—THE WORTHINGTON PUMPING-ENGINE (SECTION).

Water enters the pump through the induction-pipe, E, passes into the pump-barrel through the valves, V V, and issues through the eduction-valves, T T, and goes on to the "mains" by the pipe, G, above which is seen an air-chamber, which assists to preserve a uniform pressure on that side the pump.

The "high-duty attachment," U U, of the later engines of this type performs an exceedingly important office in a very ingenious yet simple manner. It consists of a pair of plungers working in oscillating barrels, U U, attached to a cross-head on each piston-rod common to engine and pump. Water-pressure is introduced behind these plungers and retained as nearly uniform as practicable as the engine makes its stroke. It is at once seen that this pressure resists the motion of the engine from the beginning to the middle of its stroke. At mid-stroke, the centre-lines of the plungers are perpendicular to the line of the rod; they counterbalance each other, and the action of the pair is neutral as respects the engine. Beyond half-stroke this pressure aids the steam, and the more as the end of stroke is approached. The irregular action of the expanding steam is thus met by a correspondingly variable opposite action of "equalizers," and it is easy, with high ratios of expansion, even, to thus secure a very uniform pressure in excess of the resistance of the water-column, by careful proportioning of parts and of pressures.

By this simple and ingenious device, due to Mr. C. C. Worthington, it is possible to increase the ratio of expansion in the direct-acting engine very greatly with corresponding gain in duty; the engine thus entering the class known as "high-duty engines." This attachment thus does the duty of a fly-wheel, often, of enormous weight, and thus increases effectively the efficiency of the engine as a machine. It works properly, with the same variations of pressures, at all speeds, and is also, at times, a safety-attachment, stopping the engine in case of a breakage in the mains.

In the "equalizer" system, let

A = total area of section of plungers;

p = pressure admitted upon them;

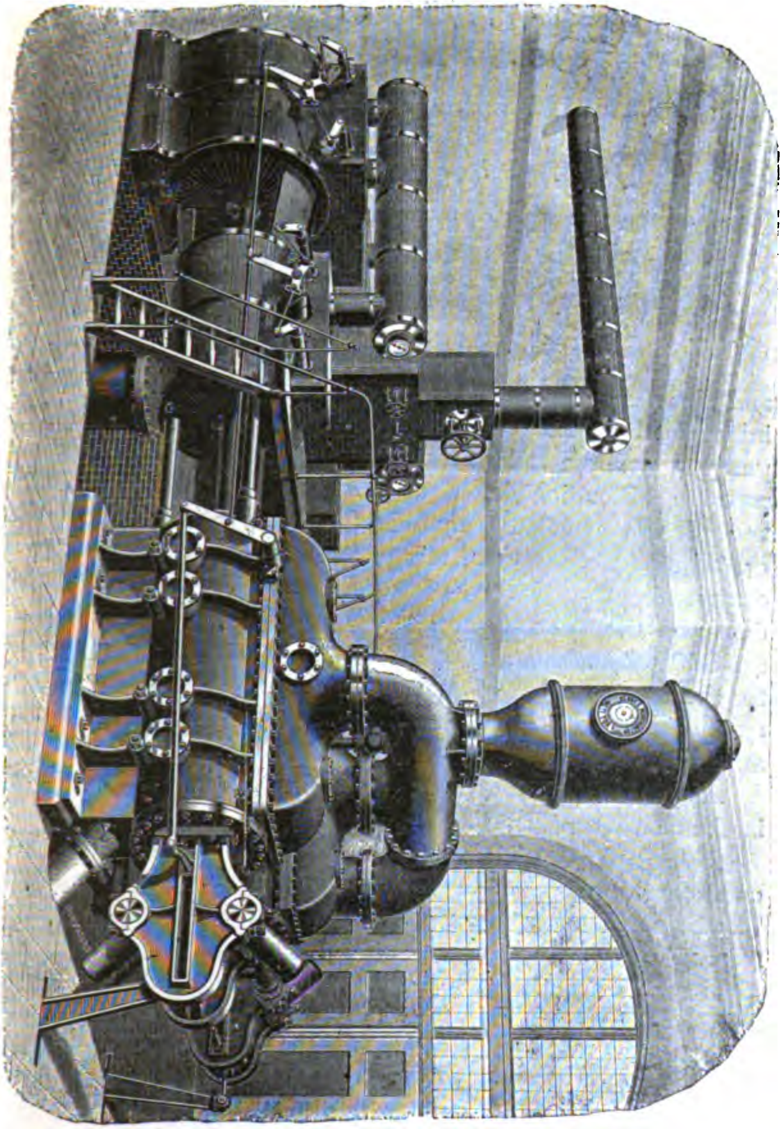


FIG. 76.—THE WORTHINGTON HIGH-DUTY PUMPING-ENGINE.

L = their full, joint, load;
T = thrust in line with piston-rod;
θ = angle of axis of equalizer with vertical.

Then the total load and the stress on the two equalizer-rods is

$$L = Ap = T \operatorname{cosec} \theta;$$

$$T = Ap \sin \theta = L \sin \theta;$$

At mid-stroke $\theta_0 = 0$ and $T = 0 = L$.
At the extreme positions, θ_1, θ_2,

$$T = Ap \sin \theta_1, = Ap \sin \theta_2;$$

and these values should be made to approximately equal the initial load on the engine-piston, less the resistance in the pump at starting, and to the latter quantity, less the terminal pressure of engine-piston, at the end of stroke. The altitude of the equalizer-trunnions above the centre-line of the engine, and the length of stroke thus fixed, are the elements determining the quantity of work done in the equalizer-cylinders and the completeness of equalization. The stroke, s, should have such extent that the work per stroke may be equal to the alternate excess and deficiency of the work of the engine, in the earlier and the later half-stroke, respectively, above and below that demanded, in the same time, at the pump.

A small sketch illustrating this equalization will be found in the chapter on Engine-trials.

Beam pumping-engines are now almost invariably built with crank and fly-wheel, and very frequently are compound engines. The illustration on page 169 represents an engine of the latter form.

A and *B* are the two steam-cylinders, connected by links and parallel motion, *CD*, to the great cast-iron beam, *EF*. At the opposite end of the beam, the connecting-rod, *G*, turns a crank, *H*, and fly-wheel, *LM*, which regulates the motion of the engine and controls the length of stroke, averting all danger of accident occurring in consequence of the piston striking either cylinder-head. The beam is carried on handsomely

shaped iron columns, which, with cylinders, pump, and fly-wheel, are supported by a substantial stone foundation. The pump-rod, *I*, works a double-acting pump, *J*, and the resistance to the issuing water is rendered uniform by an air-chamber, *K*, within which the water rises and falls when pressures tend to vary greatly. A revolving shaft, *N*, driven from the fly-wheel shaft, carries cams, *O P*, which move the lifting-rods seen directly over them and the valves which they actuate. Between the steam-cylinders and the columns which carry the beams is a well, in which are placed the condenser and air-

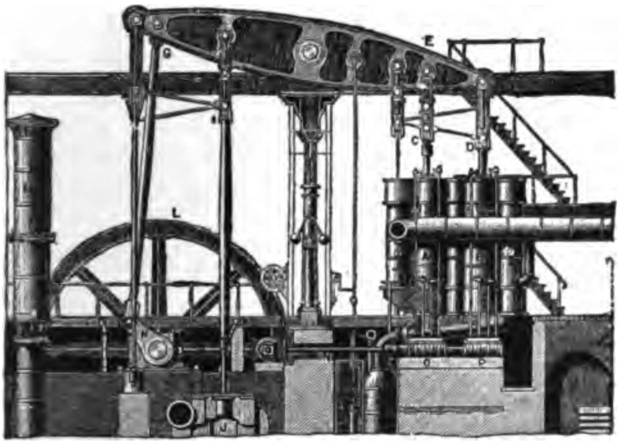


FIG. 77.—DOUBLE-CYLINDER PUMPING-ENGINE.

pump. Steam is carried at 60 or 80 pounds pressure, and expanded from 6 to 10 times.

A later form of double-cylinder beam pumping-engine is that invented and designed by E. D. Leavitt, and shown in Figs. 78 and 79. The two cylinders are placed one on each side the centre of the beam, and are so inclined that they may be coupled to opposite ends of it, while their lower ends are placed close together. At their upper ends a valve is placed at each end of the connecting steam-pipe. At their lower ends a single valve serves as exhaust-valve to the high-pressure and as steam-valve to the low-pressure cylinder. The pistons move

in opposite directions, and steam is exhausted from the high-pressure cylinder directly into the nearer end of the low-pressure cylinder. The pump, of the "Thames-Ditton" or "bucket-and-plunger" variety, takes a full supply of water on

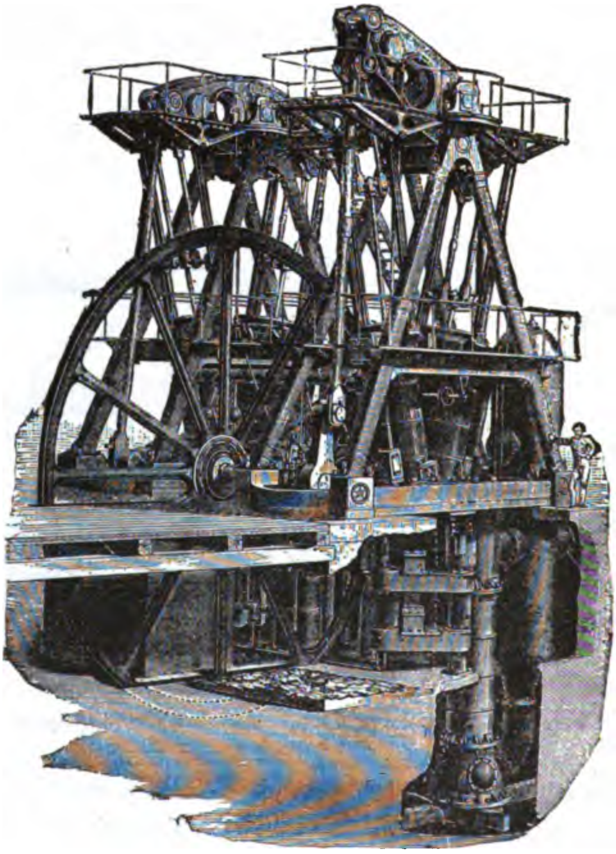


FIG. 78.—THE LEAVITT WATER-WORKS ENGINE.

the down-stroke, and discharges half when rising and half when descending again. The duty of this engine is reported as exceeding 110,000,000 foot-pounds for every 100 pounds of coal burned. The duty of a moderately good engine is usually considered to be from 90 to 95 millions; while 120,000,000 is a high figure.

The Wolff and Receiver types are the two most familiar forms of pumping-engine. The Wolff engine is so designed that the motions of the two pistons are coincident in time, as when both are attached to the same end of a working-beam, Fig. 77. It is often found advantageous to add a second, high-pressure, cylinder to a low-pressure engine, thus converting it into a compound engine. This is usually done by placing the new cylinder beside the old and connecting it to the beam through the old air-pump links. This compounding system is

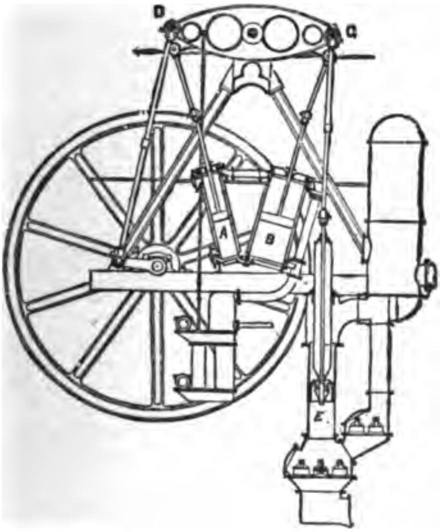


FIG. 79.—THE LEAVITT PUMPING-ENGINE

commonly known as "McNaughting," from the first engineer to practise it. In such cases, the steam-passages lead from either end of the one cylinder to the opposite end of the other, and no intermediate receiver is needed. Where, as in the Gaskill engine of the Holly Mfg. Co., Figs. 80, 81, and 82, the proportions of the cylinder are such, the diameter being great in proportion to the stroke, that it is possible to introduce a beam in the manner shown, and to secure alternation of movement, the intervening steam-passages become of minimum length, and "dead-space" is made comparatively small, with

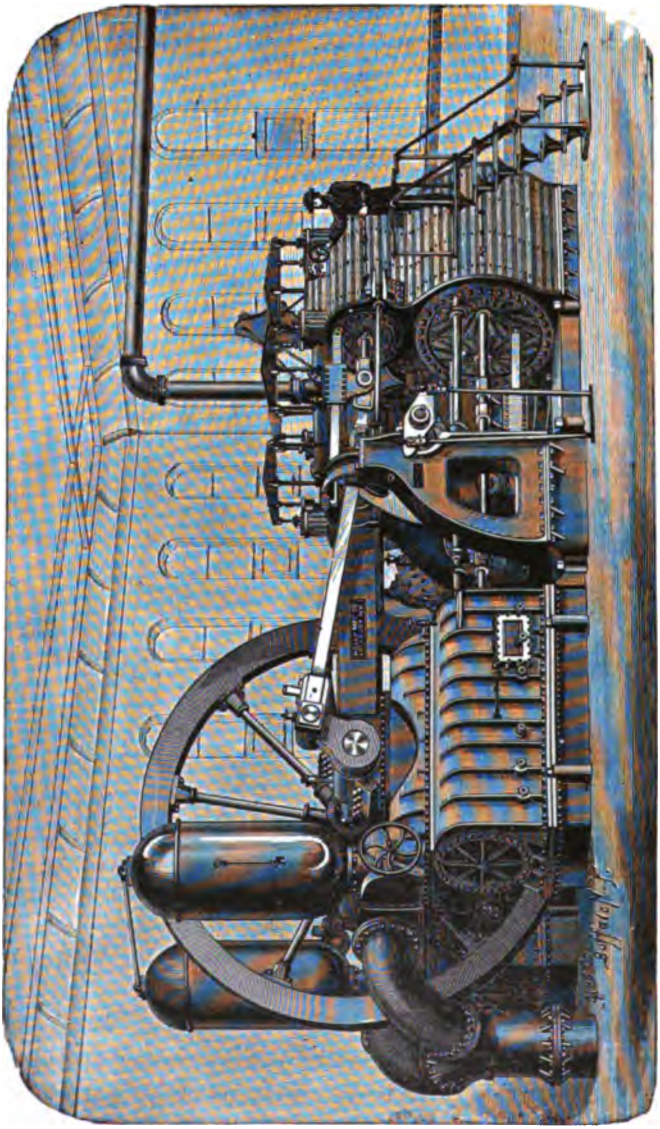


FIG. 80.—GASKILL ENGINE.

very advantageous economical results; while the engine becomes very compact.

When, as is sometimes the fact, the two cylinders are

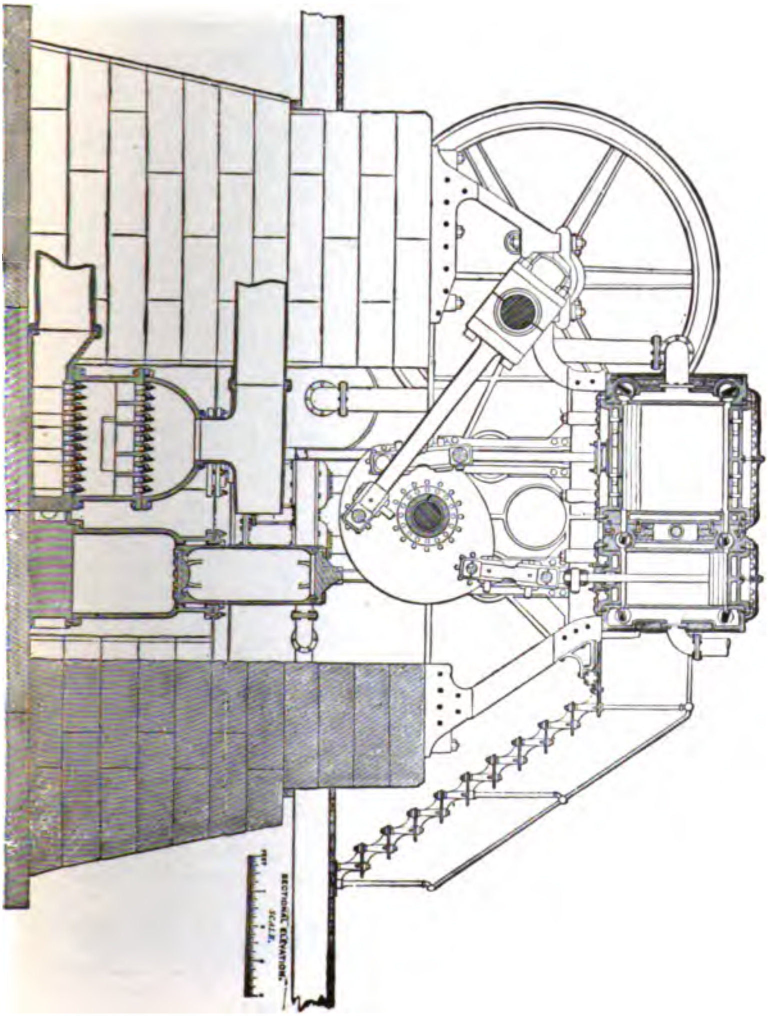


FIG. 81.—GASKILL VERTICAL COMPOUND ENGINE.

placed at opposite ends of the beam, the latter being of common proportions and the engine of long stroke, the centre-lines of the cylinders are separated by a distance equal to from

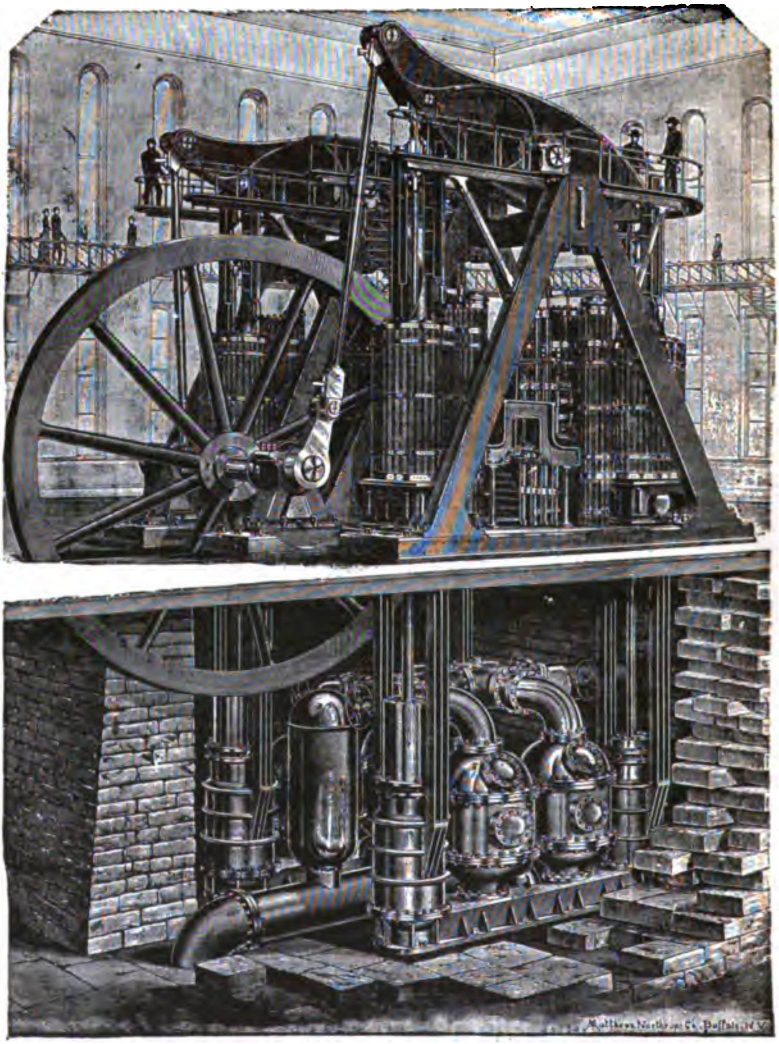


FIG. 82.—COMPOUND PUMPING-ENGINE.

two to three times the length of stroke, and the steam-passages and dead-spaces become seriously large. This objection has been met by Dr. Leavitt by inclining the cylinders, as in Figs. 78 and 79, and throwing their lower ends in under the main beam-centre, thus considerably shortening the connecting pipes.

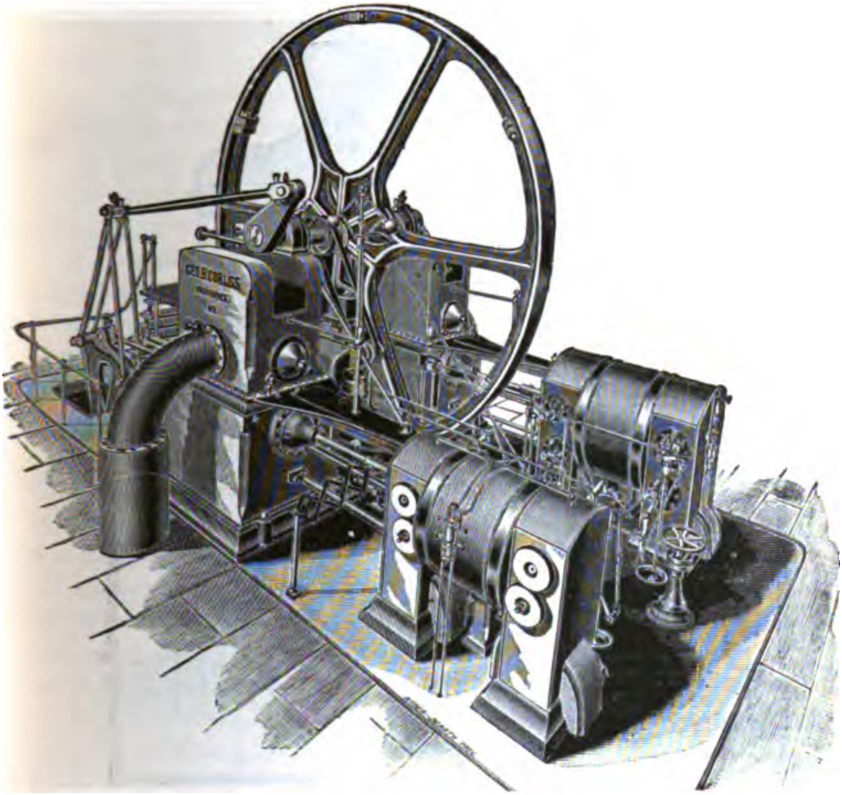


FIG. 83.—CORLISS PUMPING-ENGINE.

A Corliss Pumping-engine, designed by that great engineer for the water-works of Pawtucket, R. I., has been reported as doing, continuously, a "duty" of over 120,000,000. This engine, Fig. 83, consists of a pair of horizontal steam-cylinders, side by side, driving a pair of double-acting pumps, each in line with one of the engine-cylinders and the two having a common

piston-rod. A bell-crank lever and suitable links connect the engines with the single balance-wheel, placed between and above them. The smaller cylinder takes steam of about ten

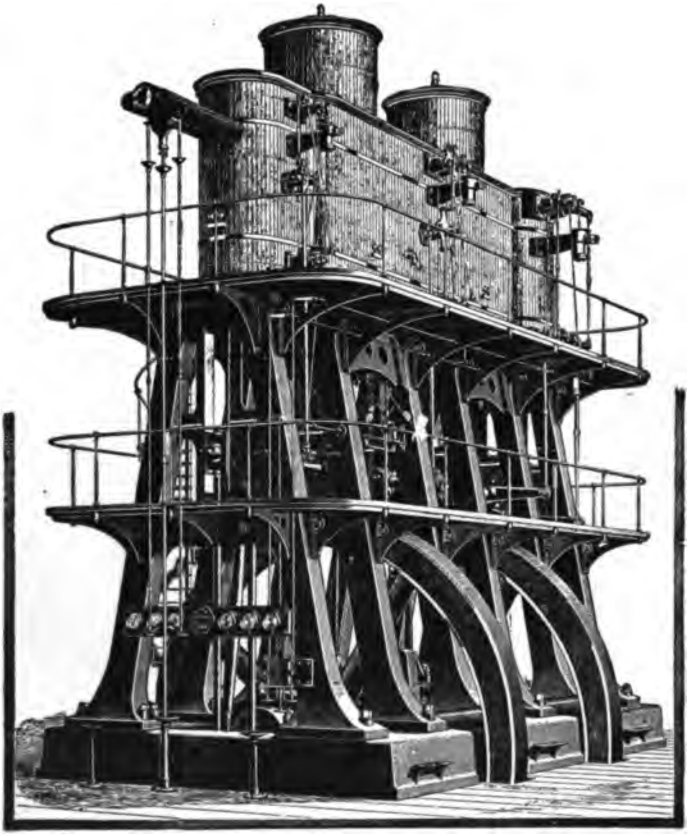


FIG. 84.—VERTICAL TRIPLE-EXPANSION ENGINES.

atmospheres absolute pressure (127 pounds by gauge), and its exhaust passes across to the larger cylinder, whence it is passed into the condenser, below the engine. The ratio of expansion is from 15 to 20, and the speed of engine 50 revolutions per minute.

The pumps are fitted with a large number of small and light valves and thus are subject to very little waste by leakage or back-flow of water while they are seating, and demand very little power in their operation.

As in all engines of this kind, a receiver is attached between the engine-cylinders, and in this case the steam is superheated both before entering the engine and while passing from the one cylinder to the other. The valve-gear is of the usual Corliss type, the expansion variable on both cylinders.

Fig. 84 illustrates a type of vertical triple-cylinder compound engines for water-works, such as have been designed by Mr. Reynolds and the Allis Co. for a number of large cities. Their capacity averages about 20,000,000 gallons per day.

The cylinders are attached to heavy A-frames which are secured to the bed-plates. In the A-frames, the guides are formed for the cross-heads. The plungers move with the cranks, which are set 120 degrees apart to insure a constant and steady flow of water in the delivery-mains. The pumps have outside-packed plungers of the single-acting type, one plunger being located under and operated by each piston. Each plunger is connected to its steam-piston by four rods attached to the cross-heads. The condenser and pumps are placed in a pit below the engine-room floor. The pump-valves are mounted in cages, and so arranged that any series of valves can be easily removed or replaced. This engine has attained a "duty" of 140,000,000; demanding but 11.678 lbs. steam per h. p. per hour (Trans. A. S. M. E., Dec. 1893; R. H. T.). (See Notes.)

The engraving following illustrates a pair of vertical triple-cylinder pumping-engines which were designed by Mr. Reynolds for the city of Allegheny, Pa., to pump six million gallons of water, each twenty-four hours, against a head of 220 feet, and develop a duty of ninety-five million foot-pounds for each one thousand pounds of water fed to the boilers. The duty obtained by a twenty-four-hour run was over 107,000,000 foot-pounds.

This type is a favorite with many builders, as it brings all parts within a small floor-plan, yet gives accessibility of parts,

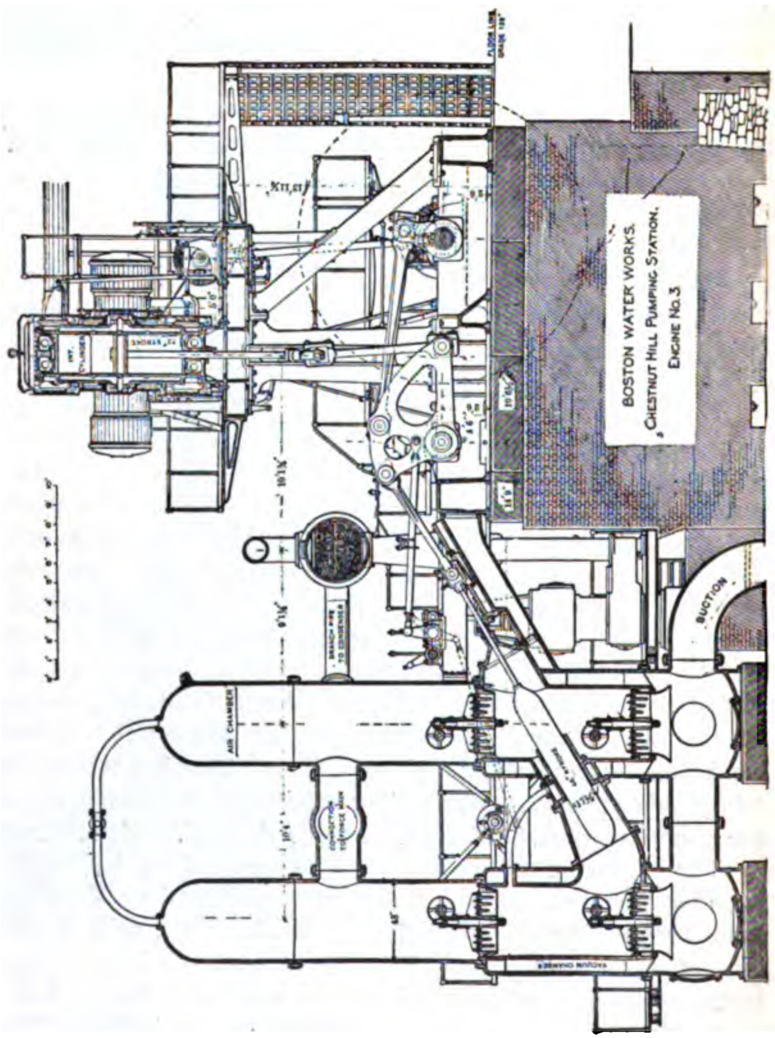


FIG. 85.—LEAVITT ENGINES WITH RIEDLER PUMPS.

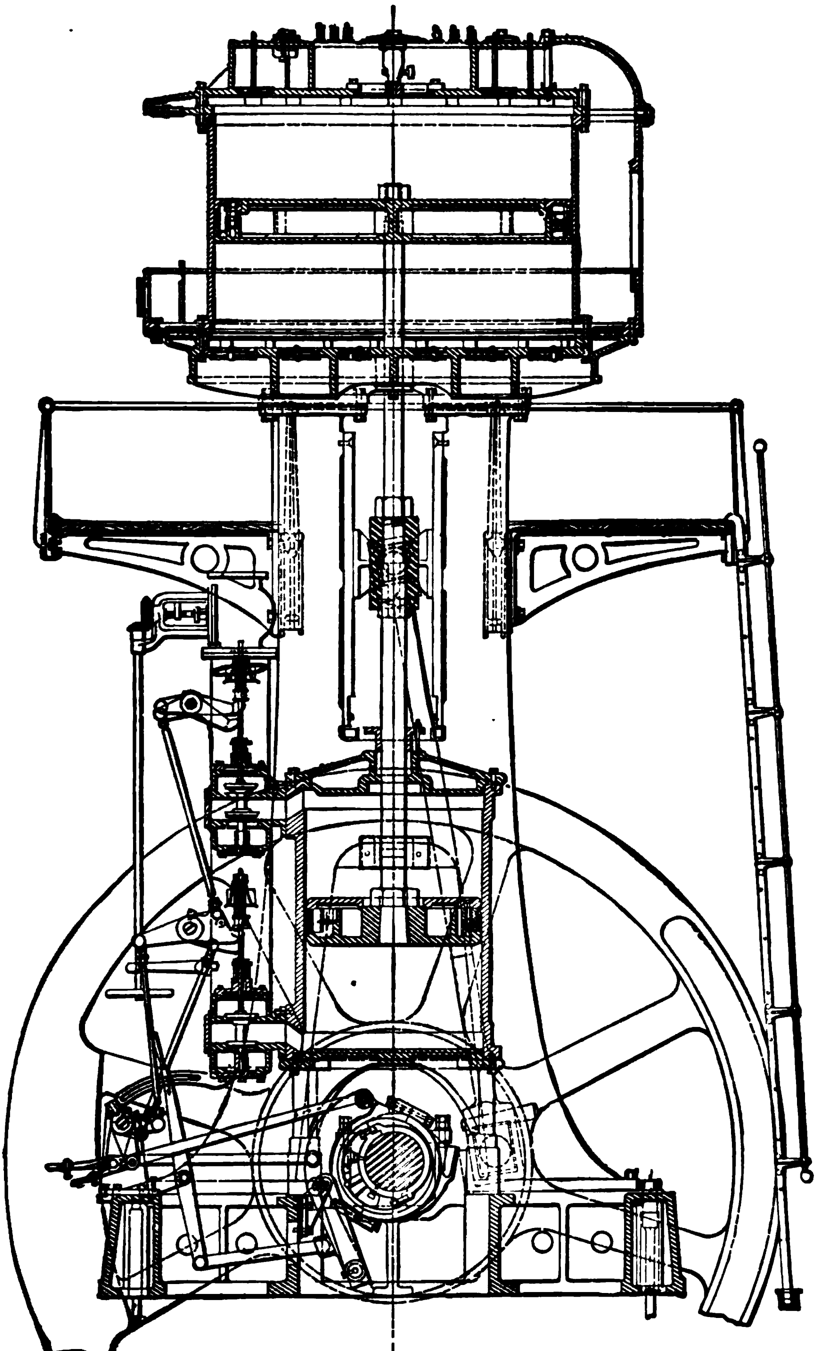


Fig. 86.—Blowing-engine (Section).

a moderate size and cost of foundation for a given capacity, and direct connection of the cylinders in series and the pumps. The section at the left is so made as to give a good idea of the arrangement of steam-passages and of water-connections. This designer built a quadruple-expansion engine in 1887.

A design of blowing-engine, of air-pumping, for large blast-furnaces, illustrating well the compactness, stiffness, and neatness attainable in such designs, and also the form of valve-motion adopted with the poppet-valve, is shown in the outline engraving on page 179.* This design is by Messrs. Gordon, Strobel & Laureau. The steam-cylinders are 42 and the blast-cylinder 84 inches in diameter, their common stroke 4 feet. The box-form of frame permits great stiffness and admits the placing of the two cylinders in line, the main (15-inch) shaft beneath, and a convenient general arrangement of valve-gearing. In the latter, as seen, a rock-shaft, actuated directly by the eccentric-rod, produces the vibration of the "wipers" raising the "toes," which, in turn, raise and depress the valves. A trip-arrangement permits a variable cut-off from one to three-fourths stroke, and a constant lead is maintained. The lift of the steam-valve varies from $\frac{5}{8}$ inch to $2\frac{1}{8}$ inches, as the expansion decreases. The action of the exhaust is unaffected by that of the steam-valve. The air-valves are so large in total area that, at the working-speed, no observable loss of pressure occurs at their ports. The depth of piston is one fourth the diameter in the steam-cylinder and one eighth in the blast-cylinder. This engine makes about 35 revolutions per minute, with 60 pounds of steam and cut-off at $\frac{1}{4}$.

38. **Portable Engines** are such as may be conveniently moved from place to place. They are generally of small size, moderate power, compact construction, non-condensing, employing steam of high pressure in cylinders worked at high piston-speed, and produced in boilers of the tubular class and which commonly serve, also, as engine-frames. In some cases,

* Reproduced by permission from the Iron Age of Feb. 26, 1891.

they consist of engine and boiler mounted on a common bed. Often they are mounted on wheels, in which case they are usually known as "agricultural engines."

Road-locomotives, which are self-impelling portable engines, are much used in some parts of the world, and the steam "road-roller" is a road-locomotive which has heavy rollers in place of wheels, and which may be used in rolling the surface

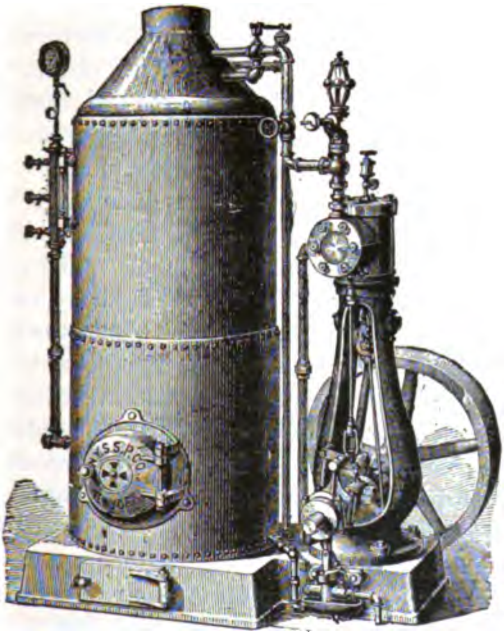


FIG. 87.—SEMI-PORTABLE ENGINE. (Scale $\frac{1}{16}$.)

of macadamized or other roads. Similarly, a "steam fire-engine" is a portable engine carrying a steam-pump which may be used in extinguishing fires.

The "semi-portable" engine in Fig. 87 is not fastened to the boiler, and is therefore not affected by expansion, nor are the bearings overheated by conduction or by ascending heat

from the boiler. The fly-wheel is at the base, which arrangement secures steadiness at the high speed which is a requisite for economy of fuel. The boilers are of the upright tubular style, with internal fire-box, and are intended to be worked at 150 pounds pressure (10 atmospheres) per inch. These boilers are fitted with a baffle-plate and circulating-pipe, to prevent priming, and also with a fusible plug, which will melt and prevent the crown-sheet of the boiler burning, if the water gets low.

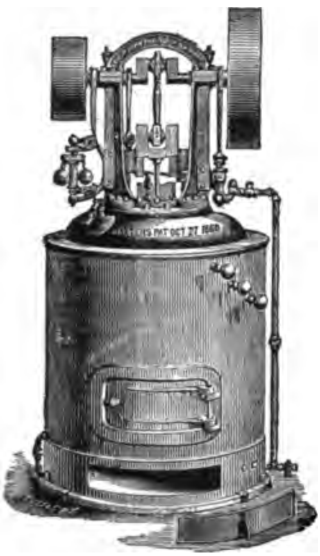


FIG. 88.—SEMI-PORTABLE ENGINE. (Scale $\frac{1}{16}$.)

Another illustration of this class of engine, as built in small sizes, is seen in Fig. 88. The peculiarity of this engine is that the cylinder is placed in the top of the boiler, which is upright. By this arrangement the engine is constantly drawing from the boiler the dryest steam, and there is thus no liability of serious loss by condensation, which is rapid, even in a short pipe, when the engine is separate from the boiler. The engine illustrated is rated at 10 horse-power.

Among the earliest of American engineers to turn attention to this department of construction were Messrs. Babcock & Wilcox. The style of engine which was designed and introduced by them has now become almost as generally accepted as standard among builders of small engines as has the Corliss engine among constructors of drop cut-off engines. It has been copied in all parts of Europe, as well as in the United States. It may be taken as representative of the best methods of construction of this class of machinery in this country, and as exhibiting the elegance in proportions, and that excellence of material and workmanship, which are now becoming recognized as desirable in steam-engines of even the smallest size.

Figs. 30, 31, 87 exhibit the form of the engine here to be described. It is a "vertical engine" mounted upon a base-plate of neat and strong form, and with the steam-cylinder bolted by the lower head to a very strong and very graceful frame. The main journals are carried in bearings constructed in the frame, and consequently free from liability to loss of perfect alignment, or to unequal wear. The valve is either a plain locomotive-slide or, preferably, a piston-valve. The latter is fitted in a detachable seat, which can be easily removed for renewal of seat and valve, should accident or wear ever make it necessary.

The vertical position of the engine prevents wear within the cylinder becoming serious or unsymmetrical. The pistons are hollow, and are packed with rings set with sufficient spring to keep them up to a bearing. The cross-head has its gibs turned to fit the guides in the frame, which latter are part of the casting of the frame and are bored out in line with the cylinder, and cannot possibly get out of line.

The engine above referred to is of small size—4 or 5 horse-power—and has been especially designed for electric-lighting purposes. The governor regulates by adjusting the supply of steam passing to the engine through a throttle-valve—a method which seems to have been here more successful than is usual in engines having to perform so exacting a kind of work. The speed of this engine is usually 250 to 300 revolutions per minute.

Larger engines of this style are often constructed, ranging up to 100 horse-power. These engines, when of 15 to 100 horse-power, are properly classed as stationary engines; they are given an independent crank-shaft pillow-block and a counterbalanced disk-crank. In these engines, of all sizes, the modern innovation of the use of steel for running parts is very generally introduced. The rods, pins, and minor parts are of this metal; the bearings are usually of bronze lined with Babbitt-metal, and are given large area. Crank-shafts are either of steel or of hammered iron.

The later work of the best English builders has given

remarkable economical results. Some of these portable engines have exhibited, at competitive trials, an economical efficiency equal to that of the largest marine engines. The causes of this remarkable economy are readily learned by an inspection of the engines, and by observation of the method of managing them at test-trials. The engines are very carefully designed. The pistons travel at high speed. Their valve-gear consists usually of a plain slide-valve, supplemented by a separate expansion-slide, driven by an independent eccentric, and capable of considerable variation in the point of cut-off. This form of expansion-gear is very effective at the usual ratio of expansion, which is not far from four or five. The governor is usually attached to a throttle-valve in the steam-pipe, an arrangement which is not the best possible under variable loads, but which produces no serious loss of efficiency when the engine is driven, as at competitive trials, under the very uniform load of a brake and at very nearly maximum capacity. The most successful engines have steam-jacketed cylinders with high steam and considerable expansion. The boilers are, as are also all other heated surfaces, carefully clothed with non-conducting material, and well lagged over all. The details are carefully proportioned, the rods and frames are strong and well secured together, and the bearings have large rubbing-surfaces. The connecting-rods are long and easy-working, and every part is capable of doing its work without straining and with the least friction.

In handling the engines at the competitive trial, experienced and skilful drivers are selected. The difference between the performances of the same engine in different hands has been found to amount to from 10 to 15 per cent, even where the competitors were both considered exceptionally skilful men. In manipulating the engine, the fires are attended to with the utmost care; coal is thrown upon them at regular and frequent intervals, and a uniform depth of fuel and a perfectly clean fire are secured. The sides and corners of the fire are looked after, especially. The fire-doors are kept open the least possible time; not a square inch of grate-surface is left

unutilized, and every pound of coal gives out its maximum of calorific power, and in precisely the place where it is needed. Feed-water is supplied as nearly as possible continuously, and with the utmost regularity. In some cases the engine-driver stands by his engine constantly, feeding the fire with coal in handfuls, and supplying the water to the heater by hand by means of a cup. Heaters are invariably used in such cases. The exhaust is contracted no more than is absolutely necessary for draught. The brake is watched carefully, lest irregularity of lubrication should cause oscillation of speed with the changing resistance. The load is made the maximum which the engine is designed to drive with economy. Thus all conditions are made as favorable as possible to economy, and they are preserved as invariable as the utmost care on the part of the attendant can make them.

These trials are usually of only three or five hours' duration, and terminate before it becomes necessary to clean fires.

Agricultural Engines.—The next illustration represents the portable, "agricultural," steam-engine as built by one of the earliest and best manufacturers of such engines in the United States. In the boilers of these engines the heating-surface is given less extent than in the stationary engine-boiler, but much greater than in the locomotive, and varies from 10 to 20 square feet per horse-power. The boilers are made very strong, to enable them to withstand the strains due to the attached engine, which are estimated as equivalent to from one tenth to one eighth that due to the steam-pressure. The engine is mounted, in this example, directly over the boiler, and all parts are in sight and readily accessible to the engineer.

Compound Portable Engines have been found to exhibit great economy as compared with the simple engine, notwithstanding the fact that the advantages of compounding are generally supposed to be less on small than on large engines. The plan adopted is usually that of placing the two engines side by side, connecting them to cranks, on a common crank-shaft, set at right-angles, and providing a receiver of moderate size to take the exhaust of the smaller and to supply steam to the

larger cylinder. In some instances, the Wolff system of two pistons having simultaneous opposite motions and without receiver is adopted, a plan admissible with small engines, but less suitable for large powers. The compounding of engines

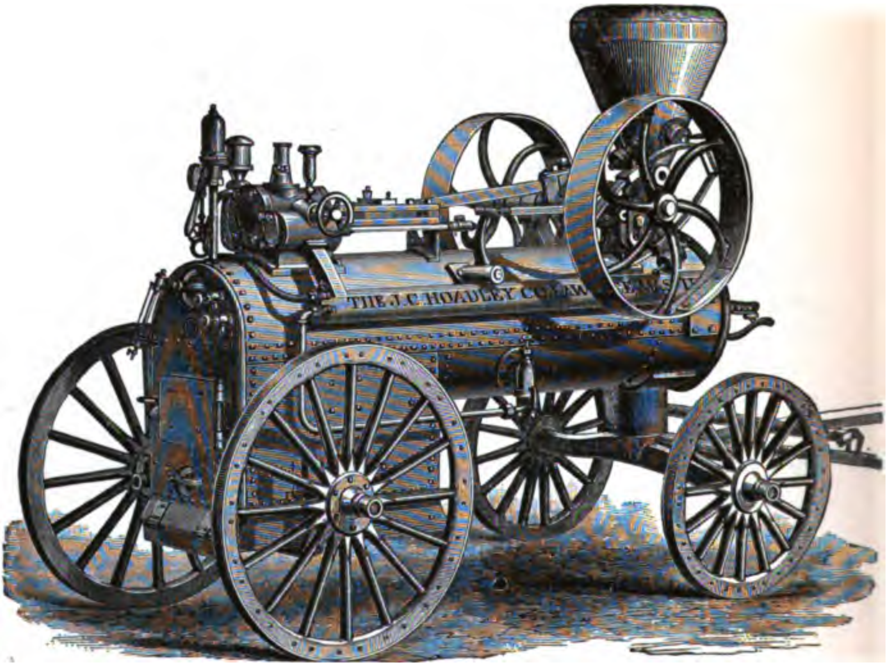


Fig. 89.—The Agricultural Engine. (Scale $\frac{1}{16}$.)

of this class, which are usually of less than 25 horse-power, has been found to produce a saving of, often, twenty-five per cent of the fuel and steam.

Steam Fire-engines have become standard in general plan and arrangement of details. These are probably the best illustrations of extreme lightness, combined with strength of parts and working power, which have ever been produced in any branch of mechanical engineering. By using a small boiler crowded with heating-surface, very carefully proportioned and arranged, and with small water-spaces; by adopting steel for running-gear and working parts wherever possible; by working at high

[*To face page* 186.]

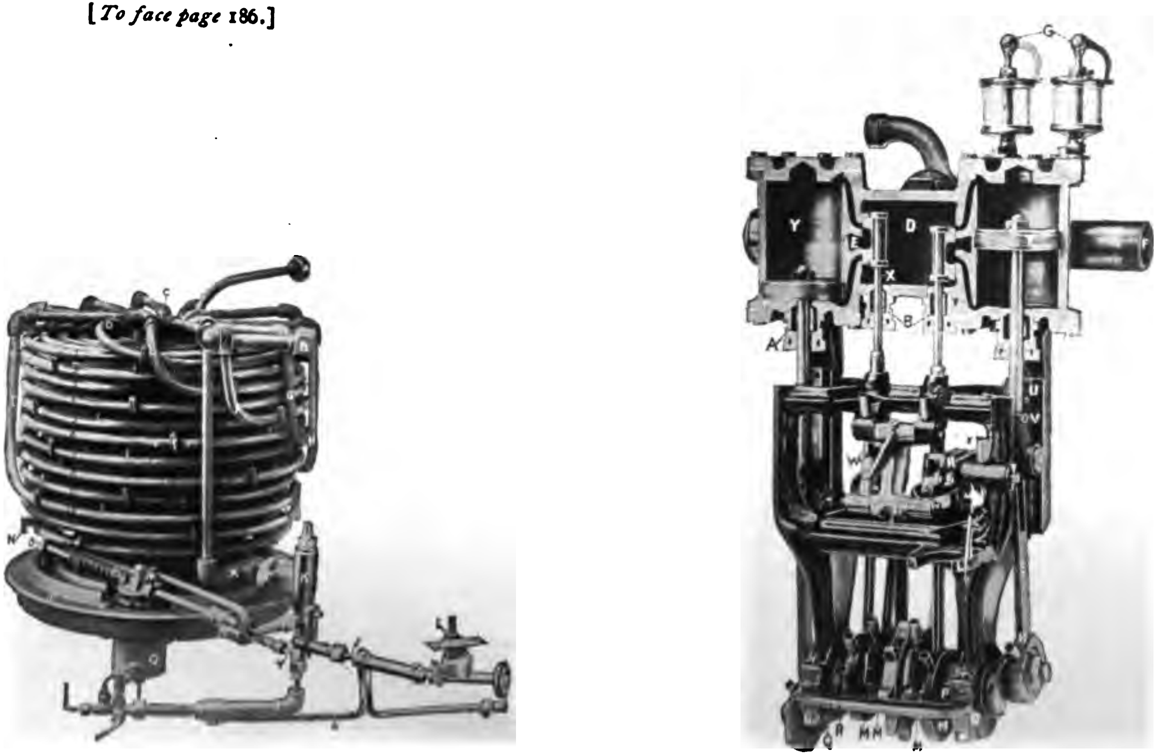


FIG. 89*a*.—AUTOMOBILE ENGINE AND BOILER. (R. H. WHITE.)

Boiler:—*A*, pump-connection; *B*, steam-pipe; *P*, burner; *K*, fire-regulator; *N*, thermostat for superheat; *D*, *E*, *S*, *G*, *H*, coils through which, successively, water and steam descend. *Engine:*—*F*, steam-pipe; *E*, exhaust; *D*, steam-chest; *Y*, cylinder; *O*, main bearing; *W*, valve-gear.

FIG. 89*b*.—STEAM AUTOMOBILE.—WHITE.

[*To face page* 187.]

piston-speed and with high steam-pressure; by selecting fuel with extreme care—by all these expedients, the steam fire-engine has been brought, in this country, to a state of efficiency far superior to anything seen elsewhere. Steam is raised with wonderful promptness, even from cold water, and water is thrown from the nozzle at the end of long lines of hose to great distances. But this combination of lightness with power is only attained at the expense of a certain regularity of action which can only be secured by greater water and steam capacity in the boiler.

The Steam "Automobile" is the modern form of the steam-carriage of 1825–35. It embodies the same principles of construction and illustrates the same general characteristics in design. The usual type includes a tubular boiler crowded with even smaller tubes than those of the steam fire-engine, light direct-acting engines and ingenious details of carriage construction. The "safety-type" of boiler has also occasionally found place, although the difficulty of handling a boiler with so little water-space has retarded its use. Serpollet made a "flash-boiler" successful, however, about 1890 or earlier. An American construction on this general plan is that of White, who employs a coil-boiler, with light petroleum or gasolene as fuel, with automatic regulation of high superheat and of pressure. Engine and boiler are shown in Fig. 89*a*.

39. **Road Locomotives and Rollers** are built, necessarily, with even greater care and of greater strength than the ordinary portable engine; since they are exposed to rougher usage and more serious strain.

In this, as in the class of engines last described, the draught is obtained by the blast of the exhaust-steam which is led into the chimney. The usual consumption of fuel is from 4 to 6 pounds per hour and per horse-power, burning from 15 to 20 pounds on each square foot of grate, and each pound evaporating about 8 pounds of water. A usual weight is, for the larger sizes, 500 pounds per horse-power.

Road-engines are arranged to propel themselves, as in the Mills road-engine or locomotive, of which the accompanying

engraving is a representation. This engine is proportioned for hauling a tank containing 10 barrels, or more, of water and a grain-separator over all ordinary roads, and to drive a thrashing-machine or saw-mill, developing 20 or 25 horse-power. This example of the road-engine has a boiler built to work at 250

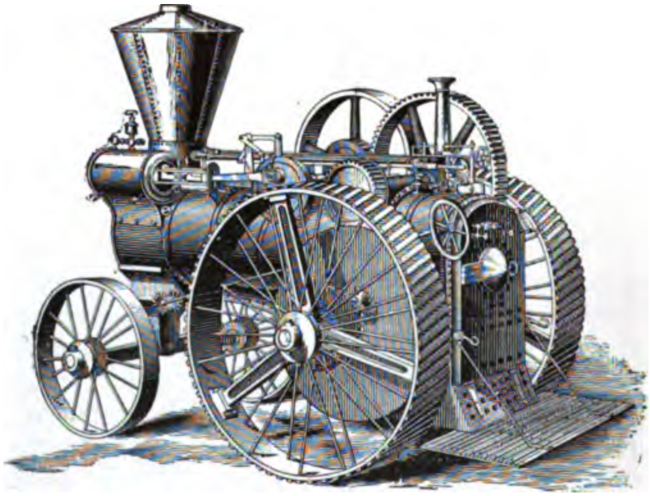


FIG. 90.—THRASHER'S ROAD-ENGINE. (Scale $\frac{1}{16}$.)

pounds of steam; the engine is designed for a maximum power of 30 horses. It has a balanced valve and automatic cut-off, and is fitted with a reversing-gear for use on the road. The driving-wheels are of wrought-iron, 56 inches diameter and 8 inches wide, with cast-iron driving-arms. Both wheels are drivers on curves as well as on straight lines. The engine is guided and fired by one man, and the total weight is so small that it will pass safely over any good country bridge. A brake is attached, to insure safety when going down-hill. Although designed to move at a speed of about three miles per hour, the velocity of the piston may be increased so that four miles per hour may be accomplished when necessary.

This is an excellent example of this kind of engine as constructed at the present time. The strongly-built boiler, with

its heater, the jacketed cylinder, and light, strong frame of the engine, the steel running-gear, the carefully-covered surfaces of cylinder and boiler, and excellent proportions of details, are illustrations of good modern engineering.

Fig. 91 is an engraving of a road-roller as built by one of the most successful among the firms engaged in this work.

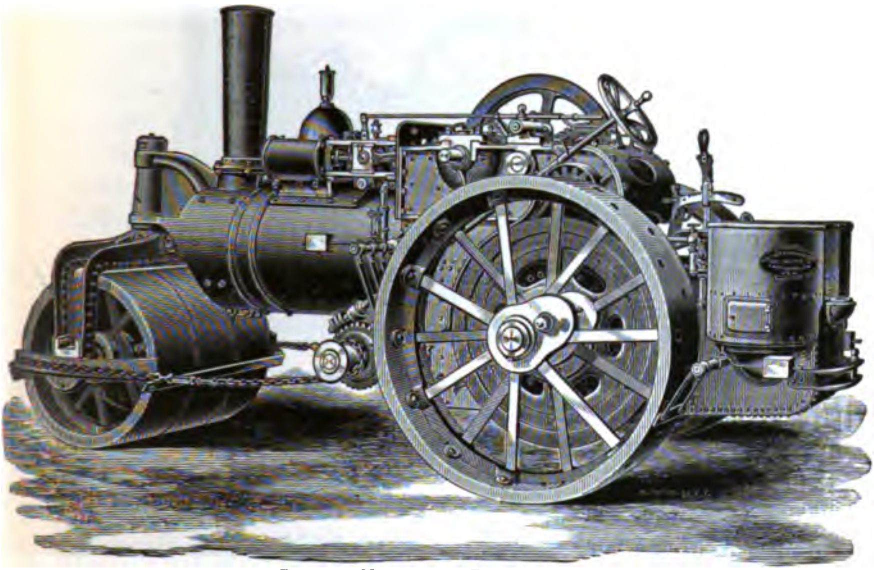


FIG. 91.—HARRISBURG ROAD-ROLLER.

The structure of such an engine, if of the better class, illustrates many specially interesting features of modern construction. They are often made with single engines; but, as in this case, a pair coupled at right-angles, as in the locomotive, is preferable; and it may often be advisable to compound them. There should be no danger of the machine getting stalled by reason of the engine "catching on the centre." These machines are made of from ten to fifteen tons weight; the valve-motion is usually the common locomotive gear; the best have steel running parts and steel boilers; a brake is fitted to the driving-wheels; and special noiseless safety-valves are used. The gearing should be of annealed cast-steel, and the driving-

wheels are best made of a mixture of peculiarly strong iron, as "car-wheel" iron with new No. 1 foundry-iron.

This class of road-locomotive was brought into use about

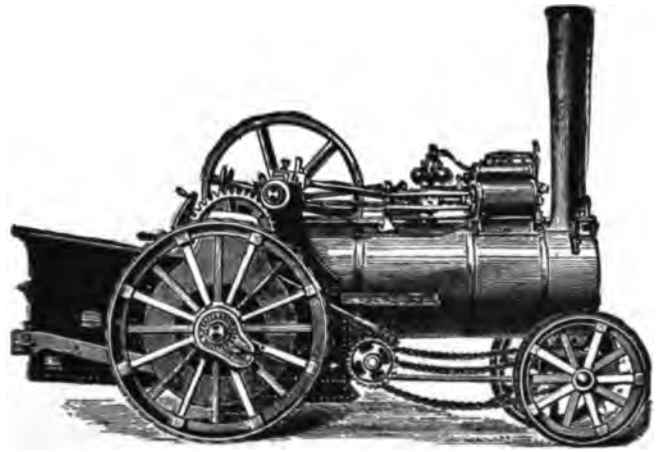


Fig. 92.—Road and Farm Locomotive.

1829 on French roads, and about 1865 in England and her colonies.

The Author has made a trial of one of these machines constructed by very successful British builders (see above figure), to determine its power, speed, and convenience of working and manœuvring. The following were the principal dimensions:

Weight of engine, complete, 5 tons 4 cwt.	11,648	pounds.
Steam-cylinder—diameter	$7\frac{3}{4}$	inches.
Stroke of piston	10	"
Revolutions of crank to one of driving-wheels	17	
Driving-wheels—diameter	60	inches.
" breadth of tire	10	"
" weight, each	450	pounds.
Boiler—length over all	8	feet.
" diameter of shell	30	inches.
" thickness of shell	$\frac{7}{16}$	inch.
" fire-box sheets, outside, thickness	$\frac{1}{2}$	"
Load on driving-wheels, 4 tons 10 cwt.	10,080	pounds.

The "automobile" is in this class. An automobile engine designed by Professor Thomson has four cylinders, each 2½ inches diameter and 3 inches stroke, single acting and worked single expansion; the output of the four being about 5 H.P. The economy obtained with this small engine reached 20½ lbs. of water per H.P.-hour, the steam pressure being between 160 and 200 lbs., with a considerable degree of superheating. The steam is admitted to the cylinder by a simple poppet-valve, and expands until the piston uncovers the exhaust ports. The conditions are such that there is no retraversing of any of the passages by the steam.*

As the marine engine illustrates the highest result of application of invention and engineering talent to production of economy of fuel, and the most elaborate and perfect type of engine, so the steam fire-engine exemplifies the result of the same application of genius to the production of a machine in which everything is subordinated to quickness and power in action. Thus, referring to Fig. 93, that of an engine designed by the Manchester Locomotive Works, we find that, in this class of engine, the demand for lightness, strength, compactness, quick action, and large and concentrated power is met, generally, by the use, as here seen, of the vertical tubular boiler, with the exhaust-blast of the locomotive, with tubes crowded in more thickly than would be desirable or safe with the horizontal form; large steam and water pipes, double-acting pumps, set vertically, as a rule, in the larger sizes, large steam-cylinders, a large air-chamber, and a steel or wrought-iron frame. The whole is mounted on springs of great strength and flexibility combined. Large fire engines of this kind will weigh three tons, and will throw 1000 gallons a minute, in a 2-inch stream, to a distance of 300 or 325 feet, or to a height of 200 feet or more. Their steam-cylinders are as large as 9 or 10 inches diameter, and pumps 5½ or 6, with a stroke of

* American Electrician, vol. XI, 1899.

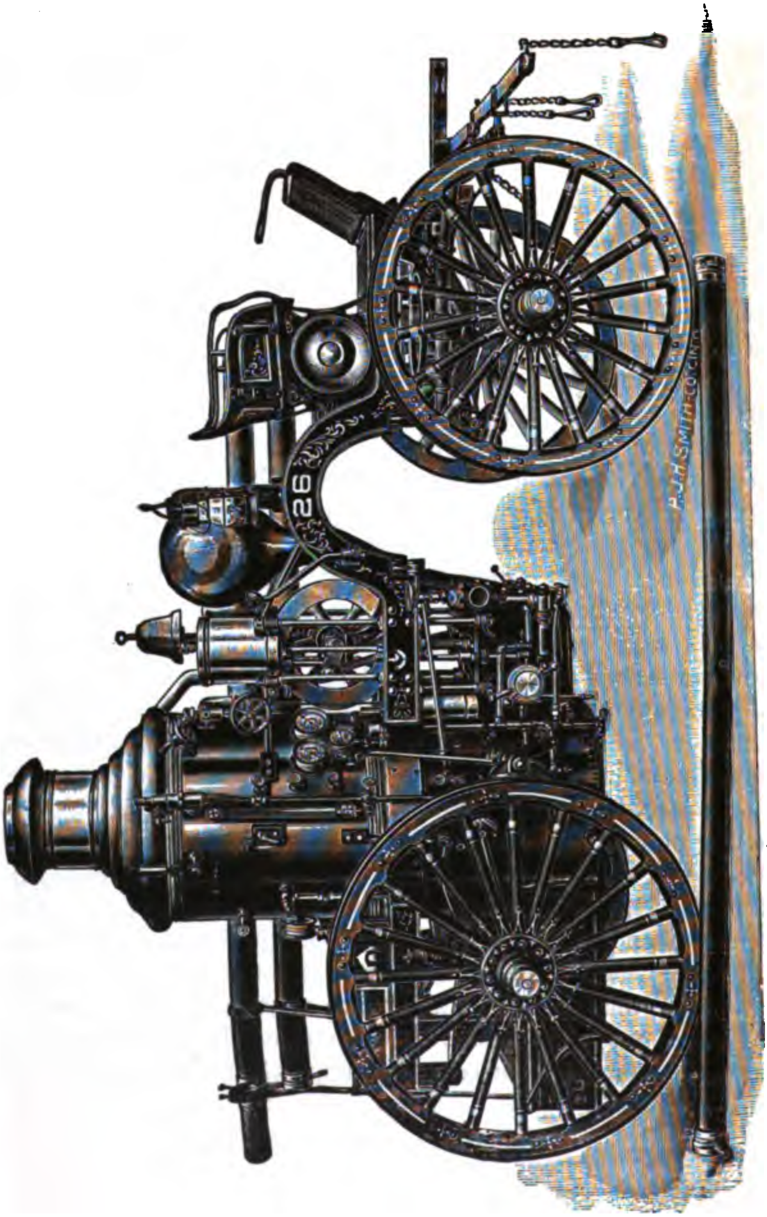


FIG. 93.—STEAM FIRE-ENGINE. (HALL.)

6 to 9 inches. Only the best of materials can be used in such machinery as this.

The balance obtainable by the use of three engines is especially useful in the case of the steam fire-engine; where smoothness and steadiness of action is necessary on so unsubstantial a base. Here, also, the use of three attached pumps, as in Fig. 94, gives a very valuable gain in smooth-working of the water-side of the machine. With skilful designing, the added weight is comparatively unimportant; since it only affects the engine and is to probably usually a sensible extent compensated by reduced size, or by greater efficiency of the boiler and by decided gain in reduction of friction and greater "throw" of the stream leaving the hose-nozzle. The details of the design by Mr. Knaust, here shown, so far as concerns the peculiarities of this class of engine, can be readily seen and need no special description.

The steam fire-engine is sometimes constructed as a "fire-boat" of enormous power, the whole steam-power of the main boilers being there available. The New Yorker, designed by Mr. Cowles, for example, displaces 351 tons, has a speed of 15 knots, has four steam-pumps, each of 16-inch steam- and 10-inch water-cylinders, capable of discharging 10,000 gallons per minute to a maximum distance of 250 feet in a 5-inch stream, or to less distances in a number of smaller streams.

40. **The Locomotive Engine** is the best known example of sustained power, with minimum weight, which has yet been produced by the mechanical engineer.

A locomotive has two steam-cylinders, either side by side within the frame, and immediately beneath the forward end of the boiler, or on each side and exterior to the frame. The engines are non-condensing, and of the simplest possible construction. The whole machine is carried upon strong but flexible steel springs. The steam-pressure is usually more than 100 pounds. The pulling-power is generally about one fifth the weight under most favorable conditions, and becomes as low as one tenth on wet rails. The fuel employed is wood in new countries, coke in bituminous-coal districts, and anthracite coal

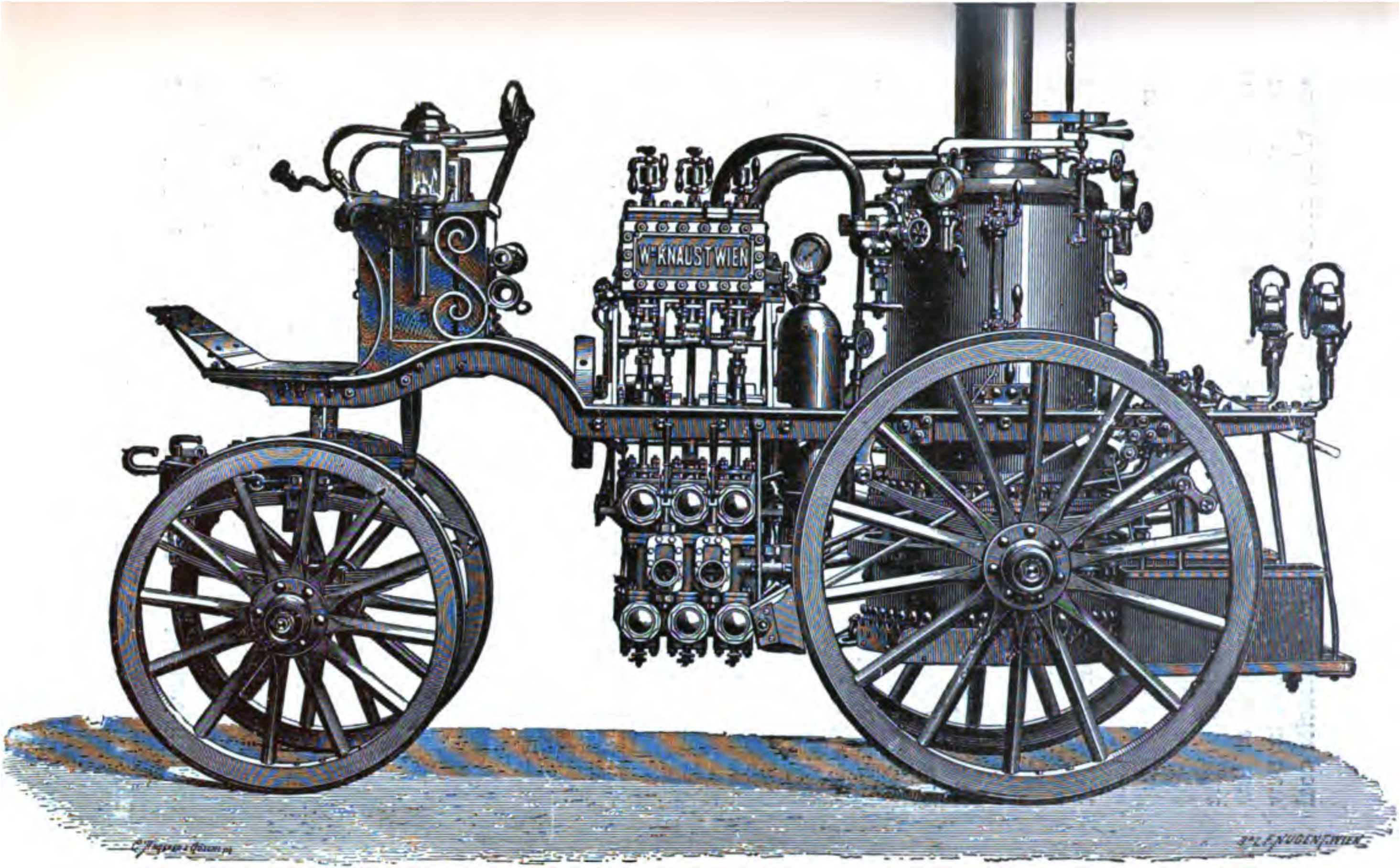


FIG. 94.—THREE-CYLINDER FIRE-ENGINE.

in the eastern part of the United States. The general arrangement and the proportions of locomotives differ somewhat in different localities. The peculiarities of the American type (Fig. 95) are the truck, *I J*, or bogie, supporting the forward part of the engine, the system of equalizers, or beams which distribute the weight of the machine equally over the several axles, and minor differences of detail. The cab or house, *r*, protecting the engine-driver and fireman, is an American device, which is gradually coming into use abroad also. The American locomotive is distinguished by its flexibility and ease

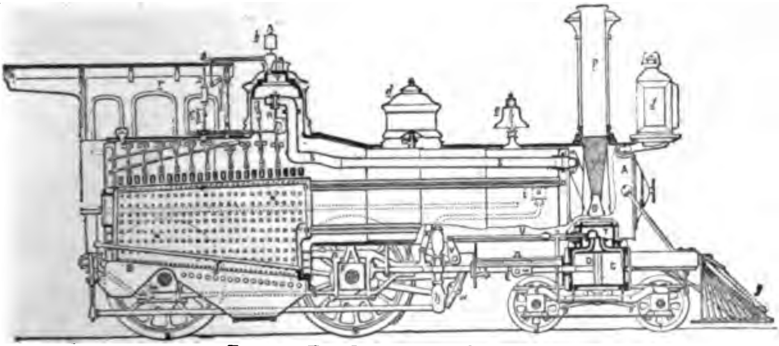


FIG. 95.—THE LOCOMOTIVE. Scale ($\frac{1}{1_{80}}$.)

of action upon even roughly-laid roads. In the sketch, which shows a standard American engine in section, *A B* is the boiler, *C* one of the steam-cylinders, *D* the piston, *E* the cross-head, connected to the crank-shaft, *F*, by the connecting-rod, *G H* the driving-wheels, *I J* the truck-wheels, carrying the truck, *K L*; *M N* is the fire-box, *O O* the tubes, of which but four are shown. The steam-pipe, *R S*, leads the steam to the valve-chest, *T*, in which is seen the valve, moved by the valve-gear. *U V*, and the link, *W*. The link is raised or depressed by a lever, *X*, moved from the cab. The safety-valve is seen at the top of the dome, at *Y*, and the spring-balance by which the load is adjusted is shown at *Z*. At *a* is the cone-shaped exhaust-pipe, by which a good draught is secured. The attachments *b*, *c*, *d*, *e*, *f*, *g*—whistle, steam-gauge, sand-box, bell, head-light, and "cow-catcher"—are nearly all peculiar, either

in construction or location, to the American locomotive. The locomotive is furnished with a tender, which carries its fuel and water. A standard passenger-engine on railways in the United States has four driving-wheels, 6 feet diameter; steam-cylinders, 21 inches diameter and 2 feet stroke; grate-surface 25 square feet, and heating-surface 2000 square feet. It weighs 90,000 pounds, of which 60,000 pounds are on the drivers and 30,000 on the truck. The freight-engine has six driving-wheels, 54⅝ inches in diameter. The steam-cylinders are 18 inches in diameter, stroke 22 inches, grate-surface 14.8 square feet, heating-surface 1096 feet. It weighs 68,500 pounds, of which

FIG. 96.—THE AMERICAN TYPE OF PASSENGER-ENGINE.

48,000 are on the drivers and 20,500 on the truck. The former takes a train of five cars up an average grade of 90 feet to the mile. The latter is attached to a train of 11 cars. On a grade of 50 feet to the mile, the former takes 7 and the latter 17 cars. Tank-engines for very heavy work, such as on grades of 320 feet to the mile, which are found on some of the mountain lines of road, are made with five pairs of driving-wheels, and with no truck. The steam-cylinders are 20½ inches in diameter, 2 feet stroke; grate-area, 15¾ feet; heating-surface, 1380 feet; weight with tank full, and full supply of wood, 112,000 pounds; average weight, 108,000 pounds. Such an engine has hauled 110 tons up this grade at the speed of 5 miles an hour,

the steam-pressure being 145 pounds. The adhesion was about 23 per cent of the weight.

In checking a train in motion, the inertia of the engine itself absorbs a seriously large portion of the work of the brakes. This is sometimes reduced by reversing the engine and allowing the steam-pressure to act in aid of the brakes. To avoid injury by abrasion of the surfaces of piston, cylinder, and the valves and valve-seats, M. Le Chatelier introduced a jet of steam into the exhaust-passages when reversing, and thus prevented the ingress of dust-laden air and the drying of the rubbing surfaces. This method of checking a train is rarely resorted to except in case of danger. The introduction of the "continuous" or "air" brake, which can be thrown into action in an instant on every car of the train by the engine-driver, is so efficient that it is now almost universally adopted. It is one of the most important safeguards which American ingenuity has yet devised. In drawing a train weighing 200 tons at the rate of 90 miles an hour, about 1800 effective horse-power is required. A speed of 100 miles an hour has been sometimes attained. At 75, the pull is 5 lbs. per H.P.

The standard locomotive-engine has a maximum life which may be stated at an average of about 30 years. The annual cost of repairs is from 10 to 15 per cent of its first cost. On moderately level roads, the engine requires a pint of oil to each 25 miles, and a ton of coal to each 40 or 50 miles run.

The compound locomotive engine is now coming to be adopted. This involves considerable changes of proportions, increasing the volume and weight of steam-cylinders, but enabling the designer to more than proportionally decrease the weight of boiler and the quantity of fuel carried. No serious objection to their use has been experienced, however, and no difficulty in the construction of the "double-cylinder" type of engine for the locomotive. Many such engines have been constructed. They will be referred to again.

The increasing demands upon the railways of the United States have recently brought about considerable changes in the forms of engine employed. The standard "American" type of

locomotive is much less generally employed for slow and heavy traffic, and its place has, on the trunk lines, been taken by 8-, 10-, and 12-wheeled engines of great weight. Even in passenger service, engines with six and ten coupled wheels are displacing it in many cases. For "switching" or "shunting" heavy trains, engines of 40 tons weight, with six coupled wheels and 17- to 19-inch cylinders of 24 inches stroke, are used. The weights on the drivers are usually 5 to 7 times the adhesion demanded. In Europe, with lighter trains and shorter runs, as a rule, but with higher speeds, the single pair of drivers, the opposite extreme of practice, seems preferred. On both continents the compound locomotive is rapidly coming into use.

The modern developments of the locomotive-engine, which have been seen to involve no change of general construction, have been mainly the refinement of details, the introduction

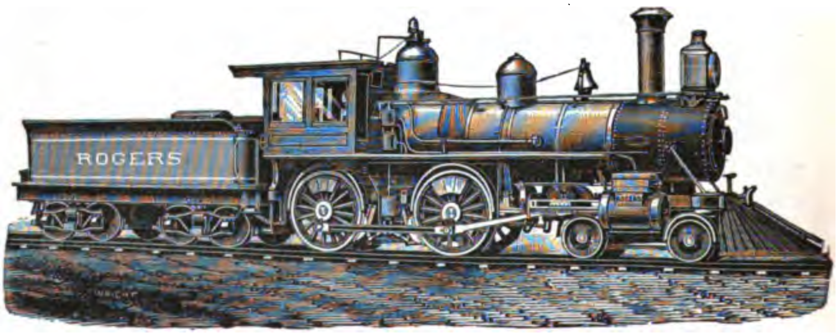


FIG. 97.—STANDARD PASSENGER-ENGINE.

of a few recent inventions, as the extended smoke-box, and the application of the air-brake. The engine is to-day the locomotive of George and Robert Stephenson.

But while the type remains unchanged in its essentials, there are now in use a great number of designs of engine differing among each other in proportions and often widely in external appearance, designs which have been produced in the endeavor to adapt the machine to specific kinds of work or to special localities and purposes. Thus the fast passenger and

the slow-freight, or "goods," engine have very different proportions and appear like quite different machines.

The common standard passenger-engine is of the type illustrated in the accompanying figure, as built by the Rogers Works, in which a comparatively recent device, the "extension smoke-box," is shown, acting as a trap and temporary receptacle for hot ashes and cinders carried through the tubes and formerly thrown out to set fire to buildings or vegetation or to annoy the people on the train.

Ten- and twelve-wheeled engines are employed for the heaviest kinds of work. These locomotives weigh from 45 to 75 tons, and occasionally even more, of which nearly all is carried on coupled driving-wheels of not far from 4 feet diameter. The cylinders are 20 to 22 inches in diameter, and stroke of piston usually about 2 feet. They have 25 to 35 square feet

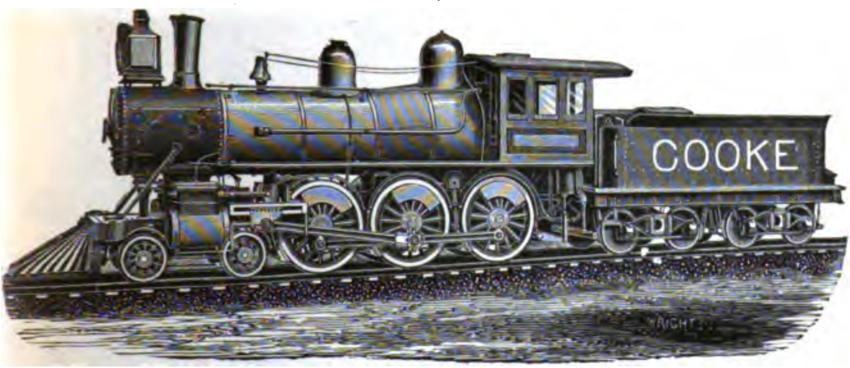


FIG. 98.—COOKE'S TEN-WHEELED LOCOMOTIVE.

of grate-surface and 1500 to 2500 feet of heating-surface. The alternate pairs of wheels have "blank," or unconed, tires, to permit easy movement around curves. Their details are similar to those elsewhere described as made for standard passenger-engines.

Where the line is of narrow gauge, as often in new countries, or wherever it is found desirable to concentrate more hauling power than the usual forms of engine would give, special designs have been sometimes adopted. The Fairlie engine is one of these. This plan unites two engines, back to

back, in effect, giving a twin arrangement of engines and on boiler, united at the fire-box. The plan is costly but effective. A simpler system of concentration of power is that of Forney, which unites engine and tender on one frame and thus secures increased weight and adhesion, as seen in the engraving here given; which gives a total weight of 60,000 pounds on a narrow and comparatively short wheel-base, and makes an exceptionally handy and easily worked engine.

The "tank-engine," of which the last illustrates one form, is sometimes constructed on a very large scale. Thus, locomotives built at the Baldwin Locomotive Works, Philadelphia, for the Grank Trunk Railway, to be used in the St. Clair tunnel, under the bed of the St. Clair River, between

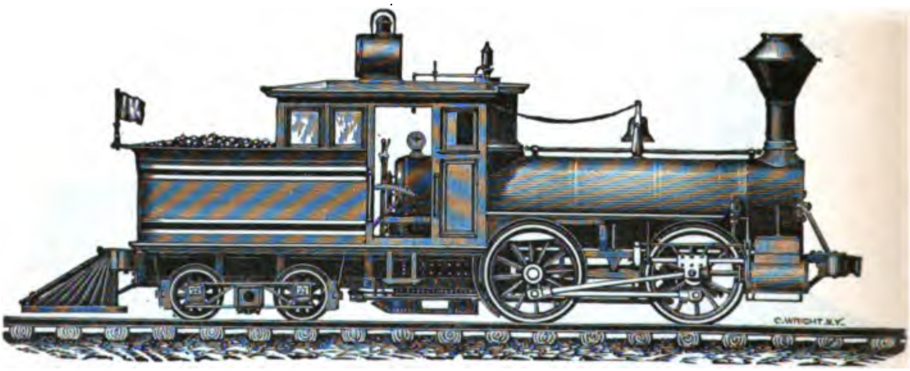


FIG. 99.—FORNEY LOCOMOTIVE.

Port Huron, Mich., and Sarnia, Ont., have five pairs of 50-inch driving-wheels on each side of the boilers, the cab in the middle of the boiler extending out over the two tanks, one each side of the boiler. The cylinders are 22.28 inches, and the boiler 74 inches in diameter, to carry 160 pounds of steam. Each locomotive with tanks filled weighs 200,000 pounds, the average weight in running order, with tanks half filled, being 180,000 pounds.

Compound Locomotives are less common than compound stationary engines. They are, however, gradually becoming used where fuel is expensive, and give, when well designed, very marked economical advantages. The usual system

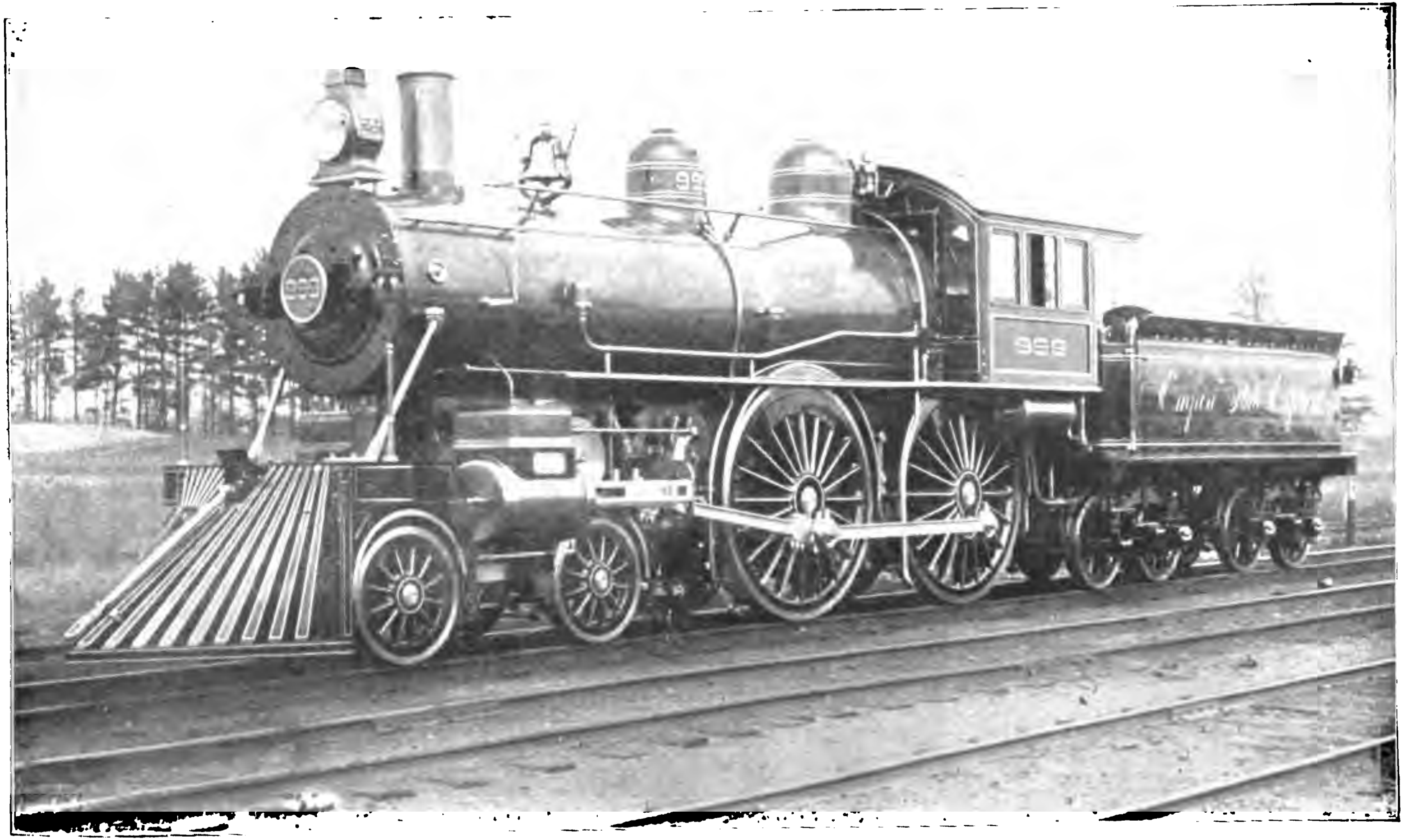


FIG. [illegible].—ENGINE "999"; N. Y. C. & H. R. R.

This engine has a record of 1 mile in [illegible] seconds, [illegible] miles an hour. Weight, [illegible] lbs.; "tender," [illegible]; train, [illegible]. Steam-cylinders, 19 in. diameter; 24 in. stroke of piston; steam, [illegible]; driving wheels, [illegible] in. diameter; truck wheels, [illegible]; H. S., [illegible]; [illegible]

[To face page [illegible].]

places a high-pressure cylinder on one side and a low-pressure cylinder on the other, the latter being commonly arranged to take steam direct from the boiler when starting or whenever, for any reason, it is desirable.

Some of the more interesting and successful designs of compound locomotive-engine are those of which outline illustrations follow, selected from Professor Woods' monograph.* That of Von Borries is exemplified by Figs. 100 and 101; the one exhibiting the arrangement adopted in a heavy engine on the Prussian State Railways,† the other a Spanish engine of less power.

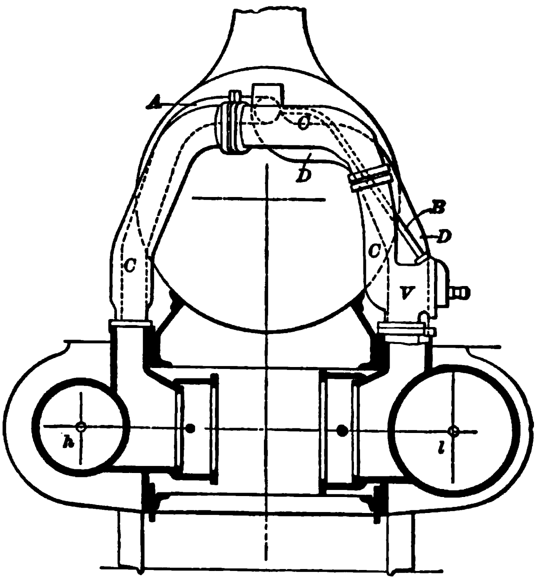


FIG. 100.—PRUSSIAN COMPOUND ENGINE.

The former has cylinders 18.1 and 25.6 inches diameter, 24.8 inches stroke, weighs 88,250 pounds, and has 1420 square feet of heating-surface and 16 feet grate-surface. The driving-wheels are 52.4 inches diameter, and the steam-pressure 175 pounds by gauge.

* Compound Locomotives; A. T. Woods, M.M.E.; N. Y., Van Arsdale, 1891.
† Engineering; Feb. 1, 1889.

The second engine is of 86,200 pounds weight, with 16- and 23-inch cylinders, 24 inches stroke of pistons, 5½ feet diameter of drivers, the pressure 170 pounds.

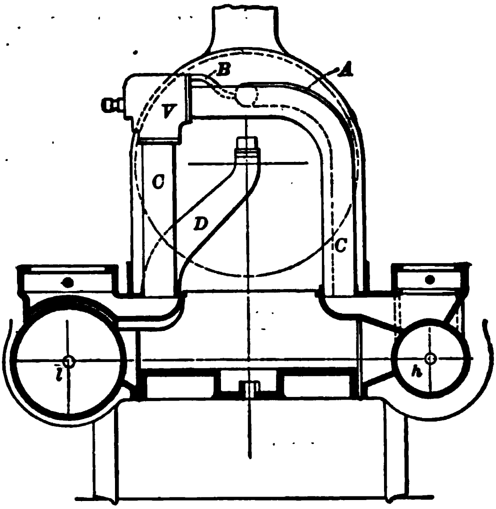


FIG. 101.—SPANISH ENGINE.

The arrangement of both engines involves the peculiar form of starting-valve devised by Von Borries, which is seen in the next figure. In the sketch, *a* is the receiver-pipe to the

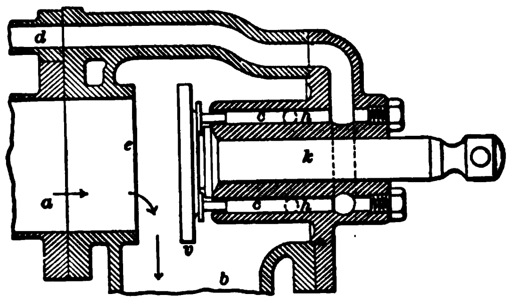


FIG. 101*a*.—VON BORRIES VALVE.

high-pressure, *b* that to the low-pressure cylinder. The valve, *v*, is seen as in ordinary working when "under way," and the

arrows show the course of the steam. Attached to the back of this valve are two plungers, *c c*, constituting the starting-valve. When the throttle-valve is opened, steam enters the pipe *d*, passing back of the plungers, forcing the valve to its seat, *e*, at the same time opening the ports *h h*, through which, and the passage *b*, it goes on to the large cylinder.

When the engine starts, the exhaust occurs from the small cylinder and the receiver-pressure rises, this valve becomes equilibrated, returns to the position shown, and, once thus started, the engine acts as compound, and so continues until, after shutting off steam, this equilibrium is lost and the engine starts again, later, as a simple machine. This device is in extensive use.

In the Worsdell form of engine, Fig. 102, the construction is as seen in the sketch.* *A* is the steam-pipe, *B* the starting-valve connection, *C* the receiver, *D* the exhaust-pipe, and *v* and *V* are the starting and the intercepting valves. The engine here taken for illustration is an English passenger-locomotive, having 16- and 26-inch cylinders, 24 inches stroke, drivers 80½ inches in diameter. The steam-pressure the same as the preceding, and the weight of engine 97,000 pounds, of which 68,000 rests on the driving-wheels. The areas of heating and grate surface are, respectively, 1323½ and 17¼ square feet. Joy's valve-gear is employed.

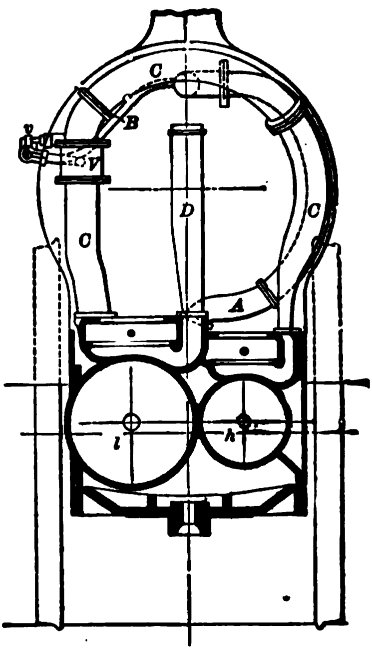


FIG. 102.—THE WORSDELL ENGINE.

Mr. Vauclain employs, on Baldwin locomotives, the construction seen in Fig. 103, adopting a high and a low pressure cylinder on each side the engine driving a single crosshead.

* Engineering; March 30, 1888.

His single valve is made to serve both cylinders. In designing compound locomotives the aim should be, according to Vau clain:

(1) To produce a compound locomotive of the greatest efficiency, with the utmost simplicity of parts.

(2) To develop the same amount of power on either side of the locomotive.

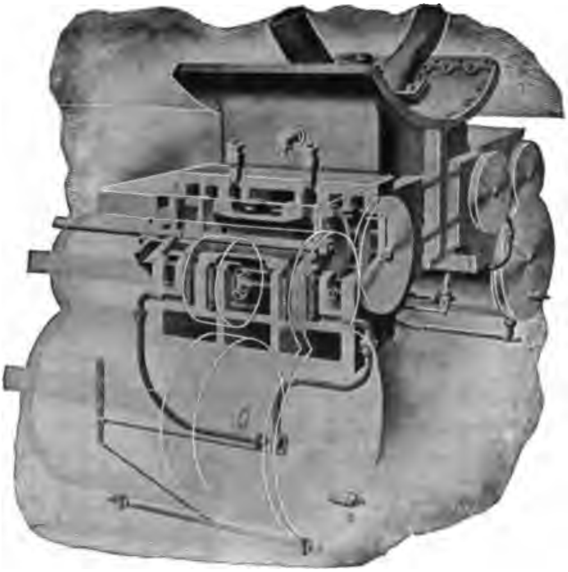


FIG. 103.—ARRANGEMENT OF CYLINDERS, STEAM-CHEST, VALVE, AND PORTS.

(3) To insure as great effectiveness as in a single-expansion locomotive of similar weight and type.

(4) To insure the least possible cost of repairs.

(5) To insure the least possible departure from the method of handling single-expansion locomotives; to apply equally to passenger or freight locomotives for all gauges of track, and to withstand rough usage.

The plan, more usual in marine engineering, of employing one high-pressure and two low-pressure cylinders is illustrated in the next sketch, that adopted on the Northern Railway of France.* In the figure, *A* is the main steam-pipe, *B* the valve-

* Engineering; Dec. 6, 1889.

chest, C C the receiver, all attached to the small cylinder, and D D are the two low-pressure exhaust-pipes. The cylinders, h and l l, are high- and low-pressure, respectively, and the whole plan is readily traced out. The cranks of the latter are set at right angles, and the high-pressure crank at 135 degrees with each. All are on one shaft, the middle one of three driving-axles.

The high-pressure valve-gear is the Rider modification of the Meyer system, permitting any desired expansion in the high-pressure cylinder. When thrown completely over, the cut-off

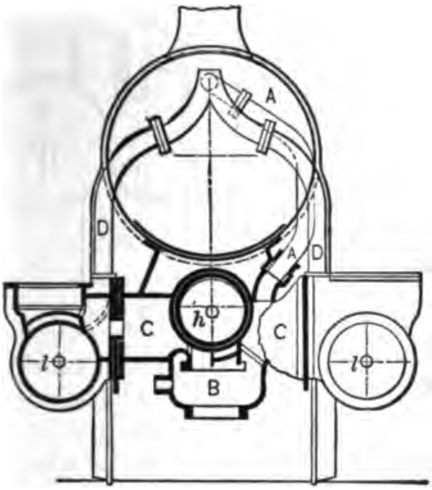


FIG. 104.—DIVIDED L. P. CYLINDER.

valve permits the steam to blow through the small cylinder, and thus the engine is converted into the common form, the two low-pressure becoming the driving-engines.

This engine has cylinders of 17 and 19.7 inches diameter and 27.6 inches stroke, driving-wheels (six) 64.9 inches diameter, 1225 square feet of heating-surface, 13 feet grate-surface, weighs 106,176 pounds, of which 91,000 rests on the drivers, and the steam-pressure by gauge is 199 pounds.

The Mallet system, now much employed and well known, is exhibited in the next figure.

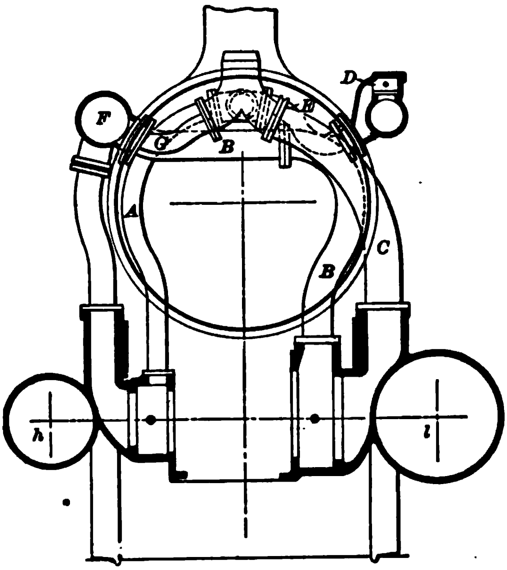


FIG. 105.—THE MALLET SYSTEM.

Here *A* and *B* are the steam-pipe and receiver, and *C* the exhaust-pipe. *D* is a starting-valve, taking steam through *E*, and *F* is the "intercepting-valve." The pipe *G* serves to convey the exhaust from the small cylinder when working non-compound. A pressure-reducing valve is placed between starting-valve and receiver. When in ordinary operation as a compound engine, the pressure of boiler-steam keeps the intercepting-valve closed against receiver-pressure. On starting from rest, however, this valve is relieved and steam passes over into the low-pressure cylinder, the pair then working as simple engines. The engine can thus start any load that the standard machine can take. Once started, this and other compounds have less hauling power than the simple type; but no such reduction occurs as to interfere with any ordinary work.

In all these engines, automatic relief-valves are desirable, on the large cylinder especially, since they must be expected to add to the priming the water of cylinder-condensation in larger proportion than in cases of restricted ratios of expansion.

The Webb system, as introduced on the London and Northwestern Railway of Great Britain for both passenger and heavy traffic, is exhibited in the accompanying illustration. It precisely reverses the arrangement last described, there being two high-pressure and one low-pressure cylinder, their relative position being the reverse of the preceding.

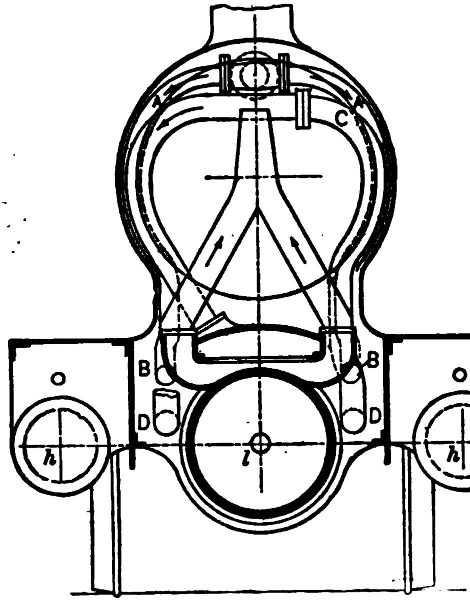


FIG. 106.—THE WEBB COMPOUND.

The pipes $A\ A$ and $B\ B$ take steam to the small cylinders, and C and $D\ D$ convey the rejected steam to the large cylinder. The former are placed ahead of the latter and are connected to an independent axle, no coupling or parallel rods being used, and the two axles "keeping time" only through the automatic adjustment produced by their own operation.

Where more than two pairs of coupled drivers are employed, the added axles are coupled to the small engines and their axles by means of parallel rods.

This engine has the following dimensions: diameter of cylinders, 14, 14, 30 in.; stroke, 24 in.; wheels, diameter, 75 in.; steam-pressure, 175 lbs.; weight, 99,350 lbs.; heating-surface, 1457 sq. ft.; grate, 20.55 sq. ft. Two thirds the total weight is on the drivers. The valve-motion is that of Joy. An engine of this type, experimentally tried between New York and Philadelphia, making regularly 87 miles in 2 hours, with 7 stops, and 200 or 225 tons weight of train, excelled the simple engine by 25 per cent in economy of fuel-consumption. The parallel rod is always felt to be a source of danger and of waste of power in the locomotive, and this plan is considered decidedly advantageous in this respect. On the other hand, the low-pressure cylinder produces a comparatively irregular "torque" on the axle to which it is coupled.

The Pitkin system, as introduced by Mr. A. J. Pitkin of the Schenectady Works, is seen in Fig. 107, below.

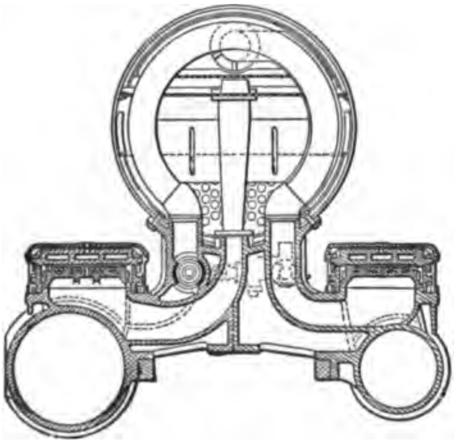


FIG. 107.—THE PITKIN COMPOUND.

It includes one high- and one low-pressure cylinder, with an ingenious intercepting-valve, seen in the next illustration. The receiver has a volume fifty per cent greater than that of

the small cylinder, and the clearance in the latter is about ten per cent, a proportion shown by the indicator to be desirable with the proportions of valves employed. The valves are arranged and the general disposition of parts is as in the standard engine of the old form.

The intercepting-valve, as here seen in section, is as at the

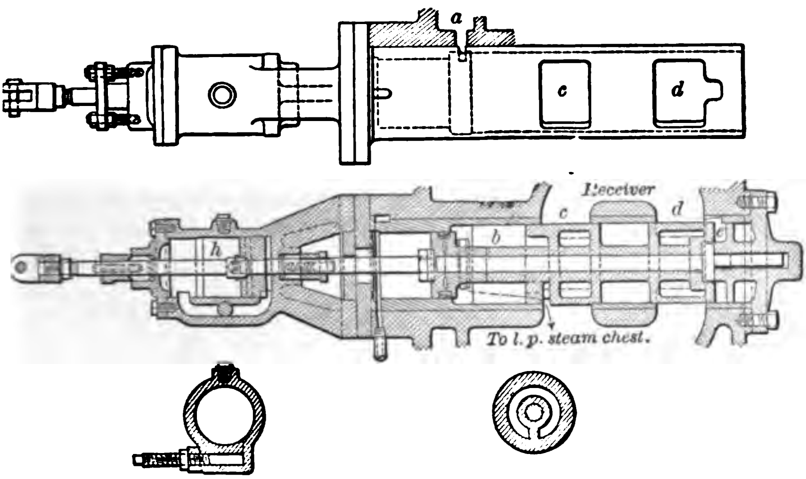


FIG. 108.—PITKIN'S INTERCEPTING-VALVE.

instant of starting and before compound working begins, the ports *c* and *d* closed and no connection existing between the receiver and the large cylinder; while the latter receives steam through a reducing-valve and the port *a* and the passage *b*.

On starting, the exhaust from the small cylinder fills the receiver, and the back-pressure taking effect, through *e*, on the intercepting-valve and destroying its equilibrium, it at once moves over and the large cylinder takes its steam properly for compound working.

The dash-pot, *h*, prevents too sudden movement.

This engine has the following dimensions: cylinders, diameter, 20 and 29 in.; stroke of piston, 24 in.; ratio of cylinders, 2.1; diameter drivers (6), 68 in.; weight of engine, 126,800 lbs.; heating-surface, 1677 sq. ft.; grate-surface, 28.57 ft. About 80 per cent of the total weight is on the drivers.

Mr. Von Borries estimates a saving of 15 per cent and upward as an offset to an increase of first cost amounting to 2 or 3 per cent. He also finds his engines to exceed the common type in hauling power by from 5 per cent on heavy engines to 10 per cent, or more, in fast passenger-service; a conclusion sustained by Mr. Lapage. The increased weight of cylinders and accessories, for a given power, is more than compensated by the decreased weight of boiler required.

The compound locomotive engine has been sometimes found to use as little as 22 pounds (10 kilos) of steam per hour and per horse-power. Power and economy increase with speed, contrary to the action of simple engines.

M. Mallet communicated to the French Society of Engineers (1883) a note from M. Borodin, giving the results of experiments to determine the relative economy of the simple

FIG 109.—BRITISH EXPRESS ENGINE.

and the compound system of engine for locomotives. The engines experimented with were those designed for the railway from Bayonne to Biarritz by M. Mallet. The trials extended over a considerable period of time, and the comparisons were made fairly complete. The result showed the compound system to have an economy of from 10 to 20 per cent, according to the conditions under which they are carried out. The practicable variation in the ratio of expansion is often very greatly restricted in the compound engine. The use of the steam-jackets with which the engines were provided did not prove to be of advantage. The expenditure of steam was greater when they were in use than when they were shut off.

12-WHEEL FREIGHT LOCOMOTIVE.

Weight, 205,000 lbs. Weight on drivers, 166,000 lbs. Drivers, 54″. Steam-cylinders, 21″ × 32″. H. S., 3168 sq. ft. G. S., 82.4 sq. ft.
Wooten Firebox. Shell, 78″. Steam-pressure, 200.

(Brooks Locomotive Works, 1899.)

[Page 210a.]

"CONSOLIDATION." (10 wheels, 8 coupled.)

Weight, 239,000 lbs. Weight on drivers, 170,000 lbs. Drivers, 58". Cylinders, 22" × 28". Steam-pressure, 200. H. S., 2689 sq. ft. G. S., 33.2 sq. ft. Boiler, 72".

(Schenectady Locomotive Works, 18[illegible].)

[Page 2106.]

Fig. 109 represents the type of engine often adopted on English roads for very high speeds and with comparatively light loads. This engine has regularly made 200 miles in four hours, and somewhat similar engines have made 250 miles in five hours, and even 400 miles in eight hours. The diameter of the drivers, in this example, is 8 feet, the steam-cylinder 18 inches, and the stroke of piston 28 inches. This type was in use even earlier than 1880; at which date the performance just stated had been attained.

British engines of this last-described type have done extraordinary work. Such locomotives on the longer main lines, between London and Glasgow, make an average of 50 miles an hour for 400 miles. The Midland Railway employs engines with cylinders 18x26, a single pair of drivers 7 feet 4 inches diameter; with 1240 feet of heating-surface and 20 feet of grate, to haul trains of 225 to 250 tons weight, at nearly 50 miles an hour, and with a fuel-expenditure of 26 pounds per mile. Compound engines of recent construction have wheels $7\frac{1}{2}$ feet in diameter, and have made nearly ninety miles an hour. One of Mr. Worsdell's engines has, for illustration, 20 and 28 by 24 inch cylinders, 7 feet $7\frac{1}{4}$ inch drivers (single pair), 1140 feet of heating and 20 feet of grate, and has attained an average of over 50 miles an hour on 26.4 pounds of coal per mile; the train, engine included, weighing something over 300 tons. The steam-pressure carried is 175 pounds.

41. The Marine Engine, on the rivers of the United States, remains largely as it was left by the earlier engines. It is a beam-engine, of moderate steam-pressure, driving the radial paddle-wheel; the details are little, if at all, altered. The pressure of steam is now sometimes as high as 60 pounds per square inch or even more. The valves are of the disk or poppet variety, rising and falling vertically. They are four in number, two steam and two exhaust valves being placed at each end of the steam-cylinder. The beam-engine is a peculiarly American type, seldom if ever seen abroad.

Fig. 110 is an outline sketch of this engine as built for a steamer plying on the Hudson River. This class of engine is

usually adopted in vessels of great length, light draught, and high speed. But one steam-cylinder is commonly used. The cross-head is coupled to one end of the beam by means of a pair of links, and the motion of the opposite end of the beam is transmitted to the crank by a connecting-rod of moderate

FIG. 110.—BEAM-ENGINE.

length. The beam has a cast-iron centre surrounded by a wrought-iron strap of lozenge shape, in which are forged the bosses for the end-centres, or for the pins to which the connecting-rod and the links are attached. The main centre of the beam is supported by a "gallows-frame" of timbers so arranged as to receive all stresses longitudinally. The crank and shaft are of wrought-iron. The valve-gear is very usually of

the form known as the Stevens valve-gear, the invention of Robert L. and Francis B. Stevens. The condenser is placed immediately beneath the steam-cylinder. The air-pump is placed close beside it, and worked by a rod attached to the beam. Steam-vessels on the Hudson River have been driven by such engines at the rate of 20 miles an hour. This form of engine is remarkable for its smoothness of operation, its economy and durability, its compactness, and the latitude which it permits in the change of shape of the long, flexible vessels in which it is generally used, without injury by "getting out of line."

For paddle-engines of large vessels, the favorite type, which has been the side-lever engine, is now rarely built. For smaller vessels, the oscillating engine with feathering paddle-wheels is still largely employed in Europe. It is very compact, light, and moderately economical, and excels in simplicity. The usual arrangement is such that the feathering-wheel has the same action upon the water as a radial wheel of double diameter. This reduction of the diameter of the wheel, while retaining maximum effectiveness, permits a high speed of engine, and therefore less weight, volume, and cost. The smaller wheel-boxes, by offering less resistance to the wind, retard the progress of the vessel less than those of radial wheels. Inclined engines are sometimes used for driving paddle-wheels. In these the steam-cylinder lies in an inclined position, and its connecting-rod directly connects the crank with the cross-head. The condenser and air-pump usually lie beneath the cross-head guides, and are worked by a bell-crank driven by links on each side the connecting-rod, attached to the cross-head. Such engines are used to some extent in Europe, and they have been adopted in the United States navy for side-wheel gunboats. They have also been used on ferry-boats plying between New York and Brooklyn.

The non-condensing direct-acting engine is used principally on the Western rivers of the United States, is driven by steam of from 100 to 150 pounds pressure, and exhausts into the atmosphere. It is the simplest possible form of direct-acting en-

gine. The valves are usually of the "poppet" variety, and are operated by cams which act at the ends of long levers having their fulcra on the opposite side of the valve, the stem of which latter is attached at an intermediate point. The engine is hori-

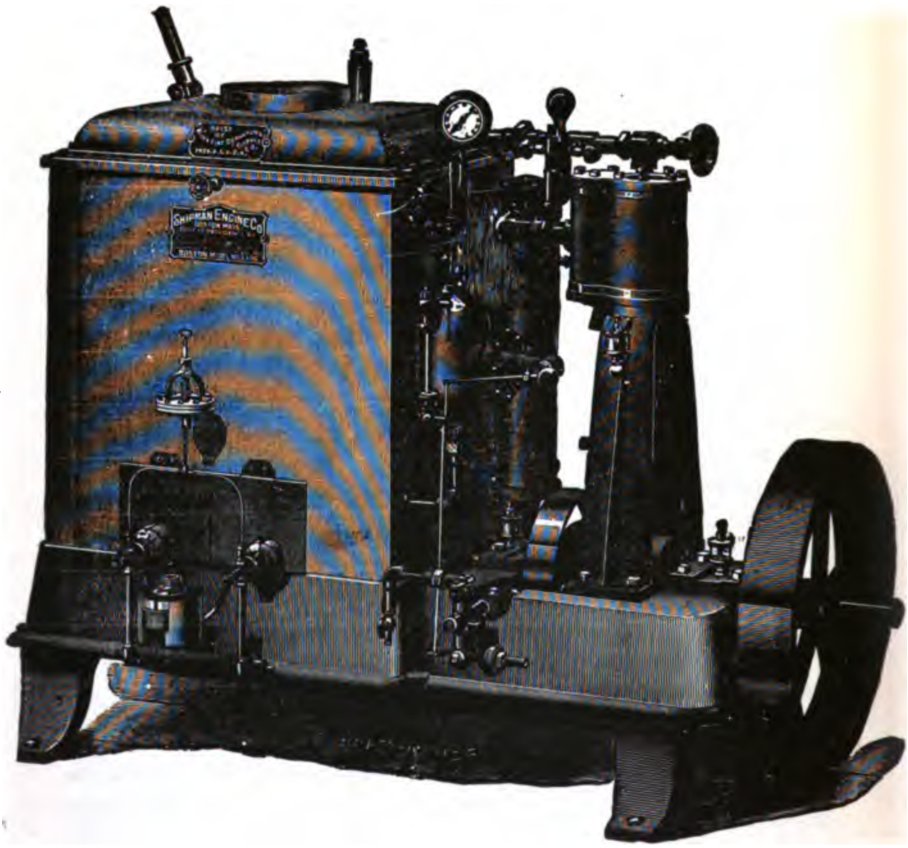
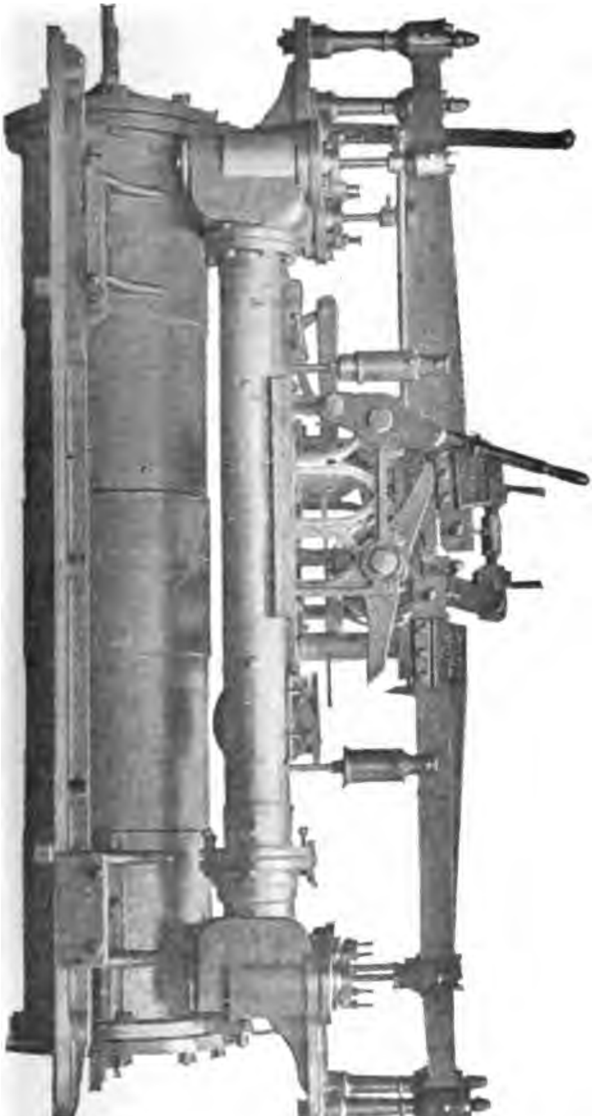


FIG. 111.—SHIPMAN ENGINE.

zontal, and the connecting-rod directly attached to cross-head and crank-pin without intermediate mechanism. The paddle-wheel is used, sometimes as a stern-wheel, as in the plan of Jonathan Hulls of 1737, sometimes as a side-wheel, as is most usual elsewhere.



ENGINE USED IN STERN-WHEEL STEAMERS.

Special designs of marine engine are sometimes found desirable for small powers. That here illustrated, for example, as designed by Shipman, is very similar in general arrangement to some forms of semi-portable engine, the engine and boiler having a common base. Larger sizes, however, are separated. The boiler is water-tubular, of the general form of that first used by Stevens. The engine, either simple or compound, is vertical and of the usual standard type, with link-motion, when used as a yacht-engine, and having a reverse-lever.

The essential feature of this motor is that it is an automatic petroleum-burning engine, designed for use where a moderate amount of power is required. When steam has been generated, no further attention is required beyond that of opening and shutting the steam-valve whenever the engine is started or stopped, the fire, speed, and water-feed being arranged as to adjust themselves automatically.

Two small aspirators or atomizers, taking steam from the boiler, take up the petroleum fuel, from a chamber below, and drive it into the furnaces in fine spray. Torches ignite this spray as it passes inward. The steam and petroleum supply of the atomizers is regulated by a diaphragm connected to a valve in the steam-pipe.

This diaphragm is exposed to steam-pressure on the one side, and is held down by a spring, loaded to a certain pressure, on the other. Its movement is conveyed to the valve by a rod, and it thus regulates the amount of steam passing to the atomizers.

The water in the boiler is kept at a constant level by means of a float, connected to a tap in the suction-pipe of the pump. This float is placed in a chamber which is joined to the top and bottom of the boiler, and rises or falls with the level of the water. The movement is conveyed, by means of levers, to the tap in the suction-pipe, which it opens or closes as the water-level changes.

The speed of the engine is regulated by means of a governor. When once steam is up, the fires, the water-supply, the oiling, and the speed of the engine require no further attention.

When first starting, a sufficient pressure is required in the boiler to work the atomizers, and for this a hand air-pump is provided.

In vessels, in nearly all cases, the ordinary screw-engine is adopted, and is direct-acting. Two engines are placed side by side, with cranks on the shaft at an angle of 90° with each other. In merchant-steamers, the steam-cylinders are usually

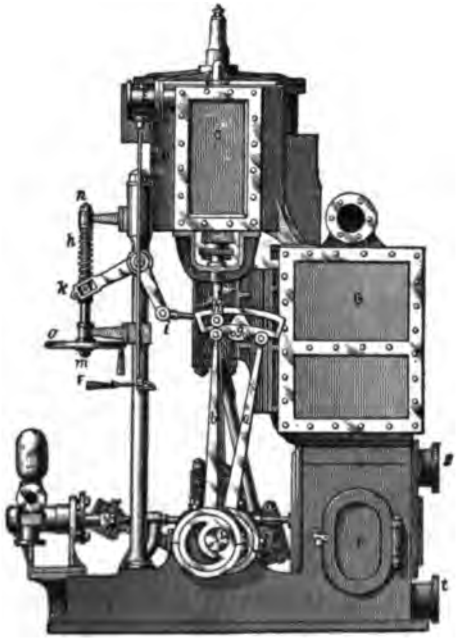


FIG. 112.—COMPOUND MARINE ENGINE.

vertical and directly over the crank-pins, to which the cross-heads are coupled. The condenser is placed behind the engine-frame, or, where a jet-condenser is used, the frame itself is sometimes made hollow, and serves as a condenser. The air-pump is worked by a beam connected by links with the cross-head. The general arrangement is like that shown in Fig. 112. For naval purposes such a form is objectionable, since its height is so great that it would be exposed to injury by shot. In naval engineering the cylinder is placed horizontally.

The trunk-engine, in which the connecting-rod is attached directly to the piston and vibrates within a trunk or cylinder secured to the piston, moving with it, and extending outside the cylinder, like an immense hollow piston-rod, has been frequently used in the British navy. It has rarely been adopted in the United States.

42. **Standard Forms** of marine engines, in nearly all steam-vessels built for the merchant-service, and in some naval vessels, have come to be some modification of the compound engine. Figs. 112 and 113 represent the usual form of the two-cylinder compound engine. Here *A A*, *B B* are the small and the large, or the high-pressure and the low-pressure, cylinders respectively. *C C* are the valve-chests. *G G* is the

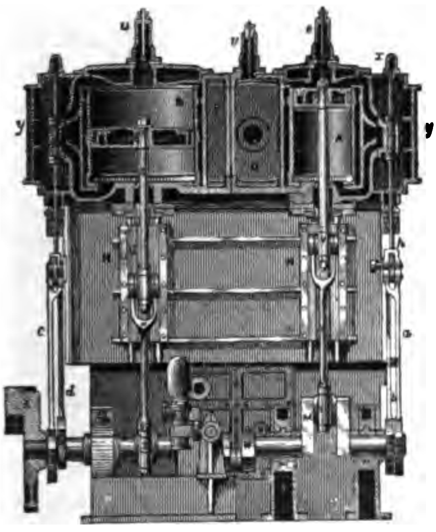


FIG. 113.—COMPOUND MARINE ENGINE (SECTION).

condenser, which is invariably a surface-condenser. The condensing water is sometimes directed around the tubes contained within the casing, *G G*, while the steam is exhausted around them and among them, and sometimes the steam is condensed within the tubes, while the injection-water which is sent into the condenser to produce condensation passes around

the exterior of the tubes. In either case, the tubes are usually of small diameter, varying from five eighths to half an inch, and in length from four to seven feet. The extent of heating-surface is usually from one half to three fourths that of the heating-surface of the boilers.

The air and circulating pumps are placed on the lower part of the condenser-casting, and are operated by a crank on the main shaft at *N*; or they are sometimes placed as in the style of engine last described, and driven by a beam worked by the cross-head. The piston-rods, *T S*, are guided by the cross-heads, *V V*, working in slipper-guides, and to these cross-heads are attached the connecting-rods, *X X*, driving the cranks, *M M*. The cranks are now usually set at right-angles; in some engines this angle is increased to 120°, or even 180°. Where it is arranged as here shown, an intermediate reservoir, *P O*, is placed between the two cylinders to prevent the excessive variations of pressure that would otherwise accompany the varying relative motions of the pistons, as the steam passes from the high-pressure to the low-pressure cylinder. Steam from the boilers enters the high-pressure steam-chest, *X*, and is admitted by the steam-valve alternately above and below the piston as usual. The exhaust steam is conducted through the exhaust passage around into the reservoir, *P*, whence it is taken by the low-pressure cylinder, precisely as the smaller cylinder drew its steam from the boiler. From the large or low-pressure cylinder the steam is exhausted into the condenser. The valve-gear is usually a Stephenson link, *g e*, the position of which is determined, and the reversal of which is accomplished, by a hand-wheel, *o*, and screw, *m n p*, which, by the bell-crank, *k i*, are attached to the link, *g e*. The "box-framing" forms also the hot-well. The surface-condenser is cleared by a single-acting air-pump, inside the frame, at *T*. The feed-pump and the bilge-pumps are driven from the cross-head of the air-pump.

The "tandem compound" marine engine, Fig. 114, is a simpler and less expensive construction, but it is so subject to uncertainty in starting and so liable to become fixed "on the

centre," that if adopted at all for marine work, it is very generally duplicated, the two engines having cranks at right angles, and thus its special advantage sacrificed. Such a combination is, however, excellent as a "quadruple-expansion" engine, the second set of steam-cylinders taking steam from the first, and

FIG. 114.—TANDEM COMPOUND ENGINE. (Scale $\frac{1}{48}$.)

a pair of two-cylinder compound engines of different size being thus grouped to give four cylinders "in series."

The latest types of Marine Engine are those compounded engines in which the number of engines in series is three, or even more, usually driving three equidistant cranks, and those which are designed to drive two, or even three, screws independently. In the extension of the principle of compounding

ments of three- and of four-cylinder engines driving but two cranks and in which the "tandem" disposition of cylinders is adopted with good results.

The engraving represents one set of the triple-expansion engines of the twin-screw sister-ships, the City of Paris and the City of New York. Their general arrangement is well shown. Each set drives one screw. The magnitude of these great engines is indicated by the altitude of the working platforms and the reversing wheel. This may be taken to represent a standard and very satisfactory disposition of parts and general proportion of engines.

A good sample set of figures for the proportions and performance of these engines are:

Steam-cylinders, diameter, inches	45, 71, 113
Stroke of pistons, feet	5
Ratios of volumes	1 ; 2.489 ; 6.304 or 0.402 ; 1 ; 2.53
Steam-pressure, per gauge, lbs	148
Rev. per min	87
Vacuum, inches	26
Mean pressures, lbs	64 ; 32 ; 14
Indicated power, H. P.	19,175
Temp. feed-water, Fahr	119°
" sea-water	54°
Area H. S., sq. ft	50,250
" G. S., " "	1,294
" cond. surf	33,000
I. H. P. per sq. ft. G. S	14.8
" " " " H. S	$\frac{1}{2.62} = 0.38$
" " " " condens. surface	$\frac{1}{1.72} = 0.58$
Ratio H. S. to G. S	38.8
" " " C. S	1.52

These figures are given by the engineer officers of the ship for a passage across the Atlantic made in 5 days, 19 hours, 34

minutes, at its date the quickest on record. The Lucania has crossed in 5 days, 7 hours, 23 minutes.

The arrangement of these engines in twin-screw steamers is seen in the next figure, which exhibits the machinery of the steamer Columbia of the Hamburg-American Line, a ship

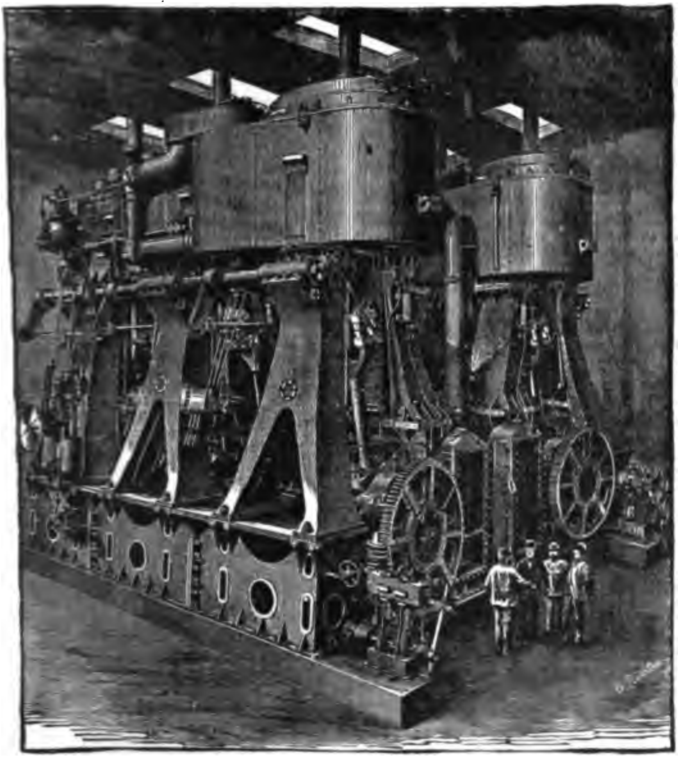


FIG. 117.—TRIPLE-EXPANSION ENGINE.

of 12,000 tons displacement and about 15,000 horse-power; each set of triple-expansion engines, as shown, having half that power. The cylinders are 40, 66, and 101 inches diameter, and the stroke of piston 66 inches. The shafts are of steel, 20½ inches diameter, driving screws of manganese bronze 18 feet diameter and of 32 feet pitch. These engines have driven the

Columbia 3045 knots—New York to Southampton—in 6 days, 15 hours, or 19.15 knots per hour.

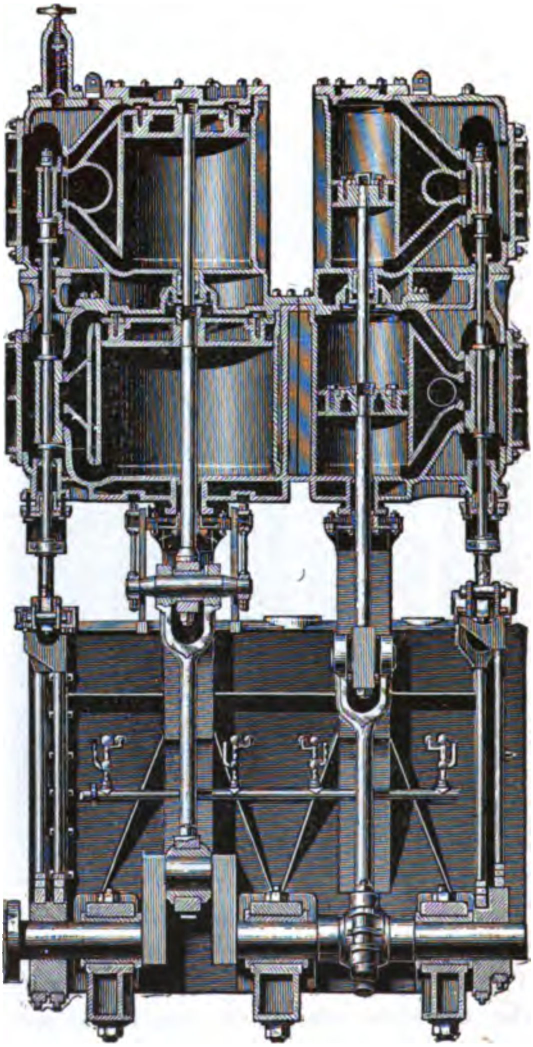


FIG. 118.—QUADRUPLE-EXPANSION ENGINE.—FIG. 119.

An illustration of a standard type of quadruple-expansion marine engine is seen in the section herewith given. This is a

Scotch engine of 350 horse-power, cylinders 10¼, 14, 20, 28 inches diameter, and 20 inches stroke of piston. Only the low-pressure cylinder is jacketed. The cranks being set at right angles, the two pairs of pistons have not synchronous motion, and a receiver or large connecting pipes must be adopted in this arrangement to insure good pressure-changes between the second and third cylinders.

The most extraordinary concentration of steam-power is illustrated in the more recent constructions of torpedo-boats. These little craft are given the lightest possible hulls, fine lines, unincumbered decks, and maximum power, everything being made subordinate to speed. That here figured, the Aviete (page 226), a Thorneycroft boat built for the Spanish Government, has made 26 knots an hour (over 30 statute miles). The hull is but 147½ feet long, of 14½ feet beam, and 5 feet draught, or but a trifle longer than Fulton's Clermont. Speeds exceeding 30 knots are reached with this class of boat. This type of boat has been given as much as 3000 indicated horse-power, steam being worked at 180 pounds pressure in water-tubular boilers, driving a hull displacing 200 tons, at speeds of from 25 to 32 knots. The coal consumed at standard half-speed—10 knots—is about 7½ tons as a minimum. Twin-screws are used. On long runs these, the "measured-mile trial," results are not usually approached very closely. In this work, the water-tube boiler is, in many cases, substituted for the ordinary "shell" fire-tube form with good results. The coal-consumption ranges not far from 2 pounds per I. H. P. per hour. The "Turbinia" (p. 62) engines weigh but 30 lbs. per H. P. The Viper has made 35 knots an hour.

The naval engine of recent times is distinguished by a combination of strength, lightness, compactness, and power, which makes it the most remarkable of all the achievements of modern engineers and mechanics. This is exemplified by the later engines built for boats of the class here illustrated. The triple-compound engine has cylinders 14, 20, and 31½ inches diameter, 16 inches stroke, and taking steam, at 200 pounds pressure, from water-tube boilers rated at 1300 horse-power, and sometimes actually exceeding that figure. "White metal"

FIG. 120.—IRON CLAD AT 20 KNOTS.

is used in all bearings, and both oil and water are supplied to all especially important journals. The low-pressure valve is balanced by an adjustable arrangement, and the piston-rings are made of an alloy requiring no lubrication, thus securing, among other advantages, a better action of the condenser. The frames and all parts not necessarily cast are of forged steel.

General experience, in brief, indicates the compound engine as customarily employed to exhibit an increase in economy over the simple engine which it displaced amounting to about 30 per cent, and a superiority of 20 or 25 per cent of the triple-expansion engine, with steam at 140–160 pounds, over the compound at 90–100; while the latter has not been found to show much advantage with increasing pressures. The reasons for the facts are readily seen on studying, as elsewhere, the theory of the engine. Similarly the quadruple-expansion engine exhibits superiority, in less degree, at 200 pounds, over the triple-expansion.

The three-crank engine also is found to possess advantages over the two-crank, in efficiency of mechanism and smoothness of operation, and to demand, often, even less repair. With similar size of low-pressure cylinder, this form of triple-expansion engine may give considerably greater power than the compound, with less serious stresses on the working parts; and this difference, again, makes it practicable to build the former at as small cost as the latter, when of equal power.

The latest type of river and sound steamers is illustrated by the Plymouth of the Fall River Line between New York and New England, traversing Long Island Sound.

The dimensions of the Plymouth are as follows:

	Feet.	Inches.
Length over all	366	
Length on water-line	351	8
Breadth over guards	87	
Breadth of hull	50	
Depth at lowest point of sheer	21	
Draught of water, light	11	
Distance from keel to topmast-head	119	
Distance from keel to dome-deck	55	3
Distance from keel to top of house on dome	59	3

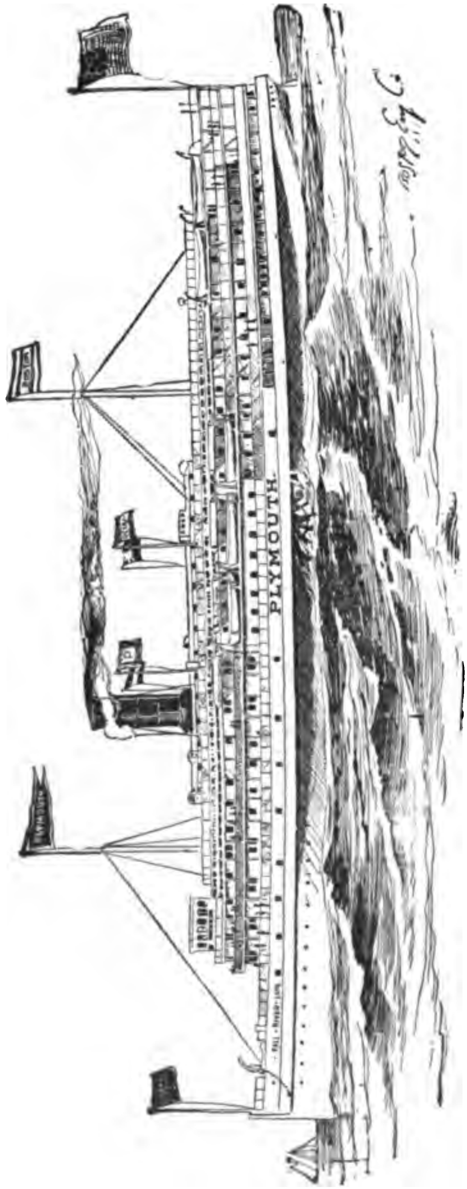


FIG. 121.—RIVER AND SOUND STEAMER.

The ship is constructed on the double hull, bracket plate and longitudinal system, securing safety for the ship as regards either sinking or destruction by fire. The designers and constructors were the same as of the Puritan, previously described.

The Plymouth is fitted with a four-cylinder, double-inclined, triple-expansion, direct-acting engine of 5500 indicated horse-power. The high-pressure cylinder, 47 inches diameter, takes steam at a pressure of 160 pounds per square inch. The intermediate cylinder is 75 inches in diameter. The high-pressure and intermediate eylinders are placed forward of the centre of the shaft, and are connected to crank-pins, placed at right-angles. Abaft the shaft are two low-pressure cylinders, each $81\frac{1}{2}$ inches in diameter. One low-pressure is connected to the same crank-pin as the high-pressure cylinder, and the other to the same crank-pin as the intermediate cylinder. All pistons have a stroke of 8 feet 3 inches. Each of the two low-pressure cylinders is supplied with its own air-pump and surface-condenser, with independent centrifugal circulating pump. The high-pressure cylinder alone has an adjustable drop cut-off. On all the other cylinders the cut-off is fixed.

The engine keelsons and frames are made of steel, strengthened in the usual manner with angles and intercostals. The wheels are of the feathering type, 30 feet diameter outside the buckets. Each wheel has 12 curved steel buckets, each being 4 feet wide and 13 feet 3 inches long.

43. **Adaptation of Structure** to economical requirements is evidently one of the essential elements of successful application of the steam-engine to best advantage. As will be shown elsewhere, for every pressure and every engine there is always a certain best ratio of expansion, all things considered; and the proper number of steam-cylinders in series in the multiple-cylinder engine is fixed by the steam-pressure adopted. It thus happens that, as pressures have risen, the compound engine has displaced the simple engine, at sea, and the "triple-expansion" engine has, at pressures exceeding about ten atmospheres (135 lbs. by gauge) begun to displace the older double-cylinder compound engine; and even the "quadruple-expan-

sion" engine, with its four cylinders in series, has been adopted for pressures considerably exceeding the latter.

The structure of these engines is essentially similar to that

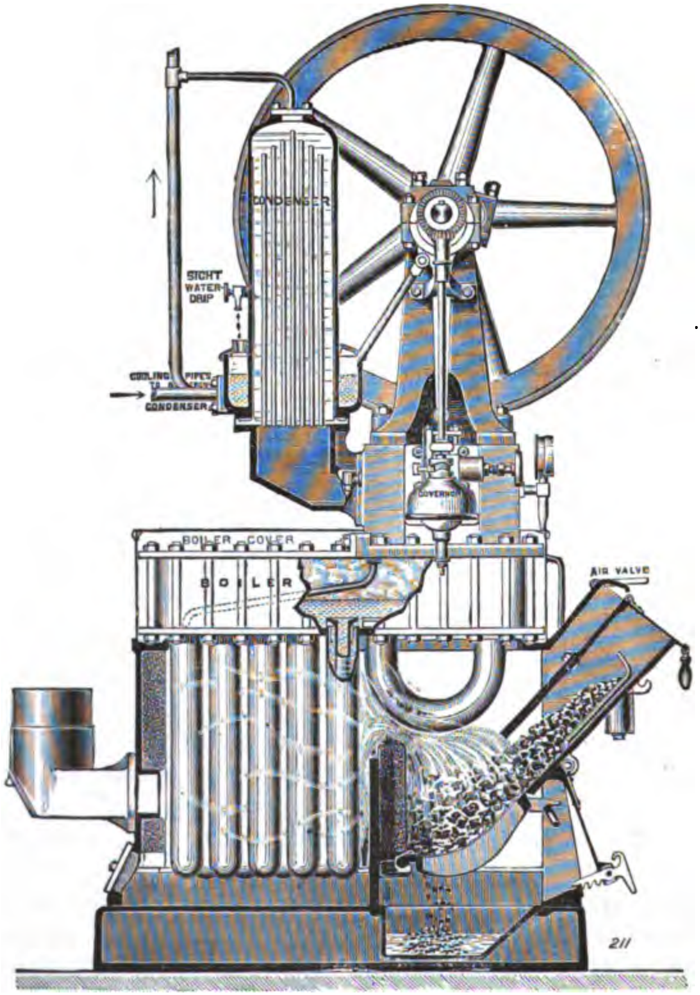


FIG. 122.—BAILEY-FRIEDRICH MOTOR.

of the older compound, the high-pressure cylinder of the latter becoming, in turn, an intermediate cylinder. These forms will be described more fully later.

In illustration of the above: The "domestic" or other small motors are often given peculiar and ingenious forms to secure automatic operation and relief from cost of attendance.

The Friedrich Motor is a combined engine and boiler. The engine is a high-expansion engine, fitted with a governor determining the amount of expansion automatically, according to the work.

A surface-condenser condenses the exhaust steam, and a feed-pump returns it to the boiler. Thus the water is used over and over again, and no incrustation takes place in the boiler.

It is stated that a four-horse engine of this kind requires about 135 pounds of coke in six hours.

The boiler generates its steam mainly in tubes suspended in a furnace extending the full width and length of the boiler. The boiler-top consists partly of the lower part of the engine-frame, which there forms a steam-dome, with the steam-cylinder suspended in it, and the remainder is a plate which is readily removed for access to the interior for inspection and cleaning.

The furnace is fitted with a fuel-hopper or magazine, and above this is an air-valve acted upon automatically by the steam, in such a manner as to lift it and pass air over the fire whenever the generation of steam is too rapid and the pressure too high, thus regulating the consumption of the fuel according to the demand. The whole is mounted upon a base-plate, fitted below the fireplace with an ash-pan, as seen, charged with water to keep the floor cool and preserve the grate-bars.

44. **Special Types** of steam-engine are occasionally used experimentally and temporarily, or are permanently employed where found to be specially adapted to some peculiar purpose. Thus the single-acting engine has been found to have its own special field; the Cornish engine was long used exclusively for a mine-pump; the rotary engine finds its place, and even a steam-turbine is successfully applied to driving machinery in which an enormous speed of rotation is demanded. The superiority of a rotary motion for a steam-engine is ap-

parently so evident that many attempts have been made to overcome the practical difficulties to which it is subject. One of these difficulties, and the principal one, has been the packing of the part which performs the office of the piston in the straight cylinder. The often claimed advantages of the rotary engine are the reduction in the size of the engine, claimed to result from the great velocity of rotation; the avoidance of great accidental strains, especially noticed in propelling ships; and a great saving of the power which is, erroneously, asserted to be expended in the reciprocating engine in overcoming the inertia while changing the direction of the motions. These advantages, so far as they exist, adapt the rotary engine, in an especial manner, to the purposes of steam fire-engines.

In the Holly steam fire-engine, seen in Fig. 125, eccentrics and sliding-cams, which are frequently used in rotary engines, are avoided. Corrugated pistons, or irregular cams, are adopted, forming chambers within the cases. In the engine the steam enters at the bottom of the case, and presses the cams apart. The only packing used is in the ends of the long metal cogs, which are ground to fit the case and are kept out by the momentum of the cams, assisted by a slight spring back of the packing-pieces. The friction on the pump, Fig. 124, is said to be less than in the engine. This is the reason given in support of the claim that the rotary engine forces water to a given distance with less steam-pressure than is necessary to drive reciprocating engines. The smaller amount of power necessary to do the work, the less strain and consequent wear and tear upon the whole machine, are said to make it durable and reliable. The pump being chambered, its liability to injury by the use of dirty or gritty water is lessened; and it is stated that it will last for years, pumping gritty water that would soon cut out a piston-pump.

This engine contains two rotating cams, each of which is also a gear having eight short teeth, arranged in pairs, with one long tooth and one deep space between. The short teeth are for the purpose of insuring that the two cams rotate

exactly together. The long teeth are abutments for the steam, forming, as they do, steam-tight joints with the walls

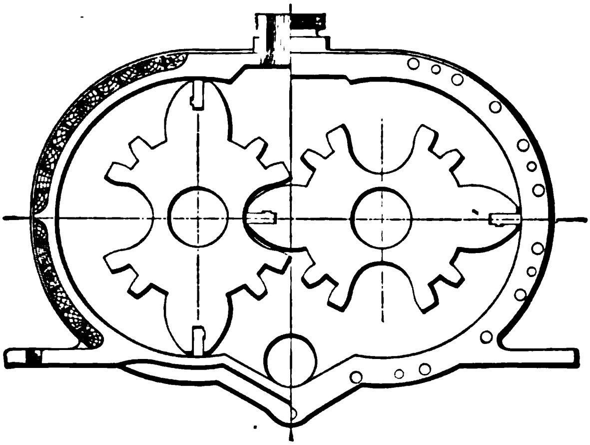


FIG. 123.—ROTARY ENGINE.

of the case in which they rotate, and with the deep spaces in which they engage. The steam entering at the bottom of the case tends to press the abutments apart and thus cause rotation of the pistons in opposite directions. The tightness of the joints of the teeth with the case is insured by packing-pieces set out by springs. The steam is discharged at the

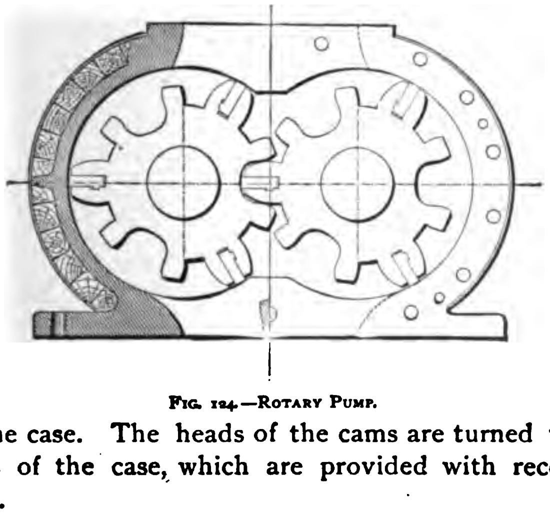


FIG. 124.—ROTARY PUMP.

top of the case. The heads of the cams are turned to fit the flat ends of the case, which are provided with recesses for lubricant.

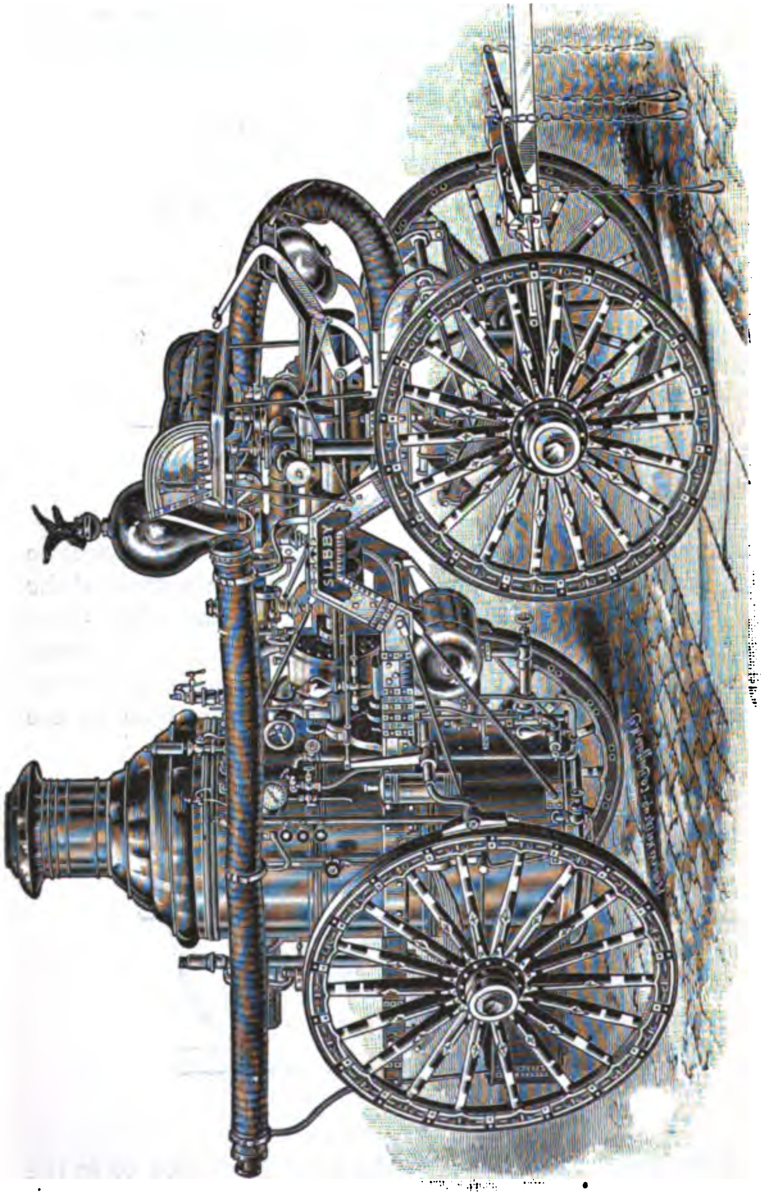


FIG. 195.—STEAM FIRE-ENGINE (ROTARY).

In the construction of the pump three long teeth are introduced to each cam, and fewer guide-teeth. The water enters at the bottom of the case, and is discharged at the top.

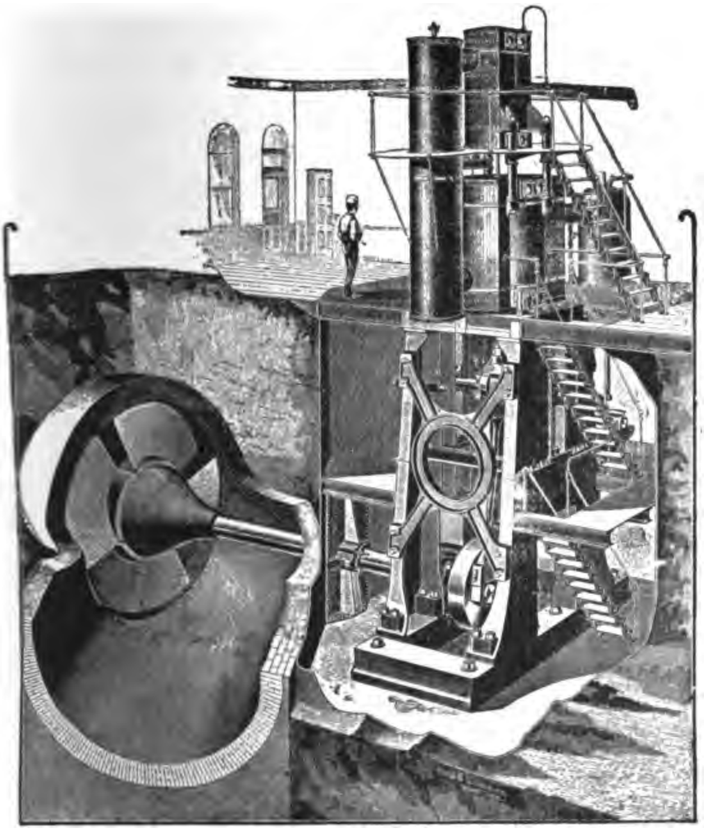


FIG. 126.—SCREW PUMPING-ENGINE.

The revolution of the pump-pistons in opposite directions causes a vacuum in the case, and the water is caught by the abutments and swept out of the case. The greater number of teeth is given in order to insure greater steadiness of stream than would be given by only two long teeth upon each piston.

The motion being continuous and the connections tight, the stream is unintermittent. The journals of the engine and pump run in long bearings. There are suitable stuffing-boxes to insure steam and water-tight joints for the shafts. The certainty of rotation of the cams is further insured by well-cut gear-wheels on the shafts outside the steam and water cases.

The steam-cams are given greater diameter than those for the water, to permit a greater water-pressure to be maintained; the steadiness of this water-pressure is further insured by an air-chamber. Engines of this class have now been in use many years.

A singular device, but one found effective for very low lifts, is illustrated in Fig. 126, as built by Allis for the city of Milwaukee. A vertical engine of economical type, and designed for a somewhat high speed of rotation, is connected to the shaft of a screw-propeller of suitable dimensions and proportions, but differing from the marine screw in the greater area of its blades. This raises water from a low level on the one side to three feet higher level with satisfactory economy. Its duty exceeds 70 millions.

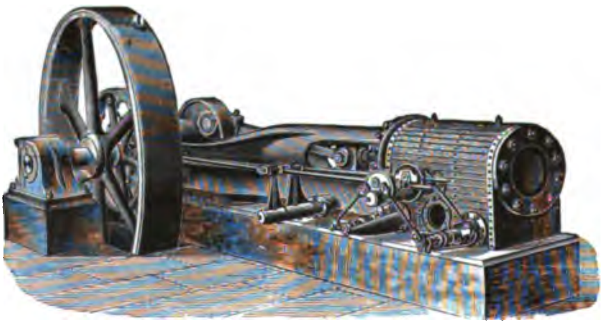


FIG. 127.—THE "AUTOMATIC" CORLISS ENGINE.

An interesting modification of the Corliss principle in the adaptation of the "automatic" system of shaft-governor regulation is illustrated in the accompanying engraving. In this arrangement, the Payne engine, the advantages of the peculiar kinematic movement of Corliss and of his form of valve,

are combined in the positive-motion system of gearing essential to the "high-speed" engine. As in some other engines, the steam- and exhaust-valves are here in the same shell, and the small clearance of this form of engine, the peculiar movement of the valves, and the exact regulation of the shaft-governor, and the high-speed system, are combined in a very compact machine.

The illustration herewith given represents a compound engine with automatic expansion-gear as designed by Fowler & Co. of Leeds, G. B., for stationary purposes. The use of rope-transmission, now in extensive use, is here exhibited, the fly-wheel being suitably grooved to carry it. The cut-off mechanism is adjusted by the governor seen on the horizontal shaft above the high-pressure steam-chest. The cut shows

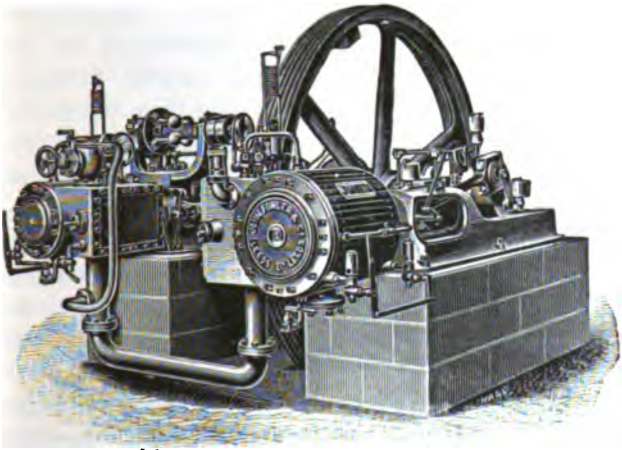


FIG. 128.—AUTOMATIC COMPOUND ENGINE.

well, also, the various important accessories of the engine: its pass-over steam-pipe, relief-valves, indicator-motion, and system of lubrication, as well as the general features of a carefully considered design.

The Steam-turbine constitutes a class of steam-engine which, although the first invented and familiar, as a type, to all engineers from the days of Hero the Younger, and known to have a high theoretical and moderately high actual effi-

ciency, has been only experimentally used until a very recent date. That of Hero has been illustrated in Fig. 1. The Atwater engine of about 1840 was of this type, and was said to be as economical as the engines of the time of equal power. Steam-turbines of the inward-flow type have been used by Gorman and others.*

The later "compound" steam-turbine has recently been somewhat extensively employed in the operation of dynamo-electric machinery. It consists of two sets of parallel-flow turbines set, in twin series, on one shaft on either side the induction-pipe, thus balancing. The passages are gradually enlarged as the volume of the steam increases with its progressive expansion.

The turbines thus alternate with their guide-blades, and both the vanes and the blades are carefully proportioned and set to secure maximum attainable efficiency at the proposed speed of rotation, their pitches and depths being suitably varied. Their theory is precisely that of piston-engines.

The computed efficiency, without allowances for wastes, is about 87 per cent. The actual consumption of steam is found to be 20 to 30 pounds per electrical horse-power produced, and per hour as steam-pressures rise from 60 to 90 pounds by gauge. The speed of rotation ranges from 5000 or 10,000 revolutions per minute upward, according to size and steam-pressure; 18,000 and 20,000 being common speeds for the smaller sizes.

Dow's turbine is an inward-flow wheel with concentric sets of guides and vanes in series, and is said to have attained 35,000 revolutions per minute, working regularly at 25,000, consuming 55 pounds of steam per horse-power per hour. Only the most perfect construction is here admissible.

The theory of this type of machine is that familiar to the hydraulic engineer, and the speeds of orifice for maximum efficiency are well known to be infinite in the Hero class of turbine and approximately one half the final velocity of flow in

* Rankine, p. 538.

the guide-blade turbine. Since these speeds are impracticable in their use, a certain loss of energy is thus inevitable. In compensation for this loss, in the steam-turbine, is the fact that it is not subject to that fluctuation of temperature of parts exposed to contact with the steam which results in large wastes by cylinder-condensation in the common forms of steam-engine. A gain of from 25 to 50 per cent, as compared with the latter, in this way, is to be counted upon.

The Dow turbine, as built for work, in connection with the Howell torpedo, gives an average of about 11 horse-power in coming up to speed in regular working, at 60 pounds steam-pressure, and weighs from 100 to 150 pounds, or not far from 13 pounds per horse-power.* Its fly-wheel rim attains a speed of nearly 7 miles an hour at 10,000 revolutions per minute. The designer estimates its power at 150 pounds steam-pressure and the same speed at 40 horse-power, or one horse-power to 3.75 pounds weight, and states that this may be still further reduced to the extraordinary minimum of 2½ pounds weight per horse-power, a figure well within the estimated allowable maximum for use in aëronautic work.

The steam-turbine of Parsons, Fig. 129, is an engine consisting of a series of turbines, the different pairs of guides and wheels being so placed that the fluid passes successively from one pair to the next. Of the two forms, radial and axial flow, only the latter have been used here. Two series of cylindrical turbines are used, arranged symmetrically to the right and left of the central steam-inlet, the exhaust taking place from the two ends. In this manner a balance is obtained, and the bearings are relieved of end-pressure. Oil is forced through the bearings by a pump. This engine has driven a torpedo-boat 100 feet long and of 44¾ tons displacement at the rate of 32¾ knots, developing 2100 horse-power with machinery weighing, with water in the boilers, 22 tons, thus producing 100 horse-power per ton and 50 per ton of displacement.

Such engines have been successfully employed in driving

* Electrical World ; April 18, 1891.

electric machinery and in "spinning" the "fly" of the Howell torpedo. For alternating electric currents, this system pos-

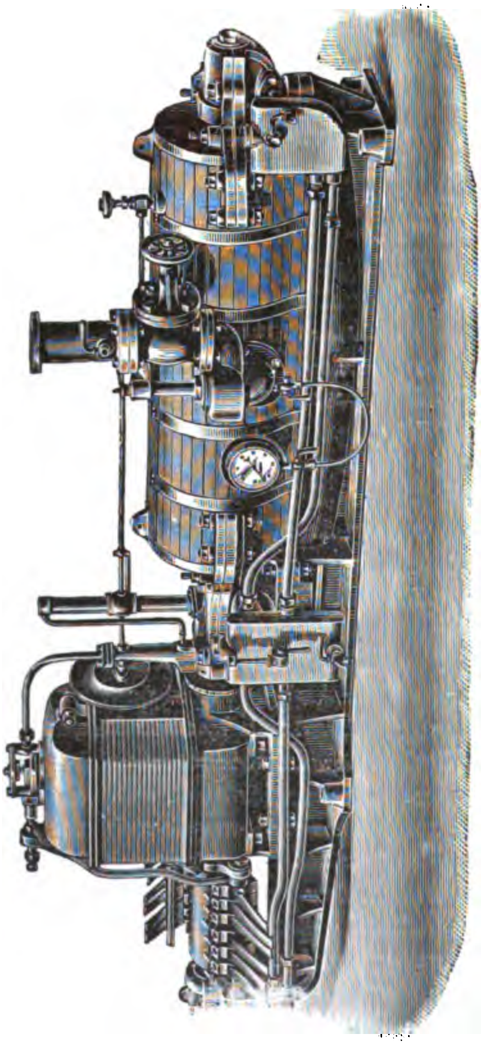


FIG. 129.—PARSONS' STEAM-TURBINE.

sesses the peculiar advantage of permitting a "dynamo" to be employed having but two poles. It may be readily driven continuously at speeds exceeding 10,000 revolutions per minute,

and, like the Dow turbine, elsewhere referred to, has been driven at 20,000 and upward. With the lower speeds of revolution usual with ordinary engines, the number of poles required generally approximates the quotient 12,000 divided by the speed of engine, if directly connected. (See pp. 62, 225.)

The best of these machines have demanded from 16 pounds of steam per horse-power per hour, upward, according to pressure employed. It may be assumed that they will require not far from the weight

$$W = \frac{a}{\sqrt{p_1}};$$

where p_1 lies between 50 and 200 pounds per square inch by gauge, and the apparatus is operated under favorable conditions; the value of a lying between 250 and 400 with dry steam.

The "Solar Engine" of Ericsson is a special design, receiving solar heat on a conical mirror which reflects it into the boiler, where it is stored in the steam. One horse-power was reported as produced by each 100 square feet section of beam in clear sunlight.

"*Experimental Engines*," or steam-engines designed especially for purposes of instruction and research, are now frequently constructed, and especially in equipping European schools. Such engines are illustrated in the frontispiece of this volume, as built for Owens College, Manchester, G. B.;* while other forms designed by American engineers and as constructed for Sibley College, Cornell University, and for the Massachusetts Institute of Technology, will be represented in a later chapter (Vol. II.) on Engine Trials.

In the design of such engines, the problem is ordinarily to make all adjustments cover a wide range; in order that the laws affecting variation of pressures, temperatures, speeds, steam-distribution, as determining efficiency, may be illustrated; as well as to secure a means of investigating problems still unsolved and of checking results previously obtained but

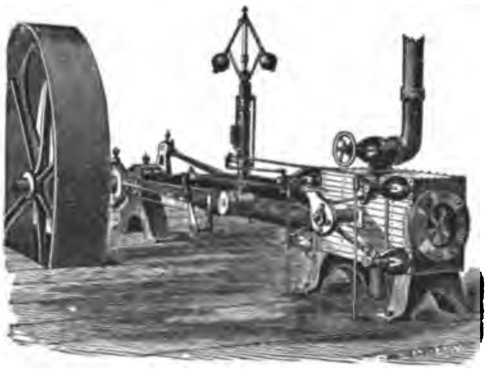
*Triple-expansion Engine Trials; by Professor Reynolds; Proc. Inst. C. E., 1889, No. 2407. See, also, Trans. A. S. M. E., 1895, No. DCXLII.

requiring confirmation. The engine illustrated consists of a triple-expansion combination so arranged that each element may be worked and tested independently, if desired, as well as either compounded or triple-expansion. The type adopted is that familiar in marine engineering, with inverted cylinders, jacketed on sides and ends, and each jacket separately piped to permit its action to be ascertained.

The working-pressure is 200 pounds as a maximum; the piston-speed may attain 1000 feet per minute; the Meyer expansion-valves give a range of expansion from $r = 1.5$ to $r = \infty$. The cylinders are 5, 8, and 12 inches diameter, 10, 10, and 15 inches stroke. The engine can be worked either condensing or non-condensing. This engine was designed under the supervision of Professor Reynolds and built by Messrs. Mather & Platt.

A surface-condenser is used containing 160 square feet surface, and is served by an air-pump, driven by the largest engine, 9 inches diameter and 4½ inches stroke. Hydraulic brakes are employed, which are simply adaptations of the centrifugal pump.

On trial the engine worked admirably and economically, demanding but 1.33 pounds of fuel per horse-power per hour; the efficiency being 0.20 at 200 pounds pressure. The efficiency of machine was about 0.80. The performance of the engine, in all respects, is reported to be eminently satisfactory. (See § 128, Chap. V.)



CHAPTER III.

THE PHILOSOPHY OF THE STEAM-ENGINE.

45. **The Scope of the Philosophy of the Steam-engine,** and a complete history of the development of the Theory of the Steam-engine, would include, first, the history of the Mechanical Theory of Heat; secondly, the history of the Science of Thermodynamics, which has been the outgrowth of that theory; third, the history of the application of the Science of Heat-transformation to the case of the Steam-engine; and, fourthly, an account of the completion of the Theory of the Steam- and other Heat-engines by the introduction of the theory of losses by the more or less avoidable forms of waste, as distinguished from those necessary and unavoidable wastes indicated by the pure theory of thermodynamics. The first and second of these divisions are treated of in works on thermodynamics and in treatises on physics. The third division is briefly considered, and usually very incompletely, in treatises on the steam-engine; while the last is of too recent development to be the subject of complete treatment, as yet, in any existing works. Our principal object is, here, simply to collect into a condensed form, and in proper relations, these several branches of the subject, leaving for an appropriate time and place that more full and complete account which might now, for the first time in history, be prepared.

46. **The Nature of the Processes** observed in the operation of the steam-engine are such as will illustrate many of the most important principles and facts which constitute the physical sciences. The steam-engine is an exceedingly ingenious but very imperfect machine for transforming the heat-energy

obtained by the chemical combination of a combustible with the supporter of combustion into mechanical energy. The original source of this energy is found far back of its first appearance in the steam-boiler. It had its origin at the beginning. When the solar system had been formed from the nebulous chaos of creation, the glowing mass which is now called the sun was the depository of a vast store of heat-energy, which was thence radiated into space and showered upon the attendant worlds in inconceivable quantity and with unmeasured intensity. During the past life of the globe, the heat-energy received from the sun was partly expended in the production of forests and the storage of an immense quantity of carbon, which had previously existed in the atmosphere, combined with oxygen, as carbonic acid. The geological changes which buried these forests resulted in the formation of coal-beds and the storage of a vast amount of carbon, of which the affinity for oxygen remained unsatisfied until finally uncovered by man. Thus we owe to the heat and light of the sun, as was pointed out by George Stephenson, the incalculable store of energy upon which the human race is dependent for life.

This coal, thrown upon the grate in the steam-boiler, takes fire, and, uniting again with the oxygen, sets free heat in precisely the same quantity that it was received from the sun and appropriated during the growth of the tree. The actual energy thus rendered available is transferred, by conduction and radiation, to the water in the steam-boiler, converts that water into steam; and its mechanical effect is seen in the expansion of the liquid into vapor against superincumbent pressure. Transferred from the boiler to the engine, the steam is there permitted to expand, doing work, and the heat-energy with which it is charged becomes partly converted into mechanical energy, and is applied to useful work.

Thus we may trace the store of energy received from the sun and contained in our coal through its several changes until it is finally set at work; and we might go still farther and observe how, in each case, it is again usually re-transformed and again set free as heat-energy.

47. **The Nature, Sources, and Transformations of Energy** in these several processes are thus easily traced. The transformation which takes place in the furnace is a chemical change; the transfer of heat to the water and the subsequent phenomena accompanying its passage through the engine are physical changes, some of which require for their investigation abstruse mathematical operations. A thorough comprehension of the principles governing the operation of the steam-engine, therefore, can only be attained after studying the phenomena of physical science with sufficient minuteness and accuracy to be able to express with precision the laws of which those sciences are constituted. The study of the philosophy of the steam-engine involves the study of Chemistry and Physics, and of the science of Energetics, of which the science of Thermodynamics is a branch. This sketch may, therefore, include an outline of the growth of the several sciences which together make up its philosophy, and especially of the science of thermodynamics, which is peculiarly the science of the heat-engines.*

48. **The Chemical Principles** involved in the action of all the steam-engines are those illustrated in the combustion of the fuels. All essential elements of this part of the philosophy of heat-engines are now at least approximately known, and it is perfectly possible for the engineer, knowing the composition and physical structure of his fuel, to compute very exactly the quantity of heat-energy stored in its mass and the amount probably to be realized in the furnace in which it is consumed and stored in the working-fluid to be sent forward into his engine.

In all cases, he is supplied, as fuel, with a certain known composition of carbon, hydrogen, and their compounds, unimportant proportions of other combustible elements, as sulphur, and a quantity of incombustible mineral matter, forming, finally, ash and clinker, or cinders. The union of these com-

* For a somewhat detailed account of the early and mediæval progress of the sciences, see the "History of the Steam-engine," by the Author, chapter VII; International Series; New York, D. Appleton & Co.

bustible elements and compounds with the oxygen of the air produces a definite and easily calculable amount of heat-energy, of which a part, equally easy of computation when the extent, nature, location, and arrangement of the absorbing, or "heating," surfaces are known, is taken up for useful purposes; while the rest is sent up-chimney or otherwise wasted. The physical, as well as the chemical, character of the fuels and the greater or less completeness of their combustion and the consequent character of the discharged furnace-gases aids in determining the final result and the total efficiency of the system.

49. **The Physical Principles** involved in the storage, transfer, and utilization of heat in the steam-engine are those which relate to the transfer, storage, and re-transfer of heat-energy in the passage of that energy from the furnace-gases to the boiler, its storage in the water and steam, and its transfer to the engine, with continuous loss and waste, until finally, a part being transformed into mechanical energy and more or less usefully applied, the remainder is finally discharged from the engine and entirely lost and wasted, as a source of power. Conduction, radiation, convection of heat, and heat conversion into other kinds of energy, are the physical phenomena involved in these operations.

In some cases these processes are somewhat obscure and remained for many years but little understood. This is especially the fact with respect to those operations which go on within the engine-cylinder in the course of the cycle there performed and which involve the introduction of the steam and its temporary storage into a cooler space in which it is partly condensed by surrender of heat to the enclosing walls; the gradual reduction of temperature and pressure of the steam with increasing expansion of volume, and with restoration of that heat, in part, to the fluid, and finally the discharge of the steam from the cylinder at a still further reduced temperature, with complete restoration of the heat previously stored in the walls of that vessel. The application of the principles of physics to this series of changes is quite as essential to a complete theory of the real heat-engine as is that

of the principles of thermodynamics to the processes of transformation of energy.

50. The Mechanical Principles which are included in a complete theory of the action of the heat-engines will be illustrated in the chapters on the design of the various parts of the engine. It is sufficient here to present a general outline of the modern science.

The science of mechanics is of comparatively recent date, and with the publication of Newton's *Principia* became thoroughly consistent and logically complete, so far as was possible without a knowledge of the modern principles of energetics. Newton's enunciations of the laws of motion were the basis of the whole science of dynamics, as applied to bodies moving freely under the action of applied forces, either constant or variable. They are as perfect a basis for that science as are the primary principles of geometry for the whole beautiful structure which is built up on them.

The three perfect qualitative expressions of dynamical law are:

1. Every free body continues in the state in which it may be, whether of rest or of rectilinear uniform motion, until compelled to deviate from that state by impressed forces.

2. Change of motion is proportional to the force impressed, and in the direction of the right line in which that force acts.

3. Action is always opposed by reaction; action and reaction are equal, and in directly contrary directions.

We may add to these principles a definition of a force, which is equally and absolutely complete:

Force is that which produces, or tends to produce, motion, or change of motion, in bodies. It is measured statically by the weight that will counterpoise it, or by the pressure which it will produce, and dynamically by the velocity which it wil' produce, acting in the unit of time on the unit of mass.

The quantitative determinations of dynamic effects o forces are always readily made when it is remembered that the effect of a force equal to its own weight, when the body is free to move, is to produce in one second a velocity of 32.2

feet per second, which quantity is the unit of dynamic measurement.

Work is the product of the resistance met in any instance of the exertion of a force, into the distance through which that force overcomes the resistance.

Energy is the work which a body is capable of doing, by its weight or inertia, under given conditions. The energy of a falling body, or of a flying shot, is about $\frac{1}{64}$ its weight multiplied by the square of its velocity, or, which is the same thing, the product of its weight into the height of fall or height due its velocity. These principles and definitions, with the long-settled definitions of the primary ideas of space and time, were all that were needed to lead the way to that grandest of all physical generalizations, the doctrine of the persistence or conservation of all energy, and to its corollary declaring the equivalence of all forms of energy, and also to the experimental demonstration of the transformability of energy from one mode of existence to another, and its universal existence in the various modes of motion of bodies and of their molecules.

Experimental physical science had hardly become acknowledged as the only and the proper method of acquiring knowledge of natural phenomena at the time of Newton; but this soon became a generally accepted principle. In physics, Gilbert had made valuable investigations before Newton, and Galileo's experiments at Pisa had been examples of similarly useful research. In chemistry, it was only when, a century later, Lavoisier showed by his splendid example what could be done by the skilful and intelligent use of quantitative measurements, and made the balance the chemist's most important tool, that a science was formed comprehending all the facts and laws of chemical change and molecular combination. We can now see how, in all the physical sciences, four primitive ideas are comprehended: matter, force, motion, and space—which latter two terms include all relations of position. These are the fundamental ideas of mechanics.

Based on these notions, the science of mechanics compre-

hends four sections, which are of general application in the study of all physical phenomena. These are:

Statics, which treats of the action and effect of forces only.

Kinematics, which treats of relations of motion simply.

Dynamics, or kinetics, which treats of simple motion as an effect of the action of forces.

Energetics, which treats of modifications of energy under the action of forces, and of its transformation from one mode of manifestation to another, and from one body to another.

51. **Energetics and Thermodynamics** are the broader and the narrower codes of similar law. Under the latter of the four divisions of mechanical philosophy is comprehended that latest of the minor sciences, of which the heat-engines, and especially the steam-engine, illustrate the most important applications—*Thermodynamics*. This science is simply a wider generalization of principles which have been established one at a time, and by philosophers widely separated both geographically and historically, by both space and time, and which have been slowly aggregated to form one after another of the sciences, and out of which we are gradually evolving wider generalizations, and thus tending toward a condition of scientific knowledge which renders more and more probable the truth of Cicero's declaration: "One eternal and immutable law embraces all things and all times." At the basis of the whole science of Energetics lies a principle which was enunciated before Science had a birthplace or a name:

All that exists, whether matter or force, and in whatever form, is indestructible, except by the Infinite Power which has reated it.

That matter is indestructible by finite power became ad- nitted as soon as the chemists, led by their great teacher Lavoisier, began to apply the balance, and were thus able to show that in all chemical change there occurs only a modification of form or of combination of elements, and no loss of matter ever takes place. The "persistence" of energy was a later discovery, consequent largely upon the experimental determination of the convertibility of heat-energy into other

forms and into mechanical work, for which we are indebted to Rumford and Davy, and to the determination of the quantivalence anticipated by Newton, shown and calculated approximately by Colding and Mayer, and measured with great probable accuracy by Joule and Rowland.

52. **The Ideal and the Real Engine** must be clearly distinguished in all that follows. The ideal engine of the earlier method is one in which only thermodynamic processes occur. Only transfer of heat from point to point in its cycle of operations, and the conversion of thermal into mechanical energy or the reverse, are assumed as possible; and the problem studied is that of determining what, under certain specified conditions, is the efficiency of the engine, the proportion of net work performed to gross energy demanded for its accomplishment. Such an engine must be constructed of materials without permeability to heat, without conducting or heat-storing capacity, absolutely free from friction, and incapable of yielding to impressed forces. It is a purely ideal case.

The real engine, on the other hand, must be composed of such materials as are available to the engineer. They must have strength, stiffness, toughness, and endurance under load and wear, and must be capable of being given the desired shape and proportions at the least possible expense. Only iron and steel, copper and the familiar alloys meet these requirements; and, practically, all engines are composed of these substances, and all have their working cylinders made of cast-iron, a substance of high conducting and storing power for heat. These facts make an enormous difference in the behavior of the engine both as respects its utilization of heat and its useful application of the energy produced within its working cylinder by heat-transformation. Large quantities of heat are necessarily wasted, in the manner already indicated above, when discussing the physical principles involved in the action of the engine; and a considerable fraction of the power exerted by the steam on the piston of the engine is, in the actual case, lost in the friction of its own journals.

Thus the real case must be carefully distinguished from the

ideal, and the pure thermodynamic theory of the latter constitutes but one element of the theory of the former.

53. **The Scientific Problem** which confronts the student of the theory of the steam-engine, as a practical case, is thus seen to be the determination of the quantity of heat-energy stored in a given fuel; the proportion which may be reasonably expected to be developed by its combustion; the amount which should be taken up and stored for useful application in a steam-generator, and the balance wasted at the chimney and elsewhere; of that which may be taken to the engine through a steam-pipe of known size and condition, of that which will be probably wasted by conduction and radiation, *en route*, or at the engine and within its cylinder; and finally, the quantity which will be converted into work and, of this, the proportion that will be capable of useful application.

The determination of the latter quantity is the measurement of a balance after all wastes are deducted; and the efficiency sought is the ratio of this quantity to the mechanical equivalent of all heat-energy supplied to the engine, or to that produced at the furnace, as the case may demand.

In detail, therefore, the problem to be solved includes the application of known chemical, physical, and mechanical principles to the determination, one by one, of all these quantities of energy, step by step, from the furnace to the driving-shaft of the engine, and the summation, at each step, of such quantities received or paid out in such manner that a final balance-sheet may be constructed exhibiting every item on both sides the account, and permitting the answering of any question that may arise respecting the receipt and expenditure of energy and mechanical power.

54. **An Outline of the Progress of Science** in the development of the philosophy of the steam-engine may be appropriately here briefly given. It properly begins with the history of the older philosophies; but its useful elements, and its actual applications, only date back to a very recent period. As will be seen, the physical sciences have all had an exceedingly slow growth until within the last two or three centuries. The abso-

lute impossibility of their promotion except through continuous experimentation, and the inability of mankind to construct the apparatus of research until modern times, would have caused this late development of sciences of this class, even had the true scientific spirit existed and the scientific method been known earlier.

The physical sciences have, since the beginning of the seventeenth century, had independent and uninterrupted growth; but it has been irregular, spasmodic, and unsymmetrical. The science of applied mechanics, as distinguished from its purely mathematical branches, had its origin with Galileo in the first and Newton in the second half of that century; chemistry may be said to have become a science under the hand of Lavoisier at the close of the eighteenth century; physics had a longer period of incubation, from the days of Gilbert, and energetics and its minor branch, thermodynamics, have only been conceived and organized into sciences in the nineteenth century.

Throughout this whole period of modern scientific work, the patient student and careful observer will see that these various sciences, now seemingly independent, are becoming established in closer and closer relations, and are gradually coming to illustrate continually more and more clearly their unquestionable mutual interdependence. All phenomena of motion and change of molecular relation, whether in physics, chemistry, or mechanical action, are subject to the laws of mechanics and of energetics, and a common science must probably sooner or later come to comprehend all.

In what follows the development of the science of thermodynamics and the gradual construction of the philosophy of the heat-engines only will be considered.* The student is, however, advised to study carefully and in a philosophical manner the development of all, and especially in their mutual relations, and in their bearing upon the science of energy, as mechanical and as molecular, and as a science of energy-transformations.

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* For a more detailed account see the encyclopædias, or consult the Author's "History of the Steam-engine," chapters VII, VIII.

55. **The Origin of the "Mechanical Theory of Heat,"** as is now well understood, dates, as a speculation, from the days of the earliest philosophies. The contest which raged with such intensity, and sometimes acrimony, among speculative men of science, during the last century, was merely a repetition of struggles of which we find evidences, at intervals, throughout the whole period of recorded history.* The closing period of this, which proved to be an important revolution in science, marked the beginning of the nineteenth century. It was inaugurated by the introduction of experimental investigation directed toward the crucial point of the question at issue. It terminated, about the middle of the century, with the acceptance of the general results of such experiment by every scientific man of acknowledged standing, on either side the Atlantic. The doctrine that heat was material, and its transfer a real movement of substance from the source to the receiver of heat, was thus finally completely superseded by the theory, now become an ascertained truth, that heat is a form of energy, and its transformation a change in the location and method of molecular vibration. The Dynamical Theory of Heat was first given a solid basis by the experiments of Count Rumford (Benjamin Thompson), in 1796–7—of which an account was given in a paper read by Rumford before the Royal Society of Great Britain in 1798,—by the experiments of Sir Humphry Davy in 1798–9, and by the later and more precise determinations of the value of the mechanical equivalent of heat by Joule and others.

James Prescott Joule, as early as 1843, obtained a series of results varying in quantity from 587 to 1026, from which he deduced an equivalent of 770 foot-pounds by the friction of water in small pipes. In the following year Mr. Joule gave a mean value of 802 foot-pounds. In 1845 he found 890 as the

* The main portion of what follows relating to this subject is abstracted from a paper read by the Author before the British Association for Advancement of Science, Montreal meeting, 1884. For the full paper see Trans. B. A. A. S., 1884—"On the Theory of the Steam-engine;" also "The Development of the Philosophy of the Steam-engine;" R. H. Thurston; N. Y., 1889.

value of the equivalent. Two years later he obtained 781.5 and 782.1 respectively; the mean of which is 781.8. He, in 1849, undertook a final determination of the equivalent, and carried out a series of forty experiments on the friction of water, fifty on the friction of mercury, and twenty on the friction of cast-iron plates, from which he deduced the value, 772 foot-pounds, which was accepted for a generation. His later determination, made for the British Association, 1876, was 774.1, with a possible error of small amount. Still later determinations indicate a higher value.

Julius Robert Mayer was engaged, at the same time, upon investigations of equal importance, carried on in an entirely different manner. In 1840, a physician on the island of Java, he noticed that the venous blood of his patients was unusually red. He concluded that it was owing to the fact that a less amount of oxidation of the tissues of the body would keep up the bodily heat in a hot country than would be required in a colder one. Following up this thought, he came to the conclusion that a fixed relation must exist between heat and work. In 1842 he made the attempt to determine this relation numerically. Professor Tyndall thus describes his reasoning: "It was known that a definite amount of air, in rising one degree in temperature, can take up two different amounts of heat. If its volume be kept constant, it takes up one amount; if its pressure be kept constant, it takes up a different amount. These two amounts are called the specific heats under constant volume and under constant pressure. The ratio of the first to the second is as 1 : 1.421." Dr. Mayer "first saw that the excess .421 was not, as then universally supposed, heat actually lodged in the gas, but heat which had been actually consumed by the gas in expanding against pressure. The amount of work here performed was accurately known; the amount of heat consumed was also accurately known; and from these data Mayer determined the mechanical equivalent of heat. Even in this first paper he is able to direct attention to the enormous discrepancy between the theoretic power of the fuel consumed in steam-engines and their useful effect." "As regards the mechani-

cal theory of heat, this obscure Heilbronn physician, in the year 1842, was in advance of all the scientific men of the time."

In a paper read before the Royal Society in 1878, Joule stated that, taking the unit of heat as that which can raise a pound of water (weighed in a vacuum) from 60° to 61° F. on the mercurial thermometer, its mechanical equivalent, reduced to the sea-level and to the latitude of Greenwich, is 772.55 foot-pounds. Favre deduced 753 from the friction of steel on steel, and 807 from the heat absorbed by an electromagnetic engine for the production of work. Hirn deduced 787 from the friction of liquids, and 775 from the compression of lead. Quintus Icilius deduced 714½ from the heat developed in an electric circuit. By comparing the work expended in revolving the plate of a Holtz electrical machine with the heat produced by the resulting current, Rosetti deduced 776.1 foot-pounds. Le Roux, from the heat produced by rotating a tube full of water in a magnetic field, found 835. Violle, by similar experiments on disks of metal in the place of water, found 793.2 with copper, 794.3 with tin, 797.3 with lead, and 792.7 with aluminium. Bartoli deduced 771.12 from the friction of mercury in small tubes. By a careful study of the velocity of sound in gases, Regnault re-determined the ratio of the two specific heats of gases used by Mayer in his first calculation. Regnault's result was 1.3945, instead of 1.421; and from this and other data Mayer's calculation, repeated, gave 794.8.

Prof. Henry A. Rowland finally made a determination of the equivalent, and his investigations involved many difficult problems in thermometry. He found that the specific heat of water is greater near the freezing-point than at and near 80°. Rowland's result gives the mechanical equivalent of heat as 778 foot-pounds at 39.2° F., the temperature being measured by a mercurial thermometer, and 783 foot-pounds if by an air-thermometer.

The value of the mechanical equivalent of heat is thus, very possibly,

778 ft.-lbs. per B. T. U.;
426.8 kilogrammetres per calorie;

and is considered probably correct to within 0.003 of its own value; i.e., it may be as low as 776 or as high as 780.

56. The Science of Thermodynamics has for its essential basis the established fact of the dynamical nature of heat, and the fact of the quantivalence of two forms of energy—heat and mechanical motion, molecular energy and mass energy. Resting, as it does, on fundamental, experimentally determined, principles, it could have no existence until, during the early part of the present century, these phenomena and these truths were well investigated and firmly established.

The first period of the development of the science was occupied almost exclusively by the exposition of the dynamical theory of heat, which lies at the bottom of the whole. Mohr, in 1837; Séguin, in 1839; Mayer, of Heilbronn, in 1842; and Colding, in 1843, each took a step into a field, the limits of which and the importance of which they could at that time hardly have imagined. Mayer had a very clear conception of the bearing of the new theory of heat upon dynamics, and exhibited remarkable insight into the far-reaching principles of the new science. He collated the facts more exactly determined later by Joule and others with the principle of the conservation of energy, and applied the rudiments of a science thus constructed to the calculation of the quantity of carbon and expenditure of heat which are unavoidably needed by a mountain-climber, doing a given quantity of work, in the elevation of his own body to a specified height. The work of Mayer may be taken as representing the first step in the production of a Science of Thermodynamics, and in the deduction of consequences of the fact which had, until his time, so seldom engaged the attention of men of science. It was only at about the middle of the nineteenth century that it began to be plainly seen that there existed such a science, and that the dynamic equivalence of heat, and energy in the mechanical form, was but a single fact, which must be taken in connection with the general principles of the persistence of energy, and applied in all cases of performance of work by expenditure of heat through the action of elastic bodies.

In 1850, Clausius* adapted Carnot's investigations of the correct theory of thermodynamics, to accord with the laws of modern thermodynamics. Clausius † then stated Carnot's principle as follows: ‡

Whenever heat is converted into work, *another quantity* of heat must, during the working cycle, be transferred from a hotter body to a colder body; the amount transferred depends only on the temperatures between which the transfer is effected, and not on the nature of the body acting as its vehicle.

This is Carnot's principle, and a direct consequence of the second law of thermodynamics.

57. **The Theory of the Steam-engine,** like every other scientific system, rests upon a foundation of facts ascertained by experiment, and of principles determined by the careful study of the laws relating to those facts, and controlling phenomena, properly classed together by that science. Like every other element entering into the composition of a scientific system, this theory has been developed subsequently to the establishment of its fundamental facts, and the history of progress in the art to which it relates shows that the art has led the science from the first. The theory of the steam-engine includes all the phenomena and all the principles involved in the production of power, by means of the steam-engine. from the heat-energy derived from the chemical combination of a combustible with the oxygen of the air acting as a supporter of the combustion. The remaining portion of this chapter will be devoted to the tracing of the growth of the theory of the steam-engine, simply as a mechanical instrument for transformation of the one form of energy into the other—of the molecular energy of heat-motion, as stored in the vapor of water, into mass-energy, or mechanical energy, as applied to the driving of mechanism. The theory thus limited includes a study of the thermodynamic phenomena, as the principal

* Poggendorff's Annalen, 1850. † Ibid., 1854, vol. XCII.
‡ See Eddy; p. 9.

and essential operations involved in the performance of work by the engine; it further includes the consideration of the other physical processes which attend this main function of the engine, and which, inevitably and unavoidably, so far as is to-day known, concur in the production of a waste of energy.

Of all the heat sent forward by the steam-boiler to the engine, a certain part, definite in amount and easily computed when the power developed is known, is expended by transformation into mechanical energy; another part, equally definite and easily calculated, also, is expended as the necessarily occurring waste which must take place in all such transformations, at usual temperatures of reception and rejection of heat; still another portion is lost by conduction and radiation to surrounding bodies; and, finally, a part, often very large in comparison with even the first and principal of these quantities, is wasted by transfer, within the engine, from the induction to the eduction side, "from steam to exhaust," by a singular and interesting process, without conversion into useful effect, and by the familiar processes of transfer. The science of thermodynamics only takes cognizance of the first and second, which are sometimes among the smallest, of these expenditures. The science of the general physics of heat takes cognizance of the others and enables us to approximately compute their magnitude.

The Science of the Steam-engine must, like every other branch of applied science, be considered as the result of two distinct processes of development: the one is what may be called the experimental development of the subject; the other is the purely theoretical progress of the science. So far as the useful application of correct principles to the improvement of the machine is concerned, the latter has always, as is usually the case elsewhere, been in advance of the former in its deduction of general principles; while, as invariably, the former has kept far in advance, in the working out of practically useful results, and in the determination of the exact facts where questions of economic importance have arisen.

58. Carnot's Work lies at the foundation of the science of the steam-engine, and its exposition may be found in his

"*Réflexions sur la Puissance Motrice du Feu.*" * He assumed the truth of the theory of substantial caloric; nevertheless, in his development of the theory of heat-engines, he enunciated some essential principles, and thus laid the foundation for a theory of the steam-engine which was given correct form, in all its details, as soon as the dynamical theory was taken for its foundation-principle. Carnot asserts that "the motive power of heat is independent of the means taken to develop it; its amount is determined, simply, by the temperature of the bodies between which the heat is transferred. Wherever there exists a difference of temperature, there may be a development of power. The maximum amount of power obtainable by the use of steam is the maximum obtainable by any means whatever. High-pressure engines derive their advantage over low-pressure engines simply from their power of making useful a greater range of temperature." He made use of the device known as the "Carnot Cycle," exhibiting the successive expansions and compressions of the working fluid in heat-engines, in the process of change of volume and temperature, while following the series of changes which gives the means of transformation of heat into power with final restoration of the fluid to its initial condition, showing that such a complete cycle must be traversed in order to determine what proportion of the heat-energy available can be utilized by conversion into mechanical energy. This is one of the most essential of all the principles comprehended in the modern science. This "Carnot Cycle" was, afterward, represented graphically by Clapeyron.

Carnot shows that the maximum possible efficiency of fluid is attained, in heat-engines, by expanding the working fluid from the maximum attainable temperature and pressure down to the minimum temperature and pressure that can be permanently maintained on the side of condensation or rejection, i.e., if we assume expansion according to the hyperbolic law,

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* Réflexions sur la Puissance Motrice du Feu; Paris, 1824; republished by Gauthier-Villars; Paris, 1878. See, also, the Author's edition: "Reflections on the Motive Power of Heat, by N.-L.-Sadi Carnot;" with notes; N. Y., J. Wiley & Sons.

by adopting, as the ratio of expansion, the quotient of maximum pressure divided by back pressure. He further shows that the expansion, to give maximum efficiency, should be perfectly "adiabatic." * He even suggests that the adiabatic expansion of steam may result in its own condensation, a fact a generation later discovered and proven by Rankine and Clausius. These principles have been recognized as correct by all authorities, from the time of Carnot to the present day, and have been, not infrequently, brought forward as new by minor later writers unfamiliar with the literature of the subject. Introducing into the work of Carnot the dynamical relation of heat and work, a relation, as shown by other writings, well understood, if not advocated publicly by him, the theory of the steam-engine becomes well defined and substantially accurate.

The Count de Pambour, writing in 1835, and later, takes up the problem of maximum efficiency of the steam-engine, shows the distinction to be drawn between the efficiency of fluid and efficiency of machine, and determines the value of the ratio of expansion for maximum efficiency of engine. He makes this ratio equal to the quotient of maximum initial pressure divided by the sum of the useless internal resistances of the engine, including back pressure and friction, and reduced to equivalent pressure per unit of area of piston. This result has been generally accepted, although sometimes questioned, and has been demonstrated anew, in apparent ignorance of the fact of its prior publication by De Pambour, by more than one later writer. De Pambour, applying his methods to the locomotive, particularly, solved the problem, since distinctively known by his name: Given the quantity of steam furnished by the boiler in the unit of time, and the measure of resistance to the motion of the engine; to determine the speed attainable. Professor Thomas Tate, writing his "Mechanical Philosophy," in 1853, gives the principle stated above a broader enunciation, thus: "The pressure of the steam, at the end of the stroke, is equal to the sum of the resistances of the unloaded engine,

* For definition of this and related terms, see chapter on the Thermodynamics of the Steam-engine. See Carnot, App. B, p. 255.

whatever may be the law expressing the relation of volume and pressure of steam."

The development of the Science of Thermodynamics into available and satisfactory form was effected mainly by Professors Rankine and Clausius, working independently but contemporaneously from 1849.

Combes, in papers presented to and published by the Académie des Sciences, was probably the first to introduce into the theory of the steam-engine the consideration of that phenomenon, discovered by Watt, to check the wasteful effects of which the latter invented the steam-jacket.* That author gradually gave shape to his ideas, as time went on, publishing them in 1845,† and, later, in 1863–67.‡ He even anticipates Rankine and Clausius in one of their most famous discoveries, saying: "*La vapeur d'eau, à l'état de saturation et entièrement sèche, se dilatait sans addition ni soustraction de chaleur; et nous avons montré que l'expansion est alors accompagnée d'une liquéfaction partielle de vapeur. C'est à peu près ainsi que les choses doivent se passer dans les machines à vapeur ordinaires.*" He goes on to describe very clearly the phenomenon of "cylinder-condensation;" but in his later works he seems to have paid less attention to this action, and may not have fully realized its importance; but his conception of the processes involved in such wastes, and in the preventive action of the jacket, was exact and well expressed.

59. **Clausius' Work** began at some time preceding 1850. He applied the modern theory of the steam-engine to the solution of the various problems which arise in the practice of the engineer, so far as they can be solved by the principles of thermodynamics. His papers on this subject were printed in 1850.§ The Count de Pambour had taken a purely mechanical mode of treatment, basing his calculations of the work

* Comptes-rendus, 1843.

† Traité d'exploration des Mines.

‡ Principes de la Théorie Mécanique de la Chaleur.

§ Poggendorff's Annalen, 1850 et seq. See, also, "The Mechanical Theory of Heat;" translated by W. R. Browne; London, 1879.

steam-engine, proves the fact of the condensation of steam during the period of expansion, estimates the amount of heat, fuel, and steam expended, and the quantity of work done, and determines thus the efficiency of the engine. He makes a special case of the engine using superheated steam, as well as that of the "jacketed" engine, considers the superheated steam-engine, and the binary-vapor engine, and reconstructs De Pambour's problem; applying the theory in the application of mechanics to general engineering. Several important text-books, a large volume on shipbuilding, and other works, with an unknown number of papers, published and unpublished, form a monument to the power and industry of this wonderful man and remarkable genius, that may be looked upon as perhaps the greatest wonder of the intellectual world. Thus, Rankine, producing, in part, the same results as Clausius, by his wonderfully condensed method of treatment, turned his attention more closely to the application of the theory to the case of the steam- and other heat-engines, giving, finally, in his "Prime Movers" (1859), a concise yet full exposition of the correct theory of those motors, so far as it is possible to do so by purely thermodynamic treatment. He was unaware, apparently, as were all the scientific men of his time, of the extent to which the conclusions reached by such treatment of the case are modified, in real engines, by the interference of other physical principles than those taken cognizance of by his science.

Sir William Thomson, partly independently, and partly working with Joule, has added much valuable work to that done by Clausius and Rankine.* In the hands of these great men the science took form, and has now assumed its place among the most important of all branches of physical science.

It was Sir William Thomson who discovered and revealed to English readers the remarkable work of Carnot and thus effectively aided in the construction of the science. As stated by Rankine :†

* Edin. Trans., 1850 et seq.; Phil. Mag.; and Mathematical and Physical Papers.

† Steam-engine; Introduction, p. xxxi.

"Professor William Thomson, adopting the true theory of heat, in 1850, not only solved some new problems in thermodynamics, and devised and carried out, jointly with Mr. Joule, some most important experiments; but he extended analogous principles to electricity and magnetism, and thereby created what may justly be styled a new science. His papers have appeared in the Transactions of the Royal Society of Edinburgh for 1851, and subsequently in the Philosophical Magazine since 1851, and the Philosophical Transactions since 1854. Numerical data, without which the theoretical researches before referred to would have been fruitless, were furnished by the experiments of Dulong, and MM. Bravais, Martins, Moll, Van Beek, and others, on the velocity of sound; by those of M. Rudberg, on the expansion of gases; by the experiments, almost unparalleled for extent and precision, of M. Regnault, on the properties of gases and vapors, made at the expense of the French Government, and published in the Proceedings and Memoirs of the Academy of Sciences, from 1847 to 1854; and by the joint experiments of Messrs. Joule and Thomson, on the thermic effects of currents of elastic fluids, made at the expense of the Royal Society, and published in the Philosophical Transactions for 1854."

Rankine concludes: "Although the mechanical hypothesis just mentioned may be useful and interesting as a means of anticipating laws, and connecting the science of thermodynamics with that of ordinary mechanics, still it is to be remembered that the science of thermodynamics is by no means dependent for its certainty upon that or any other hypothesis, having been now reduced to a system of principles, or general facts, expressing strictly the results of experiment as to the relations between heat and motive power. In this point of view the laws of thermodynamics may be regarded as particular cases of more general laws, applicable to all such states of matter as constitute *Energy*, or the capacity to perform work, which more general laws form the basis of the *science of energetics*,—a science comprehend-

ing, as special branches, the theories of motion, heat, light, electricity, and all other physical phenomena."

The physicist, as well as the engineer, is still seeking to ascertain more definitely what is the mechanism of heat-energy transmission. It is now well ascertained that both heat and light are originally, in space, methods of vibration, of oscillation, or of translation of particles of a fluid known as the "luminiferous æther;" but the physical characteristics of that fluid are not yet defined with certainty. The researches of Hertz and others seem to indicate the probability that Clerk Maxwell's suggestion that this method of transfer of energy may be electromagnetic is correct. Professor D. V. Wood, taking up the mathematical physics of the subject, deduces by a simple process, based on probably substantially accurate data,* as follows:

(1) This medium transmits energy at the rate of 186,300 miles per second.

(2) Heat-energy is transferred from the sun to the earth at the rate of 133 foot-pounds per square foot of section of the transmitted beam.

(3) The medium may be taken as possessing the characteristics of the sensibly perfect gas.

His process gives at once the essential physical characteristics of a fluid capable of transmitting this known quantity of energy at this observed velocity. It must be a medium of which one pound would occupy about twenty times the volume of the earth; its tension would be one pound, nearly, per square mile of section; and its specific heat must be about 4,600,000,000,000, that of water being taken as unity. It would weigh one pound to every 72×10^{21} cubic feet; the heat-vibrations are about 6×10^{14} per second; and it is "everywhere practically non-resisting, uniform in temperature, density, and elasticity," whether in the depths of space and at its own boundary, if it has one, or at the surface of the sun or of the largest

* Philosophical Magazine, Nov. 1885; Van Nostrand's Science Series, No. 85.

star in the universe. It would not destroy the motion of the average comet in a million of millions of years.

61. **The Thermodynamic Theory of the Steam-engine** stands, to-day, substantially as it was left by Clausius and Rankine and Thomson, at the close of their work in this field, in the decade 1850 to 1860. Many treatises have been published, some of them by men of exceptional ability; but all have followed the general line first drawn by these masters, and have only now and then found some minor point to develop.

Combes, Zeuner, and other writers have developed the subject in detail, the latter, especially, studying the theory of various working fluids, as of superheated-steam, and the phenomena of heat-transformation in relation to their effect upon the working substance. The pure theory of thermodynamics was substantially complete, however, long ago, and no important developments are to be now anticipated, except as elements in the expansion of similar principles into the broader field of energetics.

62. **The Limitations** of thermodynamic theory and of its application in the design and operation of heat-engines were first discovered by James Watt. They were systematically and experimentally investigated by Clark, in 1852 and earlier, were observed and correctly interpreted by Hirn (1855–7), and were revealed again by the experiments of Isherwood (1860), and by those of Emery and many other recent investigators on both sides of the Atlantic. These limitations are due to the fact that losses occur in the operation of such engines which are not taken into account by the hitherto accepted theory of the engine, and have no place in the thermodynamic treatment of the case.

It is assumed, in the purely thermodynamic theory of the engine, that the expansion of the working fluid takes place in a cylinder having walls impermeable to heat, and in which no losses by conduction or radiation, or by leakage, can occur. Of those losses which actually take place in the real engine, that due to leakage may be prevented, or, if occurring, can be checked; but it is impossible, so far as is now known, to secure

a working cylinder of perfectly non-conducting material. The consequence is that, since the steam or other working fluid enters at a high temperature and is discharged at a comparatively low temperature, the surfaces of cylinder, cylinder-heads, and piston are, at one instant, charged with heat of high temperature, and at the next moment, exposed to lower temperatures, are drained of their surplus heat, which heat is then rejected from the cylinder and wasted. Thus, at each stroke, the metal surfaces, exposed to the action of the expanding substance, alternately absorb heat from it, and surrender that heat to the "exhaust." As the range of temperature worked through in the engine increases, as the quantity of steam worked per stroke diminishes, and as the time allowed for transfer of heat to and from the sides and ends of the cylinder and the piston is increased, the magnitude of this loss increases. These physical phenomena are therefore no less important in their influence upon the behavior of the engine, and upon its efficiency, and are no less essential elements for consideration in the general theory of the engine than those taken into account in pure thermodynamics. Such limitations are studied in Chapter V.

63. **James Watt** not only discovered the fact of the existence of this method of waste, but experimentally determined its amount in the first engine ever placed in his hands. It was in 1763 that he was called upon to repair the little model of the Newcomen engine, then and still in the cabinets of the University of Glasgow. Making a new boiler, he set up the machine and began his experiments. He found, to his surprise, that the little steam-cylinder demanded four times its own volume, at every stroke, thus wasting, as he says, three fourths of the steam supplied, and requiring four times as much "injection-water" as should suffice to condense a cylinderful of steam. All of Watt's first inventions were directed toward the reduction of this immense waste. He proposed to himself the problem of keeping the cylinder "as hot as the steam that entered it;" he solved this problem by the invention of the separate condenser and the steam-jacket, and the discovery of

these limitations of the thermodynamic theory and their re duction was the source of Watt's fame.

John Smeaton, a distinguished contemporary of Watt seems to have been not only well aware of this defect of the steam-engine, but was possibly even in advance of Watt in attempting to remedy it. He built a large number of Newcomen engines between 1765 and 1770, in some, if not many, of which he attempted to check loss by this "cylinder-condensation" in engines, some of which were five and six feet in diameter of cylinder, by lining pistons and heads with wood.

Notwithstanding the fact that this waste was thus familiar to engineers, from the time of the invention of the modern steam-engine, and was recorded in all treatises on engine construction and management, the writers on the theory have never been aware that it gives rise to the production, in the working cylinder, of a large amount of water mingled with the steam. It has often been assumed by engineers themselves that this water is always due to "priming" at the boiler. Rankine, while correctly describing the phenomenon of cylinder-condensation, attributed the presence of the water in steam-cylinders to the fact of condensation of dry steam doing work by expansion, apparently not until later having noted the fact that this would only account for a very small proportion of the moisture actually present in the average steam-engine. He considered incomplete expansion the principal source of loss, as do usually other writers on thermodynamics.

Hirn published his *Mémoire sur l'Utilité des Enveloppes à Vapeur* in 1855.* This memorable paper gives us the first precise analysis of experiments showing the quantitative measures of the thermal action of the walls of the steam-cylinder. It presents an exact and scientific treatment of the case, and gives indisputable measures of the quantity of heat transferred to the metal, and restored to the steam when too late for transformation into its proportion of mechanical energy. In every experiment, Hirn measured the quantity of water en-

* Bulletin de la Société Industrielle de Mulhouse; t. XXVII. pp. 105–206.

tering the boiler, and there converted into steam, and compared it with the quantity of steam found, at each step in the engine-cycle, in the cylinder. He even went so far as to determine, by the use of his calorimeter, the quality of the steam entering the engine, in order that his measures of that contained in the cylinder might not be rendered uncertain by the action known as priming or foaming at the boiler. He also, for the first time, determined the weight of water leaving the condenser, and its temperature, thus securing the elements for the method of computation now known as that of Farey and Donkin. He proposed no theory, believing, as he stated expressly, that, at the time, any formulation of a theory was impossible without further knowledge.

As a result of his first series of experiments he was able to say: "The influence of the steam-jacket is now clearly explained: it consists in preventing the steam from partially condensing, and thus lessening the pressure during expansion, by that act itself. As the heat taken from the jacket is, as has been seen, a small fraction of the total heat expended, the power so gained costs very little. Were any doubt now to exist on this point, the following facts would completely remove them:

"(1) When the engine is working with jacket in use, if we suddenly shut off the steam and take it directly to the cylinder, the engine continues to work, as before, for some time, as if nothing had happened. The indicator-diagrams are precisely the same as before; it is only after 10 or 20 minutes that the power of the engine falls off $23\frac{1}{2}$ per cent in this case. It is thus evidently the heat in the walls of the cylinder, and not the simple drying of the steam, which gives us this economy of $23\frac{1}{2}$ per cent.

"(2) The jacket actually modifies very sensibly the temperature of the steam; for, while it is acting, the steam exhausted into the condenser is at 64° C., at a tension of $0^m.075$ while, in the other case, the temperature falls to 58°, although its tension rises to $0^m.095$. . . ."

Farther on he says: "Since it is the elevation of temperature of the walls of the cylinder, heated by the steam in the

jacket, which is the cause of the improvement, it is not to be doubted, for an instant, that any means of securing such temperature will be equally effective and economical." He then proposes the use of a smoke-jacket; but he finds, on trial, that it is of little value, the heat being incapable of passing with sufficient rapidity from the gases in the jacket to the metal of the cylinder.

Hirn, in this memoir, also expressly proposed the measure of the heat *consumed* by the engine as the true measure of its efficiency.

64. **The Best Ratio of Expansion** is that which gives best effect under the specified conditions. But this is obviously greater or less, accordingly as the wastes of the engine increase less or more rapidly, and as this point, known practically to exist, at which the net effect, after balancing gains and losses, is set at one ratio or another by such variations.

The limit of efficiency in heat-engines, as has been seen, is thermodynamically determined by the limit of complete expansion. The causes of the practical limitation of the ratio of expansion to a very much lower value than those which maximum efficiency of fluid would seem to demand have not been always considered, either with care or with intelligence by writers thoroughly familiar with the dynamical treatment apart from the modifying conditions here under consideration. These problems are the special subject of Chapter VII.

65. **Cylinder-condensation** is now known to produce very serious modifications of working conditions. Watt, and probably his contemporaries and successors, for many years supposed that the irregularity of motion due to the variable pressure occurring with high expansion was the limiting condition, and does not at first seem to have realized that the cylinder-condensation discovered by him had any economical bearing upon the ratio of expansion at maximum efficiency. It undoubtedly is the fact that this irregularity was the first limiting condition with the large, cumbrous, long-stroked, and slow-moving engines of his time. Nearly every accepted authority, from that day to the present, has assumed, tacitly,

that this method of waste has no influence upon the value of that ratio.

Thomas Tredgold, writing in 1827, who, but little later than Carnot, puts the limit to economical expansion at the point subsequently indicated and more fully demonstrated by De Pambour, exaggerates the losses due to the practical conditions, but evidently does perceive their nature and general effect. He also shows that, under the conditions assumed, the losses may be reduced to a minimum, so far as being dependent upon the form of the cylinder, by making the stroke twice the diameter.

Mr. D. K. Clark, however, publishing his "Railway Machinery" in 1855, was the first to discuss this subject with knowledge, and with a clear understanding of the effects of condensation in the cylinder of the steam-engine upon its maximum efficiency. Cornish engines, from the beginning, had been restricted in their ratio of expansion to about one fourth, as a maximum, Watt himself adopting a "cut-off" at from one half to two thirds. Hornblower, with his compound engine competing with the single-cylinder engines of Watt, had struck upon this rock, and had been beaten in economy by the latter, although using much greater ratios of expansion; but Clark, a half-century, and more, later, was, nevertheless, the first to perceive precisely where the obstacle lay, and to state explicitly that the fact that increasing expansion leads to increasing losses by cylinder-condensation, the losses increasing in a much higher ratio than the gain, is the practieal obstruction in our progress toward greater economy.

After a long and arduous series of trials of locomotive-engines, and prolonged experiment looking to the measurement of the magnitude of the waste produced as above described, Clark concludes: "The magnitude of the loss is so great as to defeat all such attempts at economy of fuel and steam by expansive working, and it affords a sufficient explanation of the fact, in engineering practice, that expansive working has been found to be expensive working, and that, in many cases, an absolutely greater quantity of fuel has been

consumed in extended expansion working, while less power has been developed." He states that high speed reduces the effect of this cause of loss, and indicates other methods of checking it. He states that "the less the period of admission, relative to the whole stroke, the greater the quantity of free water existing in the cylinder." His experiments revealing these facts were, in some cases, made prior to 1852. But the men handling the engines had observed this effect even before Clark; he states that they rarely voluntarily adopted "a suppression of above 30 per cent," as they found the loss greater than the gain. Describing the method of this loss, this author goes on to say that, "to prevent entirely the condensation of steam worked expansively, the cylinder must not only be simply protected by the non-conductor; it must be maintained, by independent external means, at the initial temperature of the steam." He thus reiterates the principle expressed by Watt three quarters of a century before.

The same author, writing in 1877, says: "The only obstacle to the working of steam advantageously to a high degree of expansion in one cylinder, in general practice, is the condensation to which it is subjected, when it is admitted into the cylinder at the beginning of the stroke, by the less hot surfaces of the cylinder and piston; the proportion of which is increased so that the economy of steam by expansive working ceases to increase when the period of admission is reduced down to a certain fraction of the stroke, and that, on the contrary, the efficiency of the steam is diminished as the period of admission is reduced below that fraction." The magnitude of this influence may be understood from the fact that the distinguished engineer, Loftus Perkins, using steam of 300 pounds pressure, and attaining the highest economy known, up to his time, found his engine to consume 1.62 pounds of fuel per hour and per horse-power; while this figure is now reached by engines using steam at one third that pressure, and expanding about the same amount, and sometimes less.

Mr. Humphreys, writing a little later than Clark, shows the consumption of fuel to increase seriously as the ratio of

expansion is increased beyond the very low figure which constituted the limit in marine engines of his time.

66. Hirn was the first scientific and practical investigator on the Continent of Europe.

A few writers on thermodynamics had finally come to understand the fact that such a limitation of applied theory existed, and Mons. G. A. Hirn, who, better than probably any authority of his time or earlier, combined a knowledge of the scientific principles involved, with practical experience and experimental knowledge, in his treatise on thermodynamics (1876), concludes: "*qu'il est absolument impossible d'édifier à* priori *une théorie de la machine à vapeur d'eau douce d'un charactère scientifique et exact*," in consequence of the operation of the causes here detailed. While working up his experiments upon the performance of engines, comparing the volume of steam used with that of the cylinder, he had always found a great excess, and had, at first, attributed it to the leakage of steam past the piston; but a suggestion of M. Leloutre set him upon the right track, and he came to the same conclusion as had Watt, so many years before. He explains that errors of thirty, or even up to seventy, per cent may arise from the neglect of the consideration of this loss. Combes had perceived the importance of this matter, and De Freminville suggested the now familiar expedient of compression, on the return-stroke, as nearly as possible to boiler-pressure, as a good way to correct the evil. Hirn was the first to show in detail the distribution of heat-wastes and to prove with certainty, on such grounds, that the benefit of extended expansion in real engines can only be approximated to that predicted, by the theory of the ideal engine, by special arrangements having for their object the reduction of cylinder-waste, such as superheating, "steam-jacketing," and "compounding."

His experimental work began at a very early date and in a purely scientific spirit. He had noted the discoveries of Mayer (1842) and of Joule (1846 and later) only after he had himself sought to ascertain the true nature of heat. He published his conclusions, correct conclusions, in 1848, relating to

this question as determined by his researches on heat and friction. In 1855 he was able to show that Carnot had accepted the wrong theory in his now famous work; proved, by experiment, that heat actually disappears, as heat, in the operation of the steam-engine, and showed that, in the actual engine, the steam-jacket, an element in itself wasteful, may be a very important source of economy by checking extra-thermodynamic wastes.

Hirn showed that Mayer's ideas were completely sustained, and that the Rankine and Clausius phenomenon of condensation of steam and similar vapors, during their adiabatic expansion, is actually observable in the steam-engine. His great work on Thermodynamics * was published in 1876, and in it he gave as clear an account of the physical operations taking place within the engine-cylinder as had Clark or Isherwood, and produced a theory of the real engine—an "experimental theory" as he called it—which has served as the basis for nearly all subsequent work in that direction.

Mons. V. Dwelshauvers-Dery supplemented this work of Hirn by further development of the theory and its application in fuller detail to the processes of heat-transfer in the steam-engine in the years subsequent to 1878.† From 1873, this investigator worked with Hirn and his lieutenant, Hallauer, and with M. Grossteste in the construction, upon the basis of experiment, of a correct theory of the real, as distinguished from the ideal, engine and its reduction to a practically valuable form. He gives in an "Exposé," in the Revue, in 1882, a carefully-written account of this development of the most modern form of the theory of heat-engines. This latest theory was finally completely established by a long and instructive discussion in which the ablest physicists and engineers of Europe were engaged. In its current form, its algebraic expression is that of Dwelshauvers-Dery; but it still requires further development.

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* Théorie Mécanique de Chaleur; 2 tomes; Paris, 1876.

† Revue Universelle des Mines, de Liège.

67. **Isherwood's Researches** were the first systematically conducted investigations of the latest phase of the problem of steam-engine efficiency in the United States.

Mr. B. F. Isherwood was, in 1860, a Chief Engineer in the United States navy, and Chief of the Bureau of Steam Engineering. He seems to have been the first to have attempted to determine, by systematically planned experiment, the law of variation of the amount of cylinder-condensation with variation of the ratio of expansion. Experimenting on board the U. S. S. Michigan, a naval vessel fitted with simple and unjacketed engines, he found that the consumption of fuel and of steam was greater when the ratio of expansion was carried beyond about two than when restricted to lower ratios. He determined the quantity of steam used, and the amount condensed, at expansions ranging from full stroke to a "cut-off" at one tenth. His results permit the determination of the method of variation, with practically satisfactory accuracy, for the engine upon which the investigation was made, and for others of its class. It was the first of a number of such investigations made by the same hand, and these to day constitute the principal part of our data in this particular direction. The author, studying these results, found that the cylinder-condensation there varied sensibly as the square-root of the ratio of expansion, and the method of variation is apparently substantially similar for other forms and proportions of engine. The amount of such condensation usually lies between one tenth and one fifth the square-root of that ratio, if estimated as a fraction of the quantity of steam demanded by a similar engine having a non-conducting cylinder, it being here assumed that the engine is one of fair size. The proportion of loss is some inverse function of the size of engine—probably nearly inversely as the diameter of cylinder.

Mr. Isherwood, in his works, gives admirably-expressed descriptions of the *modus operandi*, when considering this waste.* He summarizes his own work, and explains with un-

* Engineering Researches; 2 vols. 4to; Philadelphia, 1860. See especially the introduction to volume II.

exampled clearness the method of modification of the best ratio of expansion by these internal and previously unfamiliar wastes.

Professor Cotterill gives more attention to this subject than any writer up to his time. He devotes a considerable amount of space to the study of the method of absorption and surrender of heat by the metal surfaces enclosing the steam, constructs diagrams which beautifully illustrate this action, and solves the problems studied by him with equal precision and elegance of method. He summarizes the experimental work done to the date of writing, and very fully and clearly exhibits the mode of transfer of heat past the piston without transformation into work. Professor Cotterill's treatise on the steam-engine, "considered as a heat-engine," is thus most valuable to the engineer.*

Mr. Sutcliffe states, as early as 1875, that engines of approved type may sometimes exhibit losses by cylinder-waste exceeding 40 per cent.† He gives the following figures for these losses in the Corliss engines at Saltaire:

Ratio of Expansion.	Cylinder-condensation.
7.4	27 per cent.
9.04	36.37 " "
11.4	46.67 " "

These figures approach those previously obtained by Isherwood from a much less approved form of engine.

68. The Status of the Theory of the Steam-engine, about 1850, was becoming well settled as a thermodynamic system, and even the most recent phase had begun to take vague shape.

Dr. Albans, writing about 1840, says of the choice of best ratio of expansion: "Practical considerations form the best guide, and these are often left entirely out of view by mathematicians. Many theoretical calculations have been made to

* The Steam-engine considered as a Heat-engine; London, 1878–1890.

† Hopkinson on the Steam-engine Indicator; 7th ed.; 1875.

determine the point, but they appear contradictory and unsatisfactory." Renwick, in 1848, makes the ratio of initial divided by back pressure the proper ratio of expansion, but correctly describes the effect of the steam-jacket, and suggests that it may have peculiar value in expansive working, and that the steam may receive heat from a cylinder thus kept at the temperature of the "prime" steam. John Bourne, the earliest of now acknowledged authorities on the management and construction of the steam-engine, pointed out, at a very early date, the fact of a restricted economic expansion. Rankine recognized no such restriction as is here under consideration, considered the ratio of expansion at maximum efficiency to be the same as that stated by Carnot, and by other early writers, and only perceived its limitation by commercial considerations, a method of limitation of great importance, but often of less practical effect than is the waste by condensation. In his life of Elder (1871), however, he indicates the existence of a limit in practice, and places the figure at that previously given by Isherwood for unjacketed engines.

Thus the theory of the steam-engine stands, at this date, incomplete, but on the verge of completion, needing only a little well-directed experimental work to supply the doubtful elements. Even these are becoming determined. Isherwood and later engineers give facts showing waste to be proportional, very nearly, if not exactly, to the square-root of the ratio of expansion; and Escher, of Zurich, has shown the loss to be also proportional to the square-root of the time of exposure, or, in other words, to the reciprocal of the square-root of the speed of rotation; and it only remains to determine the exact method of variation of loss with variation of range of temperature and a rational basis to give the whole of the necessary material for the construction of a working theory which may enable the engineer to estimate, in advance of construction, the economic performance of his machine.

Dwelshauvers has done much to popularize the modern and accepted theory of the real engine. He has endeavored to exhibit the action of the steam-jacket, to show what is the

modification of the action of the metallic interior of the engine, by the introduction of that wasteful element, to counteract, in many cases, a greater waste; and he has sought to show the influence of the experimental philosophy of the engine upon the proportions and the working of the condenser. He has observed the fact of a maximum ratio of expansion appropriate to the condition of maximum efficiency, as determined by the variation of this waste, previously unobserved, and has engaged in the construction of its theory in accordance with his published theory of heat-expenditure, reducing all to a common basis and philosophy.

Some of the work of Dwelshauvers-Dery has been translated by Donkin, and published, from time to time in London *Engineering;* other portions remain untranslated, and are only to be found in the *Revue Universelle des Mines.* Sinigaglia has summarized it well.

It probably cannot be long before direct investigation will secure all essential knowledge. When this becomes the case, the remarks of those distinguished physicists and engineers, Hallauer and his great teacher, Hirn, will be no longer based upon apparent fact.

Says Hirn, on this subject "*Ma conviction reste aujourd'hui qu'elle était il y à vingt ans, une théorie proprement dite de la machine à vapeur est impossible; la théorie expérimentale, établie sur le moteur luimême et dans toutes les formes où il à été essayé, en mécanique appliqué peut seule conduire à des résultats rigoureux.*"

69. **Three Periods of this Philosophy** of the steam-engine may be discerned. Chronologically considered, the history of the growth of the theory divides itself distinctly into three parts: the first extending up to the middle of the present century, and mainly distinguished by the attempts of Carnot and of Clapeyron to formulate a physical theory of the thermodynamics of the machine; the second beginning with the date of the work of Rankine and Clausius, who constructed a correct thermodynamic theory; and the third beginning nearly a generation later, and marked by the introduction into the general

theory of the physics of the conduction and transfer of that heat which plays no part in the useful transformation of energy and its application.

The first period may be said to include, also, the inauguration of experimental investigation, and the discovery of the nature and extent of avoidable wastes and attempts at their amelioration by James Watt and by John Smeaton. The second period is marked by the attempt, on the part of a number of engineers, to determine the method and magnitude of these wastes by more thorough and systematic investigation, and by the exact enunciation of the law governing the necessary rejection of heat, as revealed by the science of thermodynamics. The third period is opening with promise of a complete and practically applicable investigation of all the methods of loss of energy in the engine, and of the determination, by both theoretical and experimental research, of all the data needed for the construction of a working theory.

Hirn has recognized these three periods, and has proposed to call the second the "theoretical" and the third the "experimental" stage. The Author would prefer to make the nomenclature somewhat more accordant with what has seemed to him to be the true method of development of the subject. It has been seen that the experimental stage really began with the investigations of Watt in the first period, and that the work of experimentation was continued through the second into the present, the last, period.

It is also evident that the theoretical stage, if it can be properly said that such a period may be marked off in the history of the theory of the steam-engine, actually extends into the present epoch; since the work of the engineer and the physicist of to-day consists in the application of the science of heat-transfer and heat-transformation, together, to the engine. During the second period the theory included only the thermodynamics of the engine; while the third period is about to incorporate the theory of conduction and radiation into the general theory with the already established theory of heat-transformation. The writer would therefore make the classifi-

cation of these successive stages in the progress here described thus:

1. Primary Period—that of incomplete investigation and of earliest systematic, but inaccurate, theory.

2. Secondary Period—that of the establishment of a correct thermodynamic theory, the *Theory of the Ideal Engine.*

3. Tertiary Period—that of the production of the complete theory of the engine, of the *true Theory of the Real Engine.*

70. **Work remaining to be done,** as may be now readily seen, is that of determining, by experiment, precisely what are the physical laws governing the transfers of heat between metal and vapor, in the engine-cylinder, and to apply these laws in the theory of the machine. Cotterill has shown how heat penetrates and traverses the metal, and Grashof has indicated the existence of an intermediate and approximately constant temperature between the temperatures of the initial steam and of the exhaust, and both have given us some new methods. The Author, while pointing out the nature of the true "curve of efficiency" of the steam-engine which he was so fortunate as to discover, has shown how it may be made useful in the solution of practical and of theoretical problems involved in the applied theory of heat-engines, and many able minds are now engaged upon the theory. There can be little doubt that it will soon become satisfactorily complete.

The determination of physical constants and the experimental checking of the scientific treatment of the case will undoubtedly furnish employment to able and skilful investigators for many years, and the study of the modification of the general theory in its application to the present and the coming types of engine will offer a no less important and attractive field of labor for those competent to take up the work and finding opportunities to do so. The philosophy of the construction and of the operation of the multiple-cylinder and of new forms of engine is already well understood, and the algebraic and numerical equations applying to them as a mathematical theory are now in process of development. Messrs. Hirn and Hallauer, Donkin, Dwelshauvers-Dery, Zeuner and

Kirsh have already succeeded in effecting some valuable advances in the theory of the real engine, by the introduction of data previously secured by Clark and others.

The experimental investigation of Messrs. Gately and Kletsch, to be considered later, and the more exact work since undertaken will ultimately supply all needed data. That investigation, the first attempt at systematic investigation of the methods of variation of the several main losses and wastes, in the steam-engine, with variation of the principal quantities determining their magnitudes, was made in the spring of 1884, and upon a plan schemed out by the Author some years earlier (1878). The results gave, roughly, the needed data for the provisional theory of the engine, including physical as well as thermodynamic wastes, the theory of heat-transfer and that of heat-transformation. It has now become practicable to make intelligent and useful estimates of the relative value of alternative plans of construction of proposed new engines, of probable costs of operation, and of efficiencies and best proportions of size of engine to power demanded for any given type, size, and design. Some of the most satisfactory data are those obtained by Messrs. Hill,* Willans,† Schneider,‡ English,§ Kennedy,‖ and Calendar and Nicholson.¶

71. **The Plan of this Work** thus logically includes the philosophical study of the gradual development of the modern steam engine out of the germ which existed in ancient times; the description of the machine of our own day, in its principal forms; the tracing of the evolution of scientific knowledge of its philosophy to the present time; the discussion of the scientific principles involved in the production, utilization, and wastes of energy in the apparatus and mechanisms employed; and the useful application of such principles, in the design, the proportioning, the constructing, and the economical operation of engines and transmitting machinery.

The succeeding subjects then follow in logical and natural

* Mark's Steam-engine Design. † Trans. Brit. Inst. C. E., 1888.
‡ Delaford's Report, 1884. § Trans. Brit. Inst. Mech. Engrs., 1887.
‖ Trans. Brit. Inst. Mech. Engrs., 1890.
¶ Proc. Inst. C. E., 1898.

order, thus: Chemistry of Combustion; Physics of Heat-transfer and Storage; Thermodynamics; Theory of the Steam-engine. ideal and real; Design; Construction; Operation and Management; Tests of the Machine; Theory of Efficiencies, including finance; Establishments; Specifications, Contracts, and Legal Forms and Business Principles. We thus trace the production of energy in available form and its progress in the process of its utilization, from its first appearance with the combustion of the fuel in which it had been stored, through the several steps by which it passes into the boiler, becomes stored in the steam, and is finally transferred to the engine and there converted in part into mechanical energy, to be usefully or wastefully applied to the performance of the intended task or to overcome the friction of the mechanism employed.

The *Fundamental Mechanical Principles* involved are, in brief, the following:

The object of all mechanism is to produce a certain definite motion of some part or parts—the position and form and the methods of connection of which are known and fixed—against any resistance that may be met with in the course of such movement. Every machine and every train of mechanism is therefore a contrivance by means of which energy or power available at one point, usually in definite amount and acting in a definite direction and with definite velocity, is transferred to other points, there to do work of definite amount, and there to overcome known resistances with known velocities.

The object of the engineer in designing mechanism is to effect this transfer of energy and these transformations at the least cost and with least running expense, and hence with maximum efficiency of apparatus. It is often important to secure minimum volume and weight of machine, as well as maximum effectiveness in operation.

The work of a machine is measured by the magnitude of the resistance encountered and the velocity with which it is overcome. The nature of the work, aside from its simple kinetic character, is as widely variable as are the details of human industry.

Prime Movers are those machines which receive energy directly from natural sources, and transmit it to other machines which are fitted for doing the various kinds of useful work. Thus, the steam-engine derives its power from the heat-energy liberated by the combustion of fuel; water-wheels utilize the energy of flowing streams; windmills render available the power of currents of air; the voltaic battery develops the energy of chemical action in its cells; and, through the movement of electro-dynamic mechanism, this energy is communicated to other machinery, and thus caused to do work.

Machinery of Transmission is used in the transformation of energy supplied by the prime mover into available form, for the performance of special kinds of work, or for simple transmission of power from the prime mover to machines doing that work.

The work to be done may be the raising of weights, as in hoisting and pumping machinery; the transportation of loads, as on the railway or in the steamship; the alteration of the form of solid masses, as in machine-tools; the overcoming or even the utilizing of frictional resistances, as in brakes; or any other of the numberless operations performed in mills and factories by machinery.

Machines and *Machine-tools* receive energy, derived originally from prime movers, and transferred to them through machinery of transmission, and apply that energy to special kinds of work to which they are precisely adapted by their design and construction. Thus, looms apply such energy to the weaving of cloth; lathes are especially fitted for the production of parts having circular sections; planing-machines produce straight-lined surfaces.

The power demanded by a machine is that needed to do the work for which the machine is designed, plus the additional amount expended by the machine itself, in transferring the first-mentioned quantity from the source of power to which the machine is connected, by transmitting mechanism to the point at which the work is to be done. Where the machine is subject to shock and jar sufficient to permanently distort its parts, or the bearing surfaces, a portion of the power demanded

is wasted in doing this work; where the journals heat, considerable amounts of energy are sometimes lost as heat-energy: in all cases some loss occurs in this way. Where power is transmitted by the expansion and compression of elastic fluids, also, energy is often lost in large amounts by transformation into heat.

The power demanded by any machine thus always exceeds that expended by the machine upon its proposed task. Were these wastes not to occur, the power transmitted would be the same in amount at every point in the machine.

Work, as a term in the science of engineering, may be defined as that action by which motion is produced against the resistance continuously or intermittently opposed to any moving body. It is measured by the product of the direct component of the resistance into the space traversed. Where the resistance is variable, its mean value is taken. Thus, if R be the resistance and S the space, the work is, for constant resistance,

$$U = RS, \quad \ldots \ldots \ldots \quad (1)$$

in which U is measured in foot-pounds or kilogrammetres. For a variable resistance, R, acting through a space, s,

$$U = \int R ds, \quad \ldots \ldots \ldots \quad (2)$$

which can be integrated when R is known as a function of s.

Resistances, and the forces by which they are overcome, are measured by engineers, usually, either in British or in metric units, as the pound or the kilogramme. Work, and the energy expended in doing work, are thus both measured by the product of the pounds or the kilogrammes of resistance or of effort into spaces of which the measure is usually given in feet or in metres. The unit of work and of energy is thus either the foot-pound or the kilogrammetre.

The British and metric measures have definite relations, which are given in tables to be found in all engineers' tablebooks.

Where the motion of the machine or of the part doing work is circular, the space traversed may be measured by the angular motion, a, multiplied by the lever-arm, l, and their product, multiplied by the force, R, exerted, gives the measure of the work done. Thus:

$$\left.\begin{aligned} U &= aRl \\ &= 2\pi nRl; \end{aligned}\right\} \quad . \quad . \quad . \quad . \qquad (3)$$

in which last expression n is the number of revolutions made in the unit of time.

These values are equivalent to the product of the angular motion into the moment of the resistance.

Work may also be measured, as in steam, air, gas, or water-pressure engines, by the product of the area of piston, A, the mean intensity of pressure upon it, p, the length of stroke of piston, l, and the number of strokes made. Thus,

$$\begin{aligned} U &= Apln \\ &= Aps \\ &= pV, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4) \end{aligned}$$

when V is the volume of the working cylinder multiplied by the number of strokes; in other words, the volume traversed by the piston.

Where the force acting, or the resistance, acts obliquely to the path traversed, it is evident that only the component in that path is to be considered.

Diagrams exhibiting the amount of work done and the method of its variation are often found useful. In such diagrams the ordinate is usually made proportional to the force acting or to the resistance, while the abscissas are made to measure the space traversed. The curve then exhibits the relations of these two quantities, and the enclosed area is a measure of the work performed. With a constant resistance, the figure is rectilinear and a parallelogram; with variable velocities and resistances, it has a form characteristic of the methods of operation of the part or of the machine the action of

which it illustrates. In the first case, the area can be obtained by multiplication of the difference of the ordinates by the difference between maximum and minimum abscissas; in the second case, it may be obtained by any convenient system of integration, of which systems that of mechanical integration, as by the "planimeter," is usually best.

Power is defined as the *rate of work*, and is measured by the quantity of work performed in the unit of time, as in foot-pounds or in kilogrammetres, per minute or per second. The unit commonly employed by engineers is the "horse-power," which was defined by Watt as 33,000 foot-pounds per minute, equivalent to 550 per second, or 1,980,000 foot-pounds per hour. This is considered to be very nearly the amount of work performed by the very heavy draught-horses of Great Britain; but it considerably exceeds the power of the average dray-horse of that and other countries, for which 25,000 foot pounds may be taken as a good average amount.

The metric horse-power, called by the French the *cheval-vapeur*, or *force de cheval*, is about 1½ per cent less than the British, being 542½ foot-pounds or 75 kilogrammetres per second, 4500 kilogrammetres per minute, or 270,000 per hour. These quantities are almost invariably employed to measure the power expended and work done by machines.

It is evident that power is also measured by the product of the resistance, or of the effort exerted into the velocity of the motion with which that resistance is overcome, or that force exerted. Since $s = vt$,

$$U = Rs = Rvt;$$

and when t becomes unity, the measure of the power, or of the equivalent work done in the unit of time, is

$$U' = Rv, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which the terms are given in units of force and space as above.

The power of a prime mover is usually ascertained by experimentally determining the work done in a given time, the trial

usually extending over some hours, and often several days. It is measured in foot-pounds or kilogrammetres; the total work so measured is then divided by the time of operation and by the value of the horse-power for the assumed unit of time and the mean value of the power expended thus finally expressed in horse-powers.*

The forces acting in machines are distinguished into *driving* and *resisting forces*. That component of the force, acting to produce motion in any part which lies in the line of motion only, is that which does the work; and this component is distinctively called the "Effort." Similarly, only that component of the resistance which lies in the line of motion is considered in measuring the work of resistance. In either case, if the angle formed between the directions of the motion of the piece and of the driving or the resisting force be called α, the effort is

$$P = R \cos \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The other component, acting at right angles to the path of the effort, is

$$Q = R \sin \alpha, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and has no useful effect, but produces waste of power by introducing lateral pressures and consequent friction.

Energy, which is defined as capacity for performing work, is either *actual* or *potential*.

Actual or *Kinetic Energy* is the energy of an actually moving body, and is measured by the work which it is capable of performing while being brought to rest, under the action of a retarding force; this work is equal to the product of its weight, W, into the height, $h = \frac{v^2}{2g}$, through which it must fall under the action of gravity to acquire that velocity, v, with which it is at the instant moving; i.e.,

$$E = U = Wh = W\frac{v^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

* Custom has not yet settled the proper form of the plural of this word; there is no reason why it should not follow the rule.

A change of velocity v_1 to v_2, causes a variation of actual energy, $E_1 - E_2$, and can be effected only by the expenditure of an equal amount of work—

$$E_1 - E_2 = U = W\frac{v_1^2 - v_2^2}{2g} = W(h_1 - h_2). \quad . \quad . \quad (9)$$

This form of energy appears in every moving part of every machine, and its variations often seriously affect the working of mechanism.

The total actual energy of any system is the algebraic sum of the energies, at the instant, of all its parts; i.e.,

$$E = \Sigma W\frac{v^2}{2g}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and when this energy is all reckoned as acquired or expended at any one point, as at the driving-point, the several parts having velocities, each n times that of the driving-point, which latter velocity is then v, the total energy becomes

$$E = \Sigma W\frac{n^2v^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Actual energy is usually reckoned relatively to the earth; but it must often be reckoned relatively to a given moving mass, in which case it measures the work which the moving body is capable of doing upon that mass, when brought by it to its own speed.

Potential Energy is the capacity for doing work possessed by a body in virtue of its position, of its condition, or of its intrinsic properties. Thus, a weight suspended at a given height possesses the potential energy, in consequence of its position, $E = Wh$, and may do work to that amount while descending through the height, h, under the action of gravity. A bent bow or coiled spring has potential energy, which becomes actual in the impulsion of the arrow or is expended in the work of the mechanism driven by the spring. A mass of gunpowder or other explosive has potential energy in virtue

of the unstable equilibrium of the chemical forces affecting its molecules. Food has potential energy in proportion to the amount of vital and muscular energy derivable by its consumption and utilization in the human or animal system. These potential energies are not measured by the observed actual energies derived from these substances in any case, but are the maximum quantities possibly obtainable by any perfect system of development and utilization. In practical application, more or less waste is always to be anticipated.

The law of persistence of energy affirms that the total energy, actual and potential, of the universe, or of any isolated system of bodies, is of invariable amount, and that all energy is thus indestructible, although capable of transformation into various forms of physical and chemical energy.

Every instance of disappearance of actual energy involves the performance of work, and the production of potential or of some new form of actual energy in precisely equal amount. A stone thrown vertically upward loses kinetic energy as it rises in precisely the amount—resistance of the air being neglected—by which it gains potential energy. A falling mass striking the earth surrenders the actual energy acquired by loss of potential energy during its fall, and the equivalent of the quantity so surrendered is found in the work done upon the soil; it finally passes away as the equivalent energy of heat motion produced by friction and impact. The potential chemical energy of the explosive is the equivalent of the kinetic energy of the flying projectile, and the latter has its equivalent in the work done at the instant of striking and coming to rest, and in the heat produced by the final change of mass-motion into molecular or heat motion.

Energy in all its many forms is thus transferable in definite quantivalent proportions, and in all cases changes form when work is done. Work may therefore be defined as that operation which results in a change in the method of manifestation of energy, and Energy as that which is transferred or transformed, when work is done. The motion of a projectile is the transfer of energy from one place to another. It is generated at the point of departure, stored as actual or

kinetic energy, transferred to the point of destination, and there restored and applied to the production of work.

Acceleration and retardation of masses in motion can only be produced by doing work upon them, or by causing them to do work, and thus, by the communication of energy to them or by its absorption from them, in precisely the amount which measures the variation of their actual energy as so produced. Every body which is increasing in velocity of motion thus receives and stores energy; every mass undergoing retardation must perform work, and thus must restore energy previously communicated to it. In every machine which works continuously, and in which parts are alternately accelerated and retarded, energy is stored at one period and restored at another, in precisely equal amounts.

Work done upon any machine may thus be expended partly in doing the useful work of the system, and partly in storing energy; and the same machine may do work at another instant partly by expending the energy received by it, and partly by expending stored energy previously accumulated.

Storage or restoration of energy thus always occurs when change of speed takes place. It is evident, since the storage or restoration of energy implies variation of speed, that the condition of uniform speed is that the work done upon the machine shall at each instant be precisely equal to that done by it upon other bodies. The work applied must be equal to that of resistance met at the driving-point. Thus,

$$\Sigma Pv = \Sigma Rv'; \int Pdv = \int Rdv'; \quad . \quad . \quad . \quad (12)$$

and the effort at each point in the machine will be equal to the resistance, and inversely as the velocity of the point to which it is applied; i.e.,

$$\frac{P}{R} = \frac{v'}{v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In the starting of every machine energy is stored during the whole period of acceleration up to maximum speed, and this energy is restored and expended while the machine is

coming to rest again. This latter quantity of energy is usually expended in overcoming friction.

The useful and the lost work of a machine are, together, equal to the total amount of energy expended upon the machine, i.e., to the work done upon it by its "driver." The *Useful Work* is that which the machine is designed to perform; the *Lost Work* is that which is absorbed by the friction and other prejudicial resistances of the mechanism, and which thus waste energy which might otherwise be usefully applied. These two quantities, together, constitute the *Total Work* or the *Gross Work* of a machine, or of a train of mechanism. In every case some energy is wasted, and the work done by the machine is by that amount less than the work performed in driving it In badly proportioned machines the lost work is often partly expended in the deformation and destruction of the members of the construction; in well designed and properly worked machinery loss occurs wholly through friction. In machines acting upon fluids this lost work is usually partly wasted in the production of fluid friction—i.e., of currents and eddies; thus producing new forms of actual energy in ways which are not advantageous: even this waste energy is finally converted, like the preceding form, by molecular friction into heat, and is dissipated in that form of molecular energy. Thus all wasted work is lost by conversion from the energy of mass-motion into molecular energy and ultimately disappears as heat.

The efficiency of mechanism is measured by the quantity obtained by dividing the amount of useful work performed by the gross work of the piece or of the system. It is always, therefore, a fraction, and is less than unity; which latter quantity constitutes a limit which may be approached more and more nearly as the wastes of energy and work are reduced, but can never be quite reached. If the mean useful resistance be R, and the space through which it is overcome be s', and if the mean effort driving the machine be P, and the space through which it acts be s, the total and the *net* or *useful work* will be, respectively, Ps, Rs'; the *lost work* will be $Ps - Rs'$ and the

$$\text{Efficiency} = \frac{Rs'}{Ps} < 1. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Counter-efficiency, C, is the reciprocal of the efficiency, i.e.,

$$C = \frac{Ps}{Rs'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The efficiency and the counter-efficiency of a machine, or of any train of mechanism, is the product of the efficiencies or of the counter-efficiencies of the several elements constituting the train transmitting energy from the point at which it is received to that at which the work is done, i.e., from the "driving" to the "working" point.

Friction is thus the principal cause, and usually the only cause, of loss of energy and waste of work in machinery. A given amount of energy being expended upon the driving-point in any machine, that amount will, in accordance with the principle of the persistence of energy, be transmitted from piece to piece, from element to element, of the machine or train of mechanism, without diminution, if no permanent distortion takes place and no friction occurs between the several elements of the train, or between those parts and the frame or adjacent objects. Temporary distortion, within the limit of perfect elasticity, causes no waste of energy; permanent distortion, however, causes a loss of energy equal to the total work performed in producing it. But permanent distortion is due to deficiency of strength and defective elasticity, and is never permitted in well-designed machinery properly operated; and hence the important principle:

The only cause of lost work in mechanism, which is to be anticipated in design and calculated upon in deducing the theory of special mechanism, is the friction necessarily consequent upon the relative motion of parts in contact and under pressure.

The study of the laws of friction, the construction of its theory, and the experimental investigation of the conditions which determine the loss of efficiency in machinery by friction, are thus obviously of supreme importance to the engineer who designs, the mechanic who constructs, and the operator or manufacturer who makes use of machinery.

In engineering, therefore, the principles of pure mechanism, of theoretical mechanics, and of pure theory in the science of energetics, or of thermodynamics, are to be studied as introductory to a science of application in which all actions and all calculations are to be considered with reference to the modifications produced by the wastes of energy and the alteration of the magnitudes and other properties of forces consequent upon the occurrence of friction. This is to the engineer a vitally important branch of applied science, and it is coextensive with the applications of mechanical science.

The magnitude of the lost work in machinery and millwork is variable, but is always very large. It may probably be fairly estimated that one half the power expended in the average case, whether in mill or workshop, is wasted in lost work, being consumed in overcoming the friction of lubricated surfaces. That this is true, is evident from the fact that the power demanded to drive the machinery of such establishments has been found by Cornut and others to be variable to the extent of 15 or 20 per cent by simple change of temperature indoors from summer to winter, and a reduction of 50 per cent in the work lost by friction has often been secured by change of lubricant. Mr. Fairbairn has found a change to the extent of 10 to 15 horse-power in a cotton-mill from the former cause.

The friction of shafting in mills varies, with size and loading, from 0.33 to 1.5 horse-power per 100 feet (31 m.) length, averaging for the "main line," with good lubrication, about 1 horse-power. The loss of power in mills ranges, with different machines, from 5 to 90 per cent, averaging for cotton and flax mills about 60 per cent, with good management, and in woollen mills about 40 per cent, the efficiencies being therefore about 40 and 60 per cent for the two cases. The friction of heavy iron-working tools may be taken at about $f = 0.15$, the efficiency at 0.85. The loss in the steam-engine is usually nearly constant at all powers, and ranges from 4 pounds per square inch (0.27 atmosphere) on small engines of 25 to 50 horse-power, down to 1 pound (0.07 atmosphere) in very large marine-engines: this gives efficiencies ranging from 0.84 to 95

or 97 per cent. In a "high-speed" engine intended to drive electric lights the Author found the efficiency to be

$$\text{Efficiency} = 1 - \frac{0.06}{U},$$

in which U is the work done, calling work "at full stroke" unity. Rules for calculating the magnitude of this loss will be given in later chapters.

"*Absolute*" *Power* is that measured on the indicator-diagram, taken down to the line of zero-pressure, that of perfect vacuum. Taking the steam used per horse-power, per hour, on this basis, permits a comparison to be made, irrespective of differences of back-pressure, either in determining the intrinsic merits of different types or of individual engines of the same type.

"*Nominal*" power is that at which the machine is rated. It may represent, as in the now usual rating of boilers, that which the engine may reasonably be expected to produce under usual conditions; or it may, as in old British practice, which assumes a mean effective pressure of 7 pounds per square inch, simply give a clue to the dimensions of the machine; while its actual working power may be several times greater.

The British rule for finding the nominal horse-power of an engine is: Multiply the square of the diameter by the speed of the piston, and divide the product by 6000. Thus:

Let d = diameter of piston,

l = the length of the stroke,

n = the revolutions per minute.

The speed of the piston is $= l \times 2 \times n$.

Area of piston $= d^2 \times .7854$.

Work done per minute $= d^2 \times .7854 \times 7 \times (l \times 2 \times n)$.

$$\text{H.P.} = \frac{d^2 \times 7854 \times 7 \times (l \times 2 \times n)}{33000}$$

$$= \frac{d^2 \times \text{speed of piston}}{6000}.$$

CHAPTER IV.

THERMODYNAMICS OF THE IDEAL ENIGNE.

HEAT-UTILIZATION BY TRANSFORMATION.

72. **The Thermodynamics of the Steam-engine** includes simply the science of its heat-transformations, resulting in the production of mechanical energy and the performance of work. As will be fully shown later, this constitutes but a part of the theory of the steam-engine; although it was long assumed by writers on the subject that the theory of the machine was purely thermodynamic. The progress of discovery and the growth of the elements of the complete theory have been traced at some length in an earlier chapter. The design of the present chapter is to exhibit the relations of the science of heat-transformation to the complete theory of the steam-engine.

Since the differences between the older and the more recent philosophy of the heat-engines grow out of the facts that the thermodynamic treatment of the case is thoroughly ideal and that the real engine exhibits a vastly more complicated and wasteful operation than does the ideal, the proposed treatment includes, first, the study of the ideal case as a problem in pure thermodynamics; secondly, the examination of the real engine and its comparison with the ideal; and, thirdly, the study of the real problem, as modified by all the conditions which characterize the actual engine in its ordinary operation.

73. **Thermodynamics, defined** as that science which treats of the laws and phenomena of those processes which result in the conversion of thermal and of dynamic, of heat and of mechanical energy, the one into the other, is the science of the perfect heat-engine. In this science, no other phenomena are

considered; no other thermal or mechanical processes are taken cognizance of; and all other forms of energy are, in its study, ignored.

Thus the wastes of heat in all forms of heat-engine, in so far as they consist of losses by conduction or radiation, as heat, without transformation, or in so far as they consist in, or involve, conversion into other forms of energy than heat and dynamic, are extra-thermodynamic and must be separately considered. Thermodynamics is that science which relates to systems in which but two forms of energy act by transfer and natural reaction through transformation.

74. **Thermodynamics and Energetics** are related as a part is related to a whole. As will be seen presently, the former comprehends laws which are restricted statements of the more general code which constitutes the broader science, and its phenomena are forms of energy-transformation which illustrate, in a narrow field, principles and processes as extended as the universe and which include all its effects of active or stored energies of whatever kind.

An intelligent understanding of the one science thus presupposes an understanding of the fundamental principles of the other and of their general bearing upon all physical and chemical, as well as mechanical, phenomena; in other words, upon all nature's movements; whether atomic, molecular, or mass motion be illustrated.

Energetics, and its progeny, Thermodynamics, are thus the great sciences which have grown out of the discovery of the nature of heat and the relations of the various forms of energy. They are the product of the present century and mainly of the last generation. They represent the highest achievement of man in the substitution of the deductive for the speculative methods in science, in the substitution, rather, of science for speculation. But while they are spoken of as the two sciences of energetics and thermodynamics, it would be more correct to say that the science of energetics comprehends, among other of its subdivisions, that of thermodynamics. There might also be a branch to be called that of thermoelec-

trics, or one called electrodynamics, as, in fact, is the case. But the science of energetics itself is but one division of a broader science, that of *Mechanics*—that great science, which bears more or less directly upon every phenomenon of nature and of the universe, and which is at the foundation of all the applied sciences, of all the arts of construction, of all the exact science of physics and chemistry, of astronomy, and of forces and motions.

Mechanics, as we have seen, includes four principal divisions:

(1) *Statics* treats of the relations of forces acting in any system when no motion results from their action.

(2) *Kinematics* treats of the relations of motions simply, of their composition and resolution, and of their resultant effects.

(3) *Dynamics* or *Kinetics* treats of the motions produced in bodies by the action of forces.

(4) *Energetics* treats of the measurement, the transfer, and the transformations of energy, under the action of forces, and of their result in the variation of the method of its manifestation.

75. **Energetics is defined** as that science which treats of all natural phenomena which, through the action of force upon matter, result in the production of motion; whether it be a relative motion of atoms, of molecules, or of masses. It is that science "whose subjects are material bodies and physical phenomena."* We may here repeat (§ 51):

Energetics thus treats of modifications of energy under the action of forces, and of its transformation from one mode of manifestation to another, and from one body to another; and within this broader science is comprehended that latest of the minor sciences—of which the heat-engines and especially the steam-engine illustrate the most important applications—*Thermodynamics*. The science of energetics is simply a wider generalization of principles which have been established one at a time, and by philosophers widely separated, both geographically and historically, by both space and time, and which have been slowly aggregated, to form one after another of the physi-

* Rankine: Proc. Phil. Soc. Glasgow; vol. III. No. 6.

cal sciences, and out of which we are slowly evolving wider generalizations, thus tending toward a condition of scientific knowledge which renders more and more probable the truth of a principle, still broader than this science, even, and which was enunciated before Science had a birthplace or a name ; i.e. :

All that exists, whether matter or force, and in whatever form, is indestructible by any finite power.

As already remarked, that matter is indestructible by finite power became admitted as soon as the chemists, led by Lavoisier, began to apply the balance, and were thus able to show that in all chemical change there occurs only a modification of form or of combination of elements, and no loss of matter ever takes place. The "persistence" of energy* was a later discovery, consequent largely upon the experimental determination of the convertibility of heat-energy into other forms and into mechanical work, for which we are indebted to Rumford and Davy, and to the determination of the quantivalence anticipated by Newton, shown and calculated approximately by Colding and Mayer, measured with great accuracy by Joule and Rowland.

It is now generally understood that all forms of energy are mutually convertible with a definite quantivalence; and it is not certain that even vital and mental energy do not fall within the same category.

The essentially important, as well as interesting, fact, in this connection, to the engineer as well as to the physicist, it should be noted, is that the laws of energetics apply unqualifiedly to atomic and molecular phenomena, as well as to energies of masses, and to all transformations of energy in either class and of any kind. There is, dynamically, absolutely no distinction, in this respect, between the methods and processes of chemistry, of physics, and of the mechanics of masses. All illustrate phases of one science, and all are energies of matter in motion.

76. **Matter, Force, and Energy** are the only quantities known to the departments of natural science. The science of

* The term "energy" was first used by Dr. Young as the equivalent of the work of a moving body, in his "Lectures on Natural Philosophy" (1807).

Chemistry deals with the forms which matter assumes under the action of measurable atomic molecular forces affecting dissimilar kinds of matter; *Physics* is that science which deals with all the other forms of sensible force and their effects. The science of *Energetics* treats of the action of forces when motion is produced, whatever the kind of force, whatever the kind of matter; it thus covers the whole range of chemistry and physics.

Matter is that which is capable of directly affecting the senses, and which occupies space. Nothing is known of the ultimate nature of matter, and we are acquainted with it only as it affects the organs of the body. It is usually divided into four classes: solids, liquids, gases, and imponderable matter, or that which cannot be assigned a finite specific measure of mass or weight; the luminiferous æther is an example of this last; the other three are familiar forms.

A *Body* is a limited portion of matter.

Force is that which produces, or tends to produce, motion, or change of motion, in bodies; it is measured, statically, by the weight which will counterpoise it, or by comparison with a known standard of force, and, dynamically, by the velocity which it will give to a known mass, in a stated time, i.e., by the "acceleration" which it is capable of producing.

Work is always performed by the expenditure of energy, and is the product of the resistance overcome by a force, or of the effort exerted by it, into the space through which that action takes place. That resistance may be constant, or variable, and due to an active, opposing force, to resisting pressure, to the inertia of masses, or of molecules compelled to submit to acceleration or retardation; or it may be due to any one of the physical or chemical forces. Thus, if U represents the work done by a force, F, acting through a space, s,

$$U = Fs. \qquad \qquad (1)$$

For variable motion,

$$dU = Fds. \qquad \qquad (2)$$

For variable forces,

$$dU = sdF. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For forces and motion variable,

$$dU = d(Fs) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The Unit of Work is the product of the units of its factors force and space, as the foot-pound, the kilogrammetre, the foot-ton, the gramme-centimetre.

Useful Work is that which is applied to the production of a specified useful effect; *Lost Work* is that which is incidentally wasted, in the endeavor to perform useful work, in overcoming prejudicial resistances, and in doing useless work; this waste occurs usually and principally in overcoming friction of moving parts.

Work of Acceleration is work expended in producing increased velocity in a freely-moving body. The effort exerted, and the resistance met, is dependent upon the inertia of the mass, and is measured thus: A body moving freely under the action of gravity, i.e., of a force equal to its own weight, acquires, in this latitude, a velocity of 32.2 feet (9.81 metres), nearly, in one second, and the acceleration, or retardation, of any freely-moving body is proportional to the effort applied, as to the resistance met by it. If f is the actual acceleration, if g measures that produced by gravity, if F is the statical measure of the effort, and W is the weight of the body, we have

$$F : W :: f' : g; \quad t : 1 :: v_2 - v_1 : f;$$

$$F = \frac{f}{g} W;$$

$$= \frac{v_2 - v_1}{gt} W; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

v_1 and v_2 being the initial and final velocities, and t the time of action of the accelerating force.

For variable acceleration,

$$f = \frac{dv}{dt}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$F = \frac{dv}{dt} \cdot \frac{W}{g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The space, s, is equal to $\frac{v_2 + v_1}{2} t$, and the *work* done is, for uniform acceleration,

$$U = Fs = \frac{v_2 - v_1}{t} \cdot \frac{v_2 + v_1}{2} \cdot t \cdot \frac{W}{g}$$

$$= W \frac{v_2^2 - v_1^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For variable acceleration,

$$U = d(Fs) = W \cdot d \cdot \frac{v^2}{2g} = W \frac{vdv}{g}. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Since $\frac{v_2}{2g} = h$, the height due the velocity v, the work is equal to that required to raise the body through the difference of the two heights due the initial and the final velocities, respectively.

Energy, the product of these forces acting upon this matter, may be defined as capacity for doing work, or to effect physical change; it is measured, either by the measure of the work which it can perform, Fs, or by the available *vis viva*, $W \frac{v^2 - v_1^2}{2g}$, or the work of acceleration. The quantity, $W \frac{v^2}{2g}$, is the "actual energy" of the mass, W. When the body is relatively at rest, and thus without available actual energy, but yet is so situated that it may do work by change of position, or of affecting conditions, under the action of existing or available forces, as, for example, when it may do work by falling from a height,

it possesses "potential energy"; this is measured by the product, $Wh = W\frac{v^2}{2g}$, of the weight into the height of fall, or into the height due the final velocity which may be acquired.

Energy, whether of Masses, or of Molecules, wherever existing, has the same character, quality, and measure; yet its availability for useful purposes depends very greatly upon the nature of the body through which it acts, and upon the method of its exhibition. The two methods of exhibition of energy are, thus, in the forms of energy of masses and of molecular or of atomic energy. A falling stone, flowing water, a flying shot, are illustrations of the first, and the energy of heat, of electricity, and of chemical combination of the second.

Energy may be potential as well as actual in either class, as the potential energy of a suspended weight, or of water in the reservoir in the one, or that of unignited fuel, or gunpowder or of the open voltaic circuit in the other.

Energy of the second form is often, but never necessarily, measured in other units than those customarily adopted in mechanics, as in "thermal units," in "ergs," or in "volt-amperes." All such units are capable of reduction to a common standard, which will here be taken as either British, as in foot-pounds, or metric, as in kilogrammetres. Work and energy must evidently have this same measure.

The quantity of work done, or of energy transformed, in the unit of time, is measured by the Unit of Power, which, in engineering, is usually the *horse-power;* this is, reckoned in British units,

550	foot-pounds	per	second;
33,000	"	"	"	minute;
1,980,000	"	"	"	hour;

in metric units, the horse-power is taken as

75	kilogrammetres	per	second;
4,500	"	"	minute;
270,000	"	"	hour.

These units are, however, slightly different, the British horse-power being 1.014 metric horse-power; i.e., instead of 550 foot-pounds per second, or 33,000 per minute, the latter is 542½ per second, or 32,549 per minute. Neither unit is the measure of the power of a horse, which is usually lower, averaging 20 or 25 per cent less than the above figures.

77. **The Laws of Energetics,** the basis of the science which it has been proposed to call by that name, are:

(1) *The Law of Persistence, or of Conservation of Energy,* viz.—Existing energy can never be annihilated; and the total energy, actual and potential, of any isolated system can never change.

This is evidently a corollary of that grander law, asserting the indestructibility of all the work of creation, which has already been enunciated.

(2) *The Law of Dissipation, or of Degradation of Energy,* viz.—All energy tends to diffuse itself throughout space, with a continual loss of intensity, with what seems, now, to be the inevitable result of complete and uniform dispersion throughout the universe, and consequently of entire loss of availability.

It is only by differences in the intensity of energy, and the consequent tendency to forcible dispersion, that it is possible to make it available in the production of work.

(3) *The Law of Transformation of Energy,* viz.—Energy may be transformed from one condition to another, or from any one kind or state to any other; changing from mass-energy to molecular energy of any kind, or from one form of molecular energy to another, with a definite quantivalence.

These laws lead to the conclusion that, in any isolated system of bodies, or in any isolated mass, the total of all energy present is always the same; though it may be transformed in various ways, and to an extent only limited by the special conditions affecting the system. They lead to the conclusion that energy of higher intensity than the mean must occupy a limited space, and will continually tend to dissipate itself by dissemination through a greater volume, affecting larger and larger quantities of matter, with proportional reduction of in-

tensity, until the whole system is occupied by the originally existing energy, at a finally uniform and minimum intensity. Energy confined within a limited space thus continually tends to expand, and to break through its boundaries, and, if not freed from this constraint, it produces a pressure upon the surrounding surfaces, which, e.g., is exhibited as tension of enclosed vapors and gases. Freed from confinement, it tends to indefinitely expand.

Either form of energy may produce either other form under suitable conditions.

Rankine's statement of the "General Law of the Transformation of Energy" is as follows: *

"The effect of the whole actual energy present in a substance, in causing transformation of energy, is the sum of the effects of all its parts."

The *axiom*, as Rankine calls it, that "any kind of energy may be made the means of performing any kind of work" is derived by "induction from experiment and observation," and confirmed by all experience. The science of energetics may be based either upon this principle, so derived, or, probably better, upon the fundamental law stated as underlying all existences; although the latter has, after all, the same basis. The science is one of which, as its great student has said, the subjects are boundless; and never can, by human labors, be exhausted, nor the science brought to perfection.

Professor Balfour Stewart considered the universe to be "composed of atoms with some sort of medium between them as the machine, and the laws of energy as the laws of working of this machine."

The Sources of Energy are: (1) Potential: (*a*) fuel; (*b*) food; (*c*) head of water; (*d*) chemical forces. (2) Actual: (*a*) air in motion; (*b*) gravity in waterfalls; (*c*) tides.

78. "**Newton's Laws**" follow directly from the general law of persistence of energy, a corollary to which may be stated thus: Change of energy can only be produced by the action of force, and by doing work. *Newton's Laws* are:

* Proc. Phil. Soc. of Glasgow; vol. III. No. V; 1853.

(1) A free body tends to continue in the state in which it, at any instant, exists, either of rest or of uniform rectilinear motion.

(2) All change of motion in a body free to move is proportional to the force impressed, and is in the direction of that force.

(3) The reaction of the body acted upon by the impressed force is equal, and directly opposed, to that force.

Inertia is that property, observed in all bodies, in consequence of the existence of which they are capable of exhibiting the action of these laws. The laws of Newton themselves are all easily verified by experiment. The "Atwood Machine," illustrated in nearly all works on physics, is constructed for this special purpose.

While Newton's laws are readily verified by experiment, the more general laws of energetics are accepted simply as being in accordance with universal experience. The generally accepted theory of the constitution of matter being assumed as a premise, however, the general laws of energy are all easily deducible from Newton's laws. Thus: the first law is but a differently worded statement of Newton's three laws combined.

To assert that every moving body tends to persist in its rate of motion, exerting an effort always equal to the retarding or accelerating force, and exerting such effort in the line of action of such force, is to assert that its energy can only be altered by the performance of an equivalent amount of work, and an equal amount of energy of opposite sign; and this latter assertion is a declaration of the indestructibility of energy. To assert that all bodies, whether masses or molecules, when in motion tend to move in rectilinear paths, is to assert a tendency to unlimited dissipation of energy through space. To assert that all matter in motion is subject to Newton's laws is to assert the laws of universal conservation of energy, and of the quantivalence of all transformations, as stated in the third general law. Whenever it becomes established that any phenomenon, as the transfer of heat, of light, of electricity, or of sound, is a mode of motion affecting bodies of whatever

class. Newton's laws bring that phenomenon within the scope of the general laws of energy. Every phenomenon, molecular or other, which involves relative motion of masses, vibrations of parts, or pulsations in fluid media, is now well understood to be subject to these laws.

Tait* finds in the Principia of Newton, in the scholium to his Third Law, the enunciation of the principle of D'Alembert, and also of the Law of Conservation of Energy. He paraphrases this statement thus:

"Work done in any system of bodies has its equivalent in the form of work done against friction, molecular forces, or gravity, if there be no acceleration; but if there be acceleration, part of the work is expended in overcoming resistance to acceleration; and the additional energy developed is equivalent to the work so spent."

79. **Algebraic Expressions** of the transformability of the energies are now readily deduced. If in any isolated system a certain quantity of energy exists, homogeneous in character and heterogeneously distributed; and if, by any process, other and various forms of energy appear in that system, these latter must be the result of transformations of parts of the initial stock of energy by conversion into the new forms. But every such change must be effected by a perfectly definite and exact quantivalence.

Assume this ratio of values of customary units reduced to a system of equivalents, then it becomes at once practicable to measure all these energies in the same units; as, for example, when Joule measures either heat or mechanical energy, taking $J = 772$ foot-pounds as the equivalent of a British thermal unit, or $J =$ about 423 *kilogrammetres*, as the equivalent of one metre or thermal unit alone; the thermal unit being defined as the quantity of heat or energy-equivalent demanded to raise the temperature of unit weight of water one degree from the temperature of maximum density.

Taking either kind of unit in thus measuring, we shall have

* Sketch of Thermodynamics; Revised Ed., p. 65.

the initial stock of the one kind of energy altered by the quantity which, in the same units, measures the aggregate several quantities of energy resulting from the change; and

$$dE = \frac{dE}{dT}dT + \frac{dE}{dU}dU + \frac{dE}{dV}dV +, \text{ etc.}; \quad . \quad . \quad (1)$$

where E, T, U, V, etc., are the symbols representing the several energies, initial and other.

If T measure heat-energy, and U be taken as potential energy of the molecular kind, V the potential energy of an elastic fluid varying in volume, W the work of some mechanism or a dynamic process, the total variation of the initial energy, E, will be equal to the total of all the new energies, and the new work, in proportions which become known as soon as the partial coefficients $\frac{dE}{dT}$, etc., are determined.

If two energies only, as thermal and mechanical, are affected, and if the original stock were simply heat-energy, we should have a change, dE, in the initial stock of heat-energy, which would be the precise equivalent of the sum of the two changes taking place, simultaneously, in the initial store and in the temperature, T, of the system, and in work by the change of volume, V, against a pressure of, say, the intensity p. Then, obviously,

$$dE = \frac{dE}{dT}dT + \frac{dE}{dV}dV; \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and, since $\left(\frac{dE}{dT}\right)_v$ measures the specific heat, K_v, for constant volume, and as $\left(\frac{dE}{dV}\right)_T$ must measure the intensity of pressure producing, or resisting, the change of volume,

$$dE = K_v dT + pdV. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If but one kind of transformation occurs, as by conversion of any original form of energy, E, into work,

$$dE = pdV; \quad \text{or,} \quad dE = RdS; \quad . \quad . \quad . \quad . \quad (4)$$

accordingly as the work is performed in compressing a fluid, or in overcoming a resistance, R, through a space, dS.

80. Energetics and Thermodynamics are thus seen to be sciences of similar general character, of which the first involves the second, together with all other applications of the foundation-principles of the persistence of energy, and of equivalence in transformation of energy from one form to another.

Energetics, as first defined by Rankine, comprehends all physical phenomena involving transfer, or change, of energy. *Thermodynamics* confines itself to such as involve simply transfer, or transformation of energy, in the related forms of heat and mechanical energy. The general laws of transformation of energy are here limited, in their application, to cases in which heat is transformed into mechanical energy, or by the production of mechanical work, and to instances of the opposite kind, in which mechanical energy or work produces heat by its own transformation into thermal energy.

When heat enters into any substance, the operation is a process of adding to the total energy of that mass, and it may increase either its kinetic or its potential energy, or both; the loss of heat from a body is the loss of a definitely measurable quantity of energy.

The usual effects are these:

(1) To increase the energy of molecular motion, by intensifying the energy of vibration of the particles.

(2) To separate molecules, thus producing an increase of potential energy.

(3) Expanding the whole mass against external pressure; i.e., doing external work.

The sum of all work in these three ways is the mechanical equivalent to the heat-energy transferred.

Heat being energy, there can be no restricted kinematic science of Thermotics; this science is purely thermodynamical.

The Science of Thermodynamics is defined as comprehending all facts and principles which are involved in the transformation of heat-energy into mechanical energy, or work, or in the reverse process. This science consists of a system of definitely

stated laws, based on observed facts, and united to form a consistent physical theory.

Thermodynamics is sometimes called the "Mechanical Theory of Heat;" but it is more than this; and, based on that theory, it comprehends all the physical laws and all the phenomena involved in dynamic changes in which heat-energy plays a part. The mechanical theory of heat—i.e., the theory, now considered established, that heat is a form of energy—is simply the expression of a fact which underlies the science of thermodynamics. Thermodynamics is thus a branch of the division, "Energetics," of the still broader science, "Mechanics."

81. **The Basis of the Science of Thermodynamics** is found in the fundamental laws of persistence of energy and of existing forms of matter, which have been already enunciated, which laws are here restricted to their applications in the relations of interchanging heat and work; they are, therefore, restricted statements of the more general laws of energy, and are all comprehended in the larger science. The science of thermodynamics is thus based upon the experimentally proven facts that heat is a form of energy; that "it is a kind of molecular disturbance; that the motion is, in solids, one of vibration, in fluids, of translation, of molecules; that it is possible to transfer this molecular energy from part to part of any mass, and from one body to another, by contact—i.e., by conduction—and by radiation through space, the "luminiferous æther" supplying the necessary medium; that this molecular energy may become transformed into other kinds of energy, and that such transformation is definite in its extent and in its effects.

As will be hereafter seen more fully, therefore, the Science of Thermodynamics is based, primarily, upon the great laws of the persistence of energy, of the equivalence, in transformation, of one form of energy into another, and of the tendency of all kinds of energy to indefinite expansion, with indefinite reduction of intensity; it rests directly upon two sets of well-established relations:

(1) The relation, qualitatively known, and quantitatively established with a considerable degree of accuracy, between heat, considered as one form of energy, and mechanical work and energy, either actual and kinetic or stored and potential.

(2) The relation between variations of quantities of heat and of mechanical work or energy, during a process of transfer or transformation, and the temperature at which such transformation, or transfer, takes place.

These relations being determined, equations are easily deduced from them expressing the efficiency of heat-engines, and applicable to all physical actions illustrating such changes.

The Methods of Transformation, in such thermodynamic operation in heat-engines, involve, simply, the variation of the volume and pressure of a confined "working substance," which expands with accession of heat and contracts with its rejection. The resistance to expansion by heat during the first operation is less than that met with in the second, and the mean difference measures the mean external resistance, the continuous overcoming of which constitutes the work of the system. It is evident that such changes are essential to the production of mechanical energy; as no work can be done at constant volume, either externally or internally.

This working substance may be either solid, liquid, or gaseous; is almost invariably of the latter class, and is always of this class in the familiar forms of heat-motors.

The First Law of Thermodynamics may be stated thus:

Where work is done by expenditure of heat, the quantity of heat consumed—i.e., transformed or converted—is a measure of the quantity of work done, or of energy acquired, in the new form; and, conversely, the transmutation of work into heat-energy occurs by a definite equivalence.

This first law, or fundamental principle, has several important corollaries:

(1) When mechanical energy is expended upon bodies which do not transfer it to others, or do not in any way transform it, heat is produced in equivalent amount, and the temperature of the mass is thus correspondingly elevated.

Conversely: When mechanical energy is expended by an expanding body exhibiting no mass-energy, and without transfer of heat, the substance loses an equivalent amount of heat, and its temperature is correspondingly depressed.

(2) When internal work is gained or lost during changes or transfers of energy, the amount of that work measures a corresponding external loss or gain of heat or work.

(3) No internal work being done, all isothermal changes are accompanied by a transfer of heat to or from the substance, precisely equal in amount to the work done by that substance upon other bodies, or by other bodies upon it.

(4) Whatever the character of the work done by, or upon, any substance, the actual thermal, or internal, energy, whether kinetic or potential, will remain unchanged *only* when the energy so transferred has an equivalent in the quantity of heat received by it, in the one case, or discharged from it, in the other.

The principle of equivalence of energy thus applies in thermodynamic changes as it does whenever transformation occurs between any existing forms of energy, whether mechanical, physical, or chemical; and, evidently, since the algebraic sum of all energies communicated to any substance is equal to the algebraic sum of all work done, both within the substance and by it upon other bodies, and of all energies stored within it, or transferred by it to adjacent masses, the same principle and its converse obviously hold with respect to this limited class, involving only thermal and mechanical energies.

The Second Law of Thermodynamics, which relates to the proportion of energy present in any thermodynamic operation, which may be converted from the one into the other of the two forms, and in accordance with the First Law, will be stated later.

82. Algebraic Expressions of the First Law of Thermodynamics, illustrating the operations seen wherever one of the two forms of energy is converted into the other, are readily deduced:

As illustrating the transformability of heat into mechanical energy, suppose a quantity of heat, Q, in thermal measure,

given in dynamic measure, $H = JQ$, to be expended in raising a weight W to a certain height, h, thus performing mechanical work, Wh; let the body thus raised fall again, and measure its height, h', and velocity, v', at any given altitude, thus determining the actual and potential energies at that point. We should thus find several equivalent measures of energy, taking as before $J = H \div Q$;

$$H = JQ = Wh = \frac{Wv'^2}{2g} + Wh' = \tfrac{1}{2}Mv'^2 + Wh'. \quad . \quad (1)$$

Should the falling mass strike an inelastic body on reaching the ground, transferring to it all its energy without producing movement of the mass struck; or should it be arrested by friction, the equivalent of all this energy would reappear in its original form of heat, and might be measured by the quantity $\frac{W(v_2^2 - v_1^2)}{2g}$, in which W is the weight, or $\frac{W}{g} = M$ the mass, of the heated body, v_1 the mean velocity of its molecules at the instant before and v_2 the velocity after the shock. Thus energy, originally heat, is changed from one form to another, as it passes from point to point; but it always finally eludes observation by dissipation as heat of continually decreasing intensity, extending throughout constantly enlarging space.

Every transformation of energy illustrates some one of these changes, and, in every case throughout the series, we have energy transformed by transfer from one body to another, and by change in mode of motion, until a cycle is completed and all energy originally heat becomes heat again of "lower grade"—i.e., of lower intensity, but affecting a greater mass of matter. In every step of the series, we find the equality:

Energy exerted (i.e., Energy transformed) = Work done.

In any machine, the energy exerted is partly transferred through the machine to its legitimate work, partly transformed into heat-motion by friction, and, in some cases, partly temporarily stored in the machine by acceleration of velocity of

heavy parts, in which cases it is restored when retardation takes place. In all such instances the First Law is exemplified, the work and heat observed having definite relations of quantity.

Heat, or energy, taking effect in expansion of solids, the evaporation of liquids, or expansion of vapors, is precisely equivalent to the mechanical work done in altering molecular velocity, and in producing changes of relative position among the molecules of the substance, thus doing work against external pressures and internal molecular forces. In such cases, we have a definite quantity of heat, H, or JQ, transformed, and an equally definite internal and external total mean resistance, $f+p$, overcome through a certain space v in each unit of time; then

$$H = JQ = v(f+p). \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For variable pressures and volumes, the heat transformed, and thus, as heat, expended, between configurations a, b, is

$$H = JQ = \Sigma \int_a^b pdv = \Sigma \int_a^b Pds. \quad . \quad . \quad . \quad (3)$$

The mechanical equivalent of heat is the specific heat of water at its temperature of maximum density expressed in dynamic units.

The value of the mechanical equivalent of heat has been commonly taken as first adopted by Joule, although recent and most carefully conducted investigations indicate a value higher, by perhaps one per cent, to be more accurate. Existing tables, and nearly all work done in this field to date, have, however, been based upon Joule's figure.* The First Law may be thus enunciated:

Thermal and Mechanical Energy are mutually interconvertible in the proportion of one British Thermal Unit for each 772 *or* 778 *foot-pounds, or of one* Calorie *for each* 424 *or* 427 kilogrammetres *of energy or of work.*

* $J = 778$ for the air-thermometer at 62° F. corresponds closely with 778 for the mercurial thermometer at the temperature of maximum density of water.

This "Mechanical Equivalent" of the heat-unit, or "Dynamical Equivalent" of heat, known as "Joule's Equivalent," is represented by the symbol J.

As is seen from the above, the metric unit has nearly four times the magnitude (3.968 times) of the British unit.

83. **The Steam-engine illustrates the First Law** as well as does any other apparatus or machine converting heat into work. The performance of work by heat-engines invariably results in the conversion, or destruction by transformation, of a definite quantity of heat into mechanical energy; and, conversely, the expenditure of mechanical energy will produce heat energy in the proportion of 2545 B. T. U. per h. p. per hour, 42.42 B. T. U. per minute.

When a steam-engine is in regular, steady, operation, doing its stated work, the stream of energy sent to it from the boiler, in the steam which is, to that point, its vehicle, divides into two, the one passing out, as mechanical energy, to do the prescribed work, the remainder, usually, vastly the greater part, flowing on, unchanged in kind, to be rejected into the condenser or into the atmosphere, losing, however, *en route* through the machine, a part, usually small, by conduction and radiation to surrounding objects.

Could the magnitude of these currents of energy be continually observed and measured, it would be found that the quantity of energy leaving the machine by these several routes would be, at every instant, precisely the quantity entering the engine; but that the amount rejected and lost as heat would be less by precisely the amount of mechanical energy produced.

84. **The Second Law of Thermodynamics** *asserts that the total of any single effect of any given quantity of heat acting in any thermodynamic operation is proportional to the total amount of heat-energy so acting.**

Experiment and general experience indicate that actual heat-energy is homogeneous in condition and attributes, and

* This principle is substantially that first accepted by Rankine as the second law. Actual energy of vibration is understood.

that the effect of any one portion of the total amount acting to produce any single, definite, effect, as change of pressure, or change of volume, is precisely the same as that of every other equal portion.

In other words, the units of which it may be assumed to be composed are all of precisely the same nature, and are, under similar conditions, capable of producing precisely equal effects. Since, in accordance with this law, the magnitude of any and every effect of heat-energy is proportional to the quantity of that energy acting in its production, it follows that every such effect has for its measure the product of that quantity of heat into some function; the form and magnitude of which are determined by the conditions under which the change takes place.

Thus, if we call the quantity of heat undergoing transfer H, the total heat Q, and the function above referred to, called by Rankine the "*Thermodynamic Function*," ϕ; then any elementary quantity of work, produced by transformation of heat,

$$dH = Qd\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the value of H can be determined by integration when the method and the rate of variation of heat and the Thermodynamic Function are known.

Since, in any case, the quantity of heat, Q, is known to be proportional to the absolute temperature, T, it follows, also, that

$$dH = Td\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and the value of H can be obtained when ϕ is known in terms of T and of constants, or of other independent variables so expressed as to make the above equation integrable. This expression, the basis of the whole theory of heat-engines, shows that the amount of energy transformed is measured by the product of the absolute temperatures of transformation into some function of the changes of condition of the working substance. This factor is Clausius' "Entropy." *

This Second Law is also more generally expressed by Rankine as follows: *If the total actual heat of a homogeneous and*

* This function depends solely on the state of the body.

uniformly hot substance be conceived to be divided into any number of equal parts, the effects of those parts in causing work to be performed are equal. This law is one case of a general law applicable to every kind of *actual energy;* that is, of capacity for performing work, constituted by a certain condition of each particle of a substance, how small soever, independently of the presence of other particles. The symbolical expression of the Second Law of Thermodynamics is given as follows: Let unity of weight of a homogeneous substance, possessing the actual heat Q, undergo any indefinitely small change, so as to perform the indefinitely small amount of work dU. It is required to find how much work is performed by the disappearance of heat. Conceive Q to be divided into an indefinite number of indefinitely small equal parts, each of which is δQ. Each of those parts will cause to be performed the quantity of work represented by

$$\delta Q \cdot \frac{d}{dQ} dU;$$

consequently the quantity of work performed by the disappearance of heat will be

$$U = Q \cdot \frac{d}{dQ} dU, \quad \text{or} \quad \frac{U}{dU} = \frac{Q}{dQ},$$

which quantity is known when Q, and the law of variation of dU with Q, are known.

From the mutual proportionality of actual heat and absolute temperature, there follows—

The Second Law of Thermodynamics, expressed with reference to absolute temperature. If the absolute temperature of any uniformly hot substance be divided into any number of equal parts, the effects of those parts in causing work to be performed are equal. This law is expressed algebraically as follows: From the relation between absolute temperature (τ) and actual heat (Q) it follows that

$$\frac{\tau}{d\tau} = \frac{Q}{dQ};$$

consequently the expression above, for the work performed by the disappearance of heat, is transformed into

$$\frac{U}{dU} = \frac{\tau}{d\tau}.$$

The first and second laws constitute the basis of the Theory of Thermodynamics.

Rankine has shown that the second law must follow from the hypothesis that "sensible heat consists of any kind of steady, molecular motion within limited space;" and it is now considered as well established, both that heat does consist of such molecular motion, and that the second law is correct. The magnitude of heat-energy must thus be proportioned to the weight of matter affected by it, and to the mean square of the velocity of molecular motion. Absolute temperature, properly defined, is proportional to the actual molecular energy of the matter so affected; and it thus again follows that any conversion of such energy, during any change in the dimensions of the space enclosing it, is proportional to the absolute temperature.*

Clausius' enunciation of this law is as follows:† "The work which heat is capable of performing, in any variation of the arrangement of parts of any body, is proportional to the absolute temperature at which such change occurs."

This law evidently asserts the independence of the maximum possible work and the nature of the "working substance;" and it may be taken as a corollary that—

When, in any heat-engine tracing a cycle, the working substance operates between two fixed temperatures, the work done, or the energy produced, is precisely proportional to the quantity of heat transmitted from the source of heat to the refrigerator, without regard to the nature of the substance adopted as its vehicle—as shown by Carnot in 1824.

* See Rankine, "On the Second Law of Thermodynamics;" Trans. Brit. Assoc., 1865; Phil. Mag., Oct. 1865.

† Poggendorff's Annalen, 1862.

This was demonstrated by Clausius, who made the principle "it is impossible for heat to pass, of itself, from a colder to a warmer body" the basis of his argument.

Thus, of the whole quantity of heat passing from the heater to the working substance, one part is always transmuted into mechanical work, or energy; while the remainder goes to the refrigerator, and the ratio of the one quantity to the other is perfectly definite.

Professor Wood expresses this law thus:

"If all the heat absorbed be at one temperature, and that rejected be at one lower temperature, then will the heat which is transmuted into work be to the entire heat absorbed in the same ratio as the difference between the absolute temperatures of source and refrigerator is to the absolute temperature of the source." *

85. The Steam-engine illustrates the Second Law, both in its operation as a whole and in the details of energy-transformation going on in its inner workings. Not only is it true that two perfect engines, of different power, working under the same thermodynamic conditions perform work by the conversion of precisely proportional quantities of heat; but it is also true that the work-effect of heat at any instant, in the midst of the steam so doing work by its expansion, is proportional to the quantity of heat at that instant there present and taking its part in the thermodynamic action of the fluid.

As will be seen, however, presently, the second law finds important application simply in enabling us to ascertain the total quantity of work, external and internal, required to produce changes of volume and energy in fluids, like the vapors, in which we cannot measure directly the internal forces and internal work.

86. General Algebraic Expressions for Thermodynamic Changes of Energy may be readily deduced directly from the First Law of Thermodynamics. Since only transfers of heat and transformations into mechanical energy, actual or poten-

* Thermodynamics, § 40.

tial, are considered, assuming any small variation of heat, dH measured dynamically, to take place, producing variations o the physical state of any substance; if the change of sensibl heat be called dS, that of "latent" heat. dL, and of externa work, dU, then the first law of thermodynamics is expressed b the equations:

$$dH = dS + dL + dU, \quad . \quad . \quad . \quad . \quad . \quad (A$$

and

$$dH = dS + dW, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B$$
$$dH = dE + dU, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (C$$

where, in the last two expressions, $dE = dS + dL$, and is th variation of energy, actual and potential; while $dW = dL + dU$ and is the total work done, externally and internally. Thes are primary and general equations.

The quantity E is often called the intrinsic energy of th substance; L is evidently a potential energy; while S is a form of molecular kinetic, or actual, energy, which may sometime be regarded as also in a sense potential.

The above are completely general expressions of the GENERAL FUNDAMENTAL EQUATION OF THERMODYNAMICS.

It will be observed, however, that, while the law enables u to say that, a given amount of work, dU, being done, and known quantity of sensible heat, dS, being transferred from the source without transformation, the total quantity of hea demanded for the two changes, occurring simultaneously o successively, will be precisely the sum of the thermal equivalent of the first and the thermal measure of the second that law does not enable us to say what, in any given case o heat-expenditure, will be the method of distribution of energ in the two forms, or the magnitude of either of the two part into which it is thus divided. We must evidently find a wa of determining dU; and this, when it includes internal work may be impracticable, as a matter of observation and direc measurement. It is this subsidiary problem which the secon law is called in to solve.

. **The Relations of the Two Laws of Thermodynamics** e theory of thermodynamic operations and heat-engines ow readily defined. The First Law states that, wherever ıal and mechanical energies are converted, the one into ther, such conversion takes place in the proportion of one ıal unit to each "mechanical equivalent," as previously ed; while the Second Law asserts that, during such conver- whatever proportion of the thermal energy present may converted, that proportion is equal to the product of the ity of heat, or of the absolute temperature, into another , the form and magnitude of which are determined by physical conditions. The first law gives no clue to the od of transformation, and no measure of the total quan- f energy transformed in any case; it simply asserts that uch heat-energy as *is* converted into the other form is so nuted with a definite quantivalence. The second law, merely asserting that the quantity transformed is propor- l to the total heat present, and to the absolute tempera- at which transformation takes place, enables a determina- to be made, by its combination with the first law, of the l quantity of energy so changing form.

he first law enables us to construct a general equation of odynamics; the second law, as will be more fully shown gives the form and value of its second term. Thus heat, whatever source derived, once stored within any mass of ing substance, becomes subject to these two laws; and the first law determines what amount of mechanical en- may be produced per unit of heat-energy transformed, the d law prescribes both the proportion of the total stored which, under the given conditions, may be so transformed, he proportion of utilized to unutilized heat. A reservoir ining any given amount of heat-energy, no additional nt can be transferred into it, except it be heat of higher erature; and, once the added energy enters the reservoir, nnot be again removed as a distinct quantity of heat of temperature, but becomes a part of the whole stock of y, and, in common with the original store, becomes sub-

ject, unqualifiedly, to the second law of thermodynamics, in all operations involving transformation.

88. The Thermodynamics of the Constitution of Matter and its physical and chemical changes must be considered before the heat-engines can be intelligently studied; since, in all of them, variations of temperatures, pressures, and volumes of one or another form of "working fluid" constitute the process of their action.

The physical state of matter is determined by the intensity of internal forces and by the quantity of internal heat, i.e., of heat-energy present in the mass, and it varies as transfer takes place to it or from it by communication with external bodies. The intrinsic energy of the solid body is a form of potential energy, or energy of position; equilibrium being maintained by the adjustment of volume to temperature, and this energy being developed as kinetic, or in the production of work, as temperature and the stock of sensible heat are reduced. The same is true of liquids, which, however, have a larger stock of molecular, potential, energy, and have, by expansion, lost stability of form. The change of state, traced further, passes through that of the vapors and of the permanent gases, and finally is exhibited the condition of the perfect gas, in which equilibrium exists between external confining pressures and the total tension due the pressure of heat-energy; in which, also, no internal condensing forces are observable. In the latter case the total heat-energy is exactly proportional to the absolute temperature, and is measured by the continued product of weight, real specific heat, and absolute temperature.

89. Solids, Liquids, and Gases constitute the three forms of matter into which all kinds are classed. The exact structure and constitution of matter are well understood only so far as the senses, aided by physical apparatus, can observe it; of its ultimate nature nothing is known. So far as our knowledge goes, all forms may be assigned to one or another of these classes. All known kinds of matter are probably capable of taking, under different conditions, definite for each case, either of these forms. Nearly all known liquids, for example, under certain

definite conditions of temperature and pressure, may be solidified, or may be vaporized; solids are liquefied and vaporized by elevation of temperature; and all familiar gases may be liquefied, and have even been solidified, by subjecting them to pressure and, at the same time, reducing their temperature.

All matter, so far as is known, may be considered as consisting of an aggregation of collections of "atoms," or particles, which collections are called molecules, separated by intermolecular spaces of greater or less extent, attracting each other with a force which is dependent upon the nature of the substance, and upon its volume, and yet held apart by repellent forces, which seem, usually, to have an intensity dependent principally upon temperature, and may probably be due solely, and in all cases, to the heat-energy present, stored in the substance. The attractive forces are considered to be of two kinds, the one purely attractive, the other giving permanence of form; the first is the force of "cohesion," the second that of "polarity."

Solids have stability both of form and of volume, resisting every attempt to alter either; *liquids* are stable as to volume, but destitute of stability of form; *gases* have no stability either of form or volume; and, at all measurable temperatures, constantly tend to indefinite diffusion throughout space. Solids have molecules bound together by cohesive attraction, and held in definite relations of position by polarity; in liquids polarity becomes unobservable; in gases only the repellent forces are seen, and equilibrium between attraction and repulsion of molecules can no longer exist.

In the passage from one state to another, in many cases, matter passes through intermediate states, solids becoming viscous when liquefying, and liquids becoming imperfectly gaseous before fairly attaining the perfectly gaseous state. The perfect gas is absolutely free from the influence of attractive molecular forces. All known gases are more or less imperfect; but a few, as oxygen, hydrogen, nitrogen, in their ordinary conditions, may, for all purposes of the engineer, be considered perfect.

The Fusing and Boiling Points, or the freezing and liquefying temperatures, are, as already stated, fixed for each fluid for

every pressure, but variable with change of pressure. Increased pressure usually increases the former and always elevates the latter. An exception, in the case of ice, for example, is seen when fusion is accompanied by contraction; the melting-point is lowered, in such cases, by increase of pressure. The pressure p, at which the boiling-point becomes a given temperature, T, on the absolute scale is very exactly given, for several liquids, by a formula constructed by Rankine* to represent Regnault's experiments:

$$com.\ log\ p = A - \frac{B}{T} - \frac{C}{T^2}; \quad . \quad . \quad . \quad . \quad (1)$$

and, for the reverse determination,

$$T = \frac{1}{\sqrt{\left(\frac{A - log\ p}{C} + \frac{B^2}{4C^2}\right)} - \frac{B}{2C}}, \quad . \quad . \quad . \quad (2)$$

in which, for the Fahrenheit scale and pressures in pounds on the square foot, the several quantities are:

	A	$log\ B$	$log\ C$	$\frac{B}{2C}$	$\frac{B^2}{4C^2}$
Water	8.2591	3.43642	5.59873	0.003441	0.00001184
Alcohol	7.9707	3.31233	5.75323	0.001812	0.00003282
Ether	7.5732	3.31492	5.21706	0.006264	0.00003924
Carbon disulphide	7.3438	3.30728	5.21839	0.006136	0.00003765

Regnault's own formula, as adapted to the Centigrade scale and to pressures in millimetres of mercury, for temperatures, t_m, exceeding 100° C., is as follows:

$$log\ p = a - b\alpha^x - c\beta^x, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which $x = t_m + 20$, and

$$a = 6.2640348; \qquad log\ b = 0.1397743;$$
$$log\ \alpha = 9.994049292 - 10; \qquad \text{“}\ c = 0.6924351.$$
$$\text{“}\ \beta = 9.998343862 - 10;$$

* Philosophical Magazine; 1854.

For temperatures between the freezing and boiling points,

$$log\,p = a + b\alpha^t - c\beta^t,$$

hich, as corrected by Moritz,*

$a = 4.7393707$; $log\ b = 8.1319907112 - 10$;
$\alpha = 0.006864937152$; " $c = 0.6117407675$.
$\beta = 9.996725536856 - 10$;

The temperatures of fusion of metals and those of fusion of boiling of other substances are given in works on physics on special materials.

The "luminiferous ether" which apparently pervades all e, and which transmits light and heat to us from the sun, gas of such exceeding tenuity that it opposes no measble resistance to the bodies of the solar system and of the erse, and is of such slight density and high elasticity as to smit vibrations with nearly two hundred and fifty thousand es the velocity of those traversing hydrogen gas; it has efore one five-hundredth the density of any hydrogen ch may exist in the interstellar spaces.

The Kinetic Theory of Matter is now generally accepted by of science. According to this theory, a gas consists of a ection of molecules, simple or complex, which are in exnely rapid motion, and which intermingle freely, coming collision with each other,† and with the confining surfaces, h a violence which depends upon their velocities; which ocities, in turn, are determined by the temperature of the ss. The intermolecular spaces, and therefore the free paths he molecules, are of comparatively great extent. In liquids, free paths of the molecules become very greatly restricted the action of now measurable attractive forces; and in ids, in consequence of the confining action of cohesion and polarity, brought into play by the condensation marking the

* Clausius.

† Boltzmann suggests that collisions may be rare, if not absolutely impose, the molecules swinging about each other in hyperbolic, comet-like orbits, hout contact.

further change from the liquid state, the particles can only vibrate about a fixed point without change of mean position relatively to adjacent particles.

The state, or the form, of matter is thus determined by the action of forces external and internal. The intensity of internal attractive and repulsive forces, and of external pressure, determines whether a substance may exist in the liquid or the gaseous condition, and the action of polarity produces when the particles are brought closely together, the solid state; while rise in temperature, by modifying the intensity of the molecular forces and separating molecules, causes the solid to pass through the pasty and viscous condition, and to become liquid at higher temperatures; it then vaporizes, and finally becomes gaseous, in consequence of separation of particles by the repulsion produced by heat-motion.

The size of the molecule is probably always the same in the same kind of matter; but different in different substances. Sir William Thomson estimates a molecule of glass as probably less than one twenty-five millionth and more than one two hundred and fifty millionth of an inch in diameter (less than $\frac{1}{1000000}$ and more than $\frac{1}{10000000}$ millimetre). He states that, were a drop of water as large as a pea magnified to the size of the earth, its molecules would then appear to be, in size, between that of a small leaden shot and that of a cricket-ball.* He calculates the number of molecules present in a cubic inch of any perfect gas at atmospheric pressure, and at the freezing point in temperature, to be 10^{23}, or one hundred thousand million million million. According to Avogadro's law, this number is the same for all perfect gases.

Plateau† concludes, from experiments made by him upon the tenuity of liquid bubbles, that the radius of molecular attraction is less than one seven-hundredth of an inch ($\frac{1}{28}$ millimetre). Wartmann‡ makes the range still less. Robison had-

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* Nature; 1870. Silliman's Journal of Science and Art; July 1870.

† Smithsonian Report; 1856.

‡ Trans. Soc. Geneva; 1862.

long before * inferred, from experiments with Newton's rings, that the effect of pressure is observable before actual contact, at a distance of about one five-thousandth of an inch ($\frac{1}{200}$ millimetre), and Powell † detects this action at one eleven-hundredth of an inch ($\frac{1}{40}$ millimetre).

90. **Internal and External Work,** when change of physical state occurs, are always the immediate cause of change of volume and molecular arrangement. As this alteration of condition involves, internally, the application of force to overcoming atomic or molecular resistance, or the reverse, with alteration of volume, work is consumed or is developed in the process, and an equivalent amount of energy is transformed into, or out of, work. Work so done is entirely independent of external forces and conditions, and its amount is a function, solely, of the forces acting, and of the spaces traversed, or of the alteration of volume incident to the physical changes occurring. Such work is called *Internal Work*, and energy operating in this manner is known as *Internal Energy.*

Changes of volume occurring in any mass, in the presence of other substances, involve the overcoming of external pressure, or are facilitated, to some extent, by the action of external forces. This is equivalent to the production, or to the consumption, of a certain amount of work, which is known as *External Work.*

Energy may thus be expended in the production of either internal or external work, or both; and, on the other hand, internal energy may be transformed into external work; or the reverse operation may take place.

The amount of external work so performed is evidently determined by the magnitude of the change of volume, and by the intensity of external pressures, solely, and is thus not necessarily dependent upon internal conditions. An equation involving both internal and external work, or energy, is evidently an equation involving two independent variables.

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* Mechanical Philosophy; vol. 1. † Phil. Trans.; 1834.

Thus, when steam, air, or gas expands behind the piston of a heat-engine, the internal work done is measured by the product of the mean intensity of molecular attraction into the change of volume occurring; and this quantity may be much greater than the external work; in the case of steam, it far exceeds the external work done in driving the piston; its amount is comparatively small, and is very difficult to measure, in the case of air, and it becomes indefinitely small in the case of the perfect gas; it is a function of volumes, and of molecular forces. The external work is measured by the product of the mean intensity of pressure on the piston by the volume traversed, and is limited by the resistance to the motion of the piston on the one hand, and by the intensity of molecular repulsion on the other. An equilibrium always exists between this latter force and the sum of internal attractive and external compressive forces. When the fluid expands freely into a vacuum, evidently no external work is done. External work is usually ultimately converted, through mechanical energy, into heat.

The *Internal Energy* of a body is the potential energy, or the capacity to do work, possessed in virtue of the existence of internal *repulsive* force. The potential energy of a mass capable of condensation under the action of internal attractive forces is another similar and equally important form of energy.

Rotational Motion evidently can only be produced or destroyed in a fluid by the action of a force which may be denominated internal friction, or molecular friction; hence, such motion cannot exist in a perfect fluid, or, if existing, may be neglected, as being invariable, and need not be taken into account in any accepted theory.

91. **Heat, defined and measured, as a Form of Energy,** constitutes the principal subject of treatment in the branch of applied physics here studied. The term heat may be used in either of the two senses; it may represent the sensation due to that physical action which has been described, or it may mean that phenomenon itself. It is in the latter of these two senses that the term is here used; and heat is to be here con-

sidered only as a form of the energy of molecular motion, or vibration, capable of transfer from one body to another, and of transformation into other forms of energy. In its measurement, it is necessary to consider two magnitudes, the one defining intensity, the other its quantity. Since any given amount of energy may exist, whatever its form, either as the energy of a small quantity of matter in rapid motion, or as that of a larger quantity in less violent motion, the rate of heat-motion, and the consequent "intensity" of the heat, must be observed, as well as the quantity.

Since heat is energy, and since it is measured by the product of the mass of matter pervaded by it into its intensity of action, i.e., by the quantity $\frac{1}{2}Mv^2 = \frac{Wv^2}{2g}$, it is evident that, whether it pervades one substance or another, and whatever the mode of transfer, the quantity of heat-energy is the same, when the same work is done, or the same kinetic energy is present, and that it is entirely independent of the nature of the substance, having the weight W, or the mass M, and the molecular velocity v, which simply serves as its vehicle.

The Physical Effects of Heat, as a form of energy introduced into matter, are seen in several distinct classes of phenomena:

(1) The temperature of the substance rises, the sensation of heat, produced upon the nerves of touch by contact with the body, is intensified, and the tendency to transfer heat to adjacent bodies is increased.

(2) The elasticity of volume of the substance is increased, and its stability of form is decreased.

(3) The substance is given increased volume, and, reaching certain definite points in the scale of temperature, is caused to change its physical state, as from solid to liquid, or from the liquid to the gaseous form.

(4) External work is performed, i.e., work is done against forces affecting the mass from without.

(5) As a secondary effect, the chemical composition of bodies is often altered, elements uniting more readily to form compounds, and compounds changing their constitution, sometimes at fixed, sometimes at variable, temperatures; combination being, within certain limits, usually, but not always, accelerated by increase of temperature. Dissociation sometimes occurs.

Reversal of the phenomenon, causing a reversal in the direction of movement of heat, produces heat where it had been expended, and decreases temperature where increase had previously taken place.

Dynamically, the effects of transfer of heat are:

(1) Change of temperature; i.e., variation of sensible heat-energy, or of the kinetic energy of molecular motion.

(2) Performance of internal work; i.e.:

(*a*) Molecular work, which is often considered to include the preceding.

(*b*) Intermolecular work; i.e., work done against molecular cohesion, or other attractive forces.

(*c*) Interatomic work; or similar work done within the molecules, and against the chemical forces.

Temperature measures the intensity of heat and its tendency to transfer itself to surrounding bodies. When two bodies are at the same temperature, they exhibit no tendency to change by transfer of heat from one to the other. Whenever two bodies are brought together, heat is exchanged between them, the hotter yielding to the colder more than it receives from the latter, until they attain a state of common and uniform temperature at which the flow of heat ceases, each receiving precisely as much as it loses. The higher the temperature of any body the greater the tendency to expand, and the greater its elasticity of volume, and the less its elasticity and stability of form; and the colder the mass the more marked the opposite qualities. Two dissimilar substances, however, do not exhibit, usually, the same elasticities at the same temperatures.

Temperatures are usually measured by means of thermometers of which the scales are conventional, and often differ in

different instruments. *Standard temperatures* are so chosen that they may be easily identified and that comparison may be readily effected. The temperature of fusion of ice and the boiling-point of pure water, under mean barometric pressure at the sea-level, are universally accepted standards, and are readily determined and are invariable. The scale of the Centigrade thermometer is constructed by dividing the space between these two temperatures into 100 degrees, the lower point being made the zero. The Fahrenheit scale is thus divided to 180 degrees, and the lower reference-point is called 32. To transform these scales, the one to the other, we have

$$C = \tfrac{5}{9}(F - 32) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$F = \tfrac{9}{5}C + 32; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which C and F represent the readings of a common temperature on the Centigrade and Fahrenheit scales, respectively.

The Absolute Scale of Temperature is constructed on the assumption that its zero represents the real zero of heat-energy, that point at which either the pressure of a perfect gas retained at constant volume, the volume of such a gas under constant pressure, or the product of pressure and volume, when both are variable, will vanish by complete abstraction of heat. Experiment shows that the ratio of p_0v_0, this product at the melting-point of ice, to p_1v_1, the magnitude of the same product at the boiling-point, as above, is, for nearly perfect gases,

$$\frac{p_0v_0}{p_1v_1} = \frac{1}{1.365}, \text{ nearly}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$\frac{p_1v_1 - p_0v_0}{p_0v_0} = \frac{0.365}{1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If the difference $p_1v_1 - p_0v_0$ corresponds to 100°, as on the Centigrade scale, we have, for the absolute scale,

$$\frac{T_0}{T_1} = \frac{p_0v_0 - 0}{p_1v_1 - 0} = \frac{1}{1.365}; \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and the temperatures T_0 and T_1 will have the relation

$$\frac{T_1 - T_0}{T_0} = \frac{0.365\, p_0 v_0}{p_0 v_0};\ T_0 = \frac{T_1 - T_0}{0.365} = \frac{100}{0.365} = 274^\circ. \quad (6)$$

For the Fahrenheit division,

$$T_0 = \frac{T_1 - T_0}{T_0} = \frac{180}{0.365} = 493. \quad . \quad . \quad . \quad . \quad (7)$$

Generally, any temperature, t, on the common scale, may be determined from the expression

$$t - t_0 = \frac{t_1 - t_0}{0.365} \cdot \frac{pv - p_0 v_0}{p_0 v_0}, \quad . \quad . \quad . \quad . \quad (8)$$

$$\left. \begin{aligned} &= 274\, \frac{pv - p_0 v_0}{p_0 v_0}, \text{ Centigrade;} \\ &= 493\, \frac{pv - p_0 v_0}{p_0 v_0}, \text{ Fahrenheit.} \end{aligned} \right\} \quad . \quad (9)$$

The *Absolute Zero* is thus found at

$$- 274^\circ \text{ C., or } - (493.2 - 32) = - 461^\circ.2 \text{ F.} \quad . \quad (10)$$

The freezing and boiling points, on the absolute scale, are thus found at $+ 274$ C., or $+ 493$ F., and at $+ 374^\circ$ C., or $+ 673^\circ$ F.

The absolute zero has never been reached experimentally, and its existence and its location on the scale of temperature have only been determined by theory, based upon the now universally recognized laws of transfer of heat-energy. The exact value of the coefficient of expansion for the perfect gas is not known with absolute correctness. It was made 0.3646 to 0.3648 by Rudberg, for air, between the freezing and boiling points, and by Regnault 0.3665 to 0.3670. Rankine assumes

that the perfect gas would give, approximately, 0.365, as above.

Taking Regnault's value for air at constant volume, 0.3665 $= \frac{1}{273}$, as is often done, the absolute zero would be found at $-459°$ F., or $-272°.9$ C., instead of $-461°.2$ F., or $-274°$ C. The freezing and boiling points, on the absolute scale, then become respectively $+491°.4$ F., or $+273°$ C., and $+671°.4$ F., or $+373°$ C., instead of $+493°.2$ F., or $+274°$ C., and $+673°.2$ F., or $+374°$ C. The second set of figures are obtained on the assumption that the values for air and the perfect gas are substantially the same, and both sets on the hypothesis that the coefficient remains constant throughout the scale.* M. Babinet takes 0.366 $= \frac{11}{30}$.

92. **Quantity of Heat** is thus seen to be entirely distinct from temperature, and is measured by an essentially different unit. Heat as a form of energy and the equivalent of the work, of whatever kind, expended in producing that energy, may evidently be measured by any unit of energy. But any unit of energy is a product of two factors, the one measuring a force, the other a space traversed under the action of that force. Quantity of heat, therefore, is a quantity of energy and it may be similarly measured, by either of the familiar measures of energy, or by its own peculiar unit.

The Thermal Unit, or unit of heat-energy, is that invariable quantity of heat which is found to be required to raise the temperature of unity in weight of water one degree in temperature when at the lower standard temperature at 0° Centigrade, or 32° Fahrenheit. The British Thermal Unit is the heat so required when the unit of weight is the pound, and the scale Fahrenheit; the Metric Thermal Unit, or *Calorie*, measured that demanded to heat one kilogramme of water from 0° C. to 1° C. Rankine and Maxwell take this quantity at the temperature of maximum density; but for the purposes of this

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* Professor Holman concludes that, as the absolute zero is approached, the value of this coefficient approximates $\alpha = \frac{1}{273.7}$ Cent., or $\alpha = \frac{1}{492.7}$ Fahr.

work the two measures are taken as substantially equal; they are sensibly the same. Calling the quantity of heat, in any case, measured in British units, Q, and in metric units, Q_m,

$$Q = 3.96832\, Q_m;$$
$$Q_m = 0.251996\, Q;$$
$$\log Q = \log Q_m + 0.5986065;$$
$$\log Q_m = \log Q + \bar{1}.4013935.$$

The magnitude of the thermal unit is necessarily invariable; and the number of thermal units required to produce any given change of temperature in any substance usually increases slightly as that range is higher on the scale of temperatures.

Heat is often, especially in applied thermodynamics, most conveniently measured in mechanical units. The determination of the magnitude of the "mechanical equivalent of heat" has been made by processes which involve a comparison of these units both by transformation of mechanical energy into heat and by the reverse operation, but usually by the former method.

Calorimeters are instruments constructed for the purpose of Calorimetry, or heat-measurement. They are of various forms, but that principally used in physical and engineering researches consists of an apparatus containing water and fitted with thermometers and scales for measuring variations of temperature and quantities of water. The quantity of heat passing into the instrument becomes determinable when the quantity of water flowing through it and its variation of temperature become known.

It is sometimes convenient to measure the volume, rather than the weight, of water. In this case, the density must be known to permit the calculation of the weight of the liquid.

Volkmann has compiled the results of the experiments of Hagen, Matthiessen, Pierre, Kopp, and Jolly, on the expansion of water, and has obtained the following mean results for the volume and density of water at various temperatures on the Centigrade scale:

Temp.	Volume.	Density.	Temp.	Volume.
0 degr. C.	1.000122	0.999878	15 degr. C.	1.000847
1 "	1.000067	0.999933	20 "	1.001731
2 "	1.000028	0.999972	25 "	1.002868
3 "	1.000007	0.999993	30 "	1.004250
4 "	1.000000	1.000000	40 "	1.007700
5 "	1.000008	0.999992	50 "	1.011970
6 "	1.000031	0.999969	60 "	1.016940
7 "	1.000067	0.999933	70 "	1.022610
8 "	1.000118	0.999882	80 "	1.028910
9 "	1.000181	0.999819	90 "	1.035740
10 "	1.000261	0.999739	100 "	1.043230

The law governing the expansion of water is very exactly expressed by a simple form of equation. Buel has thus calculated in British units the following table, following Watt;* the figures agree with the above to the third place of decimals.

VOLUME AND WEIGHT OF DISTILLED WATER AT DIFFERENT TEMPERATURES ON THE FAHRENHEIT SCALE.

Temperature, Fahrenheit.	Ratio of volume to volume of equal weight at the temperature of maximum density.	Difference.	Weight of a cubic foot in pounds.	Difference.
32°	1.000129		62.417	
39°.2	1.000000	.000129	62.425	.008
40°	1.000004	.000004	62.423	.002
50°	1.000253	.000249	62.409	.014
60°	1.000929	.000676	62.367	.042
70°	1.001981	.001052	62.302	.065
80°	1.00332	.001339	62.218	.084
90°	1.00492	.00160	62.119	.099
100°	1.00686	.00194	62.000	.119
110°	1.00902	.00216	61.867	.133
120°	1.01143	.00241	61.720	.147
130°	1.01411	.00268	61.556	.164
140°	1.01690	.00279	61.388	.168
150°	1.01995	.00305	61.204	.184
160°	1.02324	.00329	61.007	.197
170°	1.02671	.00347	60.801	.206
180°	1.03033	.00362	60.587	.214
190°	1.03411	.00378	60.366	.221
200°	1.03807	.00396	60.136	.230
210°	1.04226	.00419	59.894	.242
212°	1.04312	.00086	59.707	.187
220°	1.04668	.00356	59.641	.066
230°	1.05142	.00474	59.372	.269
240°	1.05633	.00491	59.096	.276

* Watt's Dictionary of Chemistry; art. Heat. Weisbach's Mechanics; vol. ii. p. 113.

Temperature, Fahrenheit.	Ratio of volume to volume of equal weight at the temperature of maximum density.	Difference.	Weight of a cubic foot in pounds.	Difference.
250°	1.06144	.00511	58.812	.284
260°	1.06679	.00535	58.517	.295
270°	1.07233	.00554	58.214	.303
280°	1.07809	.00576	57.903	.311
290°	1.08405	.00596	57.585	.318
300°	1.09023	.00618	57.259	.326
310°	1.09661	.00638	56.925	.334
320°	1.10323	.00662	56.584	.341
330°	1.11005	.00682	56.236	.348
340°	1.11706	.00701	55.883	.353
350°	1.12431	.00725	55.523	.360
360°	1.13175	.00744	55.158	.365
370°	1.13942	.00767	54.787	.371
380°	1.14729	.00787	54.411	.376
390°	1.15538	.00809	54.030	.381
400°	1.16366	.00828	53.645	.385
410°	1.17218	.00852	53.255	.390
420°	1.18090	.00872	52.862	.393
430°	1.18982	.00892	52.466	.396
440°	1.19898	.00916	52.065	.401
450°	1.20833	.00935	51.662	.403
460°	1.21790	.00957	51.256	.406
470°	1.22767	.00977	50.848	.408
480°	1.23766	.00999	50.438	.410
490°	1.24785	.01019	50.026	.412
500°	1.25828	.01043	49.611	.415
510°	1.26892	.01064	49.195	.416
520°	1.27975	.01083	48.778	.417
530°	1.29080	.01105	48.360	.418
540°	1.30204	.01124	47.941	.419
550°	1.31354	.01150	47.521	.420

93. **The Specific Heat,** or capacity for heat, of any substance is the ratio of the quantity of heat required to raise the temperature of any given weight one degree, under specified conditions, to the amount demanded to raise the temperature of an equal weight of water one degree when at the lower of the two fixed standards of temperature—usually taken as that of melting ice, or the "freezing-point." The specific heat of any substance thus determines what rise or fall of temperature will follow the introduction, or the abstraction, of any given amount of heat.

Specific heats are of several kinds. The real specific heat of any substance measures the quantity of heat producing alteration of temperature, simply. Apparent specific heat measures

ıt demanded to produce variation of temperatures, accom-ıying other physical changes involving transformations of ıt-energy. When these specific heats are measured in me-ınical units of energy, they are sometimes called, as by Ran-e, "*dynamical specific heats*," real or apparent, as the case may

The specific heat of constant volume measures the quan-of heat required to produce alteration of temperature hout variation of volume. The specific heat of constant ssure is an "apparent" specific heat, and determines the ount of heat demanded to cause variation of temperature nasses of fluid under invariable pressure. In the former of se two cases the specific heat is probably always identical h the real specific heat; in the latter, the specific heat, which ın apparent specific heat, and the heat transferred, includes ıart which does not affect the temperature of the mass, but ssential to operations involving transformations of energy m one form to another.

Either form of specific heat may be taken as the quantity heat, in thermal units, producing a variation of the tempera-re of unity in weight, of the given substance, one degree, der specified conditions. It is most convenient to make the mperature of maximum density of water (39°.1 F., or 3°.9 C.) e standard. Rankine gives* the following expressions for e specific heat of water:

$$c = 1 + 0.000\,000\,309\,(t - 39.1)^2; \text{ (Fahr.)} \quad . \quad . \quad (1)$$

$$c_m = 1 + 0.000\,001\,(t - 3.94)^2; \text{ (Cent.)}. \quad . \quad . \quad . \quad (2)$$

The total heat, in thermal units, demanded to raise the tem-rature of unity of weight from to t to t_2 is

$$= t_2 - t_1 + 0.000\,000\,103\,[(t_2 - 39.1)^3 - (t_1 - 39.1)^3]; \quad . \quad (3)$$

$$= t_2 - t_1 + 0.000\,000\,33\,[(t_2 - 4)^3 - (t_1 - 4)^3]. \quad . \quad . \quad . \quad (4)$$

The specific heat, c, as determined by experiment is obvi-sly

$$c = \frac{wt}{w_1 t_1},$$

* Steam-engine; p. 246. See Appendix, p. 989.

in which w and w_1 are the weights of the given substance, and of water, or other standard, used in the experiment, and t and t_1 are the ranges of temperature where cooling is performed by the immersion of the given body in that weight of water.

The specific heats of solids and liquids are usually so nearly the same, for constant volume and for constant pressure, that the figures are usually given for their capacity for heat without reference to these conditions; these differences are rarely, if ever, measurable.*

It was discovered by Dulong and Petit that, in certain groups, the product of the specific heat of substances and the combining weight is the same for the whole group. This product, for elementary substances, is usually not far from 6.5. Thus we have, calling the real specific heat c_v,

$$c_v = \frac{6.4}{\text{atomic weight}}, \text{ nearly.}$$

The specific heats of alloys are obtained by multiplying the weight of each constituent by its percentage in the alloy, add these products and divide by 100. Regnault finds that the specific heats of alloys far removed from their fusing-points are the means of the specific heats of their constituents.

Dulong and Petit found the specific heat of iron to increase from a mean of 0.1098 between the freezing and boiling points of water and 0.1255 for a range increasing up to 662° F. (350° C.). Copper similarly increased from 0.0927 to 0.1013, and zinc from 0.0927 to 0.1015, platinum from 0.0335 to 0.0343 and to 0.03818 at 2192° F. (1400° C.). Holman, finds for the latter,†

$$\begin{aligned} c &= 0.0328 + 0.00\,000\,3022\,(t-32) + 0.000\,000\,000\,009\,(t-32)^2; \\ &= 0.0328 + 0.00\,000\,544\,t_m + 0.000\,000\,000\,016\,t_m^2. \quad . \quad . \quad (5) \end{aligned}$$

* See Constants of Nature, Part II; Clarke; Government Print.

† Journal Franklin Institute; Aug. 1882.

for the Fahrenheit and Centigrade scales, respectively. For iron he obtains

$$c = 0.10687 + 0.000\,0304\ (t - 32) + 0.000\,000\,0238\ (t - 32)^2;$$
$$= 0.10687 + 0.000\,0547\ t_m + 0.000\,000\,0428\ t_m^2. \quad . \quad . \quad . \quad (6)$$

The law of Dulong and Petit is equivalent to the statement that the quantity of heat demanded to raise the temperature of an atom of any simple substance, in the solid state, one degree is the same for all such elements; and Neumann's law asserts that all compound solid substances of similar chemical construction require the same quantity of heat per atom, but that this amount is less than for the isolated elements. The specific heat of elementary solids is greater than that of compound solids. Woestyn and Garnier find that the specific heat of molecules is equal to the sum of the specific heats of their constituent atoms, a conclusion partly confirmed by Keep. Marked exceptions are noted, however, and Thomson and Tait* enunciate the principle that if a system of material points are acted upon by impulsive forces, more kinetic energy is generated when they are free than when in combination.

The following table, mainly from Dulong and Petit and from Pouillet, gives the specific heats of a large number of solids and liquids.

SPECIFIC HEATS OF SOLIDS AND LIQUIDS.

Alcohol (liquid)	0.61500	Chalk	0.21485
Aluminium	0.21430	Charcoal	0.24150
Ammonia (a vapor)	0.50830	Chloride of barium	0.89570
Anthracite coal	0.20100	" " calcium	0.16420
Antimony	0.05077	" " lead	0.06641
Arsenic	0.08140	" " magnesium	0.19460
Benzine	0.45000	" " manganese	0.14250
Bismuth (solid)	0.03084	" " strontium	0.11990
" (liquid)	0.03630	" " zinc	0.13618
Bituminous coal	0.20085	Cobalt	0.10696
Brass	0.09391	Copper	0.09515
Bromine (liquid)	0.10700	Corundum	0.19762

* Nat. Phil.; § 315.

Diamond	0.14687	Oil of turpentine (liquid)	0.46727
Ether (liquid)	0.50342	Olive-oil	0.30960
Galena	0.05088	Oxygen	0.21750
Glass	0.19766	Palladium	0.05928
Gold	0.03244	Petroleum	0.46840
Graphite	0.20083	Phosphorus	0.18870
Hydrochloric acid	0.18450	Platinum	0.03243
Ice	0.50400	Potassium	0.16956
Iceland spar	0.20850	Salt	0.17295
Iodide of mercury	0.04197	Sapphire	0.21737
" " potassium	0.08191	Selenium	0.07446
" " silver	0.06159	Silica	0.19132
Iodine (solid)	0.05412	Silicon	0.17740
" (liquid)	0.10822	Silver	0.05701
Iridium	0.18870	Sodium	0.29340
Iron	0.11379	Steel	0.11700
" (cast)	0.12983	Sulphide of zinc	0.12813
Lead (solid)	0.03065	Sulphur (native)	0.17760
" (liquid)	0.04020	" (purified)	0.20259
Magnesium	0.24990	" (liquid)	0.23400
Manganese	0.12170	Sulphuric acid	0.34300
Marble	0.20989	Tin (solid)	0.05695
Mercury (liquid)	0.03332	" (liquid)	0.06370
" (solid)	0.03192	Tungsten	0.03342
Nickel	0.10863	Water	1.00000
Nitrate of sodium	0.27821	Wood spirit	0.64500
" " silver	0.14352	Zinc	0.09555
Nitre	0.23875		

The specific heats of gases differ from those of the solids, not only in magnitude, but also in method of variation. The specific heat of constant volume, which may be considered as the true measure of the specific heat of the substance, differs greatly from the specific heat of constant pressure. These two specific heats are, however, constant for the perfect gas and approximately so for the, so-called, permanent gases, and their ratio, which is an important quantity in thermodynamic investigation, is also constant. This ratio is given, for air, by Rankine, by calculation from the experiments of Bravois and Martens, and of Moll and Van Beek, on velocity of sound in air, as $\gamma = 1.408$, by Clausius as 1.410, by Masson as 1.419, Weisbach 1.4025, Cazin 1.410, Röntgen 1.4053, and by Wüllner as 1.40526;

it is usually taken as 1.41. The experiments of Dulong* give closely confirmatory values, thus: Air, 1.414†; oxygen, 1.413; hydrogen, 1.409. The ratio of the specific heats of all elementary gases is probably the same, as will be seen later (§ 96, Chap. IV). The greater the specific heat of a liquid, evidently, the greater that of its vapor. For all the familiar gases, at temperatures far removed from those of liquefaction, these quantities may be assumed to be sensibly invariable. It is hence inferred, also, that the zero of the perfect gas thermometer is an absolute zero marking the absence of all heat-energy, or motion. Experiment gives, with a fair degree of accuracy, values of the specific heat of constant pressure; but it has not directly determined that of constant volume.

These specific heats are usually distinguished by the symbols c_p and c_v, or K_p and K_v, accordingly as they are measured by thermal or mechanical units, and, when J represents Joule's "mechanical equivalent,"

$$Jc_p = K_p; \quad Jc_v = K_v;$$

$$c_p = \frac{K_p}{J}; \quad c_v = \frac{K_v}{J};$$

$$\gamma = \frac{c_p}{c_v} = \frac{K_p}{K_v}.$$

It seems probable that c_v and K_v are always identical with the "real" specific heat of the substance, and that their value is invariable, and entirely independent of physical changes of state. These specific heats relate to units of weight of the fluid, and measure, in thermal units, the heat required to raise its temperature one degree.

It is often desirable, however, to refer to another specific heat related to volume, comparing the quantity of heat required to raise unity of volume one degree with that demanded to raise an equal volume of the substance taken as standard

* Annales de Chimie et de Physique; XLI. 13.

† Corrected by later determination of constants, according to Watt.

through the same range. The standard almost invariably taken, where gases and vapors are compared, is atmospheric air, and specific heats are given in the tables of books of reference for both air and water as standards. We thus have for gases and vapors two specific heats at constant pressure and two at constant volume, which may be called the densimetric and the volumetric specific heats of constant pressure and of constant volume. The following table gives a number of their values as calculated by Clausius* mainly from Regnault's data. The specific heat of air at constant pressure was predicted from determinations theoretically made by Rankine before experiment had given the correct value.

SPECIFIC HEATS OF GASES.

Name.	Symbol.	Density.	S. H. of Constant Pressure.		S. H. of Constant Volume	
			Densimetric.	Volumetric.	Densimetric.	Volumetric.
Air	—	1	0.23750	1	0.1684	1
Oxygen	O_2	1.1056	0.21751	1.013	0.1551	1.018
Nitrogen	N_2	0.9713	0.24380	0.9970	0.1727	0.996
Hydrogen	H_2	0.0692	3.40900	0.9930	2.4110	0.990
Chlorine	Cl_2	2.4502	0.12019	1.248	0.0928	1.350
Bromine	Br_2	5.4772	0.05552	1.280	0.0429	1.395
Nitric Oxide	NO	1.0384	0.2317	1.013	0.1652	1.018
Carbonic Oxide	CO	0.9673	0.2450	0.998	0.1736	0.997
Hydrochloric Acid	HCl	1.2596	0.1852	0.982	0.1304	0.975
Carbonic Acid	CO_2	1.5201	0.2869	1.39	0.172	1.55
Nitric Acid	N_2O	1.5241	0.2262	1.45	0.181	1.64
Steam	H_2O	0.6219	0.4805	1.26	0.370	1.36
Carbon disulphide	CS_2	2.6258	0.1569	1.74	0.131	2.04
Carburetted Hydrogen	CH_4	0.5527	0.5929	1.38	0.468	1.54
Chloroform	$CHCl_3$	4.1244	0.1567	2.72	0.140	3.43
Olefiant Gas	C_2H_4	0.9672	0.4040	1.75	0.359	2.06
Ammonia	NH_3	0.5894	0.5084	1.26	0.391	1.37
Alcohol	C_2H_6O	1.5890	0.4534	3.03	0.410	3.87
Ether	$C_4H_{10}O$	2.5573	0.4797	5.16	0.453	6·87

Hydrogen is seen enormously to exceed every other substance in the value of its specific heat as measured for unity of weight.

Latent Heat, so called, is not, strictly speaking, heat; its

* Mechanical Theory of Heat; § 7; 1879.

measure is the equivalent of the *quantity* of heat which, in certain classes of operations, is expended in the performance of work, internal, or external, or both; it has disappeared, as heat, by transformation into mechanical energies, kinetic or potential. Thus, in the fusion of solids and in the vaporization of liquids, and in the expansion of substances with rising temperature, increase of volume occurs, in all cases, against resistances, either external, or internal and molecular, and the product of the mean intensity, p, of such resistance, into that change of volume, dv, gives a measure of an amount of work

$$\int dU = \int pdv,$$

which, according to the general laws of energy, can only be performed by the expenditure of an equivalent amount of some form of energy—in this case heat-energy. Of all the heat transferred to the body, a portion, $L' = U'$, must be transformed from the kinetic, sensible, form; becoming "latent," in the potential forms of "energy of position" of the separated molecules, and of external work performed during their displacement. It is common, incorrectly, to state that a body, thus expanded by heat, contains a certain quantity of latent heat; this heat, which has apparently become latent, as was supposed by its discoverers, Dr. Black and James Watt, no longer exists as heat. It is this so-called latent heat, the heat-energy thus transformed, which produces all alterations of volume and all variations of internal and external energy, and which, alone, performs work. Its measure is always

$$L' = U' = \int p'dv, \quad \ldots \ldots \quad (7)$$

in which p' is the intensity of the sum of the internal and external resistances to increase of volume.

The reversal of such processes causes the restoration of this energy to the form of sensible heat; the quantity so restored also has the measure just given. Heat which has been appar-

ently rendered latent is thus always caused to reappear by such reversal.

Clausius calls heat thus transformed "*work-heat*;" whether it be applied to the performance of internal or of external work.

The latent heat of expansion is that heat which disappears by transformation into the potential energy of equivalent work whenever a body is caused to expand by communication to it of that form of energy. Thus, if unity of weight of air is caused to expand at constant pressure in such manner that its temperature rises one degree, and its volume increases to such an extent as to retain its pressure unchanged, its rate of acceptance of heat is measured by its specific heat of constant pressure, $c = 0.237$; while, if caused similarly to increase in temperature, simply, without expansion, the heat demanded is proportional to $c_v = 0.168$; the difference $c_p - c_v = 0.069$, by transformation, has disappeared as heat, and is the measure of the latent heat of expansion and of the work and energy demanded to produce the observed expansion of volume against resisting forces—in this case, mainly external work against external pressure. In other than the perfect gases, this work of expansion consists partly, and sometimes principally, of internal work done against molecular attractive forces.

The difference, $c_p - c_v$, is always found to be greatest when the mass is most expansible by heat; and the part c_v, which is probably constant for all substances, under all possible conditions, is, as already stated, the real specific heat, while the large quantity, c_p, is the real specific heat increased by the quantity demanded as latent heat of expansion. Values of $c_p - c_v = l$ may be obtained from the tables.

The Latent Heat of Fusion is that quantity of heat-energy demanded to perform that work of the expansion of solids, at constant temperature and at the point of fusion, which, being done, leaves the mass so far expanded that the mutual directional force affecting adjacent molecules becomes inappreciable, and, stability of form being thus lost, the body becomes liquid. The latent heat of fusion thus measures the work

done, externally and internally, in producing this change of volume against the resisting effort of molecular forces and external pressure; the latter is usually insignificant in amount in comparison with the former; the work is principally internal work.

M. Person finds* the latent heat of fusion of non-metallic substances to be nearly

$$l = (t + 256^\circ \text{ F.})(c_1 - c_2), \quad . \quad . \quad . \quad . \quad . \quad (8)$$

in which t is the temperature, Fahrenheit, and c_1 and c_2 are the specific heats in the liquid and solid states, respectively.

The latent heat of fusion of ice is found by experiment to be 142.5 British thermal units, nearly, or, on the metric scale, about 79 calories. During fusion, all the heat-energy applied to the substance is expended in doing the work of expansion, and none is effective in producing increase of the temperature, which remains constant during the whole period of fusion.

The introduction and transformation of heat-energy, during the process of fusion, is observed to occur under the operation of three laws, viz.:

(1) The temperatures of fusion and of solidification are the same, and are definitely fixed for each substance under any given pressure.

(2) This temperature remains constant, heat being slowly supplied, during the whole operation of change of state of the given mass.

(3) Change of volume always occurs during this change of state, and is the greater as the quantity of heat so supplied and transformed is the greater.

The temperature of fusion is raised, as a rule, by pressure; the reverse occurs to the extent of $0^\circ.0133$ F. ($0^\circ.0074$ C.) for each atmosphere, in the case of ice, the variation being generally less for substances of high cohesion, and fusing points, and greater for those of low melting points and little strength.

The Latent Heat of Evaporation is that heat-energy trans-

* Annales de Chimie et de Physique; Nov. 1849.

formed into potential energy, or into actual energy of other form, when the change of state is that of a liquid undergoing vaporization. Its amount measures the energy demanded to remove the molecules beyond that condition of equilibrium which is the boundary between the liquid and gaseous states, and at which stability of volume, as well as of form, disappears. As will be seen (§ 112) on studying the thermodynamic theory of the heat-engines, the magnitude of this quantity measures the work which can be done per unit weight, as a maximum by the substance, if used as a working-fluid in heat-engines. It does not at all affect the thermodynamic efficiency or proportion of heat transformed into work with any given range of temperature.

The three laws above given for fusion hold equally well for this change. The quantity of heat transformed is, however, usually, enormously greater, and its variation with the temperature and the pressure due the boiling-point, or the point of liquefaction, accordingly as the change is produced by the communication or the abstraction of heat, is very considerable.

Regnault obtained values of this latent heat, for water, which are very exactly expressed by one of his formulas, slightly modified by Rankine, thus:*

$$l = 1091.7 - 0.695\,(t - 32^\circ) - 0.000\,000\,103\,(t - 39^\circ.1)^2; \quad (9)$$

or, similarly applying the correction indicated by the last term,

$$l_m = 606.5 - 0.695\,t_m - 0.000\,000\,33\dot{3}\,(t_m - 4^\circ)^2; \quad . \quad . \quad (10)$$

in British and metric units, respectively. For the former, the following nearly equivalent expressions may be generally used:

$$\left.\begin{aligned} l &= 1091.7 - 0.695\,(t - 32) \\ &= 1114 \quad - 0.695\,t \\ &= \;966.1 - 0.695\,(t - 212^\circ) \end{aligned}\right\} . \quad . \quad . \quad . \quad (11)$$

* Steam-engine; p. 250. See Peabody's Thermodynamics, for these constants.

The latent heats of water are greater than those of any other substance. According to Andrews, we have the following, the latent heat of water being unity:

Substance.	Latent Heat.	Substance.	Latent Heat.
Water	1	Bisulphide of carbon	0.162
Wood-spirit	0.492	Oxalic ether	0.136
Alcohol	0.378	Bromine	0.085
Ether	0.169	Peroxide of tin	0.059

According to Mr. H. Whiting,* the application of the molecular theory of gases to liquids, in combination with the magnetic theory of cohesion, requires certain numerical relations between the physical constants which are in every case obtained, very exactly, by experiment.

The most important are the following:

(1) "The product of the latent heat, molecular weights, and coefficient of expansion is equal to 8.4 for liquids at temperature zero, expanding by ordinary law."

(2) "The product in metric measures of the mechanical equivalent of the latent heat and the density is 1.2 times the product of the coefficient of expansion, the resistance, and the absolute temperature."

The Total Heat of Evaporation is the sum of the sensible and latent heats, measured in heat-units, and is constant at any one pressure, but, like the latent heat of evaporation, is variable with change of pressure, and, consequently, of the boiling-point. This quantity is sometimes called the "total heat of vapor." Its amount is always calculated from some fixed temperature, and is defined as the total heat from that fixed temperature, and at the given temperature, or pressure, of evaporation. Thus, water, fed to a steam-boiler at 60° F., and evaporated at 70 pounds pressure according to the steam-gauge, is said to be evaporated *from* 60° F. and *at* 320° F., the boiling-point for the given pressure.

This quantity of heat, in thermal units, is

$$h = c(t_2 - t_1) + l, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

* Science Bulletin; 1884.

in which c is the specific heat of the liquid, $t_2 - t_1$ the range of temperature, and l the latent heat of evaporation at the boiling-point, t_2.

For water we have, as above, according to Regnault when $t_1 = 32$, in British measures,

$$\left.\begin{aligned} h &= 1091.7 + 0.305\,(t_2 - 32^\circ), \\ &= 1082 \;\;\; + 0.305\,t_2, \\ &= 1146.6 + 0.305\,(t_2 - 212^\circ); \end{aligned}\right\} \qquad (13)$$

and when heated from any higher temperature t_1 deduct $c\,(t_1 - 32)$, c being the mean specific heat for that range of temperature.

$$h_m = 606.5 + 0.305 t_{2m} \quad . \;\; . \;\; . \;\; . \;\; . \quad (14)$$

in metric units, the heat being measured from the freezing-point; we deduct ct_1 when the initial temperature, on that scale, is t_1.

The efficiency of steam generators is often measured by the weight of water evaporated by them "from and at" the boiling-point under atmospheric pressure. Experiment determines the weight evaporated under actual conditions; the above expressions give the total heat required per pound, and this quantity divided by the latent heat under the standard conditions, 965.7 thermal units according to Regnault, or 966.1 as corrected by Rankine, gives the equivalent weight desired. For all ordinary work this divisor may be taken as 966.

The *Total Heat of Gasification* is always

$$h = c(t_2 - t_1) + l', \quad . \;\; . \;\; . \;\; . \;\; . \;\; . \quad (15)$$

in which, for steam, $c = 0.4805$, its specific heat under constant pressure, as a gas, and l has very exactly that value found at the freezing-point—1091.7, nearly, for British, or 606.5 in metric measures.

Superheated steam, or "steam-gas," requires for its production by the change of the liquid into the vapor under a stated pressure, and elevation to any given temperature, a quantity of heat and energy which is entirely independent of the pressure and temperature at which the boiling-point occurs. The process involves two distinct operations: (1) the raising of temperature, by the transfer to it of sensible heat, from the initial temperature of the substance to its final temperature; (2) the performance of internal work by the conversion of sufficient heat to separate the molecules from that proximity which characterizes the liquid state to their final relative positions in the larger volume and at the final temperature and pressure; which latter quantities are fixed for the unit of mass by the equation $pv/T =$ constant. Hence, starting from the freezing-point

$$H = H_0 + K_p(T_1 - T_0), \quad . \quad . \quad . \quad . \quad (16)$$

where H is the total heat, H_0 the latent heat at T_1, and K_p the dynamically measured specific heat of the gas. Rankine takes for British measures: *

$$h_0 = 842,872 \text{ foot-pounds,}$$
$$K_p = 772 \times 0.475 = 366.7 \text{ foot-pounds;}$$

in thermal units,

$$h = 1092,$$
$$c = 0.475.$$

The specific heat of superheated steam is found by Zeuner to be variable, thus:

p in lbs. per sq. in.	50	100	200
C_v	0.348	0.346	0.344

* Steam-engine; p. 255.

Hirn finds the following values for its specific volume:

p in atmos.	1	3	4	5
t Cent.	141°	200°	200°	20[illegible]
Sp. vol., cu. m.	1.85	0.697	0.522	0.41[illegible]

It can be readily computed if necessary.*

94. **The Critical State** is a condition, intermediate between the liquid and gaseous states, which is sometimes observed when vaporization occurs under very high pressures. At the "critical temperature" a gaseous body may be liquefied by pressure; at any higher temperature such liquefaction has never been produced. In the ordinary process of vaporization the mass rises in temperature with but slight, and often no, observable, change of volume, until, at a certain temperature and pressure, fixed for each fluid, the temperature ceases rising with constant volume, and, heat being still introduced at a uniform rate, volume increases, with temperature constant, and g[illegible] on increasing until all the fluid has, molecule by molecule, been transformed into the state of vapor. As will be seen later, in the first part of the process the heat is simply transferred as sensible heat, and produces rise of temperature; in the second period heat is transformed, and an equivalent amount of work is done in the gradual conversion into vapor and the expansion of the mass, during the continuous process of change, against internal and external resistances. When the pressures resisting the expansion are very great, this variation of volume is greatly restricted, and a point may finally be reached at which no such expansion at constant temperature can take place; the substance all passes, suddenly and completely, into the vaporous condition. The temperature at which this occurs is the "critical temperature" of the substance, and at this point the latent heat of evaporation obviously becomes zero; the two states, the liquid and the gas, at this point have a common limit.†

* Zeuner's Wärmetheorie; also Peabody, chap. vii. p. 125.

† This has been experimentally shown by Mathias; Comptes Rendus, 1889, p. 470; and Jour. Franklin Inst., Apr. 1890; p. 297

This phenomenon was observed by M. Cagniard de la Tour in the case of water, as early as 1822. Dr. Andrews has studied this phenomenon here described with great care.* He concludes the two fluids to be merely widely-separated illustrations of one physical state; more correctly, the two states, the liquid and the vapor, have a perfect continuity. The critical temperature of carbonic acid is about 87°.7 F. (30°.9 C.) and at a pressure of 75 atmospheres; that of ether is 369° F. (187° C.) and at 37.5 atmospheres; for alcohol, 498° F. (258° C.) and 119 atmospheres, according to M. C. de la Tour; for carbon disulphide, 505° F. (263° C.) and 66.5 atmospheres; while for water the temperature rises to 773° F. (410° C.), and the pressure is not exactly known, probably nearly 150 atmospheres, as calculated by the Author. At the latter temperature water was found to dissolve glass. M. Cailletet reached the critical temperature with nitric oxide at 46°.4 F. (8° C.) under 270 atmospheres, marsh gas at 44°.6 F. (7° C.) and 180 atmospheres, oxygen and carbonic oxide at below − 20°.2 F. (− 29° C.) and at 300 atmospheres, nitrogen below 55°.4 F. (13° C.) and at 200 atmospheres; hydrogen seemingly approaches this state at − 21° F. (− 29° C.) and 280 atmospheres. The so-called permanent gases may all be reduced to the liquid state by pressure when the critical temperature is reached, and have been so condensed by M. Cailletet and by M. Pictet, the pressures applied reaching, in some cases, 800 atmospheres and the necessary decrease of temperature being attained by expansion at initially low temperatures from under these pressures.

The following table of temperatures of physical phenomena has been collated by Mr. J. J. Coleman: †

PHYSICAL CONDITIONS AND TEMPERATURE.

Deg. Fahr.	Deg. Cent.				
+698	+370	Critical point of water	= 195.5	atmos.	pressure.
+311	+155.4	" " " sulphurous anhydride	= 78.9	"	"
+285	+141	" " " chlorine	= 83.9	"	"

* Philosophical Transactions; 1869.

† Trans. Phil. Soc. Glasgow; March 18, 1885.

Deg. Fahr.	Deg. Cent.			
+266	+130	Critical point of ammonia .... = 115	atmos.	pressure.
+212	+100.2	" " " sulphuretted hydrogen = 92	"	"
+ 98	+ 37	" " " acetylene .... = 68	"	"
+ 95	+ 35.4	" " " nitrous oxide .... = 75	"	"
+ 89	+ 31.9	" " " carbonic acid .... = 77	"	"
+ 50	+ 10.1	" " " ethylene .... = 51	"	"

+ 32	− 0	Nitrous oxide boils at 32 atmos. pressure	*Faraday.*
+ 32	− 0	Carbonic acid boils at 36 " "	"
+ 14	− 10	Sulphurous anhydride boils	"
+ 15	− 10.5	" " "	*Bunsen.*
− 10	− 23	Methyl chloride boils	*Regnault.*
− 10	− 23	Carbonic acid boils at 19.38 atmos. pressure	*Faraday.*
− 20	− 29	Sulphurous anhydride boils in current dry air	*Pictet.*
− 20	− 29	Carbonic oxide and oxygen, air and nitrogen, compressed to 300 atmos. in glass tubes, and suddenly expanded, show liquefaction	*Cailletet.*
− 26	− 32	Alcohol containing 52 per cent water freezes	*Pictet.*
− 29	− 33.6	Chlorine boils	*Regnault.*
− 29	− 33.7	Ammonia boils	*Bunsen.*
− 31	− 35	Commercial paraffin oil (sp. gr. .810) freezes	*Coleman.*
− 40	− 40	Nitrous oxide boils at 8.71 atmos. pressure	*Faraday.*
− 40	− 40	Carbonic acid boils at 11 " "	"
− 40	− 40	Ethylene boils at 13.5 " "	"
− 53	− 47	Freezing point of Hollands gin and French brandy.	*Coleman.*
− 60	− 51	Nitrous oxide boils at 5 atmos. pressure	*Faraday.*
− 60	− 51	Carbonic acid boils at 6.75 " "	"
− 60	− 51	Ethylene boils at 9 " "	"
− 62	− 52	American petroleum (sp. gr. 790) freezes	*Coleman.*
− 62	− 52	Freezing point of extra strong whiskey and rum ...	"
− 62	− 52	Alcohol containing 40 per cent water freezes	"
− 80	− 61.8	Sulphydric acid boils	*Regnault.*
− 80	− 62	Nitrous oxide boils at 3 atmos. pressure	*Faraday.*
− 80	− 62	Carbonic acid boils at 3.75 " "	"
− 80	− 62	Ethylene boils at 6.5 " "	"
− 99	− 73	Critical point of marsh gas, pressure 56 atmos.	*Wroblewski.*
−103	− 75	Liquefied ammonia freezes.	
−103	− 75	Alcohol containing 20 per cent water freezes	*Coleman.*
−108	− 78	Carbonic acid boils	*Faraday and Regnault.*
−112	− 80	Solid sulphurous anhydride melts	*Mitchell.*
−123	− 86	Nitrous oxide boils	*Faraday.*
−123	− 86	Marsh gas boils at 40 atmos. pressure	*Wroblewski.*
−128	− 87.9	Liquid nitrous oxide boils	*Regnault.*

Deg. Fahr.	Deg. Cent.		
—144	— 98	Marsh gas boils at 25 atmos. pressure	*Wroblewski.*
—152	— 102	Amyl alcohol an oily liquid	*Olszewski.*
—152	— 102	Silicon fluoride a white mass	"
—152	— 102	Arseniuretted hydrogen liquid	"
—152	— 102	Hydrochloric acid boils	"
—152	— 102	Chlorine orange crystals	"
—152	— 102	Ethylene boils	*Wroblewski.*
—154	— 103	" "	*Olszewski.*
—166	— 110	Solid carbonic acid and ether in vacuo	*Faraday.*
—171	— 113	Critical point of oxygen, pressure 50 atmos.	*Wroblewski.*
—171	— 113	Marsh gas boils at 16 atmos. pressure	"
—175	— 115	Solid carbonic acid in vacuo, 25 mm. pressure	*Dewar.*
—175	— 115	Hydrochloric acid gas solid	*Olszewski.*
—177	— 116	Carbon disulphide solid.	
—180	— 118	Arseniuretted hydrogen white crystals	*Olszewski.*
—193	— 125	Nitrous oxide boils in vacuo	*Dewar.*
—200	— 129	Ether solidifies	*Olszewski.*
—202	— 130	Absolute alcohol solid.	
—209	— 134	Amyl alcohol solid	*Olszewski.*
—218	— 139	Ethylene boils in vacuo	"
—219	— 139.5	Critical point of carbonic oxide, press. 35.5 atmos.	"
—220	— 140	" " " air, pressure 39.0 atmos.	"
—220	— 140	*Calculated* temp. of carbonic acid snow in vacuo (?)	*Pictet.*
—220	— 140	Hydrogen compressed to 650 atmos. and pressure released produces momentary liquefaction and solidification	*Pictet.*
—220	— 140	Oxygen compressed to 320 atmos. and pressure released produces momentary liquefaction	*Pictet.*
—231	— 146	Critical point of nitrogen, 35 atmos. pressure	*Olszewski.*
—238	— 150	Ethylene boils in vacuo	"
—238	— 150	Carbonic oxide boils at 20 atmos. pressure	"
—242	— 152	Atmospheric air boils at 20 " "	"
—247	— 155	Marsh gas boils	*Wroblewski.*
—306	— 187	Fluorine boils	*Dewar.*
—312	— 191.4	Air boils	*Olszewski.*
—312	— 191.2	" "	*Wroblewski.*
—315	— 193	Carbonic oxide boils	"
—317	— 194	Nitrogen boils	*Olszewski.*
—337	— 205	Atmospheric air boils in vacuo	"
—348	— 211	Carbonic oxide solidifies in vacuo	"
—351	— 213	Nitrogen boils in vacuo	"
—400	— 239	Neon boils	*Dewar.*
—441	— 252	Hydrogen boils	"
—438	— 260	Helium boils	"

Deg. Fahr.	Deg. Cent.	
− 355	− 215	Calculated boiling-point of hydrogen.......... *E. J. Mills.*
460	− 273	Absolute zero.

(The critical points above freezing-point of water are quoted from Professor Dewar. See Table XXIII, Appendix, p. 990, for Melting Points, etc.)

95. **The Definition of the Perfect Gas** has been seen to be capable of expression by statement either of its physical constitution, of its physical properties, or of its thermodynamic equation. It is so constituted that its molecules exert no inherent cohesive attractions, or mutual repulsions, and it can only be confined, when acted upon by heat, if allowed to expand within a defined volume by the application of external force; and hence its effort to expand—i.e., its pressure, tension, or elasticity, as it is variously called—is supposed to be due solely to that energy of molecular motion which we call heat. Its distinguishing physical property is found in the fact that, when reduced to any given volume, and confined within any given space, its total pressure upon the confining-walls, or its total tension, is precisely equal to the sum of the pressures which any number of equal parts would produce, if each were separately enclosed in an equal space. This is equivalent to saying that, the temperature being constant, the tension is inversely as the volume, which is the law of Boyle and of Mariotte. The perfect gas is also found to vary in pressure, or in volume, or to vary in product of pressure and volume, both varying together, directly as the temperature measured from the "absolute" zero, i.e., according to the law of Charles and of Gay Lussac.

Experiment thus shows that the more nearly a gas approaches this ideal state, the more perfectly does it illustrate the law of Boyle and Mariotte, the pressure varying inversely as the volume; and the more exactly does it follow the law of Charles and Gay Lussac, according to which the variation of pressure at constant volume, or of volume at constant pressure, or of the product of pressure and volume, varies directly as the absolute temperature.

The Defining Equation of the Perfect Gas is, therefore, as already seen,

$$\frac{pv}{p_0 v_0} = \frac{T}{T_0}; \quad \frac{pv}{T} = \frac{p_0 v_0}{T_0} = \text{constant} = R. \quad . \quad . \quad . \quad (1)$$

The quantity of matter considered is commonly taken as unity of weight, and v is here, therefore, the "specific volume," or the volume of unity of weight.

The value of R is thus constant for any one gas, and, for different gases, will vary inversely as their densities at standard temperature and pressure. Thus, for the nearly perfect gases, oxygen, hydrogen, nitrogen, and for the mixture, air, the values of R are, respectively, nearly 26.5, 42.3, 30, and 29.3, in metric measures, or 48, 70, 55, and 53, in British measures. Evidently,

$$R = \frac{pv}{T} = \frac{p}{TD}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which D is the density of the gas, as measured by the weight of unity of volume, and T is absolute temperature.

96. **The Thermodynamics of the Perfect Gas** involves the determination of the methods of variation of temperature, pressure, and volume, and of variations of quantities of heat and work consequent upon those changes; in such manner that the General Thermodynamic Equation may be applied to the case. This means the measurement of the quantities entering into the equation and ascertaining their physical relations, and thus the algebraic relations of their symbols; so that numerical values may be substituted and the equation solved for any given case.

The general thermodynamic equation has been seen to be an expression in which the change of heat-energy is measured in terms of two distinct phenomena; the application of heat to alteration of temperature in any "working fluid," the transfer,

simply, of heat-energy; and that producing mechanical energy, or work. We now see that the first of these quantities is measured by the product of the range of temperature of the unit weight, always taken, and the *real* specific heat; while the second must be measured by the product of the intensity of the total pressure of the fluid by the change of volume. Hence

$$\left.\begin{aligned} dH &= dS + dW; \\ &= K_v dT + p dv; \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which the equation may be solved when, in the second term of the second member, the relation of p to v is known; or, when the values of all the quantities involved can be directly obtained by observation or other means.

The first law of thermodynamics and the experimental measure of the mechanical equivalent of heat enable us to express the specific heat, K_v, of constant volume, the "real dynamical specific heat," in terms of either thermal or dynamical units. The second law of thermodynamics asserts that the value of p in the second term may be taken as proportional to absolute temperature and, hence, the value of p, at any instant, may be obtained by multiplying the rate of variation of p with T, $\frac{dp}{dT}$, by T, the absolute temperature of the fluid; and hence

$$p = T\left(\frac{dp}{dT}\right)_v; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and we have simply to write

$$dH = K_v dT + T\left(\frac{dp}{dT}\right)_v dv; \quad . \quad . \quad . \quad . \quad . \quad (3)$$

then to obtain values of the several symbols; and to determine the value of the partial differential coefficient $\left(\frac{dp}{dT}\right)_v$, by refer-

ence to the algebraic expression of the laws of variation of the physical characteristics of the fluid.

The "*Thermodynamic Function*" is obtained by reference to the second law, also, as originally shown by Rankine, thus:*

We have seen that the second law asserts that any effect of heat, being proportional to the quantity of heat acting in its production, is proportional to the absolute temperature of the fluid, and is measured by the product of this quantity by a "thermodynamic function," the form and magnitude of which for a gas will be presently determined; that is:

$$dH = T\,d\phi; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

when ϕ represents that function.

The Thermodynamic Equations for Gases are thus obtained by inserting in the general fundamental equations the values of the partial differential coefficients obtained from the characteristic equation of the gas. The perfect gas, as has been seen, is defined by the equation

$$\frac{pv}{T} = R = \frac{p_0 v_0}{T_0}, \text{ a constant,}$$

in which the subscript, $_0$, may be taken to indicate the state of the substance at a standard temperature, as at the melting-point of ice. For all purposes of the engineer, and for nearly all the purposes of the physicist, the permanent gases, so called, may be taken as perfect. The values of the coefficients are

$$\left(\frac{dp}{dv}\right)_T = -\frac{p}{v}; \quad \left(\frac{dp}{dT}\right)_v = \frac{R}{v}; \quad \left(\frac{dv}{dT}\right)_p = \frac{R}{p}.$$

The general equations thus become, in accordance with these two laws,

$$dH = Td\phi = K_v\,dT + T\left(\frac{dp}{dT}\right)_v dv$$

* Steam-engine; p. 310.

$$= K_v dT + RT\frac{dv}{v}$$

$$= K_v dT + pdv. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Since the value of the total differential dv in (5) is

$$dv = \frac{dv}{dT}dT + \frac{dv}{dp}dp = \frac{R}{p}dT - \frac{RT}{p^2}dp,$$

$$dH = (K_v + R)dT - \frac{RT}{p}dp, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$dH = Td\phi = K_p dT - T\left(\frac{dv}{dT}\right)_p dp$$

$$= K_p dT - RT\frac{dp}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$= K_p dT - vdp.$$

Then, from (6) and (7),

$$\frac{dH}{dT} = K_p = K_v + R; \quad K_p - K_v = R; \quad . \quad . \quad . \quad (8)$$

and thus, as was first shown by Clausius, both specific heats, that of constant volume and that of constant pressure, and their difference, are found, by thermodynamic science, as well as by experiment, to be constant.

Since, from (7),

$$dT = \frac{vdp + pdv}{R},$$

$$dH = \frac{K_v}{R}vdp + \frac{K_v + R}{R}pdv. \quad . \quad . \quad . \quad . \quad (9)$$

But since the specific heat at constant pressure, K_p, is also $\left(\frac{dH}{dT}\right)_p$, we have

$$K_p = \left(\frac{dH}{dT}\right)_p = K_v + R.$$

We may, therefore, unite the three forms of the equations for perfect gases:

$$\begin{aligned} dH &= (K_p - R)dT + pdv \\ &= K_p dT - vdp \\ &= (K_p - R)\frac{dp}{p}T + K_p T\frac{dv}{v}, \quad . \quad . \quad . \quad . \quad (10) \end{aligned}$$

in which equation the specific heat at constant pressure appears instead of the specific heat at constant volume inserted in (5).

Introducing both specific heats, and eliminating R, we obtain:

$$\begin{aligned} dH &= K_v dT + (K_p - K_v)\frac{T}{v}dv \\ &= K_p dT - (K_p - K_v)\frac{T}{p}dp \\ &= \frac{K_v}{K_p - K_v}vdp + \frac{K_p}{K_p - K_v}pdv. \quad . \quad . \quad . \quad (11) \end{aligned}$$

When a perfect gas expands at constant temperature, obviously no internal work can be done, and no change occurs in the amount of sensible heat present in the mass.

Hence, under the laws of transference of energy, if no external work is done, a constant weight of such gas, freely expanding at constant temperature, requires no heat from external sources to keep its condition, with respect to heat or energy, unchanged. Internal energy depends on temperature alone.

This conclusion, based upon the law of persistence of energy, has been confirmed by experiments made by Joule and Thomson upon the permanent, or nearly perfect, gases. In the case of the non-permanent gases, such as carbonic acid, it is found by experiment, as by theory, that this conclusion does not hold. In the latter, as in any case in which internal work is done, heat must be introduced during expansion to perform that internal work, if the temperature is to be kept constant, and, reversing the process, heat must be abstracted during compression at constant temperature.

When external work is done by a perfect gas, expanding at constant temperature, it is obviously necessary to supply heat, to do that work, in exactly equivalent amount, and the heat absorbed is thus a measure of the work so done.

When imperfect gases similarly expand, heat is added, as before, in just the amount demanded for conversion into work, and its measure is also the measure of the total work done internally and externally.

The thermodynamic function, for the perfect gas, is readily derived from the general equations.

Since this function is

$$dH = Td\phi,$$

we have

Also,

$$\left.\begin{aligned} d\phi &= K_v\frac{dT}{T} + \frac{dp}{dT}dv. \\ d\phi &= \left(K_v + R\right)\frac{dT}{T} - \frac{dv}{dT}dp. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (12)$$

The latter may be deduced directly from the former, by eliminating dv and substituting its value in terms of dp. We have

$$dv = \left(\frac{dv}{dT}\right)_p dT + \left(\frac{dv}{dp}\right)_T dp, \quad . \quad . \quad . \quad . \quad (13)$$

Substituting in (12),

$$d\phi = K_v \frac{dT}{T} + \left(\frac{dp}{dT}\right)_v \left[\left(\frac{dv}{dT}\right)_p dT + \left(\frac{dv}{dp}\right)_T dp\right]$$

$$= \left[\frac{K_v}{T} + \left(\frac{dp}{dT}\right)_v \left(\frac{dv}{dT}\right)_p\right] dT + \left(\frac{dp}{dT}\right)_v \left(\frac{dv}{dp}\right)_T dp, \quad (14)$$

an equation which is perfectly general.

For perfect gases,

$$\frac{pv}{T} = \frac{p_0 v_0}{T_0}; \left(\frac{dp}{dT}\right)_v = \frac{p_0 v_0}{vT_0}; \left(\frac{dv}{dT}\right)_p = \frac{p_0 v_0}{pT_0}; \left(\frac{dv}{dp}\right)_T = -\frac{v}{p};$$

and, also, when $dv = 0$,

$$\frac{dv}{dT} = \left(\frac{dv}{dT}\right)_p + \left(\frac{dv}{dp}\right)_T \left(\frac{dp}{dT}\right)_v = 0; \quad \left(\frac{dp}{dT}\right)_v \left(\frac{dv}{dp}\right)_T = -\left(\frac{dv}{dT}\right)_p.$$

Substituting in (14),

$$d\phi = \left(K_v + \frac{p_0 v_0}{T_0}\right)\frac{dT}{T} - \frac{dv}{dT}dp, \quad . \quad . \quad . \quad (15)$$

which is the second equation (12).

Collecting the expressions for all, we have

$$d\phi = K_v \frac{dT}{T} + R\frac{dv}{v}$$

$$= K_p \frac{dT}{T} - R\frac{dp}{p}$$

$$= \left(K_v + R\right)\frac{dT}{T} - R\frac{dp}{p};$$

and integrating,

$$\phi = K_v \log_e T + R \log_e v + C$$

$$= K_p \log_e T - R \log_e p + C$$

$$= (K_v + R) \log_e T - R \log_e p + C$$

$$= \left(K_v + \frac{p_0 v_0}{T_0}\right) \log_e T - R \log_e p + C.$$

The value of C, the constant of integration, is here indeterminable; but this is a matter of no importance, since it disappears in application, differences in values of thermodynamic functions, only, being in such cases considered.

Introducing the value of $\frac{K_p}{K_v} = \gamma$, and observing that

$$K_p - K_v = \frac{p_0 v_0}{T_0},$$

$$K_p = \frac{\gamma}{\gamma - 1} \cdot \frac{p_0 v_0}{T_0}; \quad K_v = \frac{p_0 v_0}{(\gamma - 1)T_0};$$

$$\phi = \frac{p_0 v_0}{T_0}\left(\frac{\log_e T}{\gamma - 1} + \log_e v\right)$$

$$= \frac{p_0 v_0}{T_0}\left(\frac{\gamma \log_e T}{\gamma - 1} - \log_e p\right);$$

in which $\gamma = 1.405$, nearly, for air, and is usually taken as 1.41 for all permanent gases;* $\frac{1}{\gamma - 1} = 2.451$; $\frac{\gamma}{\gamma - 1} = 3.451$. The value of $\frac{p_0 v_0}{T_0}$, for air, is estimated by Rankine at 53.15 foot-pounds per degree Fahrenheit, accepting Regnault's determination of the value of $p_0 v_0$ as 26,214 foot-pounds, and taking T_0 at 493°.2 Fahr. For superheated steam, $\frac{pv}{T} = 85.5$.

The applications of the General Equations for Perfect Gases are illustrated by the following cases:

(1) Required the amount of heat demanded to produce change of volume at constant pressure.

We have

$$dH = \frac{K_p - R}{R} v dp + \frac{K_p}{R} p dv.$$

* Purely theoretic analysis indicates a possibility that this value for the perfect gas may be $1 + \frac{4}{\pi^2} = 1.405285$.—Phil. Mag., 1885. (See p. 340.)

Since p is constant, $dp = 0$, and

$$dH = \frac{K_p}{R} p_1 dv;$$

hence, integrating,

$$H = \frac{K_p}{R} p_1 (v_2 - v_1) = K_p (T_2 - T_1).$$

(2) The gas expands or contracts at constant temperature. For this case take

$$dH = K_v dT + RT\frac{dv}{v}.$$

T is constant: $dT = 0$.

$$\therefore dH = RT\frac{dv}{v};$$

$$\begin{aligned} H &= RT_1 \log_e \frac{v_2}{v_1} \\ &= p_1 v_1 \log_e \frac{v_2}{v_1} \\ &= p_1 v_1 \log_e r. \end{aligned}$$

(3) Expansion is adiabatic or isentropic, i.e., H is constant, $dH = 0$. Then

$$dH = K_v dT + (K_p - K_v) T \frac{dv}{v} = 0,$$

$$\therefore \frac{dT}{T} + \left(\frac{K_p}{K_v} - 1\right)\frac{dv}{v} = 0.$$

Integrating and calling $\frac{K_p}{K_v} = \gamma$,

$$\log_e T + (\gamma - 1) \log_e v = \text{constant},$$

$$Tv^{\gamma-1} = \text{const.} = T_1 v_1^{\gamma-1}.$$

$$\therefore \frac{T}{T_1} = \left(\frac{v_1}{v}\right)^{\gamma-1}.$$

Similarly, from the equation

$$dH = K_p dT - (K_p - K_v) T \frac{dp}{p} = 0$$

we obtain

$$\frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{\frac{1}{\gamma}-1}; \left(\frac{T}{T_1}\right)^{\gamma} = \left(\frac{p}{p_1}\right)^{\gamma-1}.$$

Combining the above, we get

$$\left(\frac{T_1}{T}\right)^{\frac{\gamma}{\gamma-1}} = \frac{p_1}{p} = \left(\frac{v}{v_1}\right)^{\gamma}; \; p_1 v_1^{\gamma} = pv^{\gamma} = \text{const.}$$

Or, from

$$dH = \frac{K_v}{K_p - K_v} vdp + \frac{K_p}{K_p - K_v} pdv = 0,$$

we have

$$\gamma \frac{dv}{v} + \frac{dp}{p} = 0;$$

and, as before,

$$\frac{p}{p_1} = \left(\frac{v_1}{v}\right)^{\gamma};$$

and $p_1 v_1^{\gamma} = pv^{\gamma} =$ constant.

(4) Required an expression for the work done by a perfect gas expanding at constant temperature, the latent heat of expansion being supplied from some external source of heat, i.e., in isothermal expansion.

We have

$$dH = pdv; \quad pv = \text{constant} = p_1 v_1,$$

and

$$p = \frac{p_1 v_1}{v}.$$

$$\therefore\ U = \int_{v_1}^{v_2} p\,dv = p_1v_1 \int_{v_1}^{v_2} \frac{dv}{v} = p_1v_1 \log_e \frac{v_2}{v_1};$$

the ratio of expansion is $r = \dfrac{v_2}{v_1}$,

$$U = p_1v_1 \log_e r = H. \quad \text{[See (2).]}$$

) To measure the work done during adiabatic expansion:
/e have

$$pv^{\gamma} = \text{constant} = p_1v_1^{\gamma}.$$

$$= \int_{v_1}^{v_2} p\,dv = p_1v_1^{\gamma} \int_{v_1}^{v_2} v^{-\gamma}\,dv = \frac{p_1v_1}{\gamma - 1}\left[1 - \left(\frac{v_1}{v_2}\right)^{\gamma - 1}\right].$$

) To find the variation of temperature when a gas ex-
s adiabatically, and without doing work; as when expand-
rom one given volume to another within a space otherwise
ous:
'rom eq. (5),

$$dH = K_v dT + RT\frac{dv}{v};$$

$$dH = 0;$$

$$RT\frac{dv}{v} = 0;$$

e the work done by the gas is zero.

$$\therefore\ 0 = K_v dT + 0;$$

$$= K_v dT; \quad dT = 0; \quad T = \text{constant}.$$

97. **The Work performed and Energy expended,** by
sfer and transformation of heat, are thus readily computed
never the method of operation is known. As already

stated, the variation of pressure with change of volume may usually be represented by some curve of the hyperbolic class, and by algebraic expressions of the general form,

$$pv^n = \text{const.} = p_0 v_0^n. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In such cases the work done, by unity of weight, is

$$U = \int_{v_1}^{v_2} p\,dv; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which then

$$pv^n = p_1 v_1^n = p_2 v_2^n; \; p = p_1 \left(\frac{v_1}{v}\right)^n; \quad . \quad . \quad . \quad . \quad (3)$$

and, thence,

$$U = p_1 v_1^n \int_{v_1}^{v_2} v^{-n} dv = \frac{p_1 v_1^n}{1-n}(v_2^{1-n} - v_1^{1-n});$$

$$= \frac{p_1 v_1 - p_2 v_2}{n-1}; \quad . \quad . \quad . \quad . \quad . \quad (4)$$

whence, for gases,

$$U = R\frac{T_1 - T_2}{n-1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Hence, the work done during expansion along a line of which the equation is $p_1 v_1^n = pv^n$ is proportional to the difference of the products of pressure and volume at the initial and terminal portions of the curve, and, in the case of gases, to the range of temperature worked through. The change of temperature is thus, in all such cases, directly proportional to the quantity of work performed by or upon the expanding or contracting fluid.

The Heat expended is, in all cases, the sum of the amounts demanded to perform internal work, to do the external work of expansion, and to produce variation of sensible heat. In the perfect gases, the internal work is zero; the external work is measured as above; and the variation of sensible heat is measured by

$$S = K_v(T_1 - T_2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

being positive for compression and negative for expansion.

Hence, the total heat demanded in any case of hyperbolic expansion, such as the above, must be $S + U$, or

$$H = \left(\frac{R}{n - 1} + K_v\right)(T_1 - T_2). \quad . \quad . \quad . \quad . \quad (7)$$

Thus it is found that the total amount of heat emitted or received in such changes is directly proportional to the range of temperature, $T_1 - T_2$, worked through during such expansion or compression. The above is also a proof that, either specific heat being found constant by experiment, the other must be constant as well.

For the case of common hyperbolic expansion, in which the law of Boyle and Mariotte is followed, $n = 1$, and the expression for work done, equation (4), becomes $H = \frac{0}{0}$, indeterminate.

In this case, unity of weight being taken, as before, $p_1v_1 = p_2v_2 = pv$, and

$$U = p_1v_1 \int_{v_1}^{v_2} \frac{dv}{v} = p_1v_1 \log_e \frac{v_2}{v_1} = p_1v_1 \log_e r, \quad . \quad . \quad (8)$$

in which r is the ratio of expansion. This case is that of isothermal expansion of gases, the heat transferred to or from the fluid being the equivalent of the work done, and wholly transformed.

When n exceeds unity, the curve falls under; and when $n < 1$, the line lies above the equilateral hyperbola. In the first case, the temperature of the gas must obviously fall; in the second case, it must as evidently rise, as expansion proceeds.

Isothermal changes are, by definition, those occurring at constant temperature.

Adiabatic changes are, by definition, those which occur without gain or loss of heat by transfer to or from the enclosing vessel; such as may take place in a vessel composed of a non-conducting substance.

Isodynamic changes are, by definition, those taking place without variation of internal energy.

The work of Isothermal and of Adiabatic Expansion of gas may evidently be now determined by assigning to n, in the expression $pv^n =$ constant, proper values, and the quantity of heat-energy transformed may be thus ascertained.

For Isothermal Expansion of gases, as already seen, $n = 1$ and

$$U = p_1 v_1 \int_{v_1}^{v_2} \frac{dv}{v} = p_1 v_1 \log_e \frac{v_2}{v_1} = p_1 v_1 \log_e r; \quad . \quad . \quad (9)$$

which measures the quantity of heat transformed into external mechanical work, or into kinetic energy. Since no change of temperature takes place, no heat is transferred to effect such a change; and, since no intramolecular forces resist or aid the change of volume, no heat is transformed in that manner. The quantity U thus measures the total amount of heat transferred, which, measured in thermal units, is, calling $\frac{1}{J} = A$,

$$Q = AU = \frac{U}{J} = \frac{p_1 v_1}{J} \log_e \frac{v_2}{v_1} = \frac{p_1 v_1}{J} \log_e r; \quad . \quad . \quad (10)$$

where Q is the quantity of heat, in thermal units; $J = \frac{1}{A}$ is "Joule's equivalent."

This determination may be effected, also, by comparison of the thermodynamic functions for the initial and final conditions of the fluid, thus:

The thermodynamic function at the beginning of expansion is

$$\phi_1 = R\left(\frac{\log_e T_1}{\gamma - 1} + \log_e v_1\right) + C; \quad . \quad . \quad . \quad . \quad (11)$$

and, at the end of the process,

$$\phi_2 = R\left(\frac{\log_e T_2}{\gamma - 1} + \log_e v_2\right) + C. \quad . \quad . \quad . \quad . \quad (12)$$

Since the temperature is constant, $T_1 = T_2$, the heat expended is

$$U = T_1(\phi_2 - \phi_1) = RT_1(\log_e v_2 - \log_e v_1)$$

$$= RT_1 \log_e \frac{v_2}{v_1} = p_1 v_1 \log_e r, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

as before, the expanding or compressed mass weighing unity.

For air, adopting $T_0 = -493°.2$ F., $R = 53.15$, and

$$U = 53.15 T_1 \log_e r, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and, in metric measures,

$$U = 29.2 T_1 \log_e r.$$

For Adiabatic Expansion of gases, there are two equations of condition:

$$(a) \qquad \frac{pv}{T} = \frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2} = \text{constant};$$

$$(b) \qquad pv^\gamma = p_1 v_1^\gamma = p_2 v_2^\gamma = \text{constant}.$$

The work done during expansion is

$$U = \int_{v_1}^{v_2} p dv = p_1 v_1^\gamma \int_{v_1}^{v_2} v^{-\gamma} dv$$

$$= \frac{p_1 v_1^\gamma}{\gamma - 1}(v_1^{1-\gamma} - v_2^{1-\gamma})$$

$$= \frac{p_1 v_1}{\gamma - 1}\left[1 - \left(\frac{v_1}{v_2}\right)^{\gamma - 1}\right]; \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$= \frac{p_1 v_1}{\gamma - 1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}\right]; \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and if the density is called $\delta = \frac{1}{v}$,

$$U = \frac{p_1}{\delta_1(\gamma - 1)}\left[1 - \left(\frac{\delta_2}{\delta_1}\right)^{\gamma - 1}\right]. \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Again,

$$U = \int_{v_1}^{v_2} p dv = p_1 v_1^{\gamma} \int_{v_1}^{v_2} v^{-\gamma} dv$$

$$= \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} = R\frac{T_1 - T_2}{\gamma - 1}. \quad . \quad . \quad . \quad . \quad (19)$$

For compression, the work is similarly measured; its value is negative, and heat is produced in place of being expended.

The variation of temperature is controlled by the laws expressed in the equations

$$p_1 v_1 = RT_1; \quad p_2 v_2 = RT_2; \quad p_1 v_1^{\gamma} = p_2 v_2^{\gamma};$$

whence

$$T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = T_1 r^{1-\gamma}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

r being the "ratio of expansion.

The Isothermal and the Isodynamic Lines on a diagram of heat-energy in cases of the expansion of gas are identical in form and location. As has been seen, the whole internal energy of the perfect gas, and, approximately, all that of the permanent gases, is the energy of heat-motion, and is manifested as sensible heat, its total amount being proportional to the absolute temperature of the fluid. A line of invariable internal energy, or the isodynamic line, is therefore, for gas, a line of uniform temperature.

The equation of this line is obtained directly from the defining equation of the gas, thus:

$$T = \text{constant}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$pv = RT = \text{constant}. \quad . \quad . \quad . \quad . \quad (22)$$

Calling abscissa and ordinate x and y, $v = ax$, $p = by$, and

$$xy = \frac{pv}{ab} = \frac{RT}{ab} = \text{constant}; \quad . \quad . \quad . \quad . \quad (23)$$

a and b being assigned values to be determined by the scale on which the curve is drawn. The isothermal and isodynamic lines, for gases, are thus again seen to be alike hyperbolic.

Since heat must be converted into mechanical energy, when the fluid expands isothermally behind a piston, it is again evident that an amount of heat must be supplied, during such expansion, precisely equal to the external work done, in order that the temperature of the gas shall not vary; and that during compression, heat must be abstracted to a similar extent.

The Adiabatic or Isentropic Line, also, represents the method of variation of pressures and volumes when the "entropy" of the fluid is constant, i.e., when no heat is communicated to, or emitted from, the gas, all change of temperature of the fluid being due to transformation of energy. Since all energy expended upon external bodies must, in this case, be produced by conversion of heat into mechanical energy, and all heat gained by the substance must be due to the reverse transformation, it is evident that the fluid must cool during expansion, and become heated by compression, when it is enclosed in a non-conducting envelope of variable volume. It thus follows, also, that the expanding fluid will give an adiabatic line which will fall more rapidly from the same initial state than its own isothermal, the adiabatic curve thus lying under the isothermal, on the diagram of energy. When compression occurs from the same initial state, the adiabatic line lies above the isothermal.

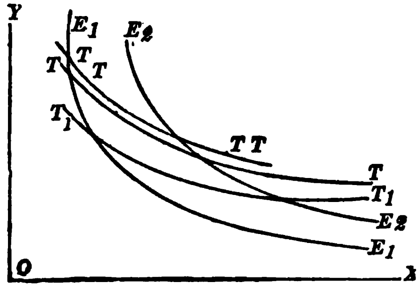

FIG. 130.—THERMAL LINES.

The relations of the two lines are shown in Fig. 130, in which T, T, T_1, T_1, are isothermals, and E_1, E_1, E_2, E_2, are adiabatics. The intersections of the latter with the former being considered as marking initial conditions, the curves are seen to separate, in the manner just indicated, with change of volume in either direction, as explained in § 81.

The equation of the adiabatic line is readily obtained from the characteristic equation of the gas, combined with the special defining conditions of the assumed change, thus:

Since no heat-energy is absorbed by, or emitted from, the fluid, in this case, during change of volume,

$$\phi = \text{constant}; \quad dH = Td\phi = 0; \quad d\phi = 0.$$

Then

$$d\phi = K_v \frac{dT}{T} + \frac{dp}{dT} dv \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$= K_v \frac{dT}{T} + \frac{p_0 v_0}{T_0} \frac{dv}{v}; \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$\phi = K_v \log_e T + K_v (\gamma - 1) \log_e v. \quad (26)$$

Then

$$\frac{\phi}{K_v} = \log_e T + (\gamma - 1) \log_e v$$

$$= \log_e (Tv^{\gamma - 1}); \quad . \quad . \quad . \quad . \quad . \quad (27)$$

and, since $\phi = \text{const.}$,

$$Tv^{\gamma - 1} = \frac{T_0}{p_0 v_0} pv^\gamma = e^{\frac{\phi}{K_v}} = \text{constant}; \quad . \quad . \quad (28)$$

and the equation of the adiabatic line is

$$yx^\gamma = pv^\gamma = \text{constant} = p_1 v_1^\gamma;$$

in which $\gamma = \frac{K_p}{K_v} = 1.41$, nearly.

The expressions, derivable as above,

$$\frac{p_1}{p} = \left(\frac{v}{v_1}\right)^k = \left(\frac{v}{v_1}\right)^\gamma, \quad p_1 v_1^\gamma = pv^\gamma,$$

are statements of Poisson's law relating pressures to volumes and specific heat ratios; the heat being constant, none being supplied or abstracted from the fluid, and expansion being adiabatic.

As, in isothermal expansion, the sole equation of condition is

$$T = \text{const.}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

so, in adiabatic expansion, the equation of condition is

$$\phi = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

98. **The Thermodynamics of the Vapors** and imperfect gases may be considered as precisely the same, in essence, as for the permanent and the perfect gases; which for the purposes of the engineer are taken to be physically identical, as well as thermodynamically. The total change of energy, in any operation in which changes of pressure, volume, heat, and temperature occur in the vapors, as working fluids in heat-engines, is always capable of representation on a balance-sheet, the gain or loss of energy as heat being equal to the sum of the quantities producing variation of temperature and acting to produce loss or gain of work or mechanical energy.

The general thermodynamic equation remains, as before,

$$dH = K_v dT + pdv;$$

but, in this case, while the first term of the second member is as easily determined and measured as in the case of the gas, this is by no means the fact as regards the second term. In this case, internal as well as external work is done, and a difficulty at once presents itself in the attempt to measure the former component, and hence the total work. We have no instrumental means of directly measuring the internal pressure, p_i, and work, $p_i dv$; which, with the external pressure, p_e,—which we easily measure, in all cases, by means of pressure-gauges,—and work, $p_e dv$, make the total work, pdv, of the equation. Then we have, as before,

$$dH = K_v dT + (p_e + p_i)\, dv;$$

$$= K_v dT + pdv.$$

Yet the computation of internal forces and work, and of external work, are readily effected. Notwithstanding the fact, as just stated, that the molecular forces, and the work performed by or against them, are beyond the reach of any physical apparatus and are incapable of direct measurement, it becomes easy to calculate both force and work from measurable data by application of the second law of thermodynamics.

The rate of variation of external pressure and work, with temperature, at constant volume, may be determined easily by experiment; this rate, according to the second law, is constant for all temperatures, and hence, being multiplied by the absolute temperature at which the total pressure or the work is to be determined, the product measures that total pressure or work. In symbols, let p, w, and T represent the total pressure and work, and the absolute temperature; then the rates of variation $\frac{dp}{dT}, \frac{dw}{dT}$, with temperature, may be ascertained by, for example, noting the change of external pressure, as measured by the steam-gauge, for a change of one degree or other small but exactly measurable range, and taking this ratio of differences, $\frac{\Delta p}{\Delta T}$, as sensibly equal to $\frac{dp}{dT}$. The value of $\frac{dp_e}{dT}$ is identical, evidently, at any given point on the scale, with $\frac{dp}{dT}$; as is $\frac{dw_e}{dT}$ with $\frac{dw}{dT}$. The work-ratio is obtained by multiplying the Δp by the volume and taking this product, $\Delta p \, . \, v = \Delta w$, as the numerator in $\frac{\Delta w}{\Delta T} = \frac{dw}{dT}$. Then the total pressure, *internal and external*, must be measured by

$$p_i + p_e = p_t = T\frac{dp}{dT}, \quad \ldots \ldots \quad (1)$$

and the total work of expansion from zero-volume

$$w = T\frac{dw}{dT} = T\frac{dp}{dT}v. \quad \ldots \quad (2)$$

Cotterill gives as ratios of internal to external pressures and works, approximately,

$$k = \frac{11{,}530}{T} - 4.7.$$

Since the heat rendered latent, in any case, is the equivalent of the work performed by it, the latent heat of vaporization must be exactly equal, dynamically, to the work just measured; and if it be called H for unity of weight,

$$H = T\frac{dw}{dT} = T\frac{dp}{dT}\Delta v, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

when Δv is the increase of volume taking place during the change of physical state. If its value is made known, as is usual, by experiment, and Δv is observed, it becomes easy to obtain

$$\frac{dp}{dT} = \frac{H}{T(v_2 - v_1)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The value of $\frac{dp}{dT}$ is sometimes found to be negative, e.g., in the case of ice. Professor James Thomson found

$$-\frac{dp}{dT} = 0^\circ.0133 \text{ Fahr.} = 0^\circ.0074 \text{ Cent.}$$

as the amount by which the melting-point of ice is lowered by every increase of one atmosphere of pressure. The latent heat of fusion is similarly measured. The *total heat* of vaporization, as it is called, from a temperature T_1 and *at* a temperature T_2, is the sum of the latent heat converted into work, as just measured, and the sensible heat demanded to raise the temperature from T_1 to T_2.

The latent heat of vaporization per unit of volume is obviously measured by

$$L = \frac{H}{v_2 - v_1} = T\frac{dp}{dT}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and this permits the ready computation of the heat demanded in supplying any steam, or other vapor, engine with the quantity of fluid required to do any given amount of work, or to drive its piston through any given space, and this without knowing the density of the fluid.

99. **The Thermodynamics of Steam** may thus be brought under the general rules of the science. The rate of variation of the *external*, or gauge, pressure of the vapor in contact with the liquid from which it is produced, or at the boiling-point, with temperature, may be obtained from the tables, or from formulas such as have been given for steam by Regnault, and for that and other vapors by Rankine.* The latter are the most general and usually the most exact; they have the form, as already seen (§ 90),

$$com.\ log\ p = A - \frac{B}{T} - \frac{C}{T^2}; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

whence

$$L = T\frac{dp}{dT} = p\left(\frac{B}{T} + \frac{2C}{T^2}\right) \log_e 10. \quad . \quad . \quad (2)$$

The density of vapor may thus be readily computed from the known value of its latent heat, and much more satisfactorily and exactly than it can be derived by any known method of experimental determination. The increase of volume of unity of weight must always be

$$v_2 - v_1 = \frac{H}{L}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which, practically, the values of v_1 may usually be neglected. Then the density† is

$$D = \frac{1}{v} = \frac{L}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

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* Steam-engine; § 206, Div. III. See Hirn, t. II. p. 211.

† Tables thus calculated for steam and for ether and other fluids are given by Rankine in his Miscellaneous Papers and in his treatise on the Steam-engine.

he specific volume, the volume of a pound of water, at
ɔmary modern working pressures and temperatures is not
rom 0.017 cubic foot. The external work of formation of
m is thus

$$U = p(v - 0.017), \text{ nearly}; \quad . \quad . \quad . \quad . \quad . \quad (5)$$

the latent heat is H foot-pounds; and the variation in internal
work, during evaporation, its increase, is

$$\Delta H = H - p(v - 0.017); \quad . \quad . \quad . \quad . \quad . \quad (6)$$

hich quantity measures its total energy, T. The heat ab-
rbed in any purely thermodynamic operation is the sum of
e accessions of internal energy and external work, i.e., of
nsible and internal latent heat and external work.

When steam is wet, if x represents its quality, as measured
the fraction of dry steam, the latent heat is

$$H' = xH;$$

nd its total heat is

$$H' + S = xH + S; \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where S is the total sensible heat of the water. The specific
volume is

$$V' = xV + (1 - x)\,0.017; \quad . \quad . \quad . \quad . \quad . \quad (8)$$

V being the specific volume of pure dry saturated steam; and
this is

$$V' = xV, \text{ nearly}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Temperature, Pressures, and Volumes of Steam are related by
natural law quite as definitely as those governing these relations
for the gases; but algebraic expressions of those laws are not
yet obtained, except empirically. There have been numerous
formulas proposed of the latter class, some of which are remark-
bly exact within a moderate range. The most accurate are

probably those of Rankine,* already given (§§ 89, 101) for vapors generally, taking p as the symbol of gauge-pressure.

The Value of $\left(\frac{dp}{dT}\right)_v$ is independent of the magnitude of the internal pressure of the fluid. This is shown thus:

At constant volume no change can occur with variation of external or of total pressure, since the mean distance of molecule from molecule remains necessarily unchanged. Hence in the expression,

$$p_t = p_e + p_i, \quad p_i = \text{const.},$$

whence

$$\left(\frac{d\,.\,(p_e - p_i)}{dT}\right)_v = \left(\frac{dp_e}{dT}\right)_v.$$

It is thus seen that it is a matter of indifference whether the value of p_i is known or not, and the value of $(dp/dT)_v$ may just as well be determined by measuring the relation of those quantities by reference to the steam-tables, for example, in the case of that fluid, where only the variations of external pressure with constant volume are indicated, as from computed values of p_t; the corresponding variations of temperature at the stated constant volumes being employed to obtain dT. It is, in fact, customary to take out the values of this differential coefficient from the steam-tables.

Thus to ascertain the value of the coefficient at the specific volume, 5 cubic feet per pound, we turn to the tables and there find this volume to correspond with the absolute pressure, 86 pounds per square inch. The corresponding temperature is 316° F. Taking, as we may, the value of

$$dp/dT = \Delta p/\Delta T;$$

where Δp and ΔT are taken between 85 and 87 pounds, and

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* Steam-engine; p. 237, § 206. Ibid.; pp. 559–564.

mean rate of variation within that range equals the varia- within the elementary range, we have

$$(dp/dT)_v = (\Delta p/\Delta T)_{87}^{85} = 2/1.629 = 1.23;$$

ere the units are pounds on the square inch and Fahrenheit rees.

Similarly, where the metric system is employed, as on e 380, by Clausius, we have the values, at six atmospheres ssure,

$$dp/dT = 116.085.$$

It is obvious that, respecting temperatures, absolute and nmon,

$$d(a - t) = dt;$$

l the value of dT may be taken as the accession of magni- le at constant volume, or otherwise, on either the absolute the common scale.

Rankine's expression for the relations of pressure to tem- ature has just been given, and for this expression we have, steam,

$$A = 8.2591; \qquad \frac{B}{2b} = 0.003441;$$

$$\log B = 3.43642; \qquad \log c = 5.59873;$$

$$\frac{B^2}{4b^2} = 0.00001184.$$

Unwin's formulas follow (§ 101).

Many simple expressions have been proposed for the rela- ns of pressure and temperature of saturated steam. These, their simplest forms, are usually of the type:

$$p^s = at;$$

which, for British measures, as the Fahrenheit scale and ab- ute pressures in pounds on the square inch, values are very

nearly $a = 0.0085$; $i = 0.22$. Thus Mr. Estler makes $a = 0.008484$, $i = 0.222$, for all customary working pressures, and obtained a sufficiently close approximation for any ordinary work of the engineer.

Internal pressure and work are computed by deducting external pressure and work from the totals. Clausius thus obtained the following values of p for steam of the pressures given, all in millimetres of mercury, of which 760 measure one atmosphere of pressure:

TOTAL PRESSURES OF STEAM.

Centigrade.		External Pressure.		Ratio $\frac{dp}{dT}$.	Total Pressure $p = T\frac{dp}{dT}$.	Ratio $\frac{p}{p_e}$.
t.	T.	p_e.	At.			
100°	374°	760	1	27.200	10146	13.3
120	394	1520	2	48.595	19150	12.6
134	408	2280	3	67.020	27277	11.9
144	418	3040	4	84.345	35172	11.5
152	426	3800	5	100.375	42659	11.2
159	433	4560	6	116.085	50149	11.0
166	440	5320	7	133.445	58502	10.8
171	445	6080	8	146.910	65228	10.7
176	450	6840	9	161.27	72410	10.6
180	454	7600	10	173.425	78561	10.4
199	473	11400	15	239.57	113077	9.9

It is seen that the rate of variation of pressure with the temperature of steam continually increases as pressures and temperatures rise, and that the proportion of internal to external work and pressure continually diminishes; but that the latter ratio is large, about ten to one, for the whole range of pressures familiar in standard practice.

The specific volume of steam, or the volume of unity of weight, and its reciprocal, the density, have been seen to be capable of easy computation when the latent heat of vaporization at the given temperature is known; since this latent heat measures the work done while the force resisting it is calculable as above. From the expressions (3) already given, § 98,

$$H = T\frac{dp}{dT}\Delta v; \quad \Delta v = \frac{H}{T} \div \frac{dp}{dT},$$

we thus obtain very exact values.

Clausius thus obtains the following values, and compares them with the somewhat uncertain figures of Fairbairn and Tate, derived experimentally. Metric measures are employed.

SPECIFIC VOLUMES OF STEAM.

t.	T.	Δv Calculated.	Δv By Experiment.
117.17	391.17	0.947	0.941
124.17	398.17	0.769	0.758
128.41	402.41	0.681	0.648
137.46	411.46	0.530	0.514
144.74	418.74	0.437	0.432

Adopting his nomenclature, let s and σ represent the specific volumes of vapor and liquid; then the change of volume in evaporation is

$$u = s - \sigma,$$

and the external and internal work are, respectively,

$$U_e = p_e(s - \sigma) = p_e u;$$

$$U_i = \rho = r - p_e u;$$

when r is the total heat, in thermal measure.

The heat which has been absorbed by one pound of water to convert it into a pound of steam at atmospheric pressure is sufficient to have melted three pounds of steel or thirteen pounds of gold. Fairbairn and Tate give the following for the specific volume of steam, as derived by experiment:

$$v = 0.41 + \frac{389}{p + 0.35},$$

v in cubic feet per pound, p pounds per square inch.

The diagram, Fig. 131, for which we are indebted to Mr. Babcock, shows graphically the relation of heat to temperature, the horizontal scale being quantity of heat in British thermal units, and the vertical temperature in Fahrenheit degrees, both reckoned from absolute zero and by the usual scale. The dotted lines for ice and water show the temperature which would

have been obtained if the conditions had not changed. The processes represented by our equations are here exhibited very clearly.

The ordinates of the diagram represent the temperatures of the substance as heat is applied, measured from absolute zero; and the abscissas measure heat supplied, in thermal units per pound of fluid, to effect the alteration of temperature and change of physical state. Every step in the process is readily traced and is clearly seen.

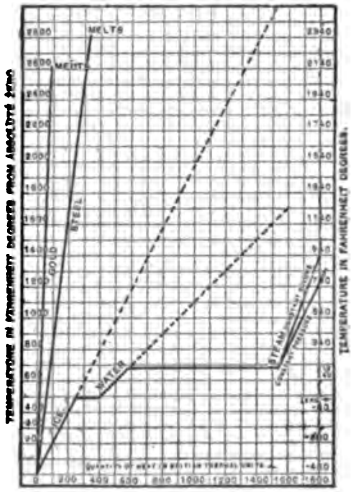

FIG. 131.—THERMODYNAMICS OF STEAM.

Factors of Evaporation measure the relative amount of heat demanded to effect the heating of water from a given temperature, t_2, and its vaporization at a higher temperature, t_1, and to simply produce vaporization at the boiling-point under atmospheric pressure, which latter is now usually taken as a standard. The value of this factor of evaporation is evidently

$$f = 1 + \frac{0.3(t_1 - 212^\circ) + (212^\circ - t_2)}{966.1}, \text{ nearly.} \quad . \quad . \quad (1)$$

The following are values of such factors, calculated as bove :

TABLE OF FACTORS OF EVAPORATION.

Boiling-point, T_1, Fahr.	Initial Temperature of Feed-water, T_2.										
	32°	50°	68°	86°	104°	122°	140°	158°	176°	194°	212°
212°	1.19	1.17	1.15	1.13	1.11	1.10	1.08	1.06	1.04	1.02	1.00
230	1.20	1.18	1.16	1.14	1.12	1.10	1.08	1.06	1.04	1.02	1.01
248	1.20	1.18	1.16	1.14	1.13	1.11	1.09	1.07	1.05	1.03	1.01
266	1.21	1.19	1.17	1.15	1.13	1.11	1.09	1.07	1.06	1.04	1.02
284	1.21	1.20	1.18	1.16	1.14	1.12	1.10	1.08	1.06	1.04	1.02
303	1.22	1.20	1.18	1.16	1.14	1.12	1.11	1.09	1.07	1.05	1.03
320	1.22	1.21	1.19	1.17	1.15	1.13	1.11	1.09	1.07	1.05	1 03
338	1.23	1.21	1.19	1.17	1.15	1.14	1.12	1.10	1.08	1.06	1.04
356	1.23	1.22	1.20	1.18	1.16	1.14	1.12	1.10	1.08	1.06	1.04
374	1.24	1.22	1.20	1.18	1.17	1.15	1.13	1.11	1.09	1.07	1.05
392	1.24	1.23	1.21	1.19	1.17	1.15	1.13	1.11	1.09	1.07	1.06
410	1.25	1.23	1.22	1.20	1.18	1.16	1.14	1.12	1.10	1.08	1.06
428	1.25	1.24	1.22	1.20	1.18	1.16	1.14	1.12	1.11	1.09	1.07

A vastly more convenient form of table is that in which the pressures at which evaporation takes place are given; such as may be found in the Appendix, Table XIII.

It is seen that the relative cost of using feed-water at any one temperature as compared with the use of water at any other temperature is as the reciprocal of their factors of vaporization. Thus if feed-water can be supplied, by means of a heater, at 210° F., where previously drawn from the mains at 50°, the relative cost of making steam will be, at 100 pounds pressure, by gauge, $\frac{1043}{1208} = 0.86$, and a gain of fourteen per cent will be effected. These tables are very useful in reducing the data obtained in trials of steam-boilers to standard conditions.

100. **Regnault's Researches and Methods** have furnished all the essential data relating to the production of steam in the boiler and the supply of stored heat-energy to the engine.

The memoir of M. Henri Victor Regnault on "The Elastic Forces of Aqueous Vapors,"* in which he described his researches, is a most magnificent exposition of a still more remarkable series of investigations. He repeated the methods

* Ann. de Chimie et de Physique, July 1844 ; Mém. de l'Institut, tome XXI. p. 465 (1847) ; Mém. de l'Académie des Sciences, XXI, XXVI.

and experiments of earlier physicists, invented new ways, and finally obtained a set of data of unexampled extent and accuracy.* Regnault found that the density of aqueous vapor *in vacuo* and under feeble pressure may be calculated according to the law of Boyle and Mariotte when the fraction of saturation is less than 0.8, while the density becomes sensibly greater when approaching saturation. He further found that the density of vapor *in air*, in a state of saturation, may be similarly calculated, and the ratio of weight of equal volumes of vapor and air is a trifle less than that obtained theoretically.

The data obtained by Regnault were carefully tabulated, and curves were constructed exhibiting the variation of pressure with temperature for saturated steam for the whole range covered by his experiments. Three formulas of interpolation were used for three different parts of the scale of temperatures; for that part below the freezing-point he adopted the formula

$$F = a + b\alpha^{\tau}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which F is the pressure, a and b constants, and α^{τ} a function of $\tau = t + 32°$, t being the temperature corresponding to F.

Between the freezing and boiling points Regnault used Biot's formula,

$$\log F = a + b\alpha^{t} - c\beta^{t}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and above the boiling-point,

$$\log F = a - b\alpha^{\tau} - c\beta^{\tau}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which $\tau = t + 20$. This last answers well, also, for the whole range. In it $a = 6.2640348$; $\log b = 0.1397743$; $\log c = 0.6924351$; $\log \alpha = \bar{1}.994049292$; $\log \beta = 1.998343862$, as given by Regnault; or, according to Dixon,

$$\begin{aligned} a &= 6.263\ 509\ 686\ 5 \\ \log \alpha &= \bar{1}.998\ 343\ 377\ 8 \\ \log \beta &= \bar{1}.994\ 048\ 173\ 7 \\ \log b &= 0.692\ 450\ 419\ 2 \\ \log c &= 0.139\ 553\ 958\ 4 \end{aligned}$$

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* *Vide* Dixon on Heat; vol. 1. § 724.

For British measures,

$$a = 4.859\ 984\ 524\ 7$$
$$\log \alpha = \bar{1}.999\ 079\ 751\ 3$$
$$\log \beta = \bar{1}.996\ 693\ 778\ 3$$
$$\log b = 0.659\ 317\ 975\ 2$$
$$\log c = 0.020\ 517\ 432\ 4$$

A break was observed by Regnault, and is exhibited by the curves and the formulas, at the freezing-point, which had been attributed to error, the two curves cutting each other at a very small but appreciable angle; but Professor James Thomson has supposed such a break to have a real existence, and to be produced by the physical change marking the freezing-point.

Regnault's Tables have been reproduced in many forms, usually with additions. The Appendix, among other tables, contains the data obtained by Regnault, and these values are accepted as standard universally. The table here given exhibits the temperatures and corresponding pressures of saturated steam throughout the full range now used in steam-boilers and far beyond; the quantity of heat, sensible and latent, in unity of weight; the total heat of evaporation, and the density of the steam. Reference to these tables is vastly more convenient than calculation. Should it be necessary, or desirable, however, to make such calculations, the formulas already given will furnish the means. They also permit the calculation of data beyond the limits of Regnault's experiment, and are probably practically correct far beyond any pressure likely to become familiar in the operation of steam-boilers. Regnault's limit was at 230° C. (446° F.). Rankine's formula has been used beyond it.*

The formulas used in these calculations are also given, Table XXII, for convenience of reference. British measures are used throughout.

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* The tables of Professor Peabody, which are more recent, may be obtained separately published. These tables, and those here given by the Author, and those reduced by Nystrom, will be found in close accordance.

The stored energy in steam at any pressure and temperature is now easily ascertained by calculation, in accordance with the first law of thermodynamics.

The first attempt to calculate the amount of energy latent in the water contained in steam-boilers, and capable of greater or less utilization in expansion by explosion, was made by Mr. George Biddle Airy,* the Astronomer Royal of Great Britain, in the year 1863, and by the late Professor Rankine † at about the same time.

Approximate empirical expressions are given by the latter for the calculation of the energy and of the ultimate volumes assumed by unit weight of water during expansion, as follows, in British and in metric measures:

$$U = \frac{772(T - 212)^2}{T + 1134.4}; \qquad U_m = \frac{423.55(T - 100)^2}{T + 648};$$

$$V = \frac{36.76(T - 212)}{T + 1134.4}; \qquad V_m = \frac{2.29(T - 100)}{T + 648}.$$

These formulas give the energy in foot-pounds and kilogrammetres, and the volumes in cubic feet and cubic metres. They may be used for temperatures not found in the tables to be given, but, in view of the completeness of the latter, it will probably be seldom necessary for the engineer to resort to them.

The quantity of work and of energy which may be liberated by the explosion, or utilized by the expansion, of a mass of mingled steam and water has been shown by Rankine and by Clausius, who determined this quantity almost simultaneously, to be easily expressed in terms of the two temperatures between which the expansion takes place.

When a mass of steam, originally dry, but saturated, so expands from an initial absolute temperature, T_1, to a final absolute temperature, T_2, if J is the mechanical equivalent of

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* "Numerical Expression of the Destructive Energy in the Explosions of Steam-boilers."

† "On the Expansive Energy of Heated Water."

the unit of heat, and H is the measure, in the same units, of the latent heat per unit of weight of steam, the total quantity of energy exerted against the piston of a non-condensing engine, by unity of weight of the expanding mass, is, as a maximum, § 101,

$$U = JT_2\left(\frac{T_1}{T_2} - 1 - \text{hyp log}\,\frac{T_1}{T_2}\right) + \frac{T_1 - T_2}{T_1}H.$$

This equation was published by Rankine previous to 1860.*

When a mingled mass of steam and water similarly expands, if M represents the weight of the total mass and m is the weight of steam alone, the work done by such expansion will be measured by the expression

$$U = MJT_2\left(\frac{T_1}{T_2} - 1 - \text{hyp log}\,\frac{T_1}{T_2}\right) + m\frac{T_1 - T_2}{T_1}H.$$

This equation was published by Clausius in substantially this form.†

It is evident that the latent heat of the quantity m, which is represented by mH, becomes zero when the mass consists solely of water, and that the first term of the second member of the equation measures the amount of energy of heated water which may be set free, or converted into mechanical energy by explosion. The available energy of heated water, when explosion occurs, is thus easily measurable.

The computers of the tables given in the Appendix were Messrs. Ernest H. Foster and Kenneth Torrance. The tables range from 20 pounds per square inch (1.4 kgs. per sq. cm.) up to 100,000 pounds per square inch (7030.83 kgs. per sq. cm.), a maximum probably falling far beyond the range of possible application, its temperature exceeding that at which the metals retain their tenacity, and in some cases exceeding their melting-points. These high figures are not to be taken as exact. The relation of temperature to pressure is obtained by the use of Rankine's equation, of which it can only be said that it

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* Steam-engine and Prime Movers; p. 387.

† Mechanical Theory of Heat; Browne's translation, p. 283.

is wonderfully exact throughout the range of pressures within which experiment has extended, and within which it can be verified. The values estimated and tabulated are probably quite exact enough for the present purposes of even the military engineer and ordnance officer.

The table presents the values of the pressures in pounds per square inch above a vacuum, the corresponding reading of the steam-gauge (allowing a barometric pressure of 14.7 pounds per square inch), the same pressures reckoned in atmospheres, the corresponding temperatures as given by the Centigrade and the Fahrenheit thermometers, and as reckoned both from the usual and the absolute zeros. The amount of the available stored energy of a unit weight of water, of the latent heat in a unit weight of steam, and the total available heat-energy of the steam, are given for each of the stated temperatures and pressures throughout the whole range in British measures, atmospheric pressures being assumed to limit expansion. The values of the latent heats are taken from Regnault, for moderate pressures, and are calculated for the higher pressures, beyond the range of experiment, by the use of Rankine's modification of Regnault's formula.*

The energy of gunpowder is somewhat variable with composition and perfection of manufacture, and is very variable in actual use, in consequence of the losses in ordnance due to leakage, failure of combustion, or retarded combustion in the gun. Taking its value at what the Author would consider a fair figure, 250,000 foot-pounds per pound, it is seen that, as found by Airy, a cubic foot of heated water, under a pressure of 60 or 70 pounds per square inch, has about the same energy as one pound of gunpowder. The gunpowder exploded has energy sufficient to raise its own weight to a height of nearly 50 miles, while the water has enough to raise its weight about one sixtieth that height. At a low red heat water has about 40 times this latter amount of energy in a form to be so ex-

* It is seen that, could we reduce steam, at atmospheric pressure, to water, without loss of heat, the energy thus stored would raise the water to the *red heat;* and if to a solid, would become hotter than molten steel.

pended. One pound of steam, at 60 pounds pressure, has about one third the energy of a pound of gunpowder.*

101. **The General Thermodynamic Equation for Vapors** must thus evidently have the same general form as that applicable to gases. The heat-energy, dH, demanded for any elementary change is, as in all other cases, composed of two portions (§ 98):

(1) That, KdT, required to effect change of temperature and of sensible heat, simply;

(2) That transformed in the performance of equivalent work, $T\frac{dp}{dT}du$; the volume u being that measuring the expansion of the fluid; which is not, in this case, equal to v, the volume of unity of weight in the gaseous state.

When this equation is applied to the change by which water is converted into steam, it is observed that the temperature remains constant, during vaporization at constant pressure, and the heat expended is simply

$$H' = T_1\frac{dp_1}{dT_1}(v_2 - v_1) = T_1\frac{dp_1}{dT_1}u, \quad . \quad . \quad . \quad . \quad (1)$$

when v_1, v_2, and u are, respectively, the volume of the liquid, that of its vapor, and the total change of volume, under the pressure p_1, and at the temperature T_1. The value of $\frac{dp_1}{dT_1}$ may be obtained either by reference to experimental data or by differentiating the algebraic expression already given (§ 99) for the relation of p to T.

The transformed equation

$$u = \frac{H'}{T\frac{dp}{dT}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

* See Manual of Steam-boilers, § 143, p. 289, for a more complete discussion of this interesting subject.

has been used to determine, from experimentally obtained values of H and of $\left(\frac{dp}{dT}\right)_v$, the density of steam; the results according very perfectly with those obtained in the researches of Messrs. Fairbairn and Tate.*

The general equation for steam and vapors thus becomes, since $K_v = J$, as before,

$$\left.\begin{aligned} dH &= K_v dT + T\frac{dp}{dT}du \\ &= JdT + T\frac{dp}{dT}du \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The thermodynamic function for vapor is, as before, in form,

$$\left.\begin{aligned} \phi = \int\frac{dH}{T} &= K_v \log_e T + \frac{dp}{dT}u, \\ &= J \log_e T + \frac{dp}{dT}u, \end{aligned}\right\} \quad . \quad . \quad . \quad (4)$$

and is similar in form to that obtained for gases. For steam, the value of K_v is the dynamically expressed measure of the specific heat of water, or "Joule's equivalent." Thus, the expression for this function becomes, for any other fluid than steam, of which the specific heat in the liquid state is C,

$$\phi = JC \log_e T + u\frac{dp}{dT}. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Professor Unwin adopts an empirical expression for the relations of external pressure and the temperature of saturated vapors, having the form,†

$$\left.\begin{aligned} \log p &= a - bT^{-n}; \\ \text{and} \qquad T &= \left(\frac{b}{a - \log p}\right)^{\frac{1}{n}}; \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

* Rankine; Miscellaneous Papers, p. 423.

† London Engineer; April 9, 1886; p. 277.

in which, when p is the pressure in pounds on the square inch, T the absolute temperature reckoned from $-461°$ F., and, for steam, $a = 5.8031$; $b = 15,900$; $n = 1.25$, common logarithms being used.

This expression gives values agreeing with those obtained by Regnault, to within 0.007, throughout a range extending up to about 25 atmospheres.

From the above equation we obtain

$$\left.\begin{aligned}\frac{1}{p}\cdot\frac{dp}{dT} &= 2.3026\frac{nb}{T^{n+1}};\\ &= \frac{45765}{T^{2.25}};\end{aligned}\right\} \quad \ldots\ldots \quad (7)$$

$$\left.\begin{aligned}\frac{T}{p}\cdot\frac{dp}{dT} &= 2.3026\frac{nb}{T^{n}};\\ &= \frac{45765}{T^{1.25}};\\ &= 2.8783(5.8031 - \log p);\end{aligned}\right\} \quad \ldots \quad (8)$$

which gives the numerical values which follow.

The ratio of total pressure, internal and external, $T\dfrac{dp}{dT}$, to external pressure, p, and of latent heat of vaporization to heat transformed into external work, is as below:

p	$\frac{T}{p}\cdot\frac{dp}{dT}$ Eq. 8	Rankine	Diff.
5	14.69	14.79	− .10
10	13.83	13.88	− .05
20	12.96	12.98	− .02
40	12.09	12.08	+ .01
70	11.39	11.36	+ .03
140	10.53	10.49	+ .04
200	10.08	10.03	+ .05
250	9.80	9.75	+ .05

The ratio of internal pressure, $T\frac{dp}{dT} - p$, or of the pressure due internal work, to the external and observed pressure, p, is

$$\left.\begin{aligned} k &= \frac{T}{p}\left(\frac{dp}{dT}\right)_v - 1 \\ &= \frac{45765}{T^{1.25}} - 1 \\ &= 2.8783(5.8031 - \log p) - 1 \end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

The specific volume of steam, $v - s$, the difference in volume of a pound of steam and of the water from which it is made, at any given pressure, p, is, as has been seen, a factor by which the total pressure, $T\frac{dp}{dT}$, being multiplied, the product measures the work expended in its evaporation, or its equivalent, the latent heat, H, of vaporization at that pressure. Thus

$$T\left(\frac{dp}{dT}\right)_v(v - s) = H = Jl; \quad . \quad . \quad . \quad . \quad (10)$$

$$v - s = \frac{Jl}{T\left(\frac{dp}{dT}\right)_v}; \quad . \quad . \quad . \quad . \quad (11)$$

and p being expressed in pounds on the square foot, and s taken as 0.016,

$$v = \frac{268.2}{p(5.8031 - \log p)} + 0.016. \quad . \quad . \quad . \quad . \quad (12)$$

An approximate expression for l is, for British units,

$$\begin{aligned} l &= 1443 - 0.71T \\ &= 1443 - \frac{1632}{(a - \log p)^{0.8}}. \end{aligned} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The following are a few calculated values of latent heats and of specific volume:

p	l	$v - s$	$D = \frac{1}{v}$
5	1000.8	73.03	0.0137
10	978.8	37.96	0.0263
20	954.0	19.73	0.0506
40	926.2	10.27	0.0972
70	900.9	6.056	0.1647
140	865.4	3.149	0.3160
200	845.0	2.248	0.4417
250	831.4	1.820	0.5447

The external work of evaporation is $p(v - s)$, or

$$p(v - s) = \frac{268.2l}{a - \log p}$$

$$= \frac{T^{1.25}l}{59.28}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

The internal work is

$$\left[T\left(\frac{dp}{dT}\right)_s - p\right](v - s) = \left(J - \frac{268.2}{a - \log p}\right)l$$

$$= \left(772 - \frac{T^{1.25}}{59.28}\right)l. \quad . \quad . \quad . \qquad (15)$$

The following table gives the value of T and of p, actual and as computed by the approximate equation. p is here given in pounds on the square inch.

F Fahr.	T	p Actual	p Computed
100	561	0.942	0.953
150	611	3.707	3.706
212	673	14.70	14.62
250	711	29.88	29.67
300	761	67.22	66.82
350	811	135.11	134.62
400	861	247.75	247.70
432	893	350.73	351.50

102. The Thermal Lines, for Vapors, differ somewhat in form from those found for gases. The exact equations of the expansion lines become, however, so difficult of application, in the theory of the heat-engines, that it has been found advisable to substitute for them approximate expressions of simple form, which may be more conveniently applied.

These approximate formulas are usually equations of hyperbolas, of the form

$$pv^n = \text{constant.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The value of the index n varies from 0 for isothermal and isopiestic expansion of moist saturated vapors, to unity, as in the isothermal expansion of gases, and to $1.33\dot{3}$ for the adiabatic expansion of steam-gas.

"*The Curve of Saturation*" and constant weight is that thermal line, on a diagram of energy, which exhibits the relations of pressure and volume (usually of unity of weight) of the fluid when expanding, and kept constantly in the saturated state. In the case of initially dry, saturated, steam, at all ordinary pressures and temperatures, it would be necessary to supply heat during expansion, and to abstract it during compression, in order that the vapor should be kept "dry and saturated;" i.e., on the point of condensation. The relations of simultaneous pressures, temperatures, and volumes of steam are given elsewhere. It will be seen that they are so complicated that the true equation of this curve becomes too cumbersome for convenient use. Comparison of numerical results has shown, however, that the curve may be very closely represented by equation (1), making $n = \frac{17}{16}$, according to Rankine, or, a little more nearly, by 1.0646, as given by Zeuner, i.e.,

$$pv^{\frac{17}{16}} = \text{const.}; \quad \text{or,} \quad pv^{1.0646} = \text{const.}; \quad . \quad . \quad (2)$$

p and v being the pressure and specific volume of the vapor. The value of this constant, in British measures, is 479, nearly; in metric measures, 1.7. An equivalent expression to the above is $p^{0.9393}v = \text{const.}$ Wood makes $pv^{\frac{17}{16}} = 484$.

The Specific Heat of Saturated Steam is that quantity demanded, during rise of temperature, to keep unity of weight in the saturated condition, and is measured by

$$\frac{dQ}{dT} = \frac{dH}{dT} - \frac{l}{T} = 0.305 - \frac{l}{T}; \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which the coefficient 0.305, representing the increment of total heat per unit rise of temperature, is obtained from data given by Regnault's experiments. Applying the formula to any familiar pressure and temperature of saturated steam, it is found that $\frac{dQ}{dT}$ is negative in all ordinary cases, and that, consequently, it is necessary to add heat to a mass of expanding steam to keep it dry and saturated, notwithstanding the fact that it continuously falls in temperature as well as in pressure. Should not heat be so supplied, the steam would become a mixture of steam and water, the proportion of steam decreasing with progressing expansion. The negative value of the specific heat of saturated steam and its consequence, partial condensation, were discovered in 1850, independently, by Rankine and Clausius. It has some importance in the theory of the engine. It is an "apparent" specific heat.

The Isothermal Line for Saturated Steam, or any other vapor, expanding in presence of the liquid from which it is formed, or containing, as is usually the case, in practice, more or less mist, is an isopiestic line, a line of constant pressure.

The pressure of saturated steam is a function of temperature, only, and remains constant and invariable so long as the temperature of the liquid and its vapor remains unchanged. The equations for isothermal expansion of steam, or other vapor, are therefore

$$n = 0; \qquad p = \text{const.} = f(t); \quad . \quad . \quad . \quad . \quad (4)$$

$$v = \frac{0}{0}; \quad pv^n = \frac{0}{0}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The line is rectilinear, parallel to the axis measuring volumes, and at a greater or less height accordingly as the temperature is higher or lower.

The Hyperbolic Expansion of Steam, or other vapor, is not isothermal. The tendency of such expansion, when produced, as it possibly may be, at times, is to dry moist vapor, and to superheat that already dry; the temperature falling at a lower rate than in expansion either adiabatically or as saturated vapor. This case thus differs greatly from that of the hyperbolic expansion of the perfect gas; which has been seen to be perfectly isothermal. In the latter case, the supply of heat must be precisely equivalent to the work done; in the former, heat must be supplied considerably in excess of the equivalent of the external work performed.

Hyperbolic expansion of steam, or other vapor, is never met with in practice, except by probably the rarest accident. The assumption made, usually, however, in computing the power of the steam-engine, that the steam expands in this manner, is often sufficiently correct for ordinary work, in practice. The differences between the several curves, as shown by the steam-engine indicator, are, in good practice, seldom important or noticeable.

The Adiabatic Line of expanding steam may be obtained by making the thermodynamic function constant, thus:

$$\phi = J \log_e T + u \left(\frac{dp}{dT}\right)_v = \text{const.}, \quad . \quad . \quad . \quad (6)$$

and

$$d\phi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

When, in such case, steam, initially dry but saturated, expands from the temperature, pressure, and volume T_1, p_1, v_1, to any other state, T, p, u, the value of ϕ remains unchanged, and

$$J \log_e T + u \left(\frac{dp}{dT}\right)_v = J \log_e T_1 + v_1 \left(\frac{dp_1}{dT_1}\right)_v . \quad . \quad (8)$$

Then its volume, and also its "quality," is

$$u = \frac{1}{\left(\frac{dp}{dT}\right)_v} \left[J \ \log_e \frac{T_1}{T} + v_1 \left(\frac{dp_1}{dT_1}\right)\right]; \quad . \quad . \quad . \quad (9)$$

which represents the volume of unity of weight of wet steam, expanding from the dry and saturated state in which its condition is p_1, v_1, T_1, to the state p, u, T; the proportion of water present, and due to expansion, as will be presently seen, increasing as expansion progresses. The value of $\left(\frac{dp}{dT}\right)_v$ may be obtained by differentiating the expressions in which p is given as a function of T, or by taking it from the "steam-tables," $\left(\frac{dp}{dT}\right)_v$ being the change of pressure due to a change, unity, of temperature, and $\frac{1}{\frac{dp}{dT}}$ being the change of temperature for a variation, unity, of pressure.*

The ratio of expansion is evidently $r = \frac{u}{v_1}$, and, since the latent heat is $H = T\left(\frac{dp}{dT}\right)_v u$,

$$\left(\frac{dp}{dT}\right)_v = \frac{H}{uT} = \frac{L}{T};$$

in which L is the latent heat when u becomes unity, or the latent heat per cubic foot, or per cubic metre,

$$r = \frac{u}{v_1} = \frac{T}{L}\left(JD_1 \log_e \frac{T_1}{T} + \frac{L_1}{T_1}\right), \quad . \quad . \quad . \quad . \quad (10)$$

when $D = \frac{1}{v}$, the density of the fluid.

Comparing the values of u as expansion progresses, with those of u for saturated steam of the same temperature, Ran-

* Care must be taken, obviously, to use correct units; e.g., in British measures pounds on the square foot, in metric measures kilogrammes on the square metre, as commonly adopted in engineering, the units of volume being, respectively, cubic feet and cubic metres, and of weight, the pound and the kilogramme.

kine found that the former quantity is the greater in all familiar cases; and it thus follows, as he first showed,* that the fluid must partially condense when expanding adiabatically. The higher the pressure, p_1, and temperature, T_1, of the initially saturated steam, the less this condensation; until a point is reached—probably at about the bright red heat of solids †—at which condensation ceases to be a consequence of adiabatic expansion of saturated steam, and beyond which adiabatic expansion may produce superheating.

The value of H for steam may be obtained from the empirical formula

$$H = a - bT; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$a = 1,117,850, \quad b = 544.5, \quad \text{for } J = 778;$$

$$a = 1,109,550, \quad b = 540.4, \quad \text{for } J = 772.$$

For Mixtures of Steam and Water, in the approximate expression for the adiabatic curve, in which, for steam initially dry and saturated, or on the point of condensation, $n = 1.135$, the equation being, in lbs. per sq. in., volumes in cu. ft.,

$$\left.\begin{aligned} pv^{1.135} &= 475, \text{ nearly}, \\ p_m v_m^{1.135} &= 1.704, \text{ nearly}, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

for British and metric measures, respectively. The value of the exponent, n, depends upon the initial condition of the steam, and Zeuner proposes the expression ‡

$$n = 1.035 + 0.1x; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

in which x is the proportion of vapor initially existing in the mixture. When $x < 0.7$ the expression becomes less exact. Rankine takes $n = \frac{10}{9} = 1.\dot{1}11$, in his treatment of the steam-engine, which corresponds to $x = 0.8$, nearly, the mixture containing over 0.20 water. The value of n is also affected

* Steam-engine; p. 384.

† Rankine; Miscellaneous Papers; p. 398.

‡ Wärmetheorie.

by variations of pressure, slightly increasing as pressures rise, the mean value being similarly affected, also, by decreasing the value of r.

When the value of x is less than about one half, evaporation, instead of condensation, goes on in the mixture.

Comparing the curve of saturation with the adiabatic curve, as represented by their equations, it is seen that the former has the lower value of n, and hence that the curve falls less rapidly than the adiabatic. But it has been seen that, in the case of the saturation curve, heat must be added to preserve the steam in the saturated state; it thus again follows that, in the case of adiabatic expansion, in which no heat can be thus supplied, a part of the steam must condense; the volume of unity of weight being less than when dry and saturated.

The adiabatic, the hyperbolic, and the saturation curves of steam have, respectively, for the approximate values of n, $n = 1.135$, $n = 1$, $n = 1.0646$; the first is therefore a curve of more rapid fall in pressure, with any given rate of expansion, than either of the others; while the saturation curve lies between the other two.

The fact independently discovered by Rankine and Clausius, in 1850, and exhibited above, that, when steam expands adiabatically, a portion must be liquefied, yielding its latent heat to assist in producing the expansion of the remainder, is important in its relation to the thermodynamics of vapors, and was at first supposed to have great importance in the operation of the steam-engine. This is not usually the case, however. The condensation observed in single-engine cylinders is mainly due to the conductivity and storing capacity of the material of which they are composed, and in but a comparatively slight degree to this cause.

Hirn, in 1853, confirmed by experiment this discovery. It is found, by experimental investigation, that vapors differ in this respect, and that while many, like steam, partially condense while expanding and doing work, some, as ether, superheat. In other words, their specific heat, under similar conditions, is positive, while, in the case of steam, it is negative; steam requir-

ing to be supplied with heat, as its temperature and pressure fall, if it is to retain the dry and saturated condition.

103. The Construction of Thermal Lines and of Diagrams of Energy illustrating the behavior of vapors acting as working substances in the transformation of heat into work is a subject of still greater importance and interest than in the working of gases. The diagram of energy, representing the cycle of operations occurring in the steam-engine, or other machine in which a vapor is employed as the working fluid, is composed of thermal lines the character and dimensions of which are determined by the construction and method of operation of the engine. Such lines may usually be referred to one or another of the classes already described, and the construction of the diagram, once its general form is so determined, becomes easy when the methods of laying down the principal thermal lines are understood.

In all cases these lines may be represented by algebraic expressions the forms of which have been given. These equations express the relations of magnitude of the simultaneous pressures and volumes of the working fluid when undergoing expansion or compression under definitely prescribed conditions. These conditions being settled by the method of operation, in any given case, the corresponding thermal line is identified for any step in the operation, and the numerical relations given by the equation of the line permit the laying down of the ordinate representing the pressure corresponding to each successive magnitude taken for the volume of the working fluid, as the piston of the engine traverses its cylinder. Thus, for the line of equal pressure produced during the entrance of steam from the boiler into the steam-engine, p_1 is fixed, and $p = p_1$ throughout its whole extent. When the supply of steam is interrupted by the closing of the induction-valve, adiabatic expansion occurs, in the ideal engine, and

$$p_1 v_1^n = pv^n = R_a,$$

and the initial state being known, and p_1 and v_1 thus given, pv^n becomes known as the value of the constant quantity R_a,

and it becomes easy to calculate the value of p corresponding to any value of v.

In illustration, for one pound of initially dry steam, expanded adiabatically,

$$p_1 v_1^{1.135} = pv^{1.135} = 475, \text{ nearly,}$$

and we obtain for successive values of $p = \frac{475}{v^{1.135}}$ to the nearest unit, volumes in cubic feet, and pressures in pounds on the square inch:

v	p	v	p	v	p
1	475	6	61	15	22
2	216	8	45	20	16
4	97	10	35	25	13

These figures measuring the co-ordinates of the curve, it may be laid down to any desired scale. The existence of sensible error in any figure is shown by the point so erroneously fixed falling outside the smooth curve passing through the other points. The graphical construction is thus a reliable check upon the computation.

The geometrical construction of curves of the class, pv^n = const., is very easy and often preferred to construction by the preceding method. When $n = 1$, the curve becomes the equilateral hyperbola and may be laid down by the following methods:

There are several methods of constructing this curve, of which the simplest are, perhaps, the following, as applied to produce the equilateral hyperbola, the curve of Mariotte, to which the expansion-line, in the best classes of engine, very closely approximates, and which is commonly taken as the standard.

Let XX, YY be given asymptotes (i.e., the clearance and true vacuum-lines of the indicator-card), and x any given point, and let xx, xY be its co-ordinates.

Extend YO until $OY' = YO$ and draw AP, making $Y'P$ equal to xY and parallel to XX.

Divide YO and OY' into similar divisions.

Assume an ordinate Om of a point to be found, and draw mx'' parallel to XX.

At Y' erect $Y'n = Om$, and draw Pnx''; the point x'' of intersection with $x''n$ is the required point.

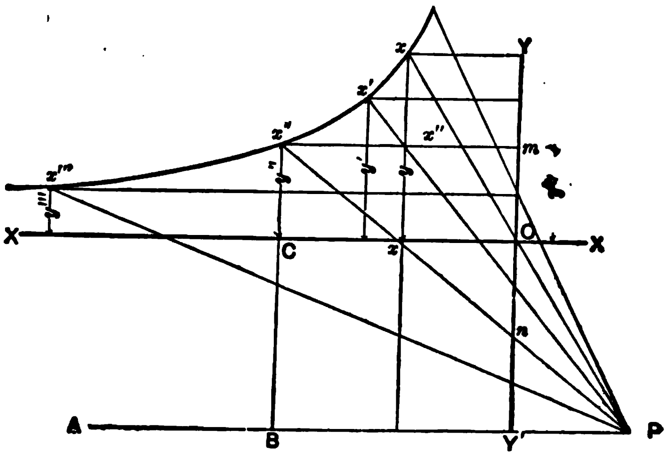

FIG. 132.—THE HYPERBOLA.

For in the triangles $nY'P$. nmx'' we shall have

$$nY' : Y'P :: mn : x''m = \frac{xy}{y''} = x'';$$

i.e., $\quad y'' : x :: y : x''.$ Q. E. D.

When the expansion-line is true to the hyperbolic curve, it becomes possible to obtain a fairly approximate measure from the diagram of the clearance-space; or, the latter being known, to determine the real locus of the hyperbolic expansion-curve, as follows:

Let S', E, E', V, S represent an indicator-card; let OX be the line of perfect vacuum; OY the line at end of cylinder plus the clearance; then OX and OY will be asymptotes of the hyperbola E, A, A', E', the curve of expansion.

Take two points on the curve AA', and AK, AC, $A'B$, and $A'H$ will be their co-ordinates.

Draw AA, and from C, the line CB parallel to AA'; the point B, where it intersects $A'B$, will be a point in the line OY.

Or, draw HK parallel to AA', and K, the intersection with AK, will be such a point.

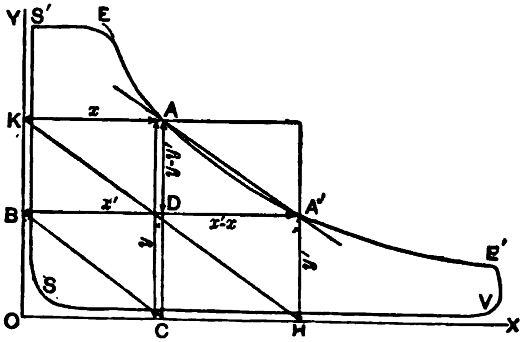

FIG. 133.—THE HYPERBOLIC EXPANSION-LINE.

For by Mariotte's law and from the properties of the hyperbola, $xy = m$; $x'y' = m$; $\therefore xy = x'y'$.

$$\therefore \quad x : x' :: y' : y; \quad x' - x : x :: y - y' : y';$$

or, $$A'D : BD :: AD : DC.$$

And, from similar triangles (by construction),

$$A'D : BD :: AD : DC'. \quad \text{Q. E. D.}$$

Conversely, having given the clearance and the scale of the indicator, with point of cut-off, *to find the expansion-line.*

In proportion $y - y' : y :: x' - x : x$, assume x' and find values of y' by constructing the triangle KPH, similar to ADA'.

Taking the point of release as a point in the hyperbolic curve, and laying down that curve on the diagram, it will be found, not only that the curve and the expansion-line of the diagram do not coincide, but that the latter falls above the

former throughout its length, in nearly all cases, indicating, usually, initial condensation and later re-evaporation, but sometimes indicating some leakage as well. If the weight of steam actually drawn from the boiler be taken as the basis of a diagram, using its volume as the initial ordinate of the hyperbolic curve, it becomes easy to trace the variations of the whole actual diagram from the ideal indicator-card, as here shown.

In any case in which the curve represented by the expansion-line is of the class of which the equation is

$$pv^n = p_1v_1^n = p_2v_2^n,$$

the co-ordinates sought, any one point, p_1v_1 or p_2v_2 being given, may be found, and any new point in the ideal curve determined by computation, thus: From the above expression,

$$n \log v + \log p = n \log v_1 + \log p_1;$$

and if p_1 and v_1 are known, for any assumed volume v, the logarithm of the corresponding new pressure must be

$$\log p = n \log v_1 + \log p_1 - n \log v;$$

which expression being used to determine several points, the curve may be drawn through them.

The values of n have been seen to be as follow:

Equilateral hyperbola, ...	1
Curve of steam; saturation $\frac{17}{16}$, or	1.0646
Adiabatic curve, steam, ...	$1.035 + 0.1x$
" " gas, ...	1.405
Isothermal " "	1.0

The variation of the actual ratios of expansion from their apparent values, in engines having large clearance-spaces, is very considerable at high ratios of expansion and in short-stroke engines.

The close approximation of the three principal steam-expansion lines is well shown by the accompanying diagram, a

set of curves shown in various publications, but probably first laid down in this form' by Mr. Porter.* *AB* exhibits the initial volume, as does also *CD*; *AD* and *BC* represent the initial pressure; *EF* is an ordinate, taken at convenience; and the terminal ordinates are *GH*, *IM*, and *LK*. *OR* is taken at half-stroke; while *CN* is the axis of the equilateral hyperbola, *AOG*, the upper curve, of which *CB* and *CH* are asymptotes. Ordinates measure absolute pressures in pounds

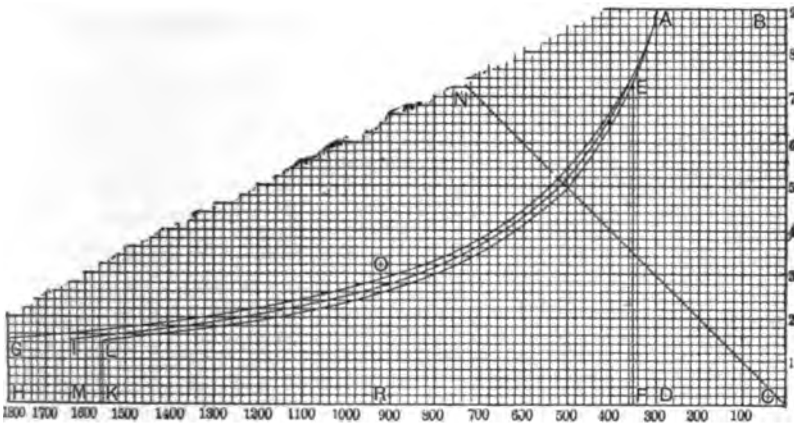

FIG. 134.—THE THREE EXPANSION-CURVES.

per square inch; abscissas represent volumes of unity of weight (1 lb.). Thus *BA* is the volume (4.73 cu. ft.) of one pound of steam at a total pressure of 90 pounds per square inch; *ABCD* is the external work done in its production. It is this curve which is commonly assumed to be that of the expansion of steam.

The curve *AOI* is the curve of dry and saturated steam, its co-ordinates representing the simultaneous pressure and volume of the fluid when in contact with the mass of water from which it is produced. The expansion is less, and the rate of fall of pressure greater, than if it were to follow the law

* Steam-engine Indicator; p. 123.

of Mariotte. It is this curve which is assumed to be described when steam expands in well-jacketed engines.

The lower line, *AOL*, is the adiabatic curve, assumed to be obtainable in engines with non-conducting cylinders and approximately in "high-speed engines." The area under this, as under the other curves, represents the work done as the steam expands, and exhibits the gain obtainable by expansion, in each case. In all real engines, however, the expansion-line falls at first more rapidly, and finally more slowly, than either of these curves. As elsewhere seen, this variation from the ideal curve is often very observable.

Cylinder Condensation and Leakage produce variations in the diagram, as obtained, which differently affect the different parts of the curve. Leakage can usually be eliminated, and always should be before the engine is set at work regularly. The first-named waste is usually irremediable. Its character, laws of variation, and magnitude will be studied in detail in the succeeding chapter. When the exact measure of the quantity of steam expended is obtained by a boiler-trial, it is easy to trace these variations, as in the indicator-diagram, Fig. 135, taken from the engine and worked up by the late Professor C. A. Smith, in which illustration the diagram which should have been produced by the same steam, had there been no initial condensation, is shown with the real diagram.*

This indicator-diagram is an unusually good sample, as to form, and was taken from the St. Louis high-service pumping-engine, a machine of 705 I.H. P., 85 inches diameter of cylinder and 10 feet stroke of piston, making $11\frac{1}{2}$ revolutions per minute. Taking measures of the abscissas of the two diagrams, it is seen that the condensation amounts to from about 30 per cent as a minimum to 50 per cent as a maximum, so far as measurable, the actual card illustrating the expansion in a metallic cylinder of the steam, which would have given the larger diagram in an ideal engine with its non-conducting cylinder. The complete ideal diagram would extend propor-

* Steam-making; p. 91.

tionally farther toward the right and beyond the limits of the actual figure. When the two lines continue so far separated, it is an indication of large initial condensation, and correspondingly great re-evaporation after the exhaust-valve opens; as the initial condensation is due to, and is proportional to, the re-evaporation. In most cases, however, the engineer, unable to determine these data, assumes the point of release, or the point of intersection of the expansion-line prolonged with the ordinate at the extreme end of the diagram, as that of coinci-

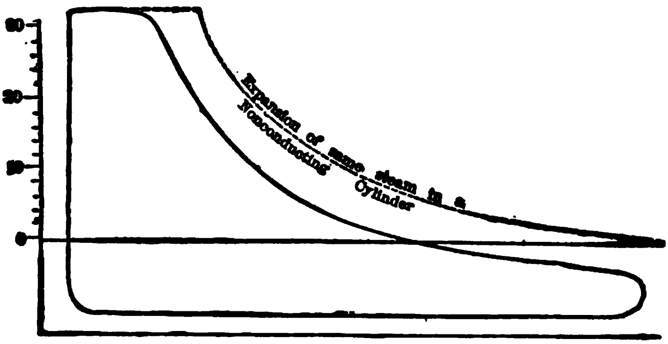

FIG. 135.—THE REAL AND THE IDEAL CARD.

dence of the ideal and the real curve, and draws the hyperbolic curve backward from that as a given point, in the manner already described. A comparison of the ideal diagram thus formed with the actual indicator-card will give a means of judging of the character of the engine studied as a thermodynamic machine.

Rankine's construction will enable the engineer conveniently to find the absolute mean pressure in the steam-cylinder and the final pressure.* Thus, in Fig. 136, draw AB and AG; take AC equal to one fourth AB; from C as a centre, strike the arc BFG; and AG then measures the stroke of piston plus clearance. Take GD proportional, on the adopted scale, to the clearance; AD then measures the stroke. Make DE the dis-

* Hutton's Handbook; p. 380.

Fig. 138 illustrates the more regular forms of thermodynamic cycle met with in the operation of heat-engines. *AB* and *CD* are two isothermal lines crossed by two adiabatics, *EF* and *GH*. The perfect engine cycle of Carnot is *abcd*; the same with the adiabatic lines replaced by lines of constant volume, which are here those of a regenerative action, is seen in *abnm*. Others are formed, as *efgh*, and *f'ijh'*, by lines of constant pressure crossing the two pairs of curves, and by lines of constant volume crossing them, as in *abnm* and *opqr*. Many other cycles are formed by other combinations. That

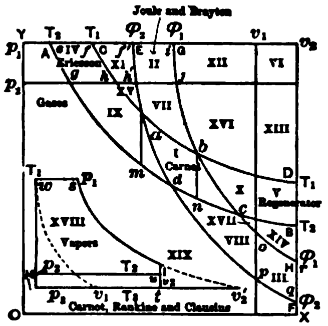

FIG. 138.—THERMODYNAMIC CYCLES.

seen in $wsIup_2$ is the ideal steam-engine cycle modified by the usual exhaust line, and drop of pressure at constant volume at Iu. HI is taken as the "atmospheric" line. Here the isothermal, ws, corresponds to that portion of AB or CD at the extreme right; where it becomes asymptotic with OX. The cycle of Carnot, *abcd*, is illustrated in the action of the now well-known type of air-engine of Stirling; in which a mass of air is permanently enclosed in a working cylinder, in which its variations of pressure, temperature, and volume are precisely such as are above described. Cycle $wsv_1'v_1$ is a Carnot steam or vapor engine cycle.

The closed cycle, $wsIup_2$, is also well illustrated in the most effective types of modern steam-engine. In the marine steam-engine, for example, the feed-water is taken from the

hot-well or the discharge of the surface-condenser, with which such engines are now always fitted; is forced into the boiler by the feed-pump; is there converted into steam by accession of heat from the fuel, with consequent expansion into the vaporous state; is next transferred to the working cylinder; where its expansion results in the conversion of a certain proportion of its heat into mechanical energy, by a process similar to that just described as in the cycle of Carnot. It first drives the piston at constant pressure and temperature, as is required in a cycle of this form; next it expands adiabatically, to a minimum temperature and pressure and a maximum volume, precisely as in that cycle; then it is compressed, at this minimum pressure, into the condenser, which removes the heat of compression and preserves its pressure and temperature constant, so as to give an isothermal change; and, finally, being thus reduced to the liquid state, once more, it is once again forced into the boiler, to enter upon another similar cycle to that now completed. The last step, compression as liquid, into the boiler, and its increase of temperature to that of the steam into which it is presently converted, corresponds to the period of adiabatic expansion at the other side of the cycle. To make it exact, however, it is evident that the last step should be the compression of the vapor into liquid, at the higher temperature and pressure, by a purely mechanical action; instead of its introduction, cold, into the boiler, and its elevation, there, to the maximum temperature by heat directly applied.

The action of the non-condensing engine is, in essence, the same as that just traced. The steam enters the engine at a maximum temperature and pressure; drives the piston, up to the point of cut-off, by an isothermal expansion; is then expanded to the back-pressure and corresponding temperature by an adiabatic process, as nearly as the nature of the case will permit; is then rejected into the atmosphere—an isothermal process,—where it is condensed, the atmosphere being here the condenser; and it finally reappears as feed-water to be once more passed through the same cycle again. Thus each revolu-

tion of the engine illustrates a cycle. Expansion is actually seldom either complete, on $s v_2'$, or adiabatic.

In the trials of engines, it is necessary, to secure a satisfactory result, that the engine should be operated steadily until its "régime" is fully established, and until it is making cycle after cycle, precisely under the same conditions, before the trial is commenced. This is here especially important because of the facts, to be more fully discussed later, that the reactions between the steam and the cylinder-surfaces are of real importance in the economics of the machine, and that it takes some time to establish uniform action in this respect.

Steam and Air being compared, as representative of two extreme types of working fluid, it will be found, as indicated by our formulas, which show maximum efficiency of fluid to be dependent upon temperature solely, that the one must be just as efficient as the other as a medium of transformation of heat into mechanical energy, provided the fluids are worked between the same initial and terminal temperatures. The formulas already given enable this comparison to be readily made. Both fluids may be assumed to work in the Carnot cycle.

Such a comparison was made by Rankine as early as 1867,* assuming for the air-engine perfect regeneration.

REGENERATIVE AIR-ENGINE *vs.* STEAM-ENGINE.

$$t_1 = 265° \text{ F.}; \quad T_1 = 727° \text{ F.}$$

$$t_2 = 104° \text{ F.}; \quad T_2 = 565° \text{ F.}$$

$$T_1 - T_2 = t_1 - t_2 = 162°.$$

$$E = \frac{T_1 - T_2}{T_1} = 0.2228.$$

$$U \text{ (assumed)} = 68{,}420 \text{ ft.-lbs.}$$

Pressure of steam .... $p_1 = 5652$; $P_1 = 39.25$
Pressure of air .... $p_1 = 5050$; $P_1 = 35.1$

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* The Engineer; Aug. 2, 1867.

Volume of steam, initial, v_1..............	1.00 cu. ft.
Weight of steam.........................	0.0956 lbs.
Weight of air...........................	1.6113 lbs.
Volume of air, initial, v_1..................	9.81 cu. ft.
Volume of air, atmospheric..............	22.87 cu. ft.

RESULTS.

	St.-engine.	Air-engine.	Diff.
Heat expended, ft.-lbs.........	68,420	68,420	0
Heat rejected, "	53,176	53,176	0
Heat transformed into work, ft.-lbs.....................	15,244	15,244	0
Work of expansion, ft.-lbs......	16,690	68,420	51,730
Work of compression, ft.-lbs....	1,446	53,176	51,730
Work per indicator, net.......	15,244	15,244	0
Efficiency...................	0.2228	0.2228	0

An enormously greater amount of work is thus seen to have been done during the forward stroke of the air-engine than in the case of the steam-engine; but this is balanced by a precisely equal excess in compression during the return-stroke; the heat of such compression being necessarily wasted. Both engines thus do exactly the same net amount of work during each cycle, expending the same quantities of heat and exhibiting the same efficiencies of fluid. But it is seen that the air-engine must have much the greater bulk, and, consequently, in nearly the same ratio, the greater weight; and this fact makes the comparison of efficiencies of engine, including both efficiency of fluid and that of mechanism, result much more favorably to the steam-engine. This advantage of the steam-engine becomes greater at high pressures, and is a vital one under its usual working conditions.

In all cases of adiabatic change of volume under pressure, doing external work, we shall have for the thermal change, in dynamic units,

$$W = (Jct + xl) - (Jct' + x'l')$$

$$= Jc(t - t') + (xl - x'l');$$

where x and l represent the proportion of fluid in the vaporous state in the mixture and the corresponding latent heat. In gases $x = x' = 1$, and $l = l' = 0$; and we have

$$W = Jc(t - t').$$

When the work is that of expansion, $t - t'$ is positive; the difference of latent heats may be either positive or negative. In compression these signs are reversed. The former case is illustrated in heat-engines; the latter in refrigerating machinery. In the former, efficiency is promoted by a wide range of expansion to a minimum temperature; in the latter, by a narrow range at maximum temperature; the measure being, for the first,

$$E = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1};$$

and, for the second,

$$\frac{JQ}{W} = E = \frac{T_2}{T_1 - T_2} = \frac{Q_2}{Q_1 - Q_2}.$$

The choice of a working fluid, in both cases, is purely a matter of extra-thermodynamic consideration, all working fluids having identical thermodynamic efficiency.

Conclusions of interest and importance relative to the thermodynamic properties of the several vapors practically most available for the operation of heat-engines, as ether, chloroform, alcohol, carbon disulphide, water, may be deduced from similar comparisons, thus:*

Where the several fluids are worked between the same temperature-limits, and consequently have the same thermodynamic efficiency, considerable differences are to be observed in their tensions, at both initial and terminal temperature, ether exhibiting highest, and steam the lowest, pressures; the one having nearly four times the tension of the other. It is ob-

* Efficiency of Fluid in Vapor-engines; Van Nostrand's Magazine; 1884. Wood's Thermodynamics: 1889.

ceived that a high tension is accompanied by a small value of he potential energy of latent heat; while great elasticity is generally an accompaniment, also, of high density; although no direct relation is yet determined.

Most interesting differences are seen in the magnitudes of work, of compression and expansion. The net effective work being the same, those vapors with which the work of expansion is greatest also demand the expenditure of most energy in their compression; the difference in energy exerted and energy received being constant throughout the list.

The variation of the ratio of expansion among the various working fluids is, in such cases, very noticeable; and the influence of this ratio, and of the magnitude of the final volumes of the several fluids, upon the size of working cylinder required is an important practical consideration, which is well illustrated by the comparison of these quantities in the steam- and the air-engines. The fact that the familiar limiting conditions of operation of real engines may produce important practical results in the modification of efficiency of fluid and of economy of working is forcibly shown by the results obtained in the other cases of comparison of vapors.

On the whole, it will be found that, if we make our comparisons within those limits of pressure found practicable with the steam-engine, the vapor of water is the most efficient of all available fluids under the conditions of use in real engines, and, since all the apparent advantages of the non-aqueous vapors may be gained by increasing pressures and, especially, of temperatures of steam, it seems probable that none of those fluids will ultimately successfully compete with steam. It is further evident that the use of air and other gases, now giving large thermodynamic efficiency, must involve comparatively low efficiency of mechanism, and that this latter disadvantage may be lessened by working a larger weight of fluid within a given volume: i.e., by working the fluid at initially greater density.

Collating expressions used in the preceding study of the thermodynamics of the ideal engine, their tabulation in compact

form will be found very convenient for reference and in doing work. The following table is thus obtained:

WORKING FORMULAS OF THERMODYNAMICS.

§ 84. (1) $dH = Qd\phi$.

§ 86. (2) $dH = Td\phi$.

$(A)\ dH = dS + dL + dU.$

$(B)\ dH = dS + dW.$

$(C)\ dH = dE + dU.$

§ 90. (3) $\log p = a - b\alpha^n - c\beta^n.$

§ 91. (3) $\dfrac{p_0 v_0}{p_1 v_1} = \dfrac{1}{1.365}.$

(4) $\dfrac{p_1 v_1 - p_0 v_0}{p_0 v_0} = \dfrac{0.365}{1}.$

(5) $\dfrac{T_0}{T_1} = \dfrac{p_0 v_0 - 0}{p_1 v_1 - 0} = \dfrac{1}{1.365}.$

(6) $\dfrac{T_1 - T_0}{T_0} = \dfrac{0.365 p_0 v_0}{p_0 v_0}.$ $T_0 = \dfrac{100}{0.365} = 274°$ Centigrade (or 493° F.).

(9) $t - t_0 = 493 \dfrac{pv - p_0 v_0}{p_0 v_0} F.$

§ 93. $Jc_p = K_p$. $Jc_v = K_v$.

$\gamma = \dfrac{c_p}{c_v} = \dfrac{K_p}{K_v}.\ K_p = \dfrac{\gamma}{\gamma - 1}R;\quad K_v = \dfrac{1}{\gamma - 1}R.$

$dU = pdv.$

(13) $H = H_0 + K_p(T_1 - T_0).$

§ 95. (1) $\frac{pv}{p_0 v_0} = \frac{T}{T_0}$ = constant $= R$, for a perfect gas.

(2) $R = \frac{pv}{T}$.

§ 96. (2′) $p = T\left(\frac{dp}{dT}\right)_v$.

(3) $dH = K_v dT + T\left(\frac{dp}{dT}\right)_v dv = Td\phi$

$= K_v dT + RT\frac{dv}{v}$

(5) $\quad = K_v dT + pdv.$

$dH = K_p dT - T\left(\frac{dv}{dT}\right)_p dp.$

(6) $\quad = K_p dT - RT\frac{dp}{p}$

$= K_p dT - vdp.$

§ 96. (7) $dH = (K_v + R)dT - \frac{RT}{p}dp.$

(8) $\frac{dH}{dT} = K_p = K_v + R. \quad \cdot R = K_p - K_v.$

(12) $d\phi = K_v\frac{dT}{T} + \frac{dp}{dT}dv.$

$d\phi = (K_v + R)\frac{dT}{T} - \frac{dv}{dT}dp.$

$d\phi = K_p\frac{dT}{T} - R\frac{dp}{p}.$

(15) $\phi = K_v \log_e T + R \log_e v + C.$

When gas expands or contracts at constant temperature:

§ 96. $$H = RT_1 \log_e \frac{v_2}{v_1} = p_1 v_1 \log_e \frac{v_2}{v_1} = p_1 v_1 \log_e r.$$

Adiabatic expansion:

§ 96. $$dH = K_v dT + (K_p - K_v) T \frac{dv}{v} = 0.$$

$$\frac{T}{T_1} = \left(\frac{v_1}{v}\right)^{\gamma - 1}. \qquad \frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{\frac{\gamma - 1}{\gamma}}. \qquad \frac{p}{p_1} = \left(\frac{v_1}{v}\right)^{\gamma}.$$

Work of perfect gas at constant temperature:

§ 96. $$U = \int_{v_1}^{v_2} p dv = p_1 v_1 \int_{v_1}^{v_2} \frac{dv}{v} = p_1 v_1 \log_e \frac{v_2}{v_1}. \quad r = \frac{v_2}{v_1}.$$

$$U = p_1 v_1 \log_e r.$$

Work of adiabatic expansion:

§ 96. $$U = p_1 v_1^{\gamma} \int_{v_1}^{v_2} v^{-\gamma} dv = \frac{p_1 v_1}{\gamma - 1}\left[1 - \left(\frac{v_1}{v_2}\right)^{\gamma - 1}\right].$$

Computation of J:

§ 96. $$J = \frac{\gamma}{\gamma - 1} \cdot \frac{R}{c_p};$$

taking Regnault's best values, for air,

$$J = \frac{1.405285}{.405285} \cdot \frac{53.37}{.2375} = 778.12.$$

For mixtures of x parts steam and $x - 1$ water:

§ 104.	Latent heat, unit weight,	xl;
	total " " "	$Q + xl$;
	volume per " "	$0.017(1 - x) + xV$;
	internal energy, unit weight,	$Q + x\rho_x$.

For hyperbolic logarithms, see Table II, p. 803.

CHAPTER V.

THERMODYNAMICS OF THE STEAM-ENGINE. WASTES OF ENERGY; EFFICIENCY.

105. The Thermodynamics of the Real Engine involves the application of the principles of science to the determination of the quantities of thermal converted into mechanical energy, the proportion wasted, the efficiency of steam as the working fluid in the machine, and the weight of steam, and—where the data permit—that of the fuel demanded per horse-power and per hour, or other unit of power or of time, in effecting such transformation in an ideal, purely thermodynamic, engine.

Since no other than thermal and the equivalent mechanical energies are taken cognizance of by this science, and no physical changes or transfers are considered, all those circumstances and conditions which distinguish the real from the ideal case, in energy-transforming machinery, in the case, for example, of the heat-engines, must be treated by extra-thermodynamic methods. The study of the thermodynamics of the steam-engine comprehends, simply, the investigation of the machine so far as it is an ideal heat-engine, subject to no other than the inevitable thermodynamic wastes.

The study of actual engines, however, involves the examination of both physics and dynamics in their applications in such machines, and the problem is thus rendered a much more complicated one than that in thermodynamics and far less easy of exact solution.

It is still generally admitted by writers on the steam-engine that, as stated by Hirn, it is impossible to construct a theory that shall be scientifically exact, and will accord perfectly with

practical experience in even the best practice.* Nevertheless, designers and builders are required to use methods and formulas in their preliminary computations and in preparation of their plans, not only for the computation of dimensions and proportions of parts, but also to obtain approximate estimates of the quantity and the cost of the work to be performed and of the heat, the steam, and the fuel to be demanded for its performance. In the following pages so much of applied theory and approximate methods as may be considered as practically useful, to-day, will be exhibited and illustrated. In all cases, however, the engineer is guided largely by, and his computations are checked by reference to, experience with as nearly as may be similar practice.

106. **The Steam-engine as a Heat-engine** is a machine in which heat-energy stored in steam is converted into the dynamical form and applied to the purposes for which the engine has been designed. In this process, the steam, produced in the steam-boiler, is supplied to the engine at such a pressure and temperature as will permit a considerable range of adiabatic, or approximately adiabatic, expansion, and the consequent transformation of a considerable fraction of its thermal energy.

This energy exists, initially, in the steam, in the form of sensible and latent heat, and is in larger proportion sensible, as has been seen, as the temperature and the pressure are more elevated. Were it practicable to use the fluid at the temperature and pressure of its "critical state," all heat-energy would be in the sensible form. In any case, a fraction of the heat supplied is converted, by the cyclical action of the machine, into work; while the other portion, usually large, is necessarily rejected at the minimum temperature reached. The larger the fraction transformed into mechanical power, the larger the efficiency of the machinery; and its maximum possible effect is measured by the proportion,

$$E = \frac{T_1 - T_2}{T_1} = \frac{H_1 - H_2}{H_1},$$

* Hirn: Thermodynamique; 1876. Sinigaglia: Machines à Vapeur; 1890.

of total heat-energy stored in the fluid at its entrance into the engine.

As elsewhere shown (§ 93, § 112), the quantity of work performed per unit weight of working fluid is determined by the quantity of energy that may be stored as latent heat of expansion and vaporization. In this respect, steam is superior to other available working substances; and the engine in which it is employed can be given smaller volume and weight than any other, the air-engine for example, in which this latent heat is less.

107. The Real and the Ideal Engines are so radically different in their conditions of action and in the nature and magnitudes of their wastes of energy that the engineer distinguishes carefully between the two cases.

The ideal engine presents a purely thermodynamic problem, capable of exact and unqualified solution. It illustrates simply the transformation of thermal into dynamic energy, with no other loss than that unavoidable waste due to the operation of the second law of thermodynamics, the magnitude of which is easily and precisely computed as soon as the conditions of the problem are definitely given. This process of computation has been fully described and its application illustrated.

The real engine is a piece of mechanism composed of substance incapable of retaining heat and permitting free transfer to and from the working fluid, wasting large quantities externally by conduction and radiation, and internally by alternate storage in its own substance and restoration to the working fluid in such manner that transfer occurs without transformation, in large proportion. It also wastes a large amount of the mechanical energy produced by transformation in the work of moving its own cumbersome parts.

The Real and the Ideal Engines in their operation are thus distinguished by a very wide difference of efficiency, resulting from the correspondingly enormous differences of physical working conditions arising out of the thermal and

mechanical operations unavoidably accompanying the thermo-dynamic phenomena.

In all real engines the departure from the ideal conditions assumed is very great, not only in steam-, but even in gas- and air-engines, and so great as, in most cases, to lead to radically different results from those attained in the ideal case.

Explosive and other gas engines are impelled by a mixture of hot gaseous and vaporous products of combustion, of which the latter portion is, like the working fluid in the steam- and other vapor-engines, subject to rapid and considerable changes of thermal state. Enclosed, usually, in a chamber the sides of which are kept cool by a water-jacket, enormous quantities of heat are lost as expansion proceeds, and the efficiency of the machine is correspondingly diminished, and both the efficiency and the most economical ratio of expansion are altered by the increased losses which accompany the higher ratios.

Steam always condenses in the steam-cylinder in consequence of the conversion of a part of its heat into work, even though the expansion be perfectly adiabatic; and, in the actual engine, this occurs to a much greater extent, unless, by superheating or by the use of an efficient jacket, considerable heat is supplied it before or during expansion. The first quantity is, however, insignificant in comparison with direct losses of heat; it probably seldom approaches ten per cent of the heat supplied, and is, usually, a very much smaller figure.

Initial condensation and later re-evaporation of steam in the steam-engine, and initial cooling without subsequent re-heating, in gas-engines, are the greatest sources of waste of heat, and give rise to losses that are both absolutely and relatively very great wherever the range of temperature during expansion is very considerable, and especially with low back-pressure.

The steam passing out of the exhaust-ports to the condenser or into the atmosphere is moist and heavy with the water of condensation, and is a good conductor of heat as well as a very greedy absorbent. It sweeps out of the

cylinder large quantities of heat abstracted from its inner surfaces, leaving those surfaces comparatively cold and wet with a chilling dew. The entering steam meets these cold metallic and liquid masses and is condensed in sufficient quantity to reheat them to the temperature of prime steam. As the piston moves forward it uncovers new surfaces, and condensation continues until, sometimes, a large fraction of the steam supplied lies in the cylinder or floats in the uncondensed steam as water and mist. Toward the end of expansion, and especially during exhaust, re-evaporation occurs, from the exposed surfaces and in the midst of the mixture of water and steam, at lower pressures and to a similarly serious extent. Thus heat is constantly transferred from the steam to the exhaust side, and, doing little or no work, is wasted, and the efficiency of the engine and the cost of fuel are greatly affected.

This loss may be greatly reduced by superheating and steam-jacketing. Loss from this cause has been found to be so great, and to increase so rapidly with increased expansion, that it practically often sets an early limit to the economical increase of the ratio of expansion.

It is thus seen that several directions of distribution and waste of energy are found in the real engine which do not exist in the ideal case, and which constitute characteristic distinctions between the two. The engineer thus observes the following facts, and bases upon them his nomenclature of the various "powers" and "efficiencies."

When steam enters the engine from the boiler, it is made the vehicle of heat-transfer and the medium of transformation of thermal into mechanical energy. The work performed in the cylinder and the power developed are called the "*indicated work and power.*"

The ratio of this work to the mechanical equivalent of heat required in a non-conducting cylinder for the same operation is the measure of thermodynamic efficiency. The ratio of this latter quantity to the actual efficiency, as measured by the ratio of mechanical energy to the total actual heat used, including

heat-wastes in the metallic cylinder, may be called the *efficiency of the working substance;* its efficiency for use as a medium of energy transfer and transformation.

When the energy applied to the piston, as measured by the indicator, is carried onward through the machine, and finally given out at the shaft to the driven machinery, it loses an amount measured by the friction of the engine, and, this lost work being taken out of the indicated work, we have the work usefully given out as measured by the dynamometer. This is called the *dynamometric power.* Its ratio to the indicated power is the *efficiency of the machine.*

Engineers usually express the quantities of power in horse-power and, in symbols, as *I. H. P.* and *D. H. P.*

108. **The Wastes of the Steam-engine** are comprised in three distinct classes: (1) the thermodynamic waste; (2) the physical, thermal, waste; (3) the friction-wastes, and other dynamic, or mechanical, losses. Of these, the first is easily computed when the thermodynamic cycle of the machine is known, and can be determined with precision. The second is divided into two parts: the waste of heat directly by immediate conduction and radiation, the heat so wasted streaming steadily out to surrounding, cooler, bodies; and the waste caused by the process described in the preceding article, that due to alternate storage of heat, without transformation, in the metal of the working cylinder, and, later, with little or no utilization, discharged from the engine. The third kind is that produced by waste of energy previously transformed, by the thermodynamic operation, from the thermal to the dynamic form, and expended in overcoming back-pressure and the friction of rubbing parts.

The sum of these wastes being deducted from the total energy supplied as heat, the remainder measures the heat-energy utilized by the engine, and delivered to the user in the form of available mechanical power.

The efficiency, as already seen, of any purely thermodynamic engine depends solely on the method of heat-supply and rejection, and in no respect upon the nature of the working

›stance, or the structural details or arrangement of the .chine, where a maximum is attained.

The heat-wastes of the real steam-engine, in the usual order magnitude and importance, may thus be considered as fol- vs:

(1) Thermodynamic loss.
(2) Internal condensation.
(3) Conduction and radiation.

In detail, wastes are due to

(1) Exhaust-wastes by action of the metal of the cylinder.
(2) Incomplete expansion and compression.
(3) Back-pressure and leakage.
(4) Clearance and restricted steam-passages.
(5) Exhaust-waste, from the expansion period on.
(6) Transmission of heat, externally.

To which may be added,

(7) Boiler and feed-water heat and other wastes.

The character and the method of these various wastes of energy in the real engine remain to be studied, and their magnitudes to be determined by experimental investigation.

As was probably first noted by Cotterill, the wastes by the exhaust include both that due "cylinder-condensation," initially, and that produced by condensation during expansion. The latter occurs, with production of a suspended mist, within the whole expanding mass, and its effect in robbing the metal of the cylinder of its heat is little or nothing; while the former measures the loss by alternate storage and restoration of heat by exchanges between the steam and the metal. There is always, as shown by Hirn, a balance, in this case, of heat stored and heat restored, of heat-waste to the condenser and heat taken out of the entering charge.

Exact computations would always require correction of estimates of energies transferred by consideration of the work of air-pump in condensing engines, and of the feed-pump in all forms; although the latter is too small a quantity to assume importance in ordinary work.

109. **The Thermodynamic Wastes** include only that pro-

portion of the heat supplied to the machine which is computed as waste in the ideal case; and which is necessarily rejected from the machine at the lower limit of temperature and pressure, during the return-stroke of the piston. In the case of those engines in which the working fluid is retained, this wasted energy is rejected as heat, by transfer to some other, cooling, substance; the work of the engine being effected by changes of volume, temperature, pressure, and heat-content of the same unchanging mass of molecules. In other engines, the heat is rejected with the discharged working fluid, during the exhaust-period. The Sterling air-engine and the non-condensing steam-engine are examples of the two classes of engine and the two methods of rejection.

In the case of the perfect, ideal, engine working in the cycle of Carnot, the proportions of heat constituting these thermodynamic utilizations and wastes have been seen to be, invariably,

$$H_a = \frac{T_1 - T_2}{T_1} H_1;$$

and

$$H_b = \frac{T_2}{T_1} H_1; \qquad H_a + H_b = H_1;$$

when H_a, H_b, and H_1 are the quantities of heat utilized and wasted, and that initially supplied. In all other cases the quantity wasted is larger, as the working cycle departs more and more from that of Carnot; as, for example, by incomplete expansion. It can always be computed, however, when the cycle is known, either by tracing the complete cycle, noting the quantities of work done positively and negatively, taking the algebraic sum as the measure of heat transformed, and the remainder as that wasted; or by simply measuring up the curve of energy, the "indicator-diagram" for the cycle, and taking it as the mechanical equivalent of the heat utilized by transformation, and the difference, between this quantity and the total supplied, as the waste.

110. **The Physical Wastes**, externally, the purely thermal ternal losses, due to conduction and radiation to adjacent dies, are not usually very large in amount in the real engine; ile they have no existence in the ideal case. In small gas-gines, the Author has found this loss to amount to, in some ses, ten or even fifteen per cent of the heat supplied; with igle-cylinder steam-engines of 100 I. H. P., and upward, this ght not, with good coverings on external surfaces, to exceed out 5 per cent; the compound engine is naturally subject greater loss. The amount can be easily computed wherever e area of exposed surfaces, their character, their tempera-res, and the nature of the covering are known. The larger e machine for a given power, the higher the steam-pressures, d the less effective the clothing and lagging of the cylinders, e greater this loss. It is also exaggerated by roughness of posed surfaces, as of cylinder-heads, or of piston-rods and lve-rods.

The internal wastes, those due to "cylinder-condensation," on e other hand, are often simply enormous, as has already en stated, and are extremely variable with all the changing nditions of the every-day operation of the engine. It is well own that the magnitude of this loss is greater as the range temperature during expansion is greater; it is increased by ow speed of engine, by reduction of the back-pressure, by in-ease in size of engine for a given amount of work done, by crease in conductivity of the surfaces of the working cylinder, d, within certain limits, probably, by wetness of steam. It reduced by low ratios of expansion, by increasing back-ressures, by reducing initial pressures, by increasing speed of gine, and by special expedients, as steam-jacketing, super-eating, and the division of the expansion between two or ore cylinders, as in "compound" or multiple-cylinder engines. ven increasing compression may reduce this loss and thus ive a higher steam-line and an altered expansion-line. The aste becomes the less, when the sides of cylinders only are cketed, the smaller their diameter; it is lessened, when both

heads and sides are jacketed, by increasing diameters, volumes being in both cases equal.

The difference in back-pressure between non-condensing and condensing engines is productive of such a wide difference in the range of temperatures worked through in usual cases, that the Author has been accustomed to consider the compensation so complete as to justify the assumption that the value of this waste, its equivalent pressure being taken as a "*virtual* back-pressure," may be assumed to be independent of the magnitude of the actual back-pressure, and to be determined solely by other conditions above noted.

This waste is found to be reduced most effectively by superheating, and somewhat by the admixture of air with the steam, or by the free use of oil in the cylinder, as well as by any expedient, in fact, which will reduce the facility of exchange of heat between the steam and the metal of the cylinder, whether by decreasing the condensing and heat-transferring power of the former, or the receiving and storing power of the latter.

This internal condensation is an exceedingly rapid process; being precisely like that occurring on the tubes of the surface-condenser, except that, instead of the difference of temperature, or head producing heat-flow, being constant, the condensing surface immediately rises in temperature, and presently reduces the condensation to that rate at which the heat received and thus stored can be transmitted into the mass of metal behind. In Emery's experiments on the Bache and the Dallas, this rate exceeded 100 pounds per square foot per hour, and is often in excess of even that rate. It is thus found that the rate of condensation exceeds that of ordinary surface-condensation very greatly; this greater activity of heat-transfer being very possibly due to the fact that the deposited water of condensation, which, unless artificially swept off, impedes this action greatly, in the engine is re-evaporated, at each exhaust; thus, perhaps, giving clean surfaces at the time of initial condensation.

III. **The Mechanical Wastes** in the real engine are commonly somewhat greater than the thermal wastes, externally;

t are not necessarily so; they have been reduced, in some stances, at least, in non-condensing engines, to as low as five r cent of the total power of the engine, and, in condensing gines, below ten per cent. Probably usual values are a half gher. This loss is measured by the difference between the wer shown on the indicator-diagram and that measured at e same time by the Prony brake, the absorption-dynamometer. Its magnitude depends on the size and proportions of the gine, and especially of its rubbing surfaces, and upon the aracter of the lubrication. Journals of sufficient size to prevent danger of overheating, and the most liberal possible continuous supply of the best lubricants, are the means to be dopted in the reduction of this waste to a minimum. Flooded urnals and a system of recovery and re-use of oil will be probably always found advisable.

The effects of clearances and of back-pressure will be studied ter (Chap. VI).

112. Transformations in the Ideal Case, those of External Work, Energy, and of Heat, by the expansion of steam, r any other vapor, are easily determined by the thermodynamic processes already enunciated and illustrated.

The external work done during isothermal expansion of apors containing, or in contact with, their liquids, since their othermal line is a line of constant pressure, is evidently, easuring from the zero line,

$$U = p_1 \int_{v_0}^{v_1} dv = p_1(v_1 - v_0) = p_1 v_1,$$

nd this amount of work demands an equivalent quantity of eat-energy for transformation into the mechanical form. A ertain additional amount of heat must also, in all cyclical perations, always be transferred, without transformation, and, t the same time, "degraded" in intensity, i.e., in temperature. his latter quantity is determined by the character of the peration of which the cycle is representative.

Still another quantity of heat will be required, for transrmation in performing the internal work of separation of

molecules—the latent heat of expansion,—the method of computation of which quantity has already been considered. As has been seen, the amount of this heat and internal work is unimportant in cyclical operations; since equal amounts are always stored and restored during the cycle.*

In isometric changes in vapors, as with gases, no work is done, and no heat is transferred, except in the production of changes of temperature; for no space is traversed against resistances only to be overcome by transformed energy.

In cases of expansion in real engines, in which the curve may be fairly represented by the equation $pv^n =$ constant the amount of external work done, and the equivalent heat transformed, is thus found:

$$pv^n = p_1v_1^n = p_2v_2^n; \quad U = \int_{v_1}^{v_2} p\,dv; \quad p = p_1\left(\frac{v_1}{v}\right)^n;$$

$$\therefore \; U = p_1v_1^n \int_{v_1}^{v_2} v^{-n}dv$$

$$= \frac{p_1v_1 - p_2v_2}{n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

When the external work of isothermal expansion, p_1v_1, is added, as in the measurement of the total work done during the forward stroke of the steam-engine,

$$U' = p_1v_1 + \int_{v_1}^{v_2} p\,dv = p_1v_1 + \frac{p_1v_1 - p_2v_2}{n-1}$$

$$= p_1v_1\left(\frac{n}{n-1} - \frac{1}{n-1}\cdot r^{1-n}\right) = p_m r v_1; \quad . \quad . \quad (2)$$

---

* Precisely as if molecule were connected to molecule by a system of coiled springs of such tension and range as would produce the observed effects.

here r is the ratio of expansion, and p_m the mean total absolute pressure. Then, for the forward stroke,

$$p_m = p_1 \frac{nr^{-1} - r^{-n}}{n - 1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When $n = 1$, the expressions just given for U, for the external work, become indeterminate; but, for this case,

$\frac{v_2}{v_1} = r$; $pv = p_1 v_1 = p_2 v_2$; and

$$U = \int_{v_1}^{v_2} p dv = p_1 v_1 \int_{v_1}^{v_2} \frac{dv}{v} = p_1 v_1 \log_e r. \quad . \quad . \quad (4)$$

The form of the expression shows, and calculation verifies the conclusion, that, as the value of $\frac{v_1}{v_2} = r$ increases in geometrical ratio, the work of hyperbolic expansion increases in an arithmetical progression.

$r = 2$, $U = 1.693\, p_1 v_1$; $\quad r = 8$, $U = 3.078\, p_1 v_1$;
$r = 4$, $U = 2.386\, p_1 v_1$; $\quad r = 16$, $U = 3.773\, p_1 v_1$.

Thus, we have a constant difference of $0.693\, p_1 v_1$.

Then, as before, for the forward stroke,

$$U = p_1 v_1 (1 + \log_e r). \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The heat demanded for transformation into external work will be the thermal equivalent of these measures of that work, and all heat supplied in excess of these amounts is waste. We now have two typical cases to examine:

The exact expression for the total work done by saturated steam in the steam-engine is obtained thus, to the zero line:

(1) The work of one stroke of the piston of the engine is

measured on the diagram of energy by $a\,a'\,b'\,x'\,O\,a$, the work of isothermal expansion being $a\,a'\,x\,O\,a$, and that of adiabatic

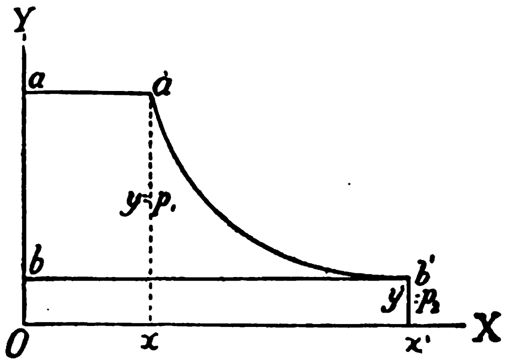


FIG. 139.—VAPOR CYCLE.

expansion $a'\,b'\,x'\,x\,a'$. The total is also composed of these two parts, the first being equal to $b\,b'\,x'\,O$, or

$$U_1 = p_2 u_2 \text{, nearly,}$$

and the second, unity of weight being taken, by $a\,a'\,b'\,b$, or

$$U_2 = \int_{p_2}^{p_1} u\,dp;$$

u being taken as the volume of the steam.

But it has been seen (§ 102) that

$$u = \frac{1}{\frac{dp}{dT}}\left(J \log_e \frac{T_1}{T} + v_1 \frac{dp_1}{dT_1}\right) = v_1 \frac{T}{H}\left(J \log_e \frac{T_1}{T} + \frac{H_1}{T_1}\right);$$

and, hence,

$$U_2 = \int_{p_2}^{p_1} u\,dp = \int_{p_2}^{p_1} dp \cdot \frac{1}{\frac{dp}{dT}}\left(J \log_e \frac{T_1}{T} + v_1 \frac{dp_1}{dT_1}\right)$$

$$= J\left[T_1 - T_2\left(1 + \log_e \frac{T_1}{T_2}\right)\right] + \left(T_1 - T_2\right) v_1 \frac{dp_1}{dT_1}; \qquad (6)$$

nd, since we have found (§ 101)

$$H' = v_1 T_1 \frac{dp_1}{dT_1}$$

:o be the latent heat of evaporation,

$$U_2 = J\left[T_1 - T_2\left(1 + \log_e \frac{T_1}{T_2}\right)\right] + \frac{T_1 - T_2}{T_1} H' . \quad . \quad (7)$$

Reckoned per unit of volume of steam admitted, since

$$\frac{dp}{dT} = \frac{L}{T}, \text{ and the density, } D = \frac{1}{v} \text{ (§§ 98, 99),}$$

$$U_2 = JD_1\left[T_1 - T_2\left(1 + \log_e \frac{T_1}{T_2}\right)\right] + \frac{T_1 - T_2}{T_1} L_1, \quad . \quad (8)$$

and $U_1 + U_2$ measures the gross work; while $U = U_1 + U_2 - U_3$, where U_3 is the negative, back-pressure, work against p_2, measures the *net* work of the ideal indicator-diagram, thus:

$$U_t = U_1 + U_2 - v_2 p_2 = U_2 + v_1(p_1 - p_2). \quad . \quad . \quad (9)$$

(2) The area of the diagram may also be measured up thus: Calling $U_1' = aa'xOa$, $U_2' = a'b'x'xa'$, and also $U_t' = U_1' + U_2'$, the total work performed, down to OX,

$$U_1' = p_1 v_1;$$

$$U_2' = \int_{v_1}^{u_2} p du = U_1' + U_2' - p_1 v_1$$

$$= \int_{p_2}^{p_1} u dp + p_2 u_2 - p_1 v_1$$

$$= J\left[T_1 - T_2\left(1 + \log_e \frac{T_1}{T_2}\right)\right] + \left(T_1 - T_2\right) v_1 \frac{dp_1}{dT_1} + p_2 u_2 - p_1 v_1; \quad (10)$$

and, *for incomplete expansion*, we evidently get

$$U = \overset{\text{I.}}{J[T_1 - T_2(1 + \log_e T_1/T_2)]}$$
$$+ \overset{\text{II.}}{H_1(T_1 - T_2)/T_1} + \overset{\text{III.}}{p_2 - p_3)u_2}.$$

This expression divides itself into three parts: a function of temperature which measures the work of compression in the Carnot vapor-cycle; a term measuring the Carnot energy-transformation, and a term measuring work obtained between terminal and back-pressures. This is illustrated in the figure,

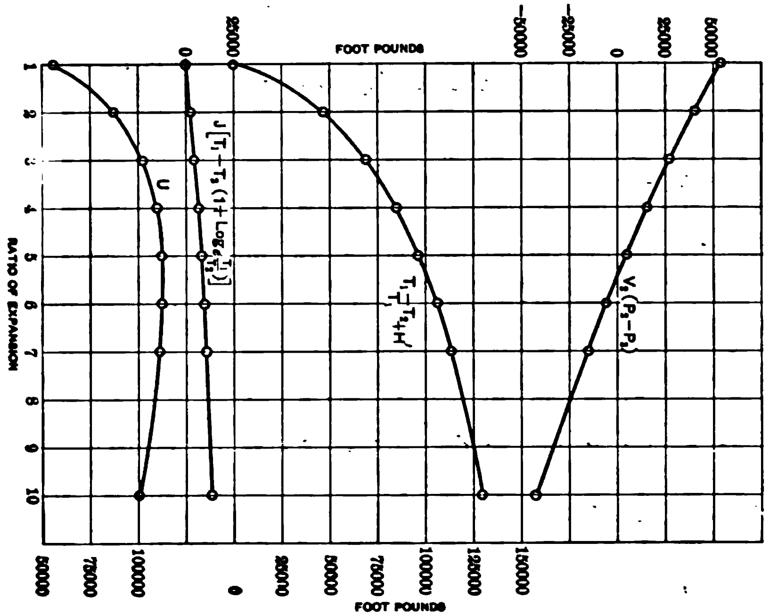


FIG. 139*a*.—DISTRIBUTION OF WORK.

exhibiting value for a non-condensing engine with steam at 100 pounds pressure, by gauge, and with a back-pressure of 2 pounds (114.7 and 16.7 absolute), with expansion varying. The engine is 18 inches diameter, 42 inches stroke of piston.

It is seen that the values of U rise rapidly between ratios of expansion from unity to 3 and 4, become nearly constant to 6 and 7, and then fall off.

The work of adiabatic expansion is the equivalent of the nsible heat stored in the fluid at its entrance into the engine, inus the work of isothermal expansion represented by the oduct of the initial absolute pressure into the volume at e "point of cut-off," a'; increased by the work-equivalent of e sensible heat at the "period of exhaust," less the work presented by the product of pressure and volume at that oint; plus the latent heat at entrance, less the proportion, $\frac{u_2}{v_2}$, f the latent heat of the same weight of vapor at the terminal mperature and pressure. The fraction $\frac{u_2}{v_2}$, or the final volume f the fluid divided by the volume it would occupy if it were ll in the state of dry and saturated steam, is the propor-ion of the initial weight of dry steam which remains nliquefied by expansion; the remainder, $\frac{v_2 - u_2}{v_2}$, being the art which, condensing, surrenders its latent heat for trans-ormation into work.

As is readily seen, the heat-energy stored as latent heat of aporization, in steam, is the principal source of transformed nergy, or work, and the difference between such a vapor and similar fluid taking up no latent heat, could such exist, may e realized on computing the respective quantities demanded or unit of useful power developed. Thus, the steam being sed in the ideal engine and a Carnot cycle, if received at 320° F.—corresponding to 75 lbs. per square inch, by gauge—and ejected into the condenser at 100° F.,

$$E = (T_1 - T_2)/T_1; \quad U = (T_1 - T_2)/T_1 H_1';$$

where U is a function of H_1', not of S.

But the sensible heat added amounts to about 80,000 foot-pounds for such a range of temperature, and the part utilized would be nearly

$$u = 0.28 \times 80,000 = 22,400 \text{ ft. lbs.}$$

It would therefore require the supply of

$$W = 1{,}980{,}000 \div 22{,}400 = 88 \text{ lbs.},$$

nearly, per horse-power and per hour; while, in the actual operation of good engines at such pressures and temperatures, it is not unusual to obtain the same quantity of work with less than one fifth this weight of feed-water supply. The use of steam in a non-conducting and frictionless, the ideal, engine would similarly demand but about one tenth the above-computed quantity.

In the case of steam-engines working, as assumed in the analysis, without compression, or where compression is neglected, the efficiency must evidently be less than if the compression be adiabatic and complete, as in the Carnot cycle. The maximum efficiency of fluid is thus reduced, in many cases, quite sensibly, and may be considerably diminished. The difference is that between the Carnot ratio and that ratio diminished by the quotient of work of adiabatic compression to whole heat supplied. For the Carnot cycles, the efficiency is

$$E = \frac{T_1 - T_2}{T_2} = \frac{H_1 - H_2}{H_2};$$

and for the assumed case,

$$E = \frac{H_1 - H_2}{H_1}$$

$$= \frac{T_1 - T_2}{T_1} - \frac{JT_2\left(\log_e \frac{T_1}{T_2} - \frac{T_1 - T_2}{T_1}\right)}{H + J(T_1 - T_2)},$$

nearly; in which latter expression H is the latent heat of evaporation.

For other vapors than steam, Jc must, of course, be substituted for J. In the case of steam this loss is usually very small, rarely amounting to an approximation to one per cent.

Adiabatic Expansion produces the liquefaction of steam, initially dry and saturated, in the proportion (§ 102, eq. 9)

$$m_e = 1 - \frac{T_2}{H_2}\left(J \log_e \frac{T_1}{T_2} + \frac{H_1}{T_1}\right).$$

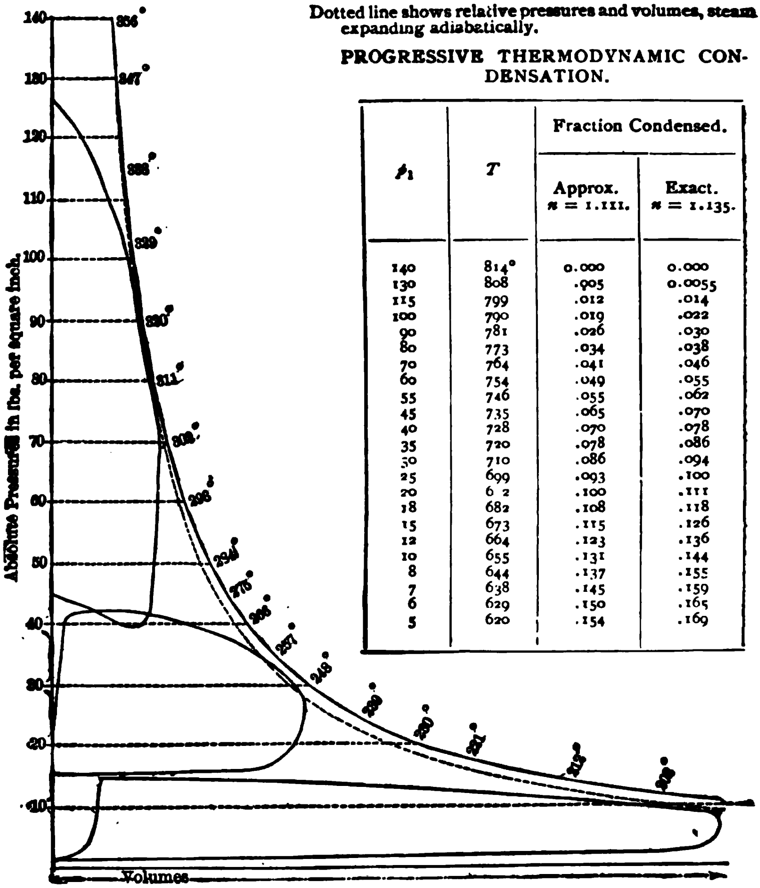
This proportion, though small in the older types of engine, with their comparatively low pressures and small ratios of expansion, becomes important in later engines with pressures ranging up toward 10 or 12 atmospheres, and ratios of expansion of 15 to 20 or more. Thus, comparing steam in the ideal non-conducting cylinder, at *absolute* pressures of 115 and of 165 pounds per square inch, as employed in modern compound and triple-expansion engines, we have, in the first case, if expanding to 8 pounds terminal pressure, over 14 per cent adiabatic condensation; in the latter, we have 17 per cent, nearly; one pound of the mixture giving $x = 0.86$ and $x = 0.83$, nearly, remanent steam. The heat utilized is, in these cases, respectively, 0.17 and 0.20, nearly—a thermodynamic gain of about 18 per cent. Raising initial pressure to 220 pounds, as in some quadruple-expansion engines, the thermodynamic gain is an additional 10 per cent, and at 250 pounds, absolute pressure, 15 per cent. This happens to correspond closely with experience, with the real engine.

The result is well shown by the illustration given on the next page, from a paper by Mr. Parker.* The diagram shows the method of expansion of steam at an absolute pressure of 140 pounds per square inch ($9\frac{1}{2}$ atmos.); (*a*) when kept dry and saturated; (*b*) when expanding adiabatically; and (*c*) as actually worked in the steam-cylinders of the S. S. "Aberdeen," designed by Mr. Kirk for the China trade, an example of exceptional economy.† It is seen that the actual expansion-line was

* Economy of Compound Engines; Trans. Brit. Inst. N. A., 1882. Thurston's Engine and Boiler Trials; p. 459.

† Engines: 30-, 45-, and 70-inch cylinders, the firs unjacketed; $4\frac{1}{2}$ feet stroke of piston. Steam 125 lbs. by gauge, in boiler; 125, 50, and 15 lbs. in jackets. H. P. 1800; fuel per h. p. per hr., 1.28 lbs.

bounded very closely by the adiabatic line, thus showing the internal condensation to be variable in a manner similar to that in a non-conducting cylinder. The jacket-wastes, however, amounting to about 4 per cent, must be added to the quantity



Dotted line shows relative pressures and volumes, steam expanding adiabatically.

PROGRESSIVE THERMODYNAMIC CONDENSATION.

p_1	T	Fraction Condensed. Approx. $n = 1.111$.	Fraction Condensed. Exact. $n = 1.135$.
140	814°	0.000	0.000
130	808	.005	0.0055
115	799	.012	.014
100	790	.019	.022
90	781	.026	.030
80	773	.034	.038
70	764	.041	.046
60	754	.049	.055
55	746	.055	.062
45	735	.065	.070
40	728	.070	.078
35	720	.078	.086
30	710	.086	.094
25	699	.093	.100
20	6 2	.100	.111
18	682	.108	.118
15	673	.115	.126
12	664	.123	.136
10	655	.131	.144
8	644	.137	.155
7	638	.145	.159
6	629	.150	.165
5	620	.154	.169

FIG. 140.—ECONOMY OF STEAM.

of steam here shown. The table accompanying the diagram exhibits the computed adiabatic condensation for the full range of expansion, varying from 0, at the start, to 14.7 per cent at the end. For such engines, with the progress of expansion,

would become, on this scale, about 4 per cent for steam at pounds absolute pressure, 6 at 47, 7½ at 35, 10 at 23, 11 at 12 at 14, and 14 per cent at 8½; or at ratios of expansion, pectively, of 2, 3, 4, 6, 8, 10, and 15.

The wastes due to action of valves, the loss in passages, d to maladjustment of the several parts of the system to ch other, are seen, as in other cases to be presented, in the riation of the real diagrams from their respective portions of e ideal curve. These wastes may be reduced somewhat by proved design and construction; but, on the whole, they crease with higher pressures and greater expansion, and thus aggerate the difficulties of securing higher economy. All ese points being considered, the gain by still higher pressure seen to be comparatively small. As will be seen later, this ndensation is cumulative in the compound engine, and can-t be reduced by arranging several cylinders in series. Insig-icant in a single cylinder, it becomes, as just seen, quite ge with the high values now usual for the total ratio of ex-nsion in such engines.

For comparison with the methods of Rankine, who prefers e computation of dynamic energies, the system of Clausius, iose method preferably considers thermal quantities, may be ken, as illustrated by the following summary of the discus-on of the ideal steam-engine cycle of Carnot:*

Let $W =$ the weight of fluid taken;
$l =$ " latent heat;
$x_a x_b x_c =$ " proportion of dry steam present;
$T =$ absolute temperature;
$t =$ any temperature;
$Q =$ quantity of heat.

The "thermodynamic function" of Rankine, ϕ, or the easure of Clausius' "entropy," has been obtained thus:

$$d\phi = \frac{dQ}{T}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

* For a complete exposition of Clausius' system, consult Peabody's "Thermodynamics of the Steam-engine;" N. Y., J. Wiley & Sons; 1889.

when Q is the measure of heat transferred, in thermal units.

When we heat water from the minimum 0° to any maximum temperature, t, in the steam-boiler, the only change noted is that of temperature and we have the change of entropy:

$$\phi' = \int_0^{t_1} \frac{dq}{T} = \int_0^{t_1} \frac{cdt}{T}; \quad . \quad . \quad . \quad . \quad (15)$$

but in vaporization, $dt = 0$,

$$\phi'' = \frac{xl}{T};$$

and the total change is measured by

$$\phi = \phi' + \phi'' = \frac{xl}{T} + \int_0^{t} \frac{cdt}{T}; \quad . \quad . \quad . \quad . \quad (16)$$

in adiabatic expansion, $d\phi = 0$, and

$$\frac{xl}{T} + \int_0^{t} \frac{cdt}{T} = \text{constant.}$$

Then, in the four operations constituting an ideal engine cycle, we have, for the case of maximum efficiency:

(A) Expansion of water into steam at constant temperature and pressure, this action occurring in the boiler; the heat demanded is

$$Q_1 = Wl_1 (x_b - x_a). \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

(B) Adiabatic expansion to back-pressure; no heat being gained or lost:

$$\frac{xl}{T_1} + \int_0^{t_1} \frac{cdt}{T} = \frac{x_2 l_2}{T_2} + \int_0^{t_2} \frac{cdt}{T}.$$

(C) Compression at constant pressure and temperature corresponding to the back-pressure, in which operation heat is rejected to the amount

$$Q_2 = Wl_2 (x_c - x_d). \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

(*D*) Adiabatic compression:

$$\frac{x_a l_1}{T_1} + \int_0^{t_1} \frac{c\,dt}{T} = \frac{x_d l_2}{T_2} + \int_0^{t_2} \frac{c\,dt}{T}. \quad . \quad . \quad . \quad (19)$$

We have, from the above,

$$x_c - x_d = \frac{T_2 l_1}{T_1 l_2}(x_b - x_a),$$

whence

$$Q_2 = Wl_1 \frac{T_2}{T_1}(x_b - x_a). \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The efficiency thus becomes, in accordance with Carnot's law:

$$\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

When, as usually assumed, $x_a = 0$, and $x_b = 1$, the work done is

$$U = JWl_1 \cdot \frac{T_1 - T_2}{T_1};$$

and the weight of fluid, for a given work,

$$W = \frac{U}{Jl_1} \div \frac{T_1 - T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The quantity of heat demanded is measured, in units of work, by

$$H_1 = \frac{T_1}{T_1 - T_2} \cdot U. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The weight of water required is thus

$$W = \frac{H_1}{Jl_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

and, per horse-power and per hour, this becomes

$$W = \frac{33000 \times 60}{Jp_1} \cdot \frac{T_1}{T_1 - T_2} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Professor Peabody computes, on the assumption $J = 778$, ideal-engine efficiencies as follow:

WEIGHTS OF STEAM DEMANDED. IDEAL CASE.

	Condensing.		Non-condensing. $p_2 = 1.5$	
p_1 (by gauge)	$\frac{T_1 - T_2}{T_1}$	M lbs.	$\frac{T_1 - T_2}{T}$	M lbs.
30	0.215	12.8	0.084	32.8
60	0.249	11.4	0.124	22.9
100	0.278	10.5	0.157	18.4
150	0.303	9.8	0.186	16.0
200	0.320	9.5	0.207	14.6
300	0.347	9.0	0.238	13.1

It can now be seen that, as already stated (§ 93), the magnitude of the quantity Q above, which is a measure, here, of the latent heat of vaporization, determines the amount of energy which can be transformed per unit of weight, and that the best working fluid, in this respect, is that having most energy thus stored; while the thermodynamic efficiency is entirely independent of Q or H and a function, solely, of temperature.*

113. **Dry, Saturated, Steam** is often assumed to be obtainable, and to be capable of being worked without condensation, the steam being kept at the point of saturation by heat supplied by a steam-jacket. The fact that dry steam, or other vapor, like gas, is a good non-conductor and non-absorber of heat makes it improbable that superheating can ever be produced, to any sensible extent at least, by the use of the jacket.

---

* See Appendix for all computation tables.

For this case, we have found, following Rankine (§ 112),

$$v = \frac{H'}{T\frac{dp}{dT}},$$

in which the latent heat of evaporation, H', may be expressed conveniently by the expression, derived from Regnault,

$$H' = a - bT, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which, in British measures, $a = 1,109,550$ foot-pounds, and $b = 540.4$, for $J = 772$; or 1,117,850 and 544.5, for $J = 778$.

$$U_2 = \int_{p_2}^{p_1} vdp = \int_{T_2}^{T_1} \left(\frac{a}{T} - b\right) dT$$

$$= a \log_e \frac{T_1}{T_2} - b(T_1 - T_2), \quad . \quad . \quad . \quad . \quad (2)$$

and, adding $U_1 = p_1 v_1$,

$$U = U_1 + U_2 = \int_{p_2}^{p_1} vdp + v_1 p_1$$

$$= a \log_e \frac{T_1}{T_2} - b(T_1 - T_2) + p_1 v_1. \quad . \quad (3)$$

The *net* work done is measured by the value of U, as above, less the work of back-pressure on the opposite side of the piston, resisting its advance, which work is

$$U_3 = v_2 p_3, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

when p_3 is the total back-pressure, and

$$U_n = U_1 + U_2 - U_3. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus the net work done, when the expansion is adiabatic, is, per unit of weight, as has been seen (p. 438),

$$U_n = J\left[T_1 - T_2\left(1 + \log_e \frac{T_1}{T_2}\right)\right] + \frac{T_1 - T_2}{T_1} H' + v_1(p_1 - p_3),$$

and, as now shown, for saturated steam, in a jacketed engine,

$$U_n = a \log_e \frac{T_1}{T_2} - b(T_1 - T_2) + v_2(p_2 - p_3); \quad . \quad . \quad (6)$$

in which expressions the pressure and corresponding temperatures are either known or may be obtained from the steam-tables.

In all cases, the total heat demanded is that required to raise all the water, used in cylinder and jacket, from the temperature at which it is received into the boiler up to that of evaporation, and to produce from it steam of the temperature and pressure, T_1, p_1, at which its expansion in the steam-cylinder begins. The heat transformed into mechanical work is always measured by the work performed, as shown by the indicator-diagram, and the difference between the total amount of heat expended and the thermal equivalent of the net work done, as thus measured by the area of the diagram exhibiting the cycle worked through, is discharged from the system as unutilized heat.

In this second typical ideal case, the steam is assumed to be maintained in the dry and saturated condition by continually supplying to it, as it expands, so much heat from the jacket as will prevent that liquefaction which would take place in the course of that adiabatic expansion which would occur in a non-conducting cylinder. Since this involves the supply of heat at all temperatures intermediate between that of the "prime" steam and that of exhaust and back-pressure, the efficiency of heat so supplied must be less than that of the heat entering with the boiler-steam. This method is therefore a method of waste of steam and of heat, as will be shown, more fully, later, by computation. This introduction of a wasteful expedient will, however, be seen, in the real engine, to have for its purpose the reduction of a greater waste; and the net result is usually found to be a sensible, and often an important, gain. More heat is supplied than in the first typical case, and more work is done, per pound of steam; but the work is increased in less proportion than the heat-supply.

114. **The Efficiency of Cyclical Operations** is evidently always measured by the ratio of the net work done by the working fluid to the work-equivalent of the total heat-energy sent into the engine, and either transformed or simply transferred with reduction of temperature. To determine this efficiency, therefore, it is only necessary to find a method of measuring the total quantity, H, of heat supplied, and the net work, U_n performed by the fluid, and the efficiency is then

$$E = \frac{U_n}{H}.$$

The total heat supplied to steam, dry and saturated, per unit of weight, is given in the "steam-tables." When superheated, additional heat of the amount

$$\begin{aligned} H &= K(T_s - T_1)w \\ &= 0.48\,(T_s - T_1)w \end{aligned}$$

is demanded, $T_s - T_1$ being the range of temperature added by superheating. The work performed is, in practice, obtained by the use of the "steam-engine indicator," and is measured by the area of its diagram. Dividing the work represented by the latter, as performed in the unit of time, by the mechanical equivalent of the total heat supplied to the steam passing through the engine in the same time, the efficiency is obtained.

In the ideal steam-engine, this usually varies, under familiar practical conditions as to temperature and pressure, from ten to about twenty per cent, and, in real engines, from about fifteen per cent down to five, or even less; the difference being due to the wastes which have been described.

The equations obtained on the assumption that $pv^n =$ const., using $n = 1.0646$ for saturated steam, $n = 1.135$ for adiabatic expansion of steam initially dry, and $n = 1.333$ for superheated steam, or steam gas, give fairly approximate results, as compared with the exact expressions just given.

The condensation of steam expanding adiabatically may be neglected at low ratios of expansion; but it becomes very considerable, as shown elsewhere, at large ratios, and the jacket must, in such cases, supply large amounts of heat. The assumption here made as to the effective operation of the jacket may be taken to be that of nearly maximum value. In the compound engine, this condensation is cumulative, and is not reduced or affected by the action which distinguished that type and gives it its efficiency in the case of the real engine.

Could the action of the jacket be made effective in the manner here assumed, and not a source of waste during the exhaust period, the ideal and the real engine would have a sensibly common efficiency. Experience indicates that a jacket of such effective action as to produce dry and saturated steam at the end of the expansion-period actually does approximate most closely to the ideal; but in any given case, this can only occur under very nicely adjusted and unstable conditions.

115. **A Standard of Efficiency** by which to measure the thermodynamic value of the steam-engine and other heat-motors, in their exceedingly various types and forms, is constantly required by the engineer, and the general adoption of a common and correct standard is one of the desirable conventions in all thermodynamic work. A number of standards, in themselves accurate and scientifically available and acceptable, have been proposed. and the question to be settled by common consent is: What one of the various possible standards shall be adopted?

The Essentials of a Satisfactory Standard are:

(1) Ideal perfection and accuracy; (2) Invariability under the conditions of its employment; (3) Convenience; (4) Special suitability to the values it is to measure.

See Chapter VIII for detailed illustration.

The Conditions Characterizing Ideal Cycles of the type-forms are as follow: In the diagram, let $ABCDO$ measure the work performed, as a maximum, without clearance, compression, or back-pressure. Let GF, KC represent back-pressure lines and AE a compression-line, where the charge is restored to an initial and stated volume. Then the three type-forms of cycle usually discussed are $ABCEA$, that of Carnot; $ABCKA$, that of Clausius; and $ABCFGA$, that of Rankine.

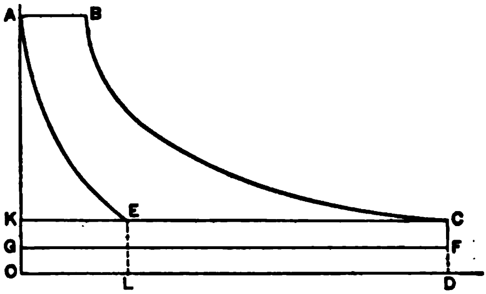


FIG. 1408.—THE TYPICAL STEAM-ENGINE CYCLES.

For these ideal cycles, the common set of conditions of maximum efficiency is that which makes the ratio of area of the diagram to the stored energy of the fluid supplied in its production a maximum.

The theory of these ideal cycles and its application is developed in the following sections and in the Appendix, p. 995 *et seq.*, and Chapter VIII.

Where, as is often the case, a number of possible standards are available, all having a common measure and fixed relations among themselves, that one will be chosen which presents the best combination of precise measures for use in the class of problems to which it is to be applied.

Each standard, absolute or relative, finds its appropriate place and purpose. The computed quantities in the tables in the Appendix, affording standards for special cases under each class, give measures of the efficiencies which the engineer

must accept as limits toward which he may approach as he improves his engines, but which he can never fully attain.

116. The Theory of the Efficiency of Ideal Engines applying steam or other vapor as the working fluid is simple and exact; but the results obtained in this case differ, usually, very widely from those practically reached in the real engine of which it is the representative. These differences are considered elsewhere; in the present article the ideal case, only, is to be illustrated.

The quantity of heat, H_1, being received by the engine, and the amount, H_2, emitted, the difference, $H_1 - H_2$, is converted into work. The efficiency is, therefore,

$$E = \frac{H_1 - H_2}{H_1}.$$

Following Rankine's method of treatment, we have (§ 112), *en résumé*, when

$\frac{u_2}{v_1} = r =$ ratio of expansion;

$p_1, p_2, p_3 =$ initial, terminal, and back pressures, absolute;

$T_1, T_2, T_3, T_4, T_5, T_6 =$ temperatures of entering steam, of steam at the end of adiabatic expansion and during the return stroke, and of feed-water, of condensation, and of atmosphere, respectively;

and when the work of unity of volume is considered,

$$r = \frac{u_2}{v_1} = \frac{T_2}{L_2}\left(JD_1 \log_e \frac{T_1}{T_2} + \frac{L_1}{T_1}\right), \quad . \quad . \quad . \quad . \quad (1)$$

for the ratio of expansion;

$$UD_1 = JD_1\left[T_1 - T_2\left(1 + \log_e \frac{T_1}{T_2}\right)\right]$$

$$+ \frac{T_1 - T_2}{T_1} L_1 + r(p_2 - p_3), \quad . \quad (2)$$

the work of the fluid per cubic foot when U is work per pound;

$$\frac{UD_1}{r} = p_m - p_s = p_e \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

is the mean effective pressure; which also measures the work done per unit of volume swept through by the piston;

$$\frac{H_1D_1}{r} = [JD_1(T_1 - T_4) + L_1] \div r, \quad . \quad . \quad . \quad (4)$$

the heat expended per unit of that volume; and the efficiency of the fluid becomes

$$E = \frac{U}{H_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The quantity of feed-water demanded will be measured by D_1; or, per unit of volume of cylinder,

$$W = \frac{D_1}{r}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The heat emitted will be

$$\frac{H_2D_1}{r} = \frac{(H_1 - U)D_1}{r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

per unit of volume of cylinder, and the volume swept over by the piston, per minute, per horse-power, must be equal to

$$\left.\begin{aligned} V &= \frac{33{,}000}{p_e} = \frac{33{,}000\,r}{UD_1}, \\ V_m &= \frac{4500}{p_e} = \frac{4500\,r}{U_mD_{1m}}, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

in British and metric measures, respectively, the heat expended per hour being

$$\left.\begin{aligned} H_t &= 1{,}980{,}000\,\frac{H_1}{U} = \frac{1{,}980{,}000}{E}, \\ H_{mt} &= 270{,}000\,\frac{H_m}{U_m} = \frac{270{,}000}{E}, \end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

in the two measures, respectively.

Also, adopting the approximate formulas for this case (§ 102), much simpler expressions become available, thus:

$$r = v_2 \div v_1; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$p_2 = pr^{-\frac{10}{9}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$p_m = p_1(10r^{-1} - 9r^{-\frac{10}{9}}); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$p_e = p_1(10r^{-1} - 9r^{-\frac{10}{9}}) - p_3; \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and the work per unit of steam admitted is

$$rp_e = p_1(10 - 9r^{-\frac{1}{9}}) - rp_3, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where Rankine takes $n = 1.111$, although, accurately, $n = 1.135$ (§ 102, p. 398).

Rankine has also shown that the heat demanded, in foot-lbs. per cubic foot of cylinder, may be taken as

$$H = \frac{H_1 D_1}{r} = \frac{13.333p_1 + 4000}{r}, \text{ nearly};$$

while the pressure which, acting through the volume of the cylinder, or the equivalent "heat-pressure," which would do the same amount of work, is, in pounds per square inch,

$$p_h = \frac{13.333p_1 + 27.5}{r}, \text{ nearly.}$$

The Efficiency of Steam kept Dry and Saturated, expanding in a cylinder, permeable to heat, and receiving just sufficient from any source, as a "steam-jacket," to keep the fluid in that condition, is computed, as already shown, in part, thus (§ 113):

$$r = \frac{v_2}{v_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$\text{Work} = U' = a \log_e \frac{T_1}{T_2} - b(T_1 - T_2) + v_2(p_2 - p_3). \quad . \quad (16)$$

The heat expended per unit weight of steam, in the cylinder,

$$H' = J(T_2 - T_4) + a - bT_2 + \int_{p_2}^{p_1} vdp, \text{ nearly,*}$$

$$= J(T_2 - T_4) + a\left(1 + \log_e \frac{T_1}{T_2}\right) - bT_1, \quad . \quad . \quad . \quad (17)$$

and, per unit of volume of piston-path, $\frac{H'}{v_2}$.

The values of p_1 and T_1 are obtained from Regnault's experiments, and are given in all "steam-tables."

$$p_e = \frac{U'}{rv_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$p_h = \frac{H'}{rv_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$E = \frac{U'}{H'} = \frac{p_m - p_3}{p_h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Or, adopting the approximate formulas,

$$pv^{\frac{17}{16}} = \text{constant}; \quad r = \frac{v_2}{v_1}; \quad p_2 = p_1 r^{-\frac{17}{16}}; \quad . \quad . \quad . \quad . \quad (21)$$

$$U' = [p_1(17r^{-1} - 16r^{-\frac{17}{16}}) - p_3]v_2; \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$p_m = p_1(17r^{-1} - 16r^{-\frac{17}{16}}); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$p_e = p_m - p_3 = \frac{U'}{v_2} = p_1(17r^{-1} - 16r^{-\frac{17}{16}}) - p_3. \quad . \quad (24)$$

* Accurately the last term should read

$$\int_{p_2}^{p_1} vdp + (p_2 - p_3)v_2 - p_1 v_1.$$

must accept as limits toward which he may approach as he improves his engines, but which he can never fully attain.

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in British and metric measures, respectively, the heat expended per hour being

$$\left.\begin{aligned} H_t &= 1,980,000\frac{H_1}{U} = \frac{1,980,000}{E}, \\ H_{mt} &= 270,000\frac{H_m}{U_m} = \frac{270,000}{E}, \end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

in the two measures, respectively.

Also, adopting the approximate formulas for this case (§ 102), much simpler expressions become available, thus:

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$$p_e = p_1(10 r^{-1} - 9 r^{-\frac{10}{9}}) - p_3 ; \quad . \quad . \quad . \quad . \quad (13)$$

and the work per unit of steam admitted is

$$rp_e = p_1(10 - 9 r^{-\frac{1}{9}}) - rp_3 , \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where Rankine takes $n = 1.111$, although, accurately, $n = 1.135$ (§ 102, p. 398).

Rankine has also shown that the heat demanded, in foot-lbs. per cubic foot of cylinder, may be taken as

$$H = \frac{H_1 D_1}{r} = \frac{13.33\dot{3}p_1 + 4000}{r}, \text{ nearly};$$

while the pressure which, acting through the volume of the cylinder, or the equivalent "heat-pressure," which would do the same amount of work, is, in pounds per square inch,

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The Efficiency of Steam kept Dry and Saturated, expanding in a cylinder, permeable to heat, and receiving just sufficient from any source, as a "steam-jacket," to keep the fluid in that condition, is computed, as already shown, in part, thus (§ 113):

$$r = \frac{v_2}{v_1} ; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

$$\text{Work} = U' = a \log_e \frac{T_1}{T_2} - b(T_1 - T_2) + v_2(p_2 - p_3) . \quad (16)$$

The heat expended per unit weight of steam, in the cylinder,

$$H' = J(T_2 - T_4) + a - bT_2 + \int_{p_2}^{p_1} v\,dp, \text{ nearly,*}$$

$$= J(T_2 - T_4) + a\left(1 + \log_e \frac{T_1}{T_2}\right) - bT_1, \quad . \quad . \quad . \quad (17)$$

and, per unit of volume of piston-path, $\frac{H'}{v_2}$.

The values of p_2 and T_2 are obtained from Regnault's experiments, and are given in all "steam-tables."

$$p_e = \frac{U'}{rv_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$p_h = \frac{H'}{rv_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$E = \frac{U'}{H'} = \frac{p_m - p_3}{p_h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Or, adopting the approximate formulas,

$$pv^{\frac{10}{9}} = \text{constant}; \quad r = \frac{v_2}{v_1}; \quad p_2 = p_1 r^{-\frac{10}{9}}; \quad . \quad . \quad . \quad . \quad (21)$$

$$U' = [p_1(17r^{-1} - 16r^{-\frac{10}{9}}) - p_3]v_2; \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$p_m = p_1(17r^{-1} - 16r^{-\frac{10}{9}}); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$p_e = p_m - p_3 = \frac{U'}{v_2} = p_1(17r^{-1} - 16r^{-\frac{10}{9}}) - p_3. \quad . \quad (24)$$

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* Accurately the last term should read

$$\int_{p_2}^{p_1} v\,dp + (p_2 - p_3)v_2 - p_1 v_1.$$

It is found that the heat demanded per pound of steam supplied is very nearly

$$H' = 15\tfrac{1}{2}p_1v_1 = 15\tfrac{1}{2}\frac{p_1v_2}{r}. \quad . \quad . \quad . \quad . \quad (25)$$

Then the efficiency

$$E = \frac{U'}{H'} = \frac{p_e}{p_h} = \frac{17 - 16r^{-\frac{1}{16}}}{15\tfrac{1}{2}} - \frac{rp_3}{15\tfrac{1}{2}p_1}. \quad . \quad . \quad (26)$$

Table III, in the Appendix, shows the values of the "cut-off," $\frac{1}{r}$, the ratio $p_1 \div rp_m$, of total work performed up to point of cut-off to the total work (inclusive of that below the back-pressure line) done at each stroke, the reciprocal, $rp_m \div p_1$, of that ratio, and the ratios, $\frac{p_1}{p_m}$ and $\frac{p_m}{p_1}$, for assumed values of r, adopting the values of n taken above in the approximate equations.

117. **Examples of Application** of principles and the theory to ideal cases of application of steam, illustrating the limit of efficiency which would be attainable at familiar pressures, could all wastes by conduction, radiation, and leakage be entirely prevented by the use of a working cylinder of non-conducting material, are the following (see Notes):

(1) Assume one cu. foot of steam, at an absolute pressure of 100 pounds per square inch, to expand adiabatically, in an engine-cylinder of perfectly non-conducting material, down to 25 pounds, and to be exhausted, on the return-stroke, into the atmosphere, the back-pressure being 15 pounds per square inch. It is desired to find the ratio of expansion, the efficiency of the fluid, and the weight of steam and of fuel demanded, per horse-power per hour, the feed-water being supplied at 110° Fahr., and the evaporation 9 pounds per pound of coal.

By the exact formulæ for this case (§ 112), the following figures are obtained (see eqs. 7, 8, 9):

DATA.

$p_1 =$	14,400 lbs. per sq. ft.;	$T_1 =$	788°.9;
$P_1 =$	100 lbs. per sq. in.;	$T_2 =$	701.7;
$p_2 =$	3,600;	$L_1 =$	157,145 ft.-lbs.;
$P_2 =$	25;	$L_2 =$	45,680 " ;
$p_3 =$	2,160;	$D_1 =$	0.2305;
$P_3 =$	15;	$D_2 =$	0.06256.

Evaporation, 9 lbs. water per lb. coal.

RESULTS.

Ratio of Expansion (eq. 1, § 116):

$$r = \frac{T_2}{L_2}\left(JD_1 \log_e \frac{T_1}{T_2} + \frac{L_1}{T_1}\right);$$

$$= \frac{701.7}{45,680}\left(772 \times 0.2305 \times 2.3 \log_{10} \frac{788.9}{701.7} + \frac{157,145}{788.9}\right);$$

$$= 3.38.$$

Work per cubic foot of Steam admitted (eq. 2):

$$U = JD_1\left[T_1 - T_2\left(1 + \log_e \frac{T_1}{T_2}\right)\right] + \frac{T_1 - T_2}{T_1}L_1 + r(p_2 - p_3);$$

taking the data, as given above,

$$= 22,547 \text{ foot-lbs.}$$

Mean Effective Pressure (eq. 3):

$$p_e = \frac{U}{r} = \frac{22,547}{3.38} = 6671 \text{ lbs. per sq. ft.};$$

$$= 46.31 \text{ lbs. per square inch.}$$

Heat expended per cubic foot of Steam admitted:

$$H = JD_1(T_1 - T_4) + L_1;$$

$$= 772 \times 0.2305\,(788.9 - 571.2) + 157.145;$$

$$= 195,884 \text{ foot-lbs.}$$

Heat per cubic foot of Cylinder and equivalent Heat-pressure:

$$H' = \frac{H_1 D_1}{r} = p_h = \frac{195,884}{3.38} = 57,954 \text{ foot-lbs. (eq. 4);}$$

$$= 57,954 \text{ lbs. per sq. ft.} = 402.4 \text{ lbs. per sq. inch.}$$

Efficiency of the Steam (eq. 5):

$$E = \frac{p_e}{p_h} = \frac{rp_e}{H_1 D_1} = \frac{46.31}{402.4} = 0.115.$$

Feed-water per cubic foot of Cylinder per stroke (eq. 6):

$$W = \frac{D_1}{r} = \frac{0.2305}{3.38} = 0.0682 \text{ lbs.}$$

Volume swept through by the Piston per indicated horse-power per hour (eq. 8):

$$V = \frac{60 \times 33,000}{p} = \frac{1,980,000}{6671} = 296.8 \text{ cu. ft.}$$

Weight of Feed-water and of Steam per I. H. P. per hour:

$$W' = 0.0682 \times 296.8 = 20.34 \text{ pounds or } 22,110 \text{ B. T. U.}$$

Fuel per I. H. P. per hour:

$$W'' = 20.34 \div 9 = 2.26 \text{ lbs.}$$

(2) The same case by the approximate formulas:—

Ratio of Expansion and "Cut-off":

$$r = 3.38; \quad \frac{1}{r} = 0.296.$$

*Mean Total Pressure:**

$$p_m = 100 \times \frac{p_m}{p_1} = 100 \times 0.634 = 63.4 \text{ lbs. per sq. inch.}$$

* See table for $\frac{p_m}{p_1}$, and interpolate.

Mean Effective Pressure:

$$p_e = p_m - p_s = 63.4 - 15 = 48.4 \text{ lbs. per sq. inch.}$$

Same by exact formula = 46.3; difference = 2.1.

Pressure equivalent to Heat expended:

$$p_h = \frac{13\frac{1}{8} \times 27.7}{r} = \frac{1361.11}{3.38} = 402.7 \text{ lbs. per sq. inch.}$$

Same by exact formula = 402.4; difference 0.3.

Efficiency:

$$E = \frac{p_e}{p_h} = 0.120.$$

Same by exact formula = 0.115; difference 0.005.

Feed-water and Steam per cubic foot of Cylinder, and per stroke:

$$W = \frac{D_1}{r} = \frac{0.2305}{3.35} = 0.0682 \text{ lb.}$$

Volume swept through by piston per I. H. P. per hour:

$$V = \frac{60 \times 33{,}000}{48.4 \times 144} = \frac{1{,}980{,}000}{6069.6} = 284 \text{ cu. ft.}$$

Feed-water and dry Steam per I. H. P. per hour:

$$W' = 0.0682 \times 284 = 19.37 \text{ lbs. or } 21{,}208 \text{ B. T. U.}$$

Same by exact formula = 20.34; difference 0.97 lb., or five per cent.

Fuel per I. H. P. per hour:

$$W'' = 19.37 \div 9 = 2.15 \text{ lbs. or } 22{,}130 \text{ B. T. U.}$$

(3) Assume one pound of dry, saturated steam to expand in a jacketed cylinder, receiving just sufficient heat from the jacket to prevent condensation by doing work. To find the

efficiency, etc., as before, when the data are as follows, the method being slightly different from the preceding:

$p_1 = 14,400;$ $v_1 = 4.35$ cu. ft. (by table);
$P_1 = 100;$ $v_2 = 37.83$ cu. ft.;
$p_2 = 1440;$ $W' \div W'' = 9;$
$P_2 = 9;$ Feed-water, 122° F.
$p_3 = 720;$
$P_3 = 5.$ (See Notes, p. 999.)

Ratio of Expansion:

$$r = \frac{37.83}{4.35} = 8.7.$$

Work per pound of Steam admitted (eq. 5, p. 445):

$$U = U_1 - U_2 = 161,826 \text{ ft.-lbs.}$$

Mean Effective Pressure:

$$p_e = p_m - p_3 = \frac{U}{rv_1} = \frac{161,826}{37.83}$$
$$= 4277.7 \text{ lbs. per sq. ft.} = 29.7 \text{ lbs. per sq. inch.}$$

Available Heat:

$$\begin{aligned} H' &= U + H_2 - h_4; \text{ (eq. 17, p. 453;)} \\ &= 144,588 + 880,756 - 69,522; \\ &= 945,822 \text{ ft.-lbs.} \end{aligned}$$

"Heat-pressure":

$$p_h = \frac{H'}{rv_1} = \frac{945,822}{37.83} = 25,502 \text{ lbs. per sq. ft., } 173.6 \text{ per sq. in.}$$

Efficiency: $E = p_e/p_h = 0.1705.$
Steam per I. H. P. per hour, for $E = 1$:

$$\frac{1,980,000}{H_1 - h_4} = \frac{1,980,000}{851,132} = 2.3.$$

The working Steam per cubic foot traversed by piston:

$$W = \frac{1}{rv_1} = \frac{D_1}{r} = \frac{1}{37.83} = 0.0264 \text{ lb.}$$

Volume traversed by piston per I. H. P. per hour:

$$V = \frac{60 \times 33{,}000}{29.7 \times 144} = 463 \text{ cu. ft.}$$

The working Steam per I. H. P. per hour is

$$W' = 0.0264 \times 463 = 12.22 \text{ lbs.}$$

Total Steam and Fuel:

$$W' = \frac{2.3}{\text{effic.}} = 13.5, \text{ including jacket-water; } W'' = \frac{13.5}{9} = 1.5 \text{ lbs.}$$

(4) Same case by approximate formulas (§ 112):

$$r = 8.7; \quad \frac{1}{r} = 0.115;$$

$$p_m = p_1 \times 0.35 = 100 \times 0.35 = 35 \text{ lbs. per sq. inch;}$$

$$p_e = p_m - p_s = 35 - 5 = 30 \text{ lbs. per sq. inch;}$$

$$p_k = \frac{15\frac{1}{2}p_1}{r} = \frac{15\frac{1}{2} \times 100}{8.7} = 178.25;$$

$$E = \frac{30}{178.25} = 0.168.$$

Cubic feet traversed per hour per I. H. P. $= \dfrac{33{,}000 \times 60}{30 \times 144}$ $= 458$.

The working Steam per I. H. P. per hour $= 458 \times 0.0264$ $= 12.09$. Total, with jacket-water, 13.4 lbs.

Fuel per I. H. P. per hour = 13.4 ÷ 9 = 1.5 ; 15,150 B. T. U.

The difference between the total and working steam, the jacket-consumptions, is seen to be about ten per cent.

(5) Assume the following data, from Rankine, as taken from an engine constructed for a somewhat famous ship, the Thetis, built by Rowan & Co., and the Messrs. Scott:*

Engine of 226 indicated horse-power, calculated by exact formulæ:

DATA.

	Bottom of Cylinder.	Top of Cylinder.
Pressure of admission, $\frac{p_1}{144}$	$108\frac{1}{2}$	104
Back-pressure, $\frac{p_3}{144}$	3.3	4.0
Ratio of expansion, r	16	14

Temperature of feed-water, T_4, about 122° Fahrenheit.

CALCULATED RESULTS.

	Bottom.	Top.
Final volume of 1 lb. of steam, $v_2 = rv_1$..	64.27	58.52
$U_1 - U_2$..............................	170,151	162,726
$v_2(p_2 - p_3)$...........................	21,286	19,382
Work of 1 lb. steam, U'................	191,437	182,108
Mean effective pressure in pounds per inch, $\frac{U'}{144v_2} = \frac{p_m - p_3}{144}$	20.7	21.6
Mean of both results....................	21.15	
Mean observed result of a series of diagrams..............................	21.00	
Difference..........................	+0.15	

Being within the limits of errors of observation.

* Steam-engine; 1859; p. 407.

	Bottom.	Top.
Heat expended per pound of steam, H'...	975,301	966,524
Equivalent pressure in pounds per square inch, $p_h \div 144$........................	105	115
Mean...........................	110	
Efficiency, $\frac{p_m - p_s}{p_h}$.....................	0.196	0.188
Mean.............................	0.192	

(6) Same case calculated by approximate formulæ:

DATA.

	Lbs. on the square inch.
Mean pressure of admission, $\frac{p_1}{144}$...................	$106\frac{1}{4}$
Back-pressure, $\frac{p_s}{144}$................................	3.65
Mean cut-off, $\frac{1}{r} = .067 = \frac{1}{15}$.	

RESULTS.

Mean gross pressure, $\frac{p_m}{144} = 106\frac{1}{4} \times .232$............	= 24.6
Mean effective pressure, $\frac{p_m - p_s}{144}$, calculated..........	20.95
observed..........	21.00
Difference............................	− 0.05
Pressure equivalent to expenditure of heat $= p_h \div 144$	110
Efficiency, 0.19.	

The engine was a two-cylinder compound, and the mean

effective pressure has reference to the larger cylinder, which was of four times the capacity of the smaller.

At $2\frac{1}{4}$ pounds of steam per hour per horse-power, for efficiency unity, this performance corresponds to 13 pounds, neglecting all wastes other than thermodynamic; and to 1.44 pounds of fuel at an evaporation of *nine* pounds steam per pound of coal. These figures would probably be increased by not less than 20 per cent by the extra thermodynamic wastes; or to 15.6 pounds of steam and 1.75 pounds of fuel, nearly.

Accepting Rankine's figures, we have the following:

CONDENSING STEAM-ENGINES WITH DRY SATURATED STEAM.

BACK-PRESSURE, $p_2 \div 144$, ASSUMED AT 4 LBS. ON THE SQUARE INCH.

Examples.	Ratio of Expansion, r, and Effective Cut-off, $\frac{1}{r}$.							
r	10.	5.	3.33	2.5	2.	1.7	1.25	1.
$\frac{1}{r}$	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0
(1) $p_1 \div 144 = 20.$								
$(p_m - p_2) \div 144$	....	....	8.8	11.1	12.8	14.0	15.5	16.0
$p_b \div 144$	....	....	93	124	155	186	248	310
Efficiency of steam	....	....	.095	.090	.083	.075	.0625	.052
(2) $p_1 \div 144 = 40.$								
$(p_m - p_2) \div 144$	....	16.2	21.9	26.2	29.6	32.0	35.0	36.0
$p_b \div 144$	....	124	186	248	310	372	496	620
Efficiency of steam	....	.131	.118	.118	.095	.086	.071	.058
(3) $p_1 \div 144 = 60.$								
$(p_m - p_2) \div 144$	14.8	26.3	34.9	41.4	46.4	50.0	54.6	56.0
$p_b \div 144$	93	186	279	372	465	558	744	930
Efficiency of steam	.159	.140	.125	.111	.100	.090	.073	.060
(4) $p_1 \div 144 = 80.$								
$(p_m - p_2) \div 144$	21.1	36.4	47.8	56.5	63.2	68.0	74.1	76.0
$p_b \div 144$	124	248	372	496	620	744	992	1240
Efficiency of steam	.170	.147	.128	.114	.102	.091	.074	.061
(5) $p_1 \div 144 = 100.$								
$(p_m - p_2) \div 144$	27.4	46.5	60.8	71.6	80.0	86.0	93.6	96.0
$p_b \div 144$	155	310	465	620	775	912	1240	1550
Efficiency of steam	.177	.150	.131	.115	.103	.092	.075	.062

See Appendix to § 149, p. 1002, for a very extensive table, following the collected formulas.

NON-CONDENSING STEAM-ENGINES WITH DRY SATURATED STEAM.

BACK-PRESSURE $p_3 + 144$, ASSUMED AT 18 LBS. ON THE SQUARE INCH.

Examples.	Ratio of Expansion, r, and Effective Cut-off, $\frac{1}{r}$.						
	5.0	3.33	2.5	2.0	1.7	1.25	1.0
	0.2	0.3	0.4	0.5	0.6	0.8	1.0
(6) $p_1 + 144 = 60$.							
$(p_m - p_3) + 144$	....	....	27.4	32.4	36.0	40.6	42.0
$p_e + 144$	....	....	372	465	558	744	930
Efficiency of steam	....	....	.074	.070	.064	.055	.045
(7) $p_1 + 144 = 80$.							
$(p_m - p_3) + 144$	....	33.8	42.5	49.2	54.0	60.1	62.0
$p_e + 144$	....	372	496	620	744	992	1240
Efficiency of steam	....	.091	.086	.080	.073	.061	.050
(8) $p_1 + 144 = 100$.							
$(p_m - p_3) + 144$	32.5	46.8	57.6	66.0	72.0	79.6	82.0
$p_e + 144$	310	465	620	775	930	1240	1550
Efficiency of steam	.105	.100	.093	.085	.077	.064	.053
(9) $p_1 + 144 = 120$.							
$(p_m - p_3) + 144$	42.6	59.8	72.8	82.8	90.0	99.2	102.0
$p_e + 144$	372	558	744	930	1116	1488	1860
Efficiency of steam	.115	.107	.098	.089	.081	.067	.055
(10) $p_1 + 144 = 160$.							
$(p_m - p_3) + 144$	62.8	85.6	103.0	116.4	126.0	138.2	142.0
$p_e + 144$	496	748	992	1240	1488	1984	2480
Efficiency of steam	.127	.115	.104	.094	.085	.070	.057

Taking the temperature of feed-water at such a point as will give nine pounds of water evaporated into dry steam per pound of fuel, for the condensing, and ten pounds for a non-condensing, engine—a heater being assumed to be used—and 2.5 pounds of steam per horse-power per hour at efficiency unity, it is easy to make a comparison of the probable ideal and the probable actual efficiencies. The computations of these and numerous other cases are given in full detail in the Appendix to § 149, p. 995 *et seq.*, which see.

The following are efficiencies computed for the perfect, ideal, engine, by Cotterill, which may afford equally interesting comparisons: *

* Steam-engine; p. 142.

Engine.	T_1 Fahr.	p_1 lbs. per sq. in.	Thermal Units per I. H. P. per min.	Lbs. Steam per I. H. P. per hour.	Lbs. Carbon per I. H. P. per hour.	Effi-ciency.
Non-condensing; $T_2=212° + 461°$ F.	401 + 461	250	195	11.4	0.806	0.219
	363 + 461	160	233	13.8	0.964	0.183
	341 + 461	120	266	15.8	1.10	0.161
	312 + 461	80	329	19.9	1.36	0.130
	287 + 461	55	427	26.0	1.77	0.100
Condensing; $T_2=100° + 461°$ F.	341 + 461	120	143	7.5	0.592	0.299
	324 + 461	95	150	8.1	0.621	0.285
	293 + 461	60	167	9.0	0.691	0.256
	250 + 461	30	203	11.2	0.840	0.211
	228 + 461	20	230	12.8	0.952	0.186
Binary-vapor; Steam and ether; $T_2 = 60° + 461°$ F.	341 + 461	120	122	6.2	0.505	0.351
	293 + 461	60	138	7.3	0.571	0.309
Air-engine; $T_2 = 60° + 461°$ F.	660 + 461	—	79.8	—	0.33	0.536

Professor Cotterill has shown that if the heat of the feed-water could be raised to boiler-temperature, by means of a heater so constructed as to receive heat by a graded system of transfer, such that the pressures of steam at transfer could be gradually varied throughout the whole range, the steam-engine might be given the efficiency of the Carnot cycle. The heat expended would be

$$H_1 = T_1 \log_e \frac{T_1}{T_2} + L_1;$$

that rejected,

$$H_2 = T_2\left(\frac{L_1}{T_1} + \log_e \frac{T_1}{T_2}\right);$$

and the efficiency would be

$$\frac{H_1 - H_2}{H_1} = \frac{T_1 - T_2}{T_1}.$$

L is the latent heat of evaporation, supplied by the boiler.*

* Cotterill; 2d ed., p. 420.

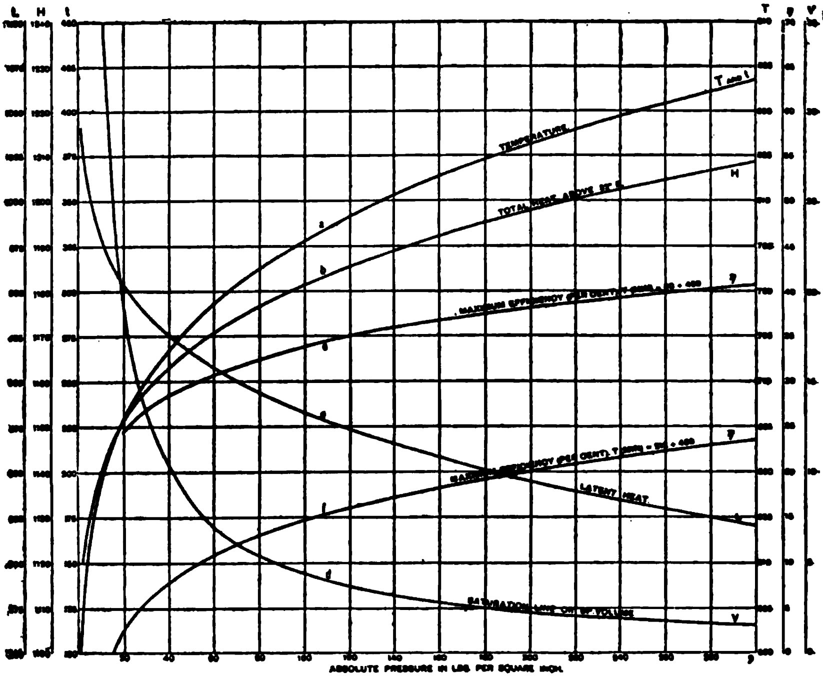
CARNOT IDEAL ENGINE PERFORMANCE. $p_1 = 180$ to 300, $p_2 = 18$.

Pressure, square inches	300	280	260	240	220	200	180
Pressure, square feet	43,200	403,200	37,440	34,000	31,700	28,800	25,920
Temperature, degrees Fahrenheit	417.371	411.05	404.37	392.80	389.73	381.64	372.88
Volume 1 lb. steam, dry and saturated	1.535	1.639	1.759	1.898	2.061	2.256	2.493
Final temperature	222.42						
Volume at end of stroke pressure = 18 (in cubic feet per pound)	18.307	18.442	18.493	18.643	18.737	18.801	18.920
Efficiency $= \frac{T_1 - T_2}{T_1}$	22.20	21.60	21.00	20.39	19.53	18.90	18.10
T_1	878.37	872.05	865.37	858.28	850.73	842.64	833.88
Ratio of expansion	11.89	11.24	10.50	9.81	9.09	8.36	7.62
Latent heat	818.305	822.97	829.89	833.10	838.64	844.57	850.96
Work U	141,200	138,000	135,000	132,000	126,800	123,600	119,500
B. T. U. per H. P. per hour	11,490	11,800	12,110	12,500	13,010	13,480	14,080
Heat, $H = \frac{U}{\text{Efficiency}}$	636,500	638,000	644,000	648,500	651,000	656,000	661,000
Pounds steam for efficiency unity	3.120	3.108	3.080	3.060	3.040	3.025	3.00
Do., actual	14.05	14.40	14.67	15.00	15.54	16.00	16.58
Pounds coal per H. P. per hour *	1.149	1.183	1.216	1.250	1.302	1.350	1.422
Volume traversed by piston per H. P. per minute	4.29	4.432	4.525	4.66	4.850	5.030	5.24

* @ 10,000 B. T. U. per pound.

In the Appendix, Tables III and IV, will be found the values of mean pressures for various methods of expansion, mean pressure ratios, and terminal pressure ratios.

118. **The Limit of Actual Efficiency** is now determined. From what has already been stated, in reference to the differences arising between the ideal and the real steam-engine, it will be understood that the quantities of steam and of fuel above given are magnitudes representing limits which may perhaps be approached, but which can never be actually reached in practice. The consumption of heat, steam, and fuel by even the best types of steam-engine, exceeds these figures by from one fourth to one half; the excess varying with circumstances, already described, affecting the physical wastes.



PROPERTIES SATURATED STEAM. (CARTER.)

The diagram exhibits the needed data relating to steam in such form as permits ready comprehension of these points.

119. **The Vapor-engine Cycles,** Figs. 137, 138, differ, often very greatly in form, from those familiar as illustrated by air- and gas-engines. The isothermal line being isopiestic, the first portion of the diagram, traced during expansion, instead of being an equilateral hyperbola, becomes a horizontal straight line, a line parallel with the axis of abscissas. Adiabatic expansion is represented by a line closely resembling that for gases, but one which falls somewhat more rapidly, and thus deviating also from the common hyperbola, as has been already indicated. The ideal cycles are usually composed of these curves, and often of these combined with lines of equal pressure and of equal volume. In the case of real steam, or vapor, engines, the actual cycles and curves approximate more or less closely to those of the corresponding ideal engine, accordingly as the engine is more or less well designed, well constructed, and well operated; but some considerable differences almost invariably exist.

The action of the metal in conducting, and in radiating, heat to and from the working substance causes changes in the form of the lines composing the diagram, and the imperfect action of the valve-gear, the mechanism controlling the introduction and discharge of the working fluid, produces considerable variations of the forms of the lines, and especially of their junctions. In the designing of engines, and in computing their probable power and efficiency, ideal diagrams are employed which are so chosen and laid down as to represent, with as close approximation as possible, the actual cycle of the engine.

120. **The Distribution of Energy in Real Engines** is vastly different from that thus far found, in the study of the ideal engine. The latter is a purely thermodynamic system; while the former illustrates not only the thermodynamic transformations and transfers of heat-energy, but also transfers and losses by every method of conduction, convection, and radiation rendered possible by the nature of the material employed, and by the structure of the machine. Of all the heat received by the engine from the boiler, and temporarily stored in the steam supplied to the engine, but a small portion is commonly transformed into useful work, even in the ideal case; while, in

the actual engine, as has been shown, wastes occur, in addition to the unavoidable thermodynamic loss, which often result in doubling, and, in small engines, much more than doubling, the consumption of heat, steam, and fuel, and the cost of their supply.

Of all these losses and wastes, that by internal, or so-called cylinder, condensation is that which offers the problem with which the engineer is now most concerned. As elsewhere remarked by the Author, "a comparison of the quantities of steam demanded to supply an engine thermodynamically 'perfect' with the actual quantities required by even the best of engines exhibits so wide a difference that it becomes obvious that the determination of the efficiency of an engine, and the solution of questions involving those of heat-expenditure, are not problems in thermodynamics, simply. The mathematical theory of the steam-engine is not yet in so satisfactory a state—and cannot be until the correct theory of this transfer of waste heat can be introduced into it—that the engineer can often use it in every-day office work, with much confidence, unless checked by direct experiment." *

The wastes in the actual engine are found, by examination of the results of many trials, to vary greatly in even what is considered good practice. The loss at the boiler, in ash, from the recorded weight of fuel is from about 6 per cent, with the best coals, to 10 and 15, with very good fuel, and up to 20 and 25 per cent with bad samples. The boiler should not "prime" more than 3 or 5 per cent, even if it does not produce dry steam; but double these figures are not uncommon. In large jacketed engines using saturated steam, the jackets may, if inefficient, condense less than five per cent of the steam made; but, if efficient, they may condense an amount which approximately measures the work done by the engine—10 per cent or more—or they may even, by extensive transfer of heat during

* "On the Several Efficiencies of the Steam-engine, and on the Conditions of Maximum Economy." Trans. Am. Soc. of Mechanical Engineers; April 1882. Journal of the Franklin Institute; May 1882.

the period of exhaust, or when the prime steam is wet, cause a waste of considerably larger magnitude. The use of a steam feed-pump may waste 3 to 5 per cent, and, in defective constructions, considerably more. Independent air-pumps, now quite common, may increase the wastes 5 or 10 per cent, and, where not efficient, may make this item as much as 15 per cent.

The magnitude of the waste of heat by internal alternate storage and restoration is variable, not only with the conditions of operation, but also with the character of the working fluid. It is comparatively small with gases, large with condensable vapors, and peculiarly large with saturated or wet steam. The ratio of work done per pound of the working fluid to that which it might perform in a non-conducting cylinder measures a certain efficiency which we may call the *Working Efficiency of the Fluid.*

The following tabular statement of the distribution of losses and of quantities of heat applied usefully, in a marine engine, as given by Hunt and Skeel,* corresponds to a consumption of 2½ pounds of coal per horse-power per hour. The best engines of the present time demand two thirds this quantity or less.

One hundred pounds of coal contain in heat-units	1,400,000
Deduct heat-units for weight of ash	200,000
Total number of heat-units in 100 pounds coal	1,200,000

	Heat-units.	Per cent.
Available heat	1,200,000	100
Loss of heat by the chimney	200,000	16⅔
Available to make steam	1,000,000	83⅓
Loss by leakage and condensation	200,000	16⅔
Available to do work on the piston	800,000	66⅔
Loss of heat rejected from cylinder	660,000	55

* "Methods of Testing Steam-engines," etc. Journal of the Franklin Institute; Dec. 1874.

	Heat-units.	Per cent.
Transformed into work	140,000	11⅔
Loss by frictional resistances	40,000	3⅓
Available to turn the screw	100,000	8⅓
Loss by useless resistances	20,000	1⅔
Balance usefully applied in propulsion	80,000	6⅔

121. **The Method of Operation,** in the process of distribution of energy, in the actual case, is the following: The supply of energy delivered to the machine is brought from its storage reservoir, the steam-boiler, by the steam which is its vehicle, in the form of heat. The engine converts a small part of this energy into the mechanical form, and applies it to the performance of work; while the remainder is wasted by transfer, untransformed, to surrounding masses, such as the atmosphere, the environing walls, and, in the case of the condensing engine, to the water by which the steam is condensed and which conveys the heat thus acquired into the water-ways of the country. Of the work developed by conversion of heat-energy, a part is expended in driving the engine itself, and is therefore a waste; while the remainder is applied to the purpose for which the engine is designed. Of the untransformed and wasted heat, the greater part, in the very best engines, is that inevitable waste which the second law of thermodynamics indicates, and the measure of the proportion of which, in the perfect engine, must always be $\frac{T_2}{T_1}$; the remainder is mainly transferred to the "exhaust" by the process of cylinder, or internal, waste to be more fully considered later; in which waste conduction, storage, restoration, and convection play the leading part; while a small portion is directly conducted, or radiated, to objects immediately adjacent to the machine.

Each of these wastes reduces the efficiency of the engine, and their total enormously restricts it; making the difference between the ideal and the real case so great as to absolutely preclude the possibility of predicting the quantity of steam

and of fuel required, or the cost of operation, of the actual engine, until all these losses can be closely estimated. Experiment and experience have supplied data on which all such estimates are now based.

122. **The Methods of Waste,** in all known forms of heat-engine, considered in detail, are the same in character, but are very different in their proportion in different types of engine. In the non-condensing steam-engine, the thermodynamic waste is greater than in the condensing engine; the loss by conduction, internal and external, is less; the waste by friction of engine is less; and the total of all losses may be either greater or less, accordingly as the gain by increased range of temperatures of operation in the latter is, or is not, compensated by the difference in the sum of wastes other than thermodynamic, and necessary losses of different kinds. In the hot-air engine, the great range of temperature worked through decreases the proportional necessary thermodynamic waste, while increasing the other losses; but the resultant actual efficiency is high. The same is true of the water-jacketed gas-engines, in which the wastes by conduction of heat are enormously increased by the action of the jacket; while the thermodynamic efficiency of fluid is high, and the unavoidable thermodynamic waste correspondingly low.

The efficiency of fluid and of engine has often been studied by standard authorities, but almost invariably as a problem in thermodynamics, simply; and the losses occurring in consequence of the working of steam in a cylinder composed of a good conductor of heat have been left unnoted, although frequently the most important of all the expenditures of heat taking place in the engine.

The process of exhaust-waste which has been described is thus seen to be one of the most serious causes of loss of heat in the modern steam-engine. It is this method of waste which prevents the engineer attaining even an approximation to the estimated gain due to considerable expansion. It is this which fixes a limit practically to our expansion of steam in a single cylinder; which limit has, as yet, in ordinary forms of engine, been

little altered by the expedients which have been adopted to extend it. It has been found by experience that with steam of 60 to 75 pounds pressure (four or five atmospheres), no gain in efficiency can usually be secured by expanding more than five or six times in the simple unjacketed engine. Passing this limit, the losses due the wasteful transfer of heat to the exhaust steam increase much more rapidly than the gain due to the increased conversion of heat into work by expansion.

When the steam is so far superheated that the mass taken into the cylinder may surrender to the metal all the heat required to warm it up to the temperature due the steam-pressure, without itself falling to the temperature of saturation at that pressure, this loss is reduced to a minimum. But any such saving is always effected at the sacrifice of some thermodynamic efficiency. Steam-jacketing produces its well-known benefit by similarly checking the waste due to this condensation and re-evaporation.

The losses by the rejection of heat from the engine without transformation have thus been seen to be due to two entirely different causes: the first, thermodynamic waste and physical heat-transfer, can evidently only be saved by some as yet unknown and radical change of type of engine; the second, which has been diminished, but has never been wholly checked by any known expedient, seems very probably to require, also, radical treatment to effect its cure.

As is so well illustrated by the investigations of Dr. Kirsch, the film of metal to which the fluctuations of temperature producing cylinder-condensation are mainly confined, receives alternating waves of high and low temperature, which rapidly traverse the iron, entering and leaving with the entrance of steam and the occurrence of the exhaust; but always, on entering, fading into the mean temperature within the mass, and always restricted, at maximum altitude, to the surface on the steam side. Their rate of alternation is that of succession of piston-strokes; and it is thus proportional to the speed of rotation of the engine, the depth affected becoming less and

less and the waste correspondingly reduced as this speed increases, indefinitely.

Thus, in Fig. 141, the method of heat-transfer between steam and cylinder is shown, as it takes place, stroke by stroke, in the heads of unclothed or inefficiently clothed, in well-covered and in well-jacketed engines, respectively.

For example, in A we have the first and second cases. (1) When the cylinder is in operation, its mean temperature

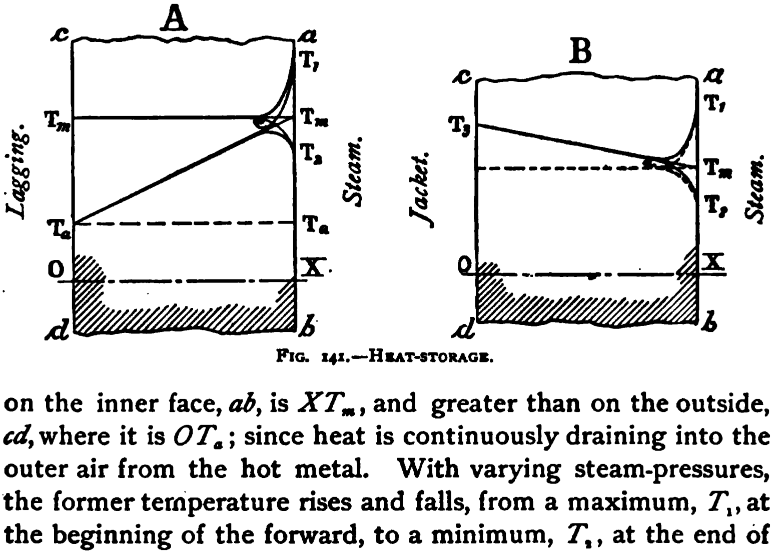


FIG. 141.—HEAT-STORAGE.

on the inner face, *ab*, is XT_m, and greater than on the outside, *cd*, where it is OT_a; since heat is continuously draining into the outer air from the hot metal. With varying steam-pressures, the former temperature rises and falls, from a maximum, T_1, at the beginning of the forward, to a minimum, T_2, at the end of the return stroke of piston. The mean for the whole cycle is represented by the line T_mT_a; and the extreme fluctuations by T_aT_1 and T_aT_2. The area $T_1eT_2T_1$ measures the heat stored per unit of area; and this, added to the waste by outflow at T_a, is the total thermal loss on the head.

(2) Similarly, if fully protected against loss, externally, T_mT_m becomes the mean line for the whole thickness; T_1eT_2 the fluctuation, and this loss, due to it, is $T_1eT_2T_1$, substantially as before; but to this loss is not, in this case, superadded the drain outward. This latter loss is, in the figures, much exaggerated. In actual work, the internal waste is usually much the greater.

(3) When the engine is well-jacketed, the temperature of the interior fluctuates as before, as seen in B, where the dotted line is that of mean temperature for the preceding case, and T_2T_m is that for the jacketed engine; but the exterior is held up to the temperature, T_2, of "prime steam" or higher by contact with jacket-steam having equal or greater pressure. The result is to produce a drain of heat inward, instead of, as in the preceding cases, outward, and to restrict the minimum temperature, T_2, in its fall, and to throw the whole area, $T_1eT_2T_1$, upward toward T_1, and to reduce its magnitude, correspondingly decreasing internal wastes by that method. Could this process be made absolutely effective, and $T_2 = T_1$, internal or "cylinder condensation" would cease, and the only internal loss would be by transfer from the jacket to the cylinder in the same manner as, in the preceding cases, heat passes outward.

Thus, with the jacketed cylinder, there are three of these wastes: loss through the lagging, externally; loss by interior storage and restoration of heat in and from the metal; internal discharge by drainage from the jacket. Calling these H_a, H_b, H_c, and the total H,

$$H = H_a + H_b + H_c;$$

and the values of the quantities vary with type and construction of engine. In case 1, H is largest; though $H_c = 0$; in case 2, H is reduced by the reduction of H_a to a small quantity, and the $H = H_b$, nearly; while in case 3, although H_c is introduced, it may, by producing a larger reduction of H_b, give a total value, H, in some cases, considerably less than in either of the other cases.

It is here obvious that the jacket will be useful, useless, or wasteful, accordingly as it reduces H_b more, an equal amount, or less than its own characteristic waste, H_c.

In Fig. 142 is shown the action of the metal during the movement of the engine through its cycle, and the fluctuations already alluded to. The inner face, *Oa*, varies in temperature from T_1 to T_2, about the mean, T_m, as before, the successive

isothermals taking the forms exhibited, as the waves of heat cross the metal towards X, the line OX representing the temperature of the outside atmosphere, and the successive lines, above, are the successive positions of the isothermals, as the flow fluctuates in the lagged but unjacketed cylinder. In the jacketed engine, the same general effect would be seen; but the line $T_m T_s$ would have the opposite inclination, as already seen.

In quick-working engines, the action of the cylinder-walls results in producing a film of water on their surfaces, and the

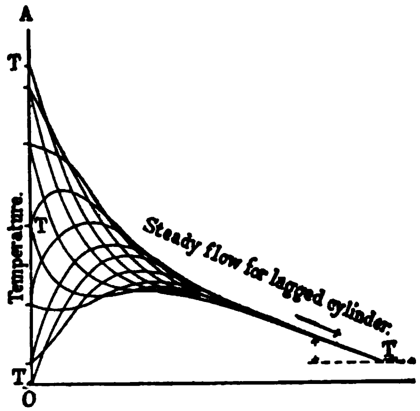


FIG. 142.—VARIABLE HEAT-FLOW.

steam remaining uncondensed is but very slightly and superficially affected; it passes out still dry. In slow engines, the mass of steam may probably be rendered comparatively wet. On the other hand, the process of expansion, after cut-off, results in producing water, diffused throughout its mass, which can neither affect nor be affected by the surrounding walls.

In the transfers of heat between engine and steam, the contact of cylinder and cooler dry steam has little effect; but moisture on the surface of the metal and its re-evaporation has a most decided and important effect.

A glance at these diagrams of heat-flow shows that, to secure usefully sustained temperatures on the working face of the

cylinder-wall, the temperature and steam-pressure in the surrounding jacket must be higher as the thickness of that wall is greater, in order to maintain any given head and inclination of the mean temperature line. Conversely: the thinner the wall, the less the necessary head and the more effective the jacket for any given pressure and temperature of its steam, in excess of the mean temperature of the inner face of the cylinder-wall.

It will be seen that the higher the speed of engine, the thinner the film of metal affected by these measurable variations of temperature and, consequently, the less useful the jacket. In other words, also, the higher the speed and the thinner this film, the higher is the temperature needed for efficient action of the jacket. Experience confirms this deduction by showing that the jacket has very slight effect, as usually applied to "high-speed" engines. To make it useful, a way must evidently be found, either to greatly reduce the thickness of the cylinder-walls—as, indeed, has been proposed, many years since—or to raise the temperature of jacket considerably, thus securing increased head.

As stated by M. Dwelshauvers-Dery, the principle of Hirn applies to all engines, thus:

Between any two given successive positions of the piston, the quantity of heat transformed in the performance of external work, plus that derived from the metal of the cylinder, gives a sum equal to that of the variation of the internal heat of the steam, plus that introduced by newly entering steam, if any, or minus that lost with rejected steam, if any.*

123. **The Magnitudes of Losses** in the steam-engine have been ascertained with considerable accuracy, for the principal types and for engines of ordinary sizes working under familiar conditions. In general, it may be said that the actual *total* efficiency of engine ranges from an average of about fifteen or sixteen per cent, in the best cases, down to five for ordinarily good engines, and, often, to much lower figures. A perfect steam-engine of efficiency unity, working under the best of

* Dwelshauvers-Dery: Exposé; § 2.

familiar conditions as to temperatures and pressures, should demand but about two and a quarter pounds of feed-water and steam, per horse-power and per hour. The best recorded figures are about five times as great; and twenty-five to thirty pounds are the figures commonly guaranteed for large sizes by good builders of simple engines. For small engines of ordinary construction, the consumption of steam and the wastes are often enormously great.

The distribution of energy in evaporation in a case of excellent performance of a boiler tested by Sir Frederick Bramwell and Mr. W. Anderson was as below:*

	B. T. U.	Per ct.
Evaporating the water in the wood........	9557	0.32
Heating wood and air....................	3884	.13
Evaporating moisture in coal.............	8374	.29
Heating coal and air.....................	129,321	4.44
Displacing atmosphere....................	53,394	1.83
Heating excess of air....................	130,980	6.34
Displacing atmosphere by ditto...........	53,509	
Making steam.............................	2,090,300	71.78
Radiation and convection.................	271,307	9.32
In ash...................................	53,915	1.85
Balance unaccounted for..................	107,552	3.70
Totals...................................	2,912,093	100.00

The heat received from the fuel, at the furnace, may be taken as distributed, in a good example, thus:

Total heat received from the fuel at the furnace, 100	Waste at chimney.......	25
	" " condenser.....	57
	" " by radiation...	3
	Useful work............	15
	Total................	100

* Thurston: Engine and Boiler Trials; p. 361.

The "working efficiency of the fluid" is here about 73 per cent, the exhaust waste being about one half cylinder-condensation.

In the case of an economical condensing engine, consuming one kilogram (2.2 lbs.) of good fuel per hour per horse-power, M. Hirsch gives the following as a fair distribution of the heat produced:*

	Coeffic.	Calories expended.	Calories remaining.
Heat of combustion.............	1.00	0.00	100.00
(1) Received from the boiler......	.60	40.00	60.00
(2) Thermodynamic efficiency....	.27	43.80	16.20
(3) Imperfection of cycle.........	.60	6.48	9.72
(4) Efficiency of machine.........	.77	2.22	7.50
(5) Total efficiency..............	0.075	92.50	7.5

In a familiar form of simple non-condensing engine, doing fair work, the Author has found the following distribution of energy received from the boiler:

Received.		Expended.	Per cent.
Heat-energy stored in steam,		Waste by external conduction, etc.........	3
dry and saturated at the en-		Waste by internal conduction..............	36
		Waste, thermodynamic	48
		" by friction.....	1
gine, in per cent.......	100	Useful work..........	12
Total..............	100	Total..............	100

Here the working value of the fluid is 70 per cent.

In the case of the best compound triple-expansion engines with steam at the same pressure (100 lbs.; atmos. absolute; nearly), cylinders jacketed and expanding about 12 times, the following is a fair distribution:

* Congrès International de Mécanique appliquée; 1891; vol. IV.

Received.		Expended.	Per cent.
Heat-energy stored in steam,		External wastes.......	5
		Internal "	40
as received from the boiler,		Thermodynamic wastes	41
		Friction wastes........	2
per cent..............	100	Useful work..........	12
Total..............	100	Total.............	100

In this case, the working efficiency of the fluid is 70 per cent.

The following are figures given by Professor Ewing, deduced from data supplied by Mr. Main:*

		B.T.U.
Heat supplied engine per rev................		1377
" " " by jackets.............		212
" total B. T. U....................	1589	
" returned to boiler................	38	
" *net* supply............................		1551
" converted into work....................		227
" rejected............................		1324
Efficiency.........................$\frac{227}{1551}$ =		0.146
Thermodynamic Efficiency.................		0.335†

In all cases assuming that the expansion may be taken as hyperbolic, the work done in a cylinder of given volume will vary nearly as $\log_e r$; but the cost of that work may vary enormously, and entirely without direct relation to the volume, v_1, of the steam at the point of cut-off.

The early trials of the Owens College experimental engine elsewhere described (see frontispiece) are reported by Professor Reynolds to have given data and an account, as deduced by Mr. Cowper, as follows: ‡

* Minutes Proc. Inst. C. E.; vol. LXX. Also, Thurston: Engine and Boiler Trials; p. 298.

† Ency. Britannica.

‡ Proceedings Brit. Inst. C. E.; 1889. Van Nostrand's Science Series; No. 99; 1890.

Dr.	B.T.U.	Per cent.
To steam in cylinders....................	14,154	81
" " " jackets....................	3,325	19
	17,479	100

Cr.	B.T.U.	Per cent.
By indicated work—efficiency..............	3,085	17.7
" heat rejected..........................	12,862	73.6
" " radiated.........................	1,176	6.7
" " lost from hot-well................	356	2.0
	17,479	100.0

The effect of *back-pressure* in limiting thermodynamic transformation and the efficiency of expansion is well exhibited by the following tables, computed by Mr. Buel:

IDEAL ENGINE; NO BACK-PRESSURE.

Point of Cut-off.	Mean Total Pressure, pounds per square inch.	Relative Area of Cylinders.	Relative Amounts of Steam used.	Per cent of Saving.
1	2	3	4	5
1	100.0	1.00	1.000	
$\frac{3}{4}$	96.4	1.04	.780	22.0
$\frac{1}{2}$	84.7	1.18	.590	41.0
$\frac{1}{3}$	70.0	1.43	.477	52.3
$\frac{1}{4}$	59.7	1.68	.420	58.0
$\frac{1}{6}$	46.5	2.15	.358	64.2
$\frac{1}{8}$	38.5	2.60	.325	67.5
$\frac{1}{12}$	29.0	3.45	.288	71.2

IDEAL ENGINE; BACK-PRESSURE 17½ LBS.

Point of Cut-off.	Mean Effective Pressure, pounds per square inch.	Relative Mean Pressure.	Relative Area of Cylinders.	Relative Am'ts of Steam used.	Percentage of Saving.
1	2	3	4	5	6
1	82.5	1.000	1.00	1.000	
$\frac{3}{4}$	78.9	.959	1.04	.780	22.0
$\frac{1}{2}$	67.2	.815	1.23	.615	38.5
$\frac{1}{3}$	52.5	.636	1.57	.523	47.7
$\frac{1}{4}$	42.2	.512	1.95	.488	51.2
$\frac{1}{6}$	29.0	.352	2.84	.473	52.7
$\frac{1}{8}$	21.0	.255	3.92	.490	51.0
$\frac{1}{12}$	11.5	.140	7.15	.596	40.4

From this table it appears that, under the assumed conditions, the most economical point of cut-off is about one sixth of the stroke, since the saving is decreased, whether the cut-off is lengthened or shortened, from this point. The conditions assumed are such as accord well with modern practice. By changing the initial or back pressure to suit a condensing engine different results will be obtained, but the table is sufficient to show the mode of application for any given data as first shown by Clark.

The cause of increased back-pressure is resistance to the escape of the steam from the cylinder, by which the mean back-pressure is raised from 1 to 3 lbs. on the square inch. There is as yet no satisfactory theory of that resistance, and it cannot be computed for any proposed engine by means of a general formula.

The back-pressure in proposed engines can be estimated roughly from the results of experience. The following is a summary of some such results:

		Mean Back-pressure, p_3. Lbs. on the square foot.	Lbs. on the square inch.
Ratio of expansion	from 1½ to 3..	720	5
" "	from 4 to 7...	648 to 504	4½ to 3½
" "	from 8 to 15..	504 to 432	3½ to 3

The diagrams show only the *effective* pressures of the steam, and not the *absolute* pressures, which are usually left to be roughly estimated by guessing the probable atmospheric pressure.*

Mr. Beer takes the back-pressure as

$$p_b = p_0 + 0.03p_1,$$
$$p_b = p_0 + 0.035p_1,$$

for non-condensing and for condensing engines, respectively; in which p_0 is the pressure of the atmosphere or in the condenser, assuming moderate engine-speed and liberal port-areas.

* Rankine: Steam-engine; chap. III.

124. The Thermodynamic Loss, which unavoidably takes place in all heat-engines, has been seen to have a magnitude which is absolutely definite, and easily determinable. Of all the heat subject to thermodynamic conditions, and not lost by conduction or radiation, one portion, never exceeding $\frac{T_1 - T_2}{T_1}$, has been found to be converted into mechanical energy; while the remainder, measured by a fraction never less than $\frac{T_2}{T_1}$, is, as has been seen, necessarily and inevitably rejected untransformed; this constitutes the "unavoidable thermodynamic loss," which, only, is considered by the pure science of thermodynamics. The part utilized, $\frac{T_1 - T_2}{T_1}$, being divided by the sum of these two parts, $\frac{T_1 - T_2}{T_1} + \frac{T_2}{T_1} = 1$, gives the measure, as already seen, of the maximum possible thermodynamic efficiency of the fluid, $\frac{T_1 - T_2}{T_1}$.

The efficiency of any real engine, operated under familiar conditions, is measured by the quotient of converted heat divided by the sum of all expenditures, whether useful or wasteful. Thus the figure for efficiency, just obtained, is reduced in proportion to the increase of the total heat-supply compelled by the aggregate of these wastes; and the proportion of thermodynamic waste is at the same time correspondingly reduced. The latter, in many cases, thus becomes forty or fifty per cent of the total, instead of, as for the ideal case, eighty-five or ninety per cent; the extra-thermodynamic wastes of the engine often equalling, or even exceeding, the quantity of heat, or of steam, demanded in the purely thermodynamic process of its operation.

In all cases, with real engines, the quantity taken as unity, with which the useful work is compared, and on which the measure of efficiency is based, is the sum of all expenditures of heat, and not simply the heat thermodynamically demanded.

125. The Conditions of Maximum Efficiency of fluid, in all real engines, other things equal, are precisely the same as with the ideal engine of thermodynamic science, viz., maximum range of temperature worked through and maximum value of the expression $\frac{T_1 - T_2}{T_1}$. The higher the pressures and the temperatures of the working fluid supplied, and the lower those of rejection, the higher the efficiency of operation of the actually working substance. But it does not follow that the actual total efficiency will be similarly increased.

It may happen that the extra-thermodynamic wastes may also increase with increased efficiency of fluid, by this change of thermodynamic conditions, and to such an extent as to produce an actual decrease of total efficiency. This, which is a common experience, if not universal, is illustrated by the familiar fact that, for every engine of ordinary construction, a ratio of expansion may always be found, beyond which the range of temperatures and pressures being increased, an actual loss is produced by the consequent increase of internal wastes due to "cylinder-condensation."

Should it be practicable, in any case, to prevent exaggeration of losses by wastes of heat through internal and external conduction and radiation, these same conditions lead to increased efficiency of heat-transformation, with the real, as with the ideal, engine. It is, in fact, in this way that all recent important improvements in the economical operation of the engine have taken place; the wastes having been checked, while the practicable range of expansion, and of working temperature, has been extended.

126. Heat-wastes by Conduction and Radiation have been classed as of two kinds: external losses of heat, and internal heat-wastes. Of these, the first take place by conduction of heat from the cylinder to the engine-frame; by radiating from the heated cylinder-heads; and from the alternately heated and cooled piston-rods and valve-rods, as they move into and out of the steam-space, and even from the carefully clothed exterior of the cylinder.

An allowance of one British thermal unit per hour, per square foot, or of nearly three calories per square metre, will usually cover the losses in the ordinary engine per degree on well-lagged surfaces. This corresponds to about 0.001 pound of steam condensed by one square foot, or to nearly 0.005 kilogramme liquefied by a square metre. For any given engine, this loss may be assumed constant, and may often be neglected, as unimportant, in presence of so many other more serious wastes. The slight leaks of steam and of hot water about the rods will, practically, often be found much more important, economically. If it be taken, in engines of moderate size and power, as five per cent, it will probably usually prove that the assumption is a safe one.

Losses of heat in this manner by external conduction and radiation have, however, been rarely measured at the engine; but the following data (p. 485) from the trials of agricultural engines at the Royal (G. B.) Society's competition, by Sir Frederick Bramwell and Mr. Anderson, will illustrate the method and extent of this waste with varying temperatures.*

The total waste from engines and boilers was thus from 3½ to 16½ per cent of all heat of combustion. Larger engines and boilers are less subject to this waste; since the area of surface is less in proportion to weight and to quantity of steam and work. As, in the cases cited, the total waste, from engine and boiler, is, in the best example, reduced to 3½ per cent, it is evident that the loss from the steam-cylinder of the engine must be very slight, and, in large engines, may be made, by careful protection, insignificant.

The Rate of Cooling is not uniform, but decreases as the temperatures and pressures of steam and metal fall, as shown by the line *c O*, in Fig. 143, and increases observably with increasing pressures, thus indicating an increasing loss which tends to set a limit to the gain otherwise attainable by this progression. The curve, *c O*, of waste gives the total loss in British thermal units for the given temperatures and corre-

* Jour. Roy. Ag. Soc.; vol. XXIII; 1887.

RATE OF COOLING OF ENGINES AND BOILERS.

	Simple Engines.				Compound Engines.			
	A.	B.	C.	D.	E.	F.	G.	H.
1. Weight of water in boilers at normal level, lbs.	1,134	329	1,317	1,477	1,346	987	1,393	1,428
2. Volume of steam-space, cubic feet	12.83	6.41	18.23	17.62	12.49	11.30	19.37	15.86
3. Estimated weight of engines and boilers affected by heat, lbs.	17,752	2,147	8,052	7,756	7,097	19,488	8,314	8,648
4. Steam-pressure above atmosphere during trials, lbs.	120	60	95	85	125	250	150	155
5. Temperature due to steam-pressure, F.°	350	307	334	327	353	406	366	368
6. Brake horse-power, including friction of brakes, h.p.	11.37	3.798	16.94	17.08	17.25	17.57	20.33	21.07
7. Coal consumed per hour, lbs.	31.48	27.01	44.03	87.389	63.43	34.19	37.61	46.02
8. Units of heat evolved from coal per hour (14,940 units per 1 lb.)	470,310	403,530	657,820	1,305,600	947,640	510,800	562,050	687,5[illegible]0
9. Rate of cooling at the working-pressure in units per hour	70,813	41,000	43,688	46,105	51,159	84,143	62,477	[illegible]6,[illegible]10
10. Ratio of heat dissipated by cooling to heat evolved by coal.	.151	.102	.066	.035	.054	.165	.111	.068
11. Units of heat dissipated by cooling per brake horse-power per hour	6,228	10,811	2,579	2,699	2,966	4,789	3,073	2,208

sponding pressures. The line, $c B$, tangent to the curve at its upper extremity, indicates the length of time, measured by its ordinate, $A B$, which would have been required for cooling down to the minimum had the rate of cooling been constant. The actual time was nearly twice as great.

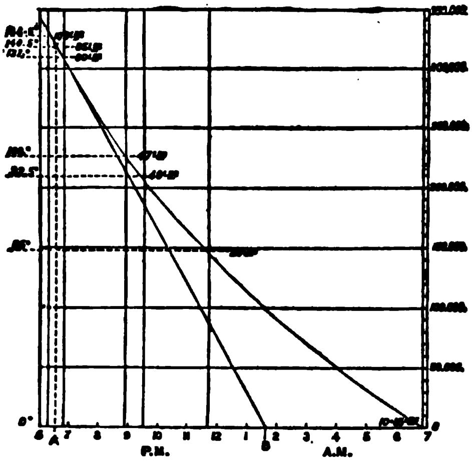


FIG. 143.—RATE OF COOLING OF ENGINES.

The zero-point is here taken at the minimum reading. The ordinates measure the total quantities of heat at each pressure and temperature and time indicated on the curve, in excess of that remaining at the end of the period of observation (364,600 B. T. U.).

It was observed, in these experiments, that the rate of cooling was very variable, ranging from about 2200 up to above 10,000 units per dynamometric, or brake, horse-power, on the "ten and twenty horse engines" compared; and that a compound engine wasted heat at a rate exceeding, by 28 per cent, that of its companion simple engine, similarly covered. These cases show clearly that, with small engines and ineffective lagging and clothing, this waste may be found, especially at high pressures, and with multiple-cylinder engines, a very observable tax upon efficiency. Wastes of from 3 to 5 pounds of steam, the ordinary range of this waste, per horse-power and per hour,

for the compound, and of $2\frac{1}{2}$ pounds and upward for simple engines, constitute very considerable percentages of the total consumption of steam.

The waste of heat by radiation from well clothed, lagged, and felted, engine-cylinders may be taken, for one hour, at from 0.50 to 1.0 British thermal unit per square foot of surface and per degree of difference of temperature, internal and external. About five times as much is lost from uncovered ends, and probably still more from that portion of the piston-rod exposed to the air and to the steam alternately. In large engines, well clothed, this loss often constitutes less than 2 per cent of the heat supplied.

127. **The Methods of Reduction of Losses of Heat** by conduction and radiation externally, as commonly practised, consist simply in carefully covering all external surfaces, where it can well be done, with hair-felt, asbestos, preparations of magnesia, or other non-conducting substances, and adding a surface-covering of painted canvas, of well-finished wooden, or Russia iron, lagging, which protects the clothing beneath it from injury. Cylinder-heads are sometimes similarly covered, but are often left bare, and are then very carefully polished, as should be all such exposed heated parts.

Conduction and radiation from piston-rods and valve-rods can only be reduced by a good polish and such effective packing as will insure their working dry. Conduction to the frame and other parts of the structure connected to the heated cylinder is checked by any packing or "joint" between the two members of the machine; but it is not usual to attempt to effect such an economy by any special device. In this respect, those engines in which the cylinder is attached to the frame only by its front head probably have some advantage.

A gain by improved efficiency of engine may usually be expected to give still greater gain in economy at the boiler. A reduction in steam-consumption results in the increase of the ratio of area of heating surface to weight of steam required, and this, in turn, effects an increase in the quantity of water evaporated per unit weight of fuel. Thus, a reported gain of 25 per

cent in a small engine, by compounding, was, in a case observed by the Author, reported, also, to be accompanied by a gain of above 35 per cent in fuel required per horse-power per hour.

128. Cylinder-condensation, or loss by internal conduction and radiation, is, in the best engines, next to the thermodynamic waste, the most serious and difficult of reduction. In ordinary cases, this is far in excess of the thermodynamic waste. Leakage produces a similar effect.

Steam-engines, as already seen, are impelled by a fluid which is a vastly better receiver and transmitter of heat than the permanent gases. Steam takes up and loses heat, in the process of formation and of condensation, with extreme rapidity. The working fluid, in all steam-engines, is readily condensable, and exchanges heat with the metallic surfaces of the working cylinder with the greatest freedom. It is usually more or less wet, and its humidity is subject to rapid and extreme variation in the course of the movement of the piston. Condensation also occurs in another way: Suppose steam to enter the steam-cylinder perfectly dry, and to expand adiabatically. As expansion progresses, after the closing of the steam-valve by the expansion-gear, the work done by the working fluid results in the transformation of so much heat into mechanical energy—which heat can now only be obtained by drawing upon the stock contained in the steam itself—that a part of the steam becomes liquefied.*

This fact was shown by Rankine and by Clausius, by the study of the thermodynamics of the case; it had, a generation earlier, been perceived by Carnot,† and by Combes as early as 1843. James Watt measured this waste in 1764 (§ 65).

The liquefaction of the steam, in consequence of transformation of heat into work, probably aggravates this evil, although, as was stated by Rankine, not itself a waste: ‡

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* On the Ratio of Expansion at Maximum Efficiency; R. H. Thurston; Trans. Am. Society M. E., 1881.

† Réflections, etc.; Thurston's Trans.; p. 255.

‡ Steam-engine and other Prime Movers, 1859; pp. 395–396.

"That liquefaction does not, when it first takes place, directly constitute a waste of heat or of energy; for it is accompanied by a corresponding performance of work. It does, however, afterwards, by an indirect process, diminish the efficiency of the engine; for the water which becomes liquid in the cylinder, probably in the form of mist and spray, acts as a distributor of heat, and equalizer of temperature, abstracting heat from the hot and dense steam during its admission into the cylinder, and communicating that heat to the cool and rarefied steam which is on the point of being discharged, and thus lowering the initial pressure, and increasing the final pressure, of the steam, but lowering the initial pressure much more than the final pressure is increased; and so producing a loss of energy, which cannot be estimated theoretically."

A fact which is rarely noted, but which should be borne in mind as a preventive of misconceptions at least, is that the so-called cylinder-condensation, the initial waste of the steam distribution, does not affect seriously the power of the engine, and especially that its effect is to increase, rather than decrease, the work performed in the engine-cycle. The area of the indicator-diagram is increased by the elevation of the expansion line above that due adiabatic expansion, which elevation is caused by the re-evaporation of water initially condensed. The period of condensation occurs during the whole period of induction practically, and the initial pressure and the steam-line are not modified. Loss of steam by condensation is instantly made up by additional flow from the steam-chest and the initial pressure is thus maintained in spite of any amount of synchronous condensation.

The surfaces affected by this action are of varying activity and efficacy in the production of wastes. The cylinder-heads, the sides of the piston, the surfaces of the port- and steam-passages, the surfaces of the clearance or "dead" spaces, often of very considerable area, and the extreme portions of the internal cylindrical surfaces, which are all exposed to the full range of temperature from boiler-steam to condenser, produce the main portion of this serious loss. Between the points of

mean cut-off at the two ends of the cylinder, this range is less. It is a minimum at the middle of the cylinder; at which point the inner surfaces are exposed to the least variation of pressure. Whatever treatment may be adopted to evade this waste will be most effective on those parts which are thus exposed to the maximum variation of temperature and pressure of the enclosed steam.

Of the absolute magnitude of this waste, some idea may be obtained from the reported results of experiment; some of which are as follows:

Mr. Clark deduces from his experiments with locomotives the following figures for usual percentages of condensation at various points of cut-off in outside connected engines. Engines with inside cylinders are observably less seriously affected; as the heat of the adjacent smoke-box and boiler, and their protection against the cooling action of the passing air, exert a favorable effect.*

CYLINDER-CONDENSATION IN LOCOMOTIVES.

Cut-off.	Actual r.	Per cent Condensation.	
		Parts of Initial Steam, per cent.	Parts of Initial Steam and Water.
0.10	4	80.0	44.0
0.15	3.40	57.0	36.0
0.20	2.85	41.0	29.0
0.25	2.50	31.0	23.6
0.30	2.20	23.0	18.7
0.35	2.00	17.5	15.0
0.40	1.83	11.0	10.0
0.50	1.60	4.5	4.3
0.70	1.25	2.75	2.7
1.00	1.00	2.0	2.0

The results of Isherwood's investigation, as summed up by himself, give the following average data: †

* Proceedings Brit. Inst. C. E., No. 1910; 1882–3.

† Experimental Researches in Steam-engineering; vol. II. p. xxxiii.

CYLINDER-CONDENSATION IN MARINE ENGINE.

STEAM 40 LBS. BY GAUGE.

Cut-off.	Actual *r*.	Lbs. Steam per I. H. P. per hour.	Relative Cost in Steam.	Internal Condensation, per cent.
100	1	46.86	1.397	10.90
90	1.11	41.57	1.239	12.43
80	1.25	37.85	1.128	14.45
70	1.43	35.54	1.059	16.95
60	1.66	34.16	1.018	20.02
50	2.00	33.55	1.000	23.94
40	2.50	33.59	1.001	28.50
30	3.33	34.52	1.029	33.56
20	5.00	36.88	1.099	38.87
10	10.00	42.83	1.277	44.46

The figures of the last columns in each of these tables show well how rapidly internal waste increases with increasing expansion.

CONDENSATION IN STEAM-CYLINDERS.

Case.	I.	II.	III.	IV.	V.
Cut-off	0.95	0.67	0.40	0.354	0.25
Fraction condensation in Ideal Case	0.004	0.026	0.056	0.061	0.081
" " " Actual Case.	0.150	0.284	0.459	0.554	0.601
Ratio of Real to Ideal	37.5	10.7	8.2	9.1	7.4

The above table presents a comparison of the actual condensations occurring in the unjacketed engine of the U. S. S. Michigan, with the condensations, resulting from the work of expansion, which would have taken place had the work been done in a non-conducting cylinder, as computed by Professor Rankine.*

Hirn's experimental work furnishes some exceedingly valuable data, as may be seen in the accompanying table, abstracted from Ledieu.† The method of loss and its distribution are here well exhibited.

* Trans. Inst. Engrs. of Scotland; Feb. 5, 1862.

† Hirn: Théorie Mécanique de la Chaleur; 1876. Ledieu: Machines à Feu; 1882; p. 383.

HEAT-WASTES.—HIRN.

Kind of engine	Simple	Same	Simple	Same	Comp. Woolf	Same	Comp. Woolf	Same
Jacketed?	No	Yes	No	No	No	Yes	Yes	Yes
Steam-pressure, atmospheres	5	5	4	4.5	4	4	5	4.5
I. H. P.	67.5	98	129.5	144	106	130	266	690
Ratio of expansion	13.5	9.1	3.9	3.9	4.4	4.4	7.5	4.8
Revolutions per minute	55	55	29	29	23	23	25	75
Proportion of cylinder-condensation	0.33	0.21	0.18	0.12	0.26	0.20	0.19	0.18
Total condensation at entrance	0.49	0.34	0.25	0.11	0.053	0.01	0.091	0.037
Heat restored during expansion	0.18	0.31	0.086	0.018	—0.067	0.075	0.14	0.011
Heat lost during exhaust by re-evaporation	0.29	0.054	0.164	0.085	0.12	0.07	0.016	0.064
Heat lost by jackets	......	0.038	......	......	......	0.098	0.10	0.061
Heat gained by piston-friction	—0.013	0.023	—0.002	0.009	0.005	0.072	0.068	0.039

These great wastes by internal transfer of heat, without transformation into mechanical energy, are evidently due to precisely those conditions which make the steam-boiler efficient. That rapidity of conduction which causes a small area of iron in the boiler to transfer a large amount of energy, in the form of heat, for useful application, is the quality which causes a small area of iron in the engine-cylinder to store and waste a considerable part of the heat entering it.

The thickness of the metallic film affected by the phenomenon here studied is probably slight. Mr. A. A. Wilson, in experiments on a large pumping-engine, found the mean temperature of the metal nearly equal that of the entering steam at a point as near the inner surface of the cylinder as he could safely place his thermometer-bulb; and Mr. Dixwell estimates, as a deduction from his own tests, that the mean variation of cylinder-temperature does not exceed 30° F. He thus takes, in the discussion of one of his engine-trials, a case in which 920,000 pounds of steam passed through the engine, while it made 223,000 strokes; giving 4.12 pounds per stroke at the point of cut-off. Taking the specific heat of steam at 0.475, and of iron at 0.114, the loss of temperature of the steam having been found to be 200° F. (steam, superheated)

$$4.12 \times 200 \times 0.475 = x \times 30 \times 0.114,$$

when x is the weight of iron; then

$$x = 114.4 \text{ lbs.}$$

of iron varying in temperature the specified amount, 30° F. The area of surface was 56.59 square feet; and the thickness, to weigh 114.4 lbs., would be but 0.054 inch, or less than one sixteenth.

In the course of experiments in the Sibley College laboratory, Mr. W. W. Churchill being the observer, the thickness of metal of cylinder-wall was reduced to 0.2385 inch, and the temperature of its outer surface observed by means of the

instantaneous action of a balanced Wheatstone bridge and a platinum-wire conductor, and without change being detected, the temperature of the entering steam being 300° F., and upward, the engine non-condensing, and the revolutions 308 per minute. The load was light, however, and compression heavy. The conductor indicated a constant temperature averaging six degrees lower than that of the steam at entrance. In another series, the same general results were obtained; but the temperature recorded was less constant.

Reducing the thickness of wall to 0.115 inch, another series of trials showed slight variation of temperature and a reduction from that of the steam of about 25 degrees. Still further reducing the wall to 0.0426 inch, a fluctuation of 17 degrees became observable. These were all tentative experiments, however, and are not considered as giving reliable quantitative values.

Mr. Willans, as a result of his own experience and research, concludes that a large proportion of the "missing quantity," due cylinder-condensation, must be ascribed to the action of water in the engine, and that "water is likely to prove a more important factor than surface at such speeds as 400 revolutions per minute," and that, as in some of his own experiments, when this condensation occurs in one thirtieth of a second, the presence of a small constant weight of water in the cylinder may account for substantially all this waste, and that its generally observed variations of magnitude may be due to changing quantities of water in the engine. His engines were therefore so designed as to avoid giving opportunity for water to lodge in the cylinder; any collecting on the piston-surface, the only place available, is swept off by the exhaust-current. A thickness of film of only about 0.008 inch of water would account for all the waste thus produced in his observed case.

The diagram opposite is given by Mr. Porter, as taken from the high-service pumping-engines at Providence, R. I., now many years ago.*

* Porter on the Indicator; p. 172.

The speed of the engine was 10 revolutions per minute; at one revolution, this action was still further, and enormously, exaggerated.*

The problem of the engineer is, evidently, *either* to render the internal surfaces as thoroughly non-conducting, and as incapable of heat-storage, as possible, or to secure similar properties for the working fluid exposed to contact with them; thus, by either

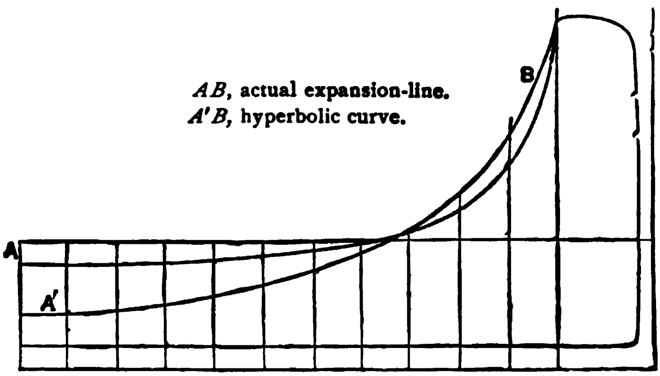


FIG. 144.—CYLINDER-WASTE.

or both methods combined, reducing the condensing power of the cylinder to a minimum.†

The cost of steam and power in engines of various sizes has been ascertained, by the researches of many investigators, to be largely dependent upon size of engine and power de-

---

* The higher the speed the more superficial the action; and, at very high speeds of rotation, a limit may be approached at which the wastes by variation of temperature of the metal of cylinder become insensible.

The more effective the jacket action, also, the thinner this film of varying temperature.

† The Westinghouse Co., about 1885, conducted a series of experiments to determine the possible gain in fuel economy to be realized from the use of non conducting surfaces in steam-engine cylinders, as far as possible. The non-conductor found to be best was porcelain. The pistons and the cylinder-heads were coated, but the difference in the fuel economy was so small as "not to be worth consideration, commercially speaking."

A device proposed by the Author consists in converting the inner surfaces into a graphitic sponge, filling it with non-conducting substances.

manded. The accompanying diagrams, prepared by Mr. Emery, and the corresponding data may be taken to exhibit this cost for engines described.* Curves in group No. 1 exhibit the results of experiments at the Novelty Works, N. Y. City, under the joint supervision of that establishment and the U. S. Navy Department. The curves, *A*, *B*, *C*, *D*, *E*, refer respectively to steam-pressures of 25, 40, 60, 80, and 100 pounds.

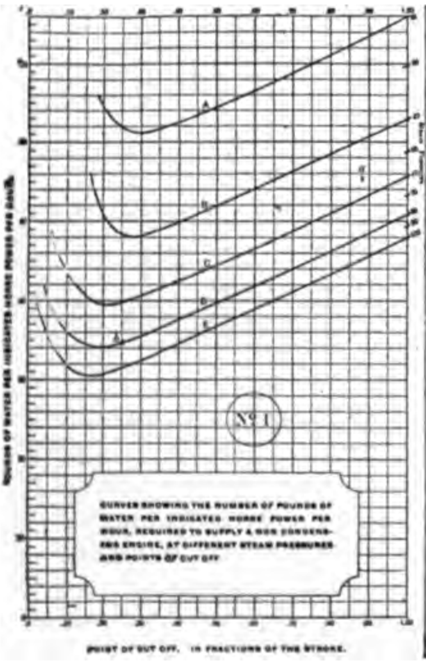


FIG. 145.—ACTUAL WATER EXPENDITURE.

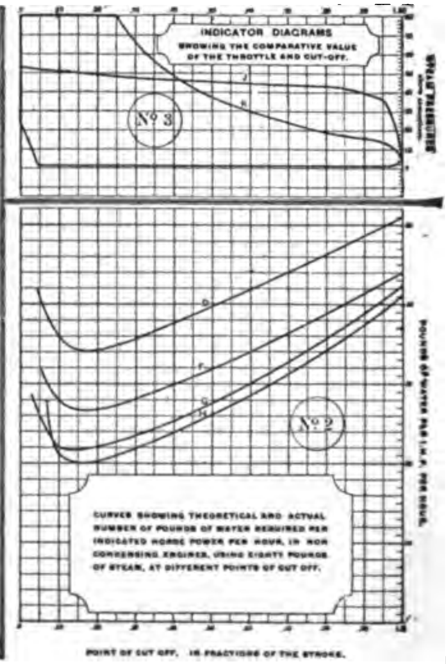


FIG. 146.—COMPUTED EXPENDITURES.

Curve *H* of the series designated No. 2 represents the calculated quantity of water required per indicated horse-power per hour in a non-condensing engine. The calculations take into consideration the weight of steam required to fill the cylinder to the point of cut-off and to supply the heat transmuted

* Trans. Am. Soc. M. E.; 1888; No. CCCXIX.

into work, but make no allowance for cylinder-condensation, for losses by clearance, or for deficiency in work due to insufficient area of passages, or to back-pressure.

Curve *G* is a similar curve based on the additional condition that the clearances and ports equal one twentieth of the cylinder volume.

Curve *D* is *D* in series No. 1, and shows the relative extent of the losses at different points of cut-off due to cylinder-condensation and other causes not included in the calculated results for an engine of 5 horse-power.

The curve *F* was originally interpolated in the position shown from such information as was available at the time to show the probable cost of using steam at 80 pounds pressure in an engine developing about 100 horse-power. Later experiments show that for conditions stated the curve should more nearly approach the curve *G*.

By means of empirical expressions conforming to the curves obtained by experiment, Mr. Emery computed probable approximate values for a somewhat wide range of conditions of operation of non-condensing engines, and tabulated them as shown on page 498.

The table shows that equal economy should be secured in non-condensing engines at somewhat higher pressures than with condensing engines. It would, however, require the use of compound, triple, and quadruple expansion engines, to secure best results. Mr. Emery would restrict the expansion ratio in each cylinder to 2½, in such engines.

The parallelism of Emery's curves indicates that we are usually safe in assuming that the probable cylinder-condensation in the regular working of ordinary unjacketed non-condensing engines is sensibly constant; and at moderate speeds Mr. Buel takes its amount as 15 pounds per hour on each square foot of total internal surface of the engine, including internal superfices of cylinder, of heads, both sides of the piston, the surface of its rod, and the internal surfaces of the steam-passages. The condensation, in ordinary forms of engine, is found, on this basis of computation, to vary somewhat on both sides the as-

STEAM-CONSUMPTION.

1	2	3	4	5	6	7	8	9	10	11
POUNDS OF WATER PER INDICATED HORSE-POWER PER HOUR.										
					Engine of proper size to develop 100 H.-P. $P = 100$.					
Gauge Pressure.	Experimental results at full stroke in small engine, extended to the higher pressures by formula.	Required to fill Cylinder. Approximate.	Required to fill Cylinder. Exact.	Required to supply heat for mechanical work.	Required for cylinder-condensation and miscellaneous losses.	Saved by expansion.	Required at full stroke.	Required with cut-off at 0.6 stroke.	Required with cut-off at 0.3 stroke.	Required at most economical cut-off.
P	E	C_1	C_m	C_3 Calc'd.	$C_3 + C_4$	S for N min.	C $1-c=1$	C $1-c=0.4$	C $1-c=0.70$	C $1-c=$ h. min.
25	74.80	57.75	57.75	2.63	2.16	8.80	62.54	53.74	....	50.44
40	62.99	47.44	47.25	2.61	1.97	14.81	51.83	43.03	....	37.28
60	52.29	41.99	41.49	2.60	1.68	16.72	45.77	36.97	30.37	29.05
80	51.00	39.33	38.56	2.58	1.48	17.82	42.62	33.82	27.22	24.80
100	48.26	....	36.32	2.57	1.41	18.57	40.30	31.50	24.30	21.73
125	45.96	....	35.24	2.56	1.22	19.16	39.02	30.22	23.62	19.86
150	44.38	....	34.19	2.55	1.15	19.60	37.89	29.09	22.49	18.29
200	42.35	....	32.79	2.53	1.05	20.15	36.37	27.57	20.97	16.22
300	40.25	....	31.21	2.51	0.98	20.75	34.70	25.90	19.30	13.95
400	39.17	....	30.28	2.49	0.96	21.03	33.78	24.93	18.33	12.70
500	38.52	....	29.65	2.47	0.96	22.33	33.08	24.28	17.68	11.35

sumed figure, for non-condensing engines, and to be somewhat greater for condensing engines; sometimes, in the latter case, exceeding 25 pounds, accordingly as the interior surface of the cylinder is a better or a worse heat-reservoir.

The performance of the best class of modern engines in this respect is illustrated by the following tables of data obtained by Professor Reynolds from the triple-expansion experimental engine of Owens College.* (§ 44; Chap. II.)

In the first three cases, in the first table, the steam-jackets were in use; in the last three, they were disconnected. The difference in result is due to the greater cylinder-condensation in the latter case; the total amounts of which are given in the

* Minutes of Proceedings of the Inst. of C. E.; 1889; No. 2407.

AREAS OF DIAGRAMS PER ROUND OF STEAM AND THERMAL EFFICIENCIES OF ENGINES.

Number of trial	44	33	56	41	35	40
Theoretical area, ft. and lbs.	238,645	233,545	228,420	235,500	233,000	221,860
Measured area, ft. and lbs.	188,096	192,067	192,000	127,545	139,546	144,350
Percentage of theoretical area	79.0	82.0	84.6	54.0	60.0	65.0
Theoretical efficiency, p. c.	23.3	23.2	22.7	23.3	23.2	23.3
Measured efficiency, p. c.	18.5	19.2	19.4	14.1	15.3	15.5
Percentage of theoretical efficiency	79.4	82.6	83.1	60.4	65.9	66.4

CONDENSATION WITHOUT JACKETS.

	Number of the Trial.	Revolutions per Minute.	Ratio of Expansion.	Proportion of Total Steam condensed at Cut-off.	Mid-stroke.	Release.
Engine No. I	41	146	2.7	0.40	0.39	0.30
	35	229	2.3	0.29	0.27	0.22
	40	322	2.0	0.22	0.21	0.17
Engine No. II	41	127	2.4	0.41	0.35	0.29
	35	215	2.4	0.38	0.34	0.26
	40	320	2.2	0.30	0.27	0.14
Engine No. III	41	109	2.7	0.51	0.48	0.37
	35	184	3.05	0.48	0.47	0.33
	40	276	2.6	0.32	0.36	0.23

second table; in which the proportion thus noted ranges from about one fourth to about one half but is usually greatest in the high-pressure and least in the low-pressure cylinder. These variations from the ideal case are well shown by the diagram in Fig. 147, in which the actual diagram is the inner and the ideal the outer, in each case; the departure from the curve for saturated steam and the modified shape of the diagram exhibiting the effect of introduction of the essential practical conditions of design and operation.

129. The Laws of Variation of Losses, internally, are not fully ascertained. The experiments of Clark, Hirn, Isherwood, and their successors, exhibit the general method of variation already indicated; but the exact law remains to be determined.

The Author has usually taken the magnitude of this loss in any given engine, other conditions invariable, to be sensibly proportional to the square-root of the ratio of expansion, and to be ordinarily measured, in engines of moderate size, as a percentage

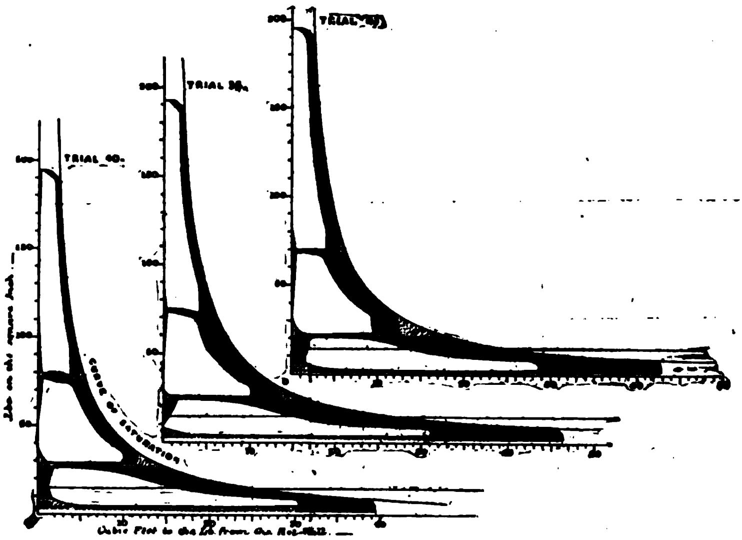


FIG. 147.—REAL AND IDEAL DIAGRAM.

of the quantity of steam or of fuel demanded by the perfect, ideal, engine under similar conditions, by from one tenth to one fifth that quantity, accordingly as this waste is more or less well provided against.

In experiments directed by the Author, the weight of steam condensed per square foot of surface exposed, up to the point of "cut-off," and per degree Fahr. and per minute, ranged from 0.015 to 0.020 pound, corresponding to from 14 to 18 British thermal units.* For ordinary single-cylinder, unjacketed engines, it may be taken, under usual conditions, at the higher

* Cylinder-condensation in Steam-engines; R. H. Thurston; Trans. Am. Assoc. for Advancement of Science; 1885. Journal Franklin Inst.: Oct. 1885.

figure; assuming the law to be capable of expression by a direct and simple function.

The experiments above referred to were made by Messrs. Gately and Kletsch, upon an unjacketed, simple, engine of 18 inches diameter of cylinder, 42 inches stroke of piston, both with and without condensation. The valve-gear was of the Corliss variety, and its action such as to secure a quick and accurate cut-off at the desired point. Four series of experiments were undertaken, in each of which two conditions were made variable, one independent, the other dependent; all others being for the time kept constant; thus ascertaining:

(1) The variation of condensation with varying ratios of expansion;

(2) Varying pressures, the condenser being in use;

(3) The same, without condenser;

(4) Variation with changing speeds of engine.

The results obtained and the deductions therefrom must be accepted as only approximate. Some irregularities will be detected in all such experimental work to date, which are probably mainly due to inevitable variations in the amount of priming and quality of steam used.

(1) To determine the amount of condensation in the steam-cylinder, up to the point of cut-off, the difference was taken between the amount of water pumped into the boiler as determined from weir measurement and the amount shown by the indicator. The ratio of this quantity to the true amount is the fraction of cylinder-condensation up to the point of cut-off. The per cent of condensation, as determined, increases as the ratio of expansion increases.

Fig. 148 shows the final results clearly, the ordinates representing cut-off, and the abscissæ the condensation expressed in per cent of the total amount of steam furnished to the engine, thus:

Cut-off	.589;	cylinder-condensation		= 22.73	per	cent.
"	.443;	"	"	= 27.08	"	"
"	.330;	"	"	= 33.87	"	"
"	.131;	"	"	= 50.07	"	"

Thus the condensation increases rapidly with expansion of steam; or, in other words, with longer exposure of the sides of cylinder, cylinder-head, and piston, to the decreasing temperatures of the expanding steam and the exhaust.

Plotting these results, we obtain the curve as represented in Fig. 148. As will be seen later, it is very probable that this

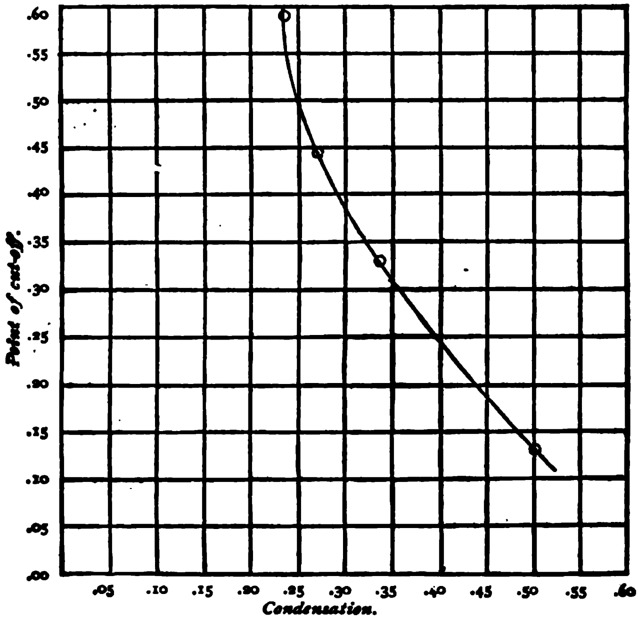


FIG. 148.—CONDENSATION WITH EXPANSION.

curve is just as well taken, as by Professor Cotterill, as logarithmic. It is, however, closely represented by the hyperbola having the equation

$$(x + 0.12)(y + 0.44) = 0.2472;$$

where x is the condensation and y the ratio of expansion; or, referring the curve to its asymptotes,

$$x'y' = 0.2472.$$

At full stroke, $y = 1$, and $x = 0.12$. and the condensation here becomes twelve per cent; a result closely corresponding with

the earlier figures obtained by Isherwood on the U. S. S. Michigan. When we approach the limit, $y = 0$; $x = 0.68$, two thirds the steam is condensed. M. Demoulin has independently reached the conclusion that this curve is hyperbolic.

In this engine, the area of internal surface exposed, up to the point of cut-off, and of which the increments are constant, with uniform variation of the cut-off, is measured by

$$A = 3.96 + 16.5x,$$

in square feet; and the variation of condensation with this varying area is shown in the next figure.

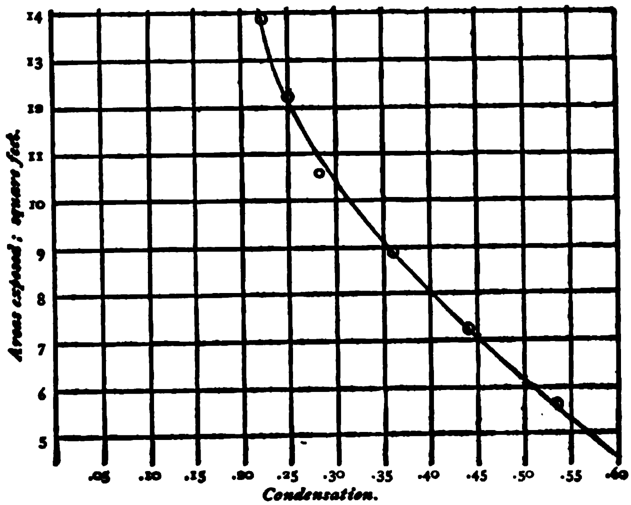


FIG. 149.—CONDENSATION WITH VARYING SURFACE.

The equation of the curve is

$$(x - 4.77)(A - 1.0266) = 216.47.$$

The Author has been accustomed to assume that the curve may be taken as parabolic, and to use the more manageable expression for the first case, above,

$$x = a\sqrt{r} = a\sqrt{\frac{1}{c}};$$

in which the condensation is expressed in terms of the reciprocal of the cut-off, $\frac{1}{c} = r$, the ratio of expansion, a, being a coefficient having a constant value in the same engine, the value of r only varying. From the data just given, and for this engine,

$$\begin{aligned} \text{when } r &= 6.66 +, & a &= 0.187; \\ r &= 4.00, & a &= 0.1987; \\ r &= 2.857, & a &= 0.1923; \\ r &= 2.222 +, & a &= 0.1812; \\ r &= 1.82, & a &= 0.174. \end{aligned}$$

From which it will be seen that the value of a is one fifth approximately, and the results of this investigation very closely coincide with those of earlier experiments and deduced by the Author previously to this investigation.

The hyperbolic equations give the following figures:

$A = 13.86$;	x = cylinder-condensation =	22.01;	error − 0.39
" 12.21;	" " "	25.00;	" ± 0.000
" 10.56;	" " "	28.50;	" ± 0.000
" 8.91;	" " "	33.50;	" − 2.50
" 7.26;	" " "	41.06;	" − 2.94

Also:

y = cut-off = 0.13;	x = cylinder-condensation	= 0.499;	error − 0.001
" = .225;	" " "	= 0.410;	" 0.000
" = .33;	" " "	= 0.338;	" − 0.002
" = .45;	" " "	= 0.274;	" + 0.004
" = .59;	" " "	= 0.222;	" + 0.002

These equations thus so closely satisfy the record obtained by direct observation that they may be taken sensibly to represent the law of condensation, as a function of the ratio of expansion for this engine under the conditions described, and show that the weight of steam condensed is sensibly constant at all ratios of expansion within these limits.

The magnitude of the coefficient, a, in the expression last given above, is obviously different with different engines, decreasing as the size of engine and its speed increase. The value, 0.20, above found will only apply to engines similar to

that here described, in size, speed, and structure, as will be seen later, when deducing the more general expression.

Taken as a function of area of surfaces producing waste, these data show the condensation to be directly proportional to that area.

(2) The variation of internal condensation with varying steam-pressures, all other conditions being as usual and retained constant, was as follows, the expansion ratio being 5:

Gauge-pressure		80	pounds;	condensation	35.24	per	cent.
"	"	66.85	"	"	37.83	"	"
"	"	52.33	"	"	36.84	"	"
"	"	37.0	"	"	41.43	"	"
"	"	22.3	"	"	41.19	"	"

The engine was here worked condensing. The equation of the curve for this case is

$$x = 45 - 0.1266y;$$

in which y is the steam-presssure and x the fraction of total steam condensed. Then

y = pressure	= 80.0 ;	x = cylinder-condensation			= 34.88 ;	error − .036
" "	= 52.33 ;	"	"	"	= 38.38 ;	+ 1.54
" "	= 37.0 ;	"	"	"	= 40.32 ;	− 1.11
" "	= 22.3 ;	"	"	"	= 42.27 ;	+ 1.08

And the equation evidently closely represents the facts for this case. It indicates that condensation would become unimportant at very high pressures; the expression giving $x = 0$ for $y = 355$ lbs. Escher's law gives $x = 300/\sqrt{p_1}$ (abs.).

(3) The non-condensing engine appears, in this case, to have a different method of variation; for, throwing off the condenser, the data obtained give, with the point of cut-off at 0.4, or a ratio of expansion $r = 2.5$, very much less condensation, and it is not, apparently, as before, directly variable with the variation of steam-pressure. Only three trials were made, owing to the impossibility of getting steam steadily for the

highest pressure attempted and the form of the curve for this case is unknown; but the figure shows the lines for both cases:

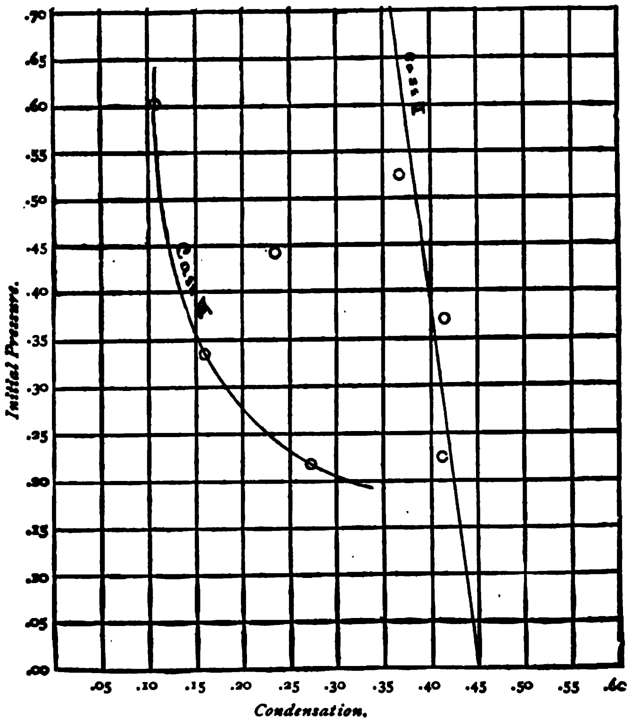


FIG. 150.—CONDENSATION WITH VARYING PRESSURES.

(4) The effect of variation in speed of engine, or time of action of the acting surface of the cylinder, is the final subject of test. Starting with an average boiler-pressure of 19.67 pounds and a cut-off of .98 of the length of stroke and the engine running at an average of 33.74 revolutions per minute, three trials were made, concluding with an average speed of 62.977 revolutions per minute; the greatest variation in the point of cut-off being .05 of the stroke, and in the pressure .63 of a pound. Differences in the condensation occurring can here be attributed purely to the variation of speed.

Difficulty was found in getting the engine to run smoothly lower than thirty-three revolutions per minute, and opportunity was not given to make a fourth test at a higher speed than sixty-three revolutions, the engine being needed for its regular work. But it will be seen by reference to the figure that the three points of the curve given by these three trials are so nearly in line that a fourth test is hardly necessary.

The conditions under which the trials were made were so strictly adhered to, and the results obtained varied so slightly, that an expression from these results determining the amount of condensation as a function of the speed may be taken as strictly representing the losses occurring by condensation in this engine. The greatest variation in the range of pressure for the three tests was three and one half per cent, and the greatest variation in the cut-off amounted to but one half of one per cent.

The per cent of condensation was:

Revolutions	per	minute,	62.977;	per cent	of	condensation,		24.37
"	"	"	50.3;	"	"	"	"	28.75
"	"	"	33.74;	"	"	"	"	33.506

From which it is seen that the condensation varies in this case sensibly inversely as the speed.

Algebraically expressed,

$$x = 45 - 0.33y, \quad x = 0.3\sqrt{t} = \frac{0.3}{\sqrt{N}};$$

and we have, as seen in Fig. 151,

y = Revolutions	per	min. =	62.977;	x = Cyl.	Con. =	24.22;	error − 0.15
"	"	"	50.3;	"	"	28.41;	" − 0.34
"	"	"	33.74;	"	"	33.86;	" + 0.504

Were the law as here expressed to hold good, the line continuing straight to its intersection with the coördinates, the loss of steam would approach 0.45 as the speed approached the zero-limit, and would itself become zero as the engine-speed approximated to 140 revolutions per minute.

Professor Cotterill's expression for this limit of speed, $N = \frac{900}{d^2}$, d taken in feet, would give for zero condensation $N = 400$, nearly, or more than twice the former figure.

Professor Marks has gone over this work to determine the

FIG. 151.—CONDENSATION WITH VARYING SPEED.

value of the so-called "condensation-constant" C, in the expression for waste,

$$W = ACt\,(T_1 - T_2),$$

for this engine, under the various conditions of its operation, accepting the assumption that wastes are proportional directly to time of exposure to the *exhaust steam*, rather than to the square-root of that quantity. Here A, t, T_1, T_2, represent area of exposed surface; time of exposure, and temperatures. The following are his results for the non-condensing engine here studied (units: lbs., ft., min.):

CYLINDER-CONDENSATION.

SIMPLE, NON-CONDENSING ENGINE.

(Experiments of Messrs. Gately & Kletzsch, Sandy Hook, 1884.)

Reference Number of Experiment.	Duration of Experiment.	Stroke of Cylinder in feet.	Diameter of Cylinder in feet.	Reciprocal of True Number of Expansions.	Number of Strokes per minute.	Quality of Steam used (British Units).	Absolute Pressure in pounds per square inch at Cut-off.	Absolute Steam-pressure of Exhaust at Mid-stroke, pounds per square inch.	Temperature of Cylinder at Cut-off (Fahr.).	Temperature of Cylinder for Exhaust.	Specific Volume of Saturated Steam at Cut-off.	Ratio of Actual to Indicated Steam at Cut-off.	Condensation per min. per 1°. In lbs. of Steam.	Condensation per min. per 1°. In British Units of Heat.
	h. m.													
(7)	1.40	3.5	1.5	0.589	136.52	Dry Sat.	61.54	4.22	294.2	155.2	430	1.294	0.01576	14.29
(8)	2.00	3.5	1.5	0.443	135.9	" "	68.34	3.91	301.14	152.15	390.13	1.371	0.01929	18.00
(9)	1.55	3.5	1.5	0.330	134.64	" "	62.10	4.48	294.80	157.56	426.35	1.512	0.01909	17.31
(10)	2.00	3.5	1.5	0.131	137.9	" "	49.11	3.65	279.84	149.4	530.9	2.003	0.01652	15.15
(11)	2.00	3.5	1.5	0.208	138.03	" "	78.80	3.24	311.02	144.58	342.2	1.544	0.01544	13.82
(12)	1.45	3.5	1.5	0.206	141.44	" "	66.89	3.83	299.69	151.88	398.1	(?)1.917	0.02563	(?)
(13)	2.00	3.5	1.5	0.244	143.46	" "	53.21	3.24	284.95	144.58	492.1	1.583	0.01416	12.94
(14)	2.00	3.5	1.5	0.210	137.82	" "	39.83	3.61	266.74	149.02	642.9	1.707	0.01528	14.16
(15)	2.00	3.5	1.5	0.242	135.85	" "	26.74	3.46	244.26	147.21	936.6	1.700	0.01372	12.94
(16)	3.00	3.5	1.5	0.412	135.96	" "	65.36	14.70	298.16	212	407.5	(?)1.122	0.00884	(?)
(17)	2.30	3.5	1.5	0.420	137.14	" "	50.42	14.82	281.30	212.4	504.5	(?)1.307	0.02288	(?)
(18)	3.00	3.5	1.5	0.401	135.02	" "	40.52	14.88	267.87	212.6	620.8	1.190	0.01416	13.11
(19)	3.00	3.5	1.5	0.466	133.04	" "	28.40	14.84	247.56	212.49	886.6	(?)1.376	0.04090	(?)
(20)	1.30	3.5	1.5	0.938	125.95	" "	27.38	3.15	245.56	146.31	916.2	1.322	0.01404	12.23
(21)	2.00	3.5	1.5	0.961	100.60	" "	28.35	3.86	247.46	151.57	886.8	1.403	0.01528	14.36
(22)	1.45	3.5	1.5	0.981	67.48	" "	28.53	4.96	247.81	161.94	881.6	1.504	0.01449	13.62
													Average....	14.33

Numbers 7–10 illustrate varying expansion; 11–15, varying steam-pressures, condensing; 16–19, varying pressure, non-condensing; 20–22, varying speed of engine.

Major English, computing the wastes in simple marine unjacketed *condensing engines*, obtains just double the above-given constants; and both sets of results correspond fairly with the experience of various other investigators.* For jacketed condensing engines he obtains nearly the mean of the two, and not far from two thirds the higher figure.

* Proceedings Inst. Mech. Engrs.; 1887.

Fourier's expression for heat-absorption, taking t as the symbol for time,

$$Q = a(T_1 - T_m) \sqrt{t},$$

shows that, as the period of exposure to the higher temperature diminishes, the amount of heat-absorption is reduced, and varies inversely as the square-root of the speed of engine, a conclusion independently derived by Escher * from direct experiment, and by the Author by observation of the results of various engine-trials, although not apparently confirmed by those just quoted.

Professor C. A. Smith, in 1880, found a variation of 120° F. in the internal temperature of the metal in a locomotive cylinder, the magnitude of the change varying inversely as the speed of the engine.† Escher found this waste to be proportional, very nearly, other things equal, to the square-root of the absolute pressure of entering steam.

The rate of transfer of heat by this condensation, in engines of large range of expansion, is very great; in average practice a dozen times as rapid as the transfer across the heating surfaces of the steam-boiler. A flow of 6000 B. T. U. per hour in the latter case and of 60,000 units in the former are not exceptional values for transfer on an area of one square foot. Mr. L. S. Marks, collating the work of earlier investigators with his own on the Sibley College experimental engines, finds the exponent of t, above, varying from 0.5 at highest pressures (140 lbs.) to 0.6 at about 100 lbs., and 0.7 at 65 lbs., 0.8 at 55, unity at 40 lbs., and 1.4 at 20 lbs., absolute; $n = 27 \div p_1 + 0.3$, nearly.

Major English finds, as previously taken by the Author,‡ and still earlier by Professor Cotterill,§ that internal, or cylinder, condensation varies, at least approximately, as the square-root

* Engineer; 1882.

† Engineering; 1880; p. 460.

‡ President's Annual Address; Am. Soc. M.E.; 1880. Efficiencies of the Steam-engine; Trans. Am. Soc. M. E.; 1880.

§ Proc. Inst. M. E.; 1871; p. 516.

of the time of action, or as $\frac{s}{\sqrt{N}}$, where N is the number of revolutions per minute and s the area of surface effecting the cooling the entering steam.* This result has been also experimentally confirmed by Escher. He proposes the formula

$$CW = \rho_1 A \frac{s}{\sqrt{N}} \cdot \frac{T_1}{T_m};$$

in which C is the initial condensation in British thermal units, per pound of steam, worked in an *unjacketed* cylinder; W is the weight of feed-water in pounds per stroke; s, the exposed surface of metal at the beginning of the stroke; and T_1 and T_m are the initial temperature of steam, and the mean temperature of the cylinder-walls, at a minimum, both on the absolute scale. ρ_1 is the density of the entering steam. A is a constant, which he finds to be, for cases studied by him, 80 in British units.

For *jacketed* engines, he takes $T_1 = T_m$ and adopts

$$CW = A\rho_1 \frac{s}{\sqrt{N}};$$

in which A becomes 56, indicating a gain of about 30 per cent in the reduction of this waste, by the use of the jacket, for the cases examined.

Major English finds, for re-evaporation, the following expression:

$$RW = B\rho_m \frac{s}{\sqrt{N}} \cdot \frac{T_m}{T_2};$$

where the total surface exposed to the point assumed is s; T_m and T_2 are the mean absolute temperatures and the final absolute temperature of the steam up to and at that point; *un-*

* Proc. Inst. M. E.; Oct. 1889.

jacketed cylinders being assumed. For *jacketed* engines, T_m is made T_1, and then

$$RW = B\rho_m \frac{s}{\sqrt{N}} \cdot \frac{T_1}{T_2}.$$

"Initial condensation and corresponding transfer of heat to the metal will of course go on upon each fresh surface exposed during the stroke; but the supply of heat to effect this is drawn by re-evaporation from that stored up in the surface already exposed;" so that the effect is simply "to distribute it over a larger area."

Thus the excess of re-evaporation over condensation will become, for any elementary movement of the piston,

$$d.RW = B\rho \frac{ds}{\sqrt{N}} \cdot \frac{T_m}{T_2},$$

or

$$d.RW = B\rho \frac{ds}{\sqrt{N}} \cdot \frac{T_1}{T_2},$$

accordingly as the unjacketed or the jacketed engine is taken; B having the value 80 or 56, as the case may be. The net condensation, up to any given point, becomes

$$(C - R)W = \frac{80}{\sqrt{N}}\left(s_1\rho_1 \frac{T_1}{T_m} - s_2\rho_m \frac{T_m}{T_2}\right),$$

or

$$(C - R)W = \frac{56}{\sqrt{N}}\left(s_1\rho_1 - s_2\rho_m \frac{T_1}{T_2}\right),$$

for unjacketed and for jacketed cylinders, respectively.

The total weights of steam per stroke become

$$W = \left[\frac{A}{\sqrt{N}} \cdot \frac{s_2 - s_1}{L} + (1 + c)(1 - n)X\right]\rho_1;$$

in which A is 80 or 56, as the case may be, and X is volume swept through, to point of cut-off, in cubic feet; c is the clear-

ance-ratio to that volume; n is the ratio of cushion to total steam per stroke; and L latent heat per cu. ft. at T_1. A comparison of these expressions with the results of test of the Willans engine, which happens to give the needed data, shows remarkably close correspondence.

It is obvious, on comparison of the data now available, and the varying conditions under which they are produced, that the precise form of the expression for this waste will be determined, as well as, probably, the magnitudes of its constants, both by the condition of the surfaces acting, and by that of the steam supplied, as well as by the variable conditions of operation of the engine itself.

The indications are, as deduced from a study of representative indicator-diagrams, that in about one second, were the time allowed, the process of absorption of heat would be practically, in such cases, complete; while for shorter periods the total absorption would be sufficient to condense steam in proportion to the square-root of the time of exposure; i.e., one half as much, for example, in one-fourth second.

Experiment, as shown by Hirn, proves the initial condensation to be progressive, as the piston advances, up to the point of cut-off, ordinarily; in cases cited by him, increasing from 1 per cent water at the beginning to 31 per cent at the end of the admission-period, and in some cases attaining very much higher proportions. Hirn has shown that, when superheated steam is used, there may exist condensation on the interior surfaces of the cylinder and superheated steam in the midst of the charge, the cylinder containing at the same time water and wet, dry, and superheated, steam.

Careful discrimination should be made between the wastes produced by heat-transfer between metal and steam during the expansion-period and those occurring during the exhaust-stroke. The latter are losses of the whole quantity so transferred; while the former are the differences between the efficiencies of transformation with maximum range of expansion and with the lesser actual ranges for the successive decrements of heat and temperature during transfer. If possible, in all

calorimetric investigations the two quantities of rejected heat should be separately recorded. The condensation at the point of cut-off is often 20 or 30 per cent; where it is reduced to 12 or 15 at the end of the expansion, and the waste by surrender of heat by the metal during the exhaust period is 10 to 20 per cent.

The advantage possessed by a valve-gear and system by which separate steam and exhaust ports are provided, in reduction of internal wastes, is obvious, and is practically found to be very considerable; especially when, as is usual, the valves are so placed at each end of the cylinder as to reduce the "dead spaces" to minimum volume. Large marine engines are now designed, in some instances, with separate steam and exhaust valves and ports; the valves being of the piston variety and double-ported.

Re-evaporation taking place during expansion gives rise to a real gain of efficiency; yet evidently a loss occurs if a comparison is made with the case in which the same steam, instead of being initially condensed, is worked from initial, maximum pressure and temperature.

While, other conditions being equal, increase of engine-speed decreases wastes, there will always be found a practical limit, if nowhere earlier, at the point at which the resulting decreased mean forward-pressure and increased back-pressure give rise to overbalancing loss. This limit is set farther away as ports are enlarged; but, again, in a new compensation, enlarged ports increase the cost, in work, of the operation of the valves and gear.

While the theory of heat-engines can only give a general knowledge of practical and of applicable principles in their design and operation, it may point the way to further improvement, may serve as a guide and check in novel constructions, and, coupled with experimental knowledge, may even, in some cases, enable a computation to be made of probable efficiencies that may be useful, if not substantially exact. In all cases, however, the engineer checks his work by reference to the experience already had with engines of as nearly as possible sim-

ilar kind and under similar conditions of operation. Experience, after the machine has been built and set at work, finally enables him to make precise adjustment, where his preliminary estimates had given him approximations.

The following table, computed by Mr. Thompson, exhibits the probable water-consumptions and the mean effective pressures for ideal cases, with a correction for initial condensation and leakage, the engines being assumed to be of good construction and of fairly large size.

These allowances for internal wastes amount to about $0.12\sqrt{r}$ and $0.15\sqrt{r}$, for non-condensing and for condensing engines respectively. The Author would allow one half greater loss in engines of common form and of one or two hundred horse-power. The figures should obviously diminish with increasing sizes and speeds, and should increase as the engines are smaller and speeds lower, as elsewhere shown.

In conclusion: The variation of condensation, with changes of pressure and temperature, under usual conditions of practice, is thus found to be very moderate, and to follow a very simple law, so far as it can be traced. The waste with varying speed of engine is found, also, to be nearly as previously indicated by the Author; but the law is less exactly determined than in the case of varying expansion. Since, however, in all ordinary cases, in practice, the speed of engine and the boiler-pressure are practically constant, in the regular operation of the engine, the most important part of the investigation is that relating to the ratio of expansion. The next most important matter is the determination of the variation of loss with varying speed of engine, and the results here reached are sufficiently exact to be very useful, both to the designer and the owner of engines, although the precise method of variation and its exact algebraic expression still remain subjects for investigation. The last investigation, relating to variation with change of pressures, is interesting as bearing upon the future of the continually progressing advance in the direction of increasing pressures. The last two lines of research demand still further exploration. The results here reached must

WATER-CONSUMPTION PER H. P. AND PER HOUR.

1-10 Cut-off. Add for Internal Condensation N. C. 35% and C. 40%.

Initial Pressure.	M. E. P. N. C.	M. E. P. C.	Terminals.	Rates. N. C.	Rates. C.
40	3.65	13.65	6.41	51.4	16.4
45	5.42	15.42	7.00	38.5	16.0
50	7.19	17.19	7.59	31.9	15.6
55	8.96	18.96	8.17	28.1	15.2
60	10.73	20.73	8.76	25.3	14.9
65	12.50	22.50	9.35	23.3	14.6
70	14.27	24.27	9.93	21.8	14.4
75	16.04	26.04	10.52	20.6	14.2
80	17.81	27.81	11.11	19.7	14.0
85	19.58	29.58	11.70	19.0	13.8
90	21.36	31.36	12.28	18.4	13.6
95	23.13	33.13	12.87	17.9	13.5
100	24.90	34.90	13.46	17.5	13.4

15-100 Cut-off. Add for Internal Condensation N. C. 30%, and C. 35%.

Initial Pressure.	M. E. P. N. C.	M. E. P. C.	Terminals.	Rates. N. C.	Rates. C.
40	9.05	19.05	9.07	31.3	16.8
45	11.32	21.32	9.87	27.7	16.4
50	13.59	23.59	10.72	25.3	16.1
55	15.86	25.86	11.55	23.4	15.8
60	18.12	28.12	12.38	22.1	15.6
65	20.39	30.39	13.20	21.1	15.4
70	22.66	32.66	14.03	20.3	15.2
75	24.92	34.92	14.86	19.5	15.0
80	27.19	37.19	15.69	18.8	14.8
85	29.46	39.46	16.51	18.4	14.6
90	31.72	41.72	17.34	18.0	14.5
95	33.93	43.93	18.17	17.6	14.4
100	36.26	46.26	19.00	17.3	14.3

1-5 Cut-off. Add for Internal Condensation N. C. 27%, and C. 31½%.

Initial Pressure.	M. E. P. N. C.	M. E. P. C.	Terminals.	Rates. N. C.	Rates. C.
40	13.46	23.46	11.79	27.9	17.7
45	16.15	26.15	12.87	25.7	17.3
50	18.85	28.85	13.94	24.0	16.9
55	21.54	31.54	15.00	22.7	16.6
60	24.24	34.24	16.08	21.7	16.4
65	26.93	36.93	17.15	20.9	16.2
70	29.63	39.63	18.23	20.2	16.0
75	32.32	42.32	19.31	19.6	15.8
80	35.02	45.02	20.39	19.1	15.7
85	37.71	47.71	21.46	18.7	15.6
90	40.41	50.41	22.54	18.4	15.5
95	43.10	53.10	23.62	18.1	15.4
100	45.80	55.80	24.70	17.8	15.3

25-100 or 1-4 Cut-off. Add for Internal Condensation N. C. 24%, and C. 28%.

Initial Pressure.	M. E. P. N. C.	M. E. P. C.	Terminals.	Rates. N. C.	Rates. C.
40	17.34	27.34	14.49	27.2	18.5
45	20.39	30.39	15.81	25.3	18.2
50	23.45	33.45	17.13	24.0	17.9
55	26.50	36.50	18.45	22.9	17.6
60	29.56	39.56	19.77	22.0	17.4
65	32.61	42.61	21.09	21.3	17.2
70	35.67	45.67	22.41	20.8	17.0
75	38.72	48.72	23.73	20.4	16.8
80	41.78	51.78	25.05	20.0	16.6
85	44.83	54.83	26.37	19.6	16.5
90	47.89	57.89	27.69	19.3	16.4
95	50.94	60.94	29.01	19.0	16.3
100	54.00	64.00	30.33	18.7	16.2

3-10 Cut-off. Add for Internal Condensation N. C. 22%, and C. 26%.

Initial Pressure.	M. E. P. N. C.	M. E. P. C.	Terminals.	Rates. N. C.	Rates. C.
40	20.75	30.75	17.11	27.0	19.4
45	24.13	34.13	18.67	25.5	19.1
50	27.50	37.50	20.24	24.3	18.8
55	30.87	40.87	21.80	23.3	18.5
60	34.24	44.24	23.37	22.5	18.3
65	37.61	47.61	24.94	21.9	18.1
70	40.98	50.98	26.51	21.4	17.9
75	44.35	54.35	28.07	21.0	17.7
80	47.72	57.72	29.64	20.6	17.5
85	51.09	61.09	31.20	20.2	17.3
90	54.46	64.46	32.77	19.9	17.2
95	57.83	67.83	34.33	19.6	17.1
100	61.20	71.20	35.90	19.4	17.0

35-100 Cut-off. Add for Internal Condensation N. C. 20%, and C. 24%.

Initial Pressure.	M. E. P. N. C.	M. E. P. C.	Terminals.	Rates. N. C.	Rates. C.
40	23.70	33.70	19.80	27.5	20.4
45	27.32	37.32	21.61	26.3	20.0
50	30.94	40.94	23.42	25.3	19.7
55	34.56	44.56	25.23	24.4	19.5
60	38.18	48.18	27.04	23.6	19.3
65	41.80	51.80	28.85	22.9	19.1
70	45.42	55.42	30.66	22.3	18.9
75	49.05	59.05	32.47	21.8	18.7
80	52.68	62.68	34.28	21.4	18.5
85	56.31	66.31	36.09	21.1	18.4
90	59.94	69.94	37.90	20.8	18.3
95	63.57	73.57	39.71	20.6	18.2
100	67.20	77.20	41.52	20.4	18.1

4-10 Cut-off. Add for Internal Condensation N. C. 18%, and C. 21½%.

Initial Pressure.	M. E. P. N. C.	M. E. P. C.	Terminals.	Rates. N. C.	Rates. C.
40	26.22	36.22	22.44	28.3	21.4
45	30.08	40.08	24.49	26.9	21.1
50	33.95	43.95	26.55	25.8	20.8
55	37.81	47.81	28.60	25.0	20.5
60	41.68	51.68	30.66	24.4	20.2
65	45.54	55.54	32.71	23.9	20.0
70	49.41	59.41	34.77	23.4	19.8
75	53.27	63.27	36.82	23.0	19.6
80	57.14	67.14	38.88	22.6	19.4
85	61.00	71.00	40.93	22.2	19.3
90	64.87	74.87	42.99	21.9	19.2
95	68.73	78.73	45.04	21.6	19.1
100	72.60	82.60	47.10	21.4	19.0

1-2 Cut-off. Add for Internal Condensation N. C. 14%, and C. 17%.

Initial Pressure.	M. E. P. N. C.	M. E. P. C.	Terminals.	Rates. N. C.	Rates. C.
40	30.50	40.50	27.78	28.5	23.4
45	34.75	44.75	30.33	27.6	23.1
50	39.00	49.00	32.88	26.9	22.8
55	43.25	53.25	35.43	26.3	22.5
60	47.50	57.50	37.98	25.8	22.2
65	51.75	61.75	40.52	25.3	22.0
70	56.00	66.00	43.07	24.9	21.8
75	60.25	70.25	45.61	24.5	21.6
80	64.50	74.50	48.16	24.2	21.5
85	68.75	78.75	50.70	23.9	21.4
90	73.00	83.00	53.25	23.7	21.3
95	77.25	87.25	55.79	23.5	21.2
100	81.50	91.50	58.34	23.3	21.1

M. E. P., N. C., = lbs. of Mean Effective Pressure, Non-Condensing. Rates, N. C., = theoretical water-consumption in lbs. per I. H. P., per hour, non-condensing.

M. E. P., C., = lbs. of Mean Effective Pressure, Condensing. Rates, C., = theoretical water-consumption in lbs. per I. H. P., per hour, Condensing.

be regarded, at present, as applicable, in the theory of the steam-engine, only provisionally, and as to be accepted finally, only after repeated experiment.

Collating the facts, so far as known, the Author has continued to employ the expressions, based on Fourier's work and on experiment,

$$x = \frac{a}{d}\sqrt{\frac{r}{N}}; \quad x = b\frac{\varDelta T}{d\sqrt{N}}, \text{ nearly};$$

in which x is the fraction of steam condensed; a a constant to be determined for each engine, or class of engines, of similar size, speed, and steam-pressure; b a constant for the general expression; $\varDelta T$ the range of temperature worked through: d the diameter of engine-cylinder or piston; r the real ratio of expansion, and N the revolutions per second.

130. **The Theory of Internal Condensation and Waste** is obviously one of exceeding difficulty; and an exact and rational theory must include so many variable and mutually interacting conditions that it cannot be expected, even if fully developed, to find application, in all cases, in the engineer's, or the designer's, work.

It is commonly assumed that cylinder-condensation will be proportional to the range of temperature between that of the entering steam and the exhaust; to the time of exposure to the exhaust, or inversely as the speed of engine, and to the area of internal surface affected, up to the point of cut-off. On this basis Professor Marks has made comparisons of data derived from a considerable number of experiments, mainly on non-condensing Corliss mill-engines, and has obtained, as already seen (§ 129), as a mean for ordinary work, a value of C, the number of pounds of steam condensed on the square foot of internal cylinder-surface, per minute and per degree range of temperature, $C = 0.02047$, equivalent to 18.13 British thermal units.* The experiments of Messrs. Gately and Kletsch, already described, give from 0.016 to 0.019 pound, or 14.5 to

* Relative Proportions of the Steam-engine; pp. 206–7.

17.4 B. T. U., with ratios of expansion varying from 2 to 7, and an average of 0.0165, nearly, equivalent to 15 B. T. U. for the whole series of trials. The first-given values may probably be found sufficiently approximate for use in estimating the waste in any similar engines.

The value of this constant being determined, the total cylinder-condensation is, approximately, in pounds of steam, per minute,

$$W=CA(T_1-T_2)t\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where C is that constant, say 0.02, A the area of internal surface covered by the steam up to the point of cut-off, in square feet; while t, the time of exposure of heat and steam observed in experiment, is in minutes and constant, at any given speed of engine, and range of pressure and temperature, irrespective of the magnitude of the varying ratio of expansion, and, as in the Sandy Hook experiments, a nearly constant product can be obtained for the product of the "cut-off" and percentage of waste. The same investigation and Professor Marks's deductions from the reported data show that it is most likely to be the time of either expansion or of exposure to the exhaust, more probably to their sum, which should be taken for t while it is not yet ascertained whether the function to be accepted is the square-root of that quantity, its first power, or some intermediate function. It is probable that the several expressions of Professors Cotterill and Marks and of the Author will find use, as at least approximate, each in its appropriate place. (See Notes.)

In producing an expression for the magnitude of internal heat-wastes by condensation and later evaporation, we may adopt, as already seen, either of several methods, which are more or less closely approximate. Following Fourier, we find that the quantity of heat thus stored and wastefully restored may be taken as proportional to the area of surface acting, the temperature range, or temperature head, and the square-root of the time of action. All these proposed expressions are based upon these assumptions or upon experimental data con-

firming them; except that it remains questionable whether this function of the time is of either constant form or value.

That the time to be taken is not that of exposure to the entering steam, up to the point of cut-off, is certain, from the fact that the total loss is never so proportional.

Experiment has indicated, however, that the waste often varies more nearly as the square-root of the time of action, and the Author has been accustomed so to take it.* It is more convenient, usually, to take this loss as a function of the ratio of expansion, as seen later, and experiment has been found to indicate this function to be the square-root, approximately, of that ratio. If the expression be taken to include all internal wastes of any engine, a single trial of that engine, or successive trials within the usually restricted range of every-day operation, will determine values of constants with such exactness as to establish the "curve of efficiency" with ample accuracy. (See Chaps. VII, VIII, and Appendix.)

It will often be sufficient, in the solution of special problems, to assume approximate data. Thus, the Marks coefficient will answer all purposes in the comparison of engines of different sizes working under otherwise similar conditions. The expression

$$W = \frac{W_1 a}{D} \sqrt{rt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

may be used in computing the probable weight of steam needed to supply cylinder-wastes; when W_1 is the weight needed, for the representative ideal case, no initial condensation occurring; a is the constant determined as already indicated, and which may be taken as not far from 4 for unjacketed and 3 for well-jacketed cylinders, for average cases. D is the diameter of cylinder in inches, r is the ratio of expansion, and t is the time of revolution, in seconds. In the Gately and Kletsch experiments, $\frac{a}{D} = 0.2$, approximately, and $t = 1$, very nearly.

* Trans. Am. Soc. M. E.; May 1881. Jour. Franklin Inst.; May 1881.

The value of the constants a and c in the expression for waste,

$$W = \frac{a}{d}\sqrt{\frac{r}{N}} = c\sqrt{r},$$

may, with superheating, or better thermodynamic conditions in other respects, be reduced considerably below the values here assumed as representing ordinarily good average practice. Mr. Barrus gives data from trials conducted by him in which they fall to, in some cases, two thirds, and in other cases to one half, those obtained in earlier work by the Author. The range would seem to be from, approximately,

$$a = 2 \text{ to } a = 3, \quad c = 0.10 \text{ to } c = 0.15;$$

where steam is used without superheating, and to as low as

$$a = 1 \quad \text{and} \quad c = 0.05,$$

or less, with steam effectively superheated; d being taken as the diameter of the cylinder in inches.* Messrs. Calendar and Nicholson find the heat-transfer 0.74 B. T. U. per second per square foot per degree difference of temperature between metal and steam.

The assumption, in our equations, that heat-transfer, in cylinder-condensation, is sensibly proportional to the range of temperature, in ordinary cases, seems justified by experiment. This conclusion of Rankine and later engineers is confirmed, not only by the cases earlier studied by the Author, but also by later investigations. Mr. Bodmer finds this fact illustrated by the engine-trials both of Willans and of Major English. His conclusions are usually substantially in accord with those previously reached. He finds the quantity of heat transferred, and of steam condensed wastefully, proportional to the total area of the walls of the clearance and port spaces, as already seen.† In the expression, for simple engines, taken by Bodmer,

$$Q = C(T-t)S \div N^{\frac{1}{2}},$$

* Trans. Am. Society Mech. Engrs.; vol. XI; 1890; pp. 170, 175; tables.

† Industries; Oct. 17, 1890; p. 385.

Q is the heat transferred in B. T. U.; $T - t$ the temperature-range; S the area of clearance and port surfaces, in square feet; and N the number of revolutions per minute. The value of C varies from about 0.1 B. T. U. in simple, unjacketed, engines, to .05 for good jacketed cylinders, and appears to be fairly constant for any one engine.

Collating a large number and great variety of experimental data, Professor C. R. Richards has proposed the following semi-empirical formula for the percentage of waste by initial cylinder-condensation, in simple unjacketed engines:

$$x = 27.7 \sqrt{r} - 6.25 \sqrt{N} - 0.25 p_1 + C;$$

in which x = percentage condensed of total steam supplied; r = real ratio of expansion; N = revolutions per minute; p_1 = absolute initial pressure in pounds per square inch; C = a constant for any one engine, but a quantity varying inversely as the diameter of the cylinder; and $C' = Cd = 1000$, nearly, for non-condensing, 1200 for condensing engines; d in inches.

Mr. A. C. Rice adopts the expression $x = C\dfrac{(r-1)^a}{dN^b}$ with pressure, and tabular values from these curves as given below:

VALUES OF CONSTANTS.

Pressure, Lbs.	a	b	C
40	0.620	0.440	168
60	0.568	0.412	133
80	0.517	0.384	106
100	0.466	0.359	87
120	0.414	0.333	78
140	0.363	0.306	75

Calendar and Nicholson deduce

$$x = \frac{100cr}{100 + cr};$$

where c is a constant, 10 for the case reported.

Both thermodynamic science and experiment on steam-turbines indicate that the quality of the steam has no economic importance in purely thermodynamic operations. The presence of water in the reciprocating engine, nevertheless, may affect

seriously the quantity of cylinder-condensation and of internal wastes at high expansions.

Professor C. A. Smith concluded, after studying the work of Isherwood and other early investigators, that, as the Author has elsewhere indicated, "the whole excess of water used over that required in a non-conducting cylinder is rudely proportional to the difference of temperature between the incoming and outgoing steam, and to the diameter of piston;" and such excess is nearly constant, and is independent of the ratio of expansion for ordinary cases, a conclusion which has been seen to be confirmed by both experiment and computation based on theory.

"Willan's law," so called, states that "when the power of a non-condensing, throttling engine is varied, the steam demanded per unit of time will be a constant plus a variable quantity proportional to the power developed." This statement is usually substantially true under the usual conditions of satisfactory operation. It is also true, that the "water-rate" varies nearly inversely as the power developed within the limits of fairly economical loads.

That the assumption of approximately constant wastes by internal cylinder-condensation, as already indicated in several ways and by reference to experiments, may be ordinarily adopted in provisional computations of efficiency and steam-consumption is also evident from an inspection of the curves given by Mr. Emery and by the results of computation by the other methods presented in this work, notably those relating to the compound engine. Mr. Buel has computed, in great detail, the probable demand of steam in an engine of about 150 horse-power, having found for such engines, by experiment, a nearly constant condensation, with varying expansion, of about 15 pounds of steam per square foot of internal surface per hour, if non-condensing, 25 lbs. if condensing.*

A tabular résumé of Buel's work will be found in the appendix with the nomenclature, the formulas, and the results for a wide range of values of the ratio of expansion.

The proportions of the cylinder affect greatly the magnitude of this waste. Thus,

* Am. Machinist; June 30, 1888.

To determine the best proportions of cylinder for any stated mean ratio of expansion, such as has been found to be desirable in designing an engine for a stated location and work:

Let $V =$ volume of cylinder;

$d =$ diameter; $l =$ length of stroke;

$r =$ ratio of expansion; $c =$ cut off $= 1/r$.

Then the surfaces of the two heads and of the two sides of the piston are, each, measured by the expression $\pi d^2/4$; while the area of cylinder, proper, exposed, up to the point of cut-off, at which point condensation practically ceases, is $c\pi dl$. But $l = 4V/\pi d^2$; and the total surface acting to condense the steam is thus, including the surface of one cylinder-head and one side of the piston, and the surfaces of the clearances or dead spaces taken as a constant fraction, e, of the latter,

$$S_1 = e\pi d^2/2 + 4cV/d.$$

Differentiating this expression with respect to d, and making the differential coefficient equal zero, we have

$$e\pi d + 4cV/d^2 = 0; \quad d^3 = 4cV/d\pi;$$

and, since $V = \pi d^2 l/4$, we finally obtain

$$d = cl/e.$$

When the clearances may be neglected, as probably in some engines of the four-valve type, this gives us $e = 1$ and

$$l/d = r.$$

In such engines, the most usual ratio of expansion is probably not far from 4, at the present time, for condensing, and 3 for non-condensing, engines; which give, as proper proportions of cylinder, for the two cases, respectively,

$$l = 4d; \quad l = 3d.$$

These proportions were not uncommon in the time of Watt. Experiment shows that, often, seven or eight tenths of the condensation occurs on the surfaces enclosing the "dead spaces."

Where the clearance spaces become too great to be neglected, as is commonly the fact in marine engines and in the simpler forms of other engines, the proportion of stroke to

diameter of cylinder becomes still greater, increasing in direct proportion to the value of e. The consideration of costs of construction and of space occupied, however, comes in to compel the adoption, more in accordance with modern practice, of shorter strokes of piston and larger proportional diameters. The main point to be kept in view is the fact that the larger the ratio of expansion, the longer, proportionally, should be the stroke; and that condensing engines, for maximum economy of fuel and steam, for highest duty, must have longer strokes than non-condensing engines; although, in all cases, these proportions should, to secure maximum commercial results, be kept far within those indicated by the above considerations alone. The higher the speed of piston and the lower the wasting effect of the interior surfaces of the cylinder, also, and the greater the proportional wastes from the exterior, the nearer do the best proportions approximate equality of stroke and diameter of piston.

To determine minimum waste, as a fraction of the steam supplied, it is thus seen that a measure of the efficiency must be found in terms of ratio of expansion and of size and proportions of cylinder. This must be differentiated for a maximum. As such an expression would, if exact, involve algebraic and transcendental functions, and both of considerable complexity, it must probably be difficult to reduce, and very likely entirely insolvable. Fortunately, however, the investigations of later engineers and physicists give us the means of securing such approximate measures and expressions as serve a good purpose, and such as are usually sufficiently accurate for the purposes of the designer.*

Where the ratio of expansion is not great, the presence of "entrained" water in the steam may not produce any important ill-effect. For the ideal case of the non-conducting engine, this was long ago shown to be true, by Combes.† This deduction was also found to be correct in practice, by Hirn.

Professor Cotterill, observing that the range of temperature

* Manual, Vol. I., Chap. VII.

† Théorie Mécanique de la Chaleur; 1863; § xxxv. p. 157.

is approximately proportional to $\log_e r$, adopts, for usual ranges of expansion in a single cylinder, the expression,

$$y = C \log_e r + d\sqrt{N};$$

in which y = condensation-ratio;
r = expansion-ratio;
d = diameter of cylinder, in feet;
N = revolutions per minute;

and finds the value of the coefficient C to average about 5; varying from 4 to 7 with the nature of the surfaces characteristic of the engine.* In this, as in all the preceding cases, the time-function is based on the period of action of the exhaust. It is nevertheless obvious that the time of action of the pro-

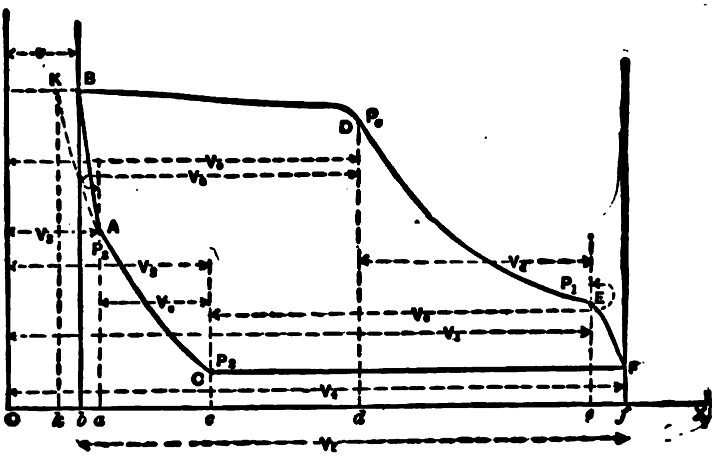


FIG. 152.—HEAT-TRANSFER.

gressive cooling during expansion must modify this effect. Since the total initial condensation, which is usually principally waste, is determined by the extent of the antecedent cooling, it would seem that these functions of time, which actually determine this waste, can be only approximately proportional to N, or to its functions as taken.

The total steam demanded is obtained by multiplying the ideal quantity by $1 + y$, the "liquefaction-factor."

* This value for the Author's work on mill-engines is 6, nearly.

Hirn's method of distinguishing the various heat and work effects, as formulated by Professor Dwelshauvers-Dery, is illustrated by the following:*

The indicator-diagram, Fig. 152, completed by marking on a proper scale the volume of the clearance, v, shows the quantities which the diagram and the data ought to furnish.

The following volumes, expressed in cubic feet, are taken directly from the engine:

v, volume of the clearance space.
V_0, " occupied by the steam at the point of cut-off.
V_1, " " " " " end of expansion.
V_2, " " " " " " exhaust.
V_3, " " " " " " the compression.
V_4, " " " " " " " stroke.
$V_a = (V_3 - v) + (V_0 - v)$ is the volume swept through during admission.
$V_d = V_1 - V_0$ is the volume swept through during expansion.
$V_e = (V_4 - V_1) + (V_4 - V_2)$ is the volume swept through during exhaust.
$V_c = V_2 - V_3$ is the volume swept through during the compression.
$V_f = V_4 - v$ is the volume swept through during a stroke.

On the diagram the pressures are measured in lbs. per square foot:

P_0, pressure at the end of admission.
P_1, " " " expansion.
P_2, " " " exhaust.
P_3, " " " the compression.

$T_a'' =$ area *bBAab* is the work in thermal units during the admission before the beginning of the stroke.
$T_c' =$ area *bBDdb* is the work in thermal units during the admission forward stroke.
$T_a = T_a'' - T_a'$ is the work in thermal units during the admission.

* Trans. Am. Society Mech. Engrs.; No. CCLIX; vol. XI; 1889.

T_d = area *dDEed* is the work in thermal units during the expansion.

T_e' = area *eEFfe* is the work in thermal units during the exhaust before the end of the forward stroke.

T_e'' = area *fFCcf* is the work in thermal units during the exhaust, backward stroke.

$T_e = T_e'' - T_e'$ is the work in thermal units during the exhaust.

T_c = area *cCAac* is the work in thermal units during the compression.

T_f = area $bBDEFfb = T_a' + T_d + T_e'$ is the work in thermal units during the forward stroke.

T_n = area $fFCABbf = T_e'' + T_c + T_a''$ is the work in thermal units during the backward stroke.

$T = T_f - T_n = ABDEFCA$ is the indicated work in thermal units corresponding to a stroke of the piston.

The steam has also done work not indicated on the diagram, represented by the area *aAKka*, which is necessary to accomplish the compression of the steam into the clearance in order to give it a pressure equal to that of the steam entering the cylinder. Since the magnitude of this work is not known, it will be reckoned in the heat exchanged between the steam and the metal during the admission.

Uncertainty exists as to the composition of the mixture of steam and water in the cylinder when exhaust ceases and compression begins. M. Hirn has shown that in general it may be assumed that the mixture contains only steam, all the water which covered the walls having been vaporized and expelled into the condenser during the exhaust, and hence our calculation gives the weight M_c pounds of steam during compression. The volume V_s of steam and its pressure P_s can be ascertained from the diagram, at this instant. From the tables the value of δ_s is deduced, the weight in pounds per cubic foot, and

$$M_c = V_s\delta_s.$$

Experiment must supply the weight M_a pounds of steam which passes into the cylinder at each stroke of the piston and

its quality x, or the weight m of pure steam contained therein at the boiler-pressure. This will be called Q thermal units. From what precedes, its value will be

$$Q = m\lambda + (M-m)q;$$

in which λ and q are the latent heat of the steam and the heat of the water at the given pressure.

During expansion the weight of the mixture in the cylinder is, therefore,

$$(M_a + M_c),$$

and during compression, M_c.

Hence if the steam is saturated, its internal heat will be as follows: During the expansion

$$U = (M_a + M_c)q + m\rho.$$

During the compression

$$U = M_c q + m\rho.$$

If there is a steam-jacket, the water which comes from the condensation is weighed, and its weight ascertained per stroke of piston. Let it be called M_j pounds. This steam is condensed under the mean pressure of the boiler. For each pound, the jacket will have furnished r thermal units. The jacket has then furnished Q' thermal units, and

$$Q' = M_j r.$$

Part of the heat brought in by the jacket will have reached the steam; another part is lost by radiation. The radiation per stroke should be evaluated experimentally. It will be called E thermal units.

When the engine is condensing, the weight of water which leaves the condenser is measured, and from this is deduced the weight of cold water, M_i pounds, introduced into the condenser for each stroke. Its initial temperature t_i is measured and its final temperature t_f. The steam-tables give the heats of the

water q_i and q_f which correspond. Hence the heat rejected per stroke by the cold water is ascertained by means of the equation

$$C = M_e(q_f - q_i).$$

Finally the weight M_a pounds of water comes from the condensed steam and is at the temperature t_f if the condensation is effectuated by injection, and at the temperature t_f' if by a surface condenser. It follows that a second part of the heat rejected into the condenser, which may be called c thermal units, will be determined by one of these two formulæ.

$$c = M_a q_f, \quad \text{or} \quad c = M_a q_f'.$$

The heat rejected in the condenser will be the sum, or

$$(C + c).$$

When there is no condensation, this heat, rejected into the atmosphere, cannot be evaluated.

The vapor in the cylinder carries there Q thermal units at each stroke. It receives from the jacket $Q' - E$ thermal units. It loses T thermal units to overcome the exterior work, and it carries into the condenser $(C + c)$ thermal units. As the *régime* is reached, the sum of all these quantities of heat is zero.

Hence follows the first fundamental equation:

$$Q + Q' - E - T - (C + c) = 0,$$

or

$$Q + Q' = T + (C + c) + E. \quad . \quad . \quad . \quad . \quad . \quad (I)$$

With condensation, all the quantities are given experimentally. This equation can only serve in this case as a check. If the second member should differ sensibly from the first, it would mean that the trial had been badly conducted. Without condensation this equation may serve to determine the value of the rejected heat $(C + c)$.

The quantities of heat received or given up by the metal, exchanged between the metal and the steam, will be designated

by R when expressed in thermal units. The subscript indicates the phase during which the exchange is measured. Hence:

R_a thermal units is the quantity of heat exchanged between the metal and the steam during admission;

R_d thermal units is the quantity of heat exchanged between the metal and the steam during expansion;

R_e thermal units is the quantity of heat exchanged between the metal and the steam during exhaust;

R_c thermal units is the quantity of heat exchanged between the metal and the steam during compression.

R_a, R_d, R_c will have positive signs when the heat passes from the steam to the metal. R_e, on the other hand, is positive when the heat passes from the metal to the steam.

In general, R_c and R_a are positive, and R_d is negative; that is to say, that, generally, the steam warms the metal during the compression and admission, and the metal gives up its heat to the steam during the expansion. The exchange which takes place while the cylinder has no communication with the condenser will be called R_f. It follows that

$$R_f = R_c + R_a + R_d.$$

The total exchange for one stroke of the piston can be called R, and

$$R = R_f - R_e = R_c + R_a + R_d - R_e.$$

This total would be zero if there were no heat denoted by E lost by radiation. $R = E$. When the jacket furnishes Q' thermal units, $Q' = E + (-R)$.

The second equation can then be written as follows:

$$R = E - Q', \quad \text{or} \quad R_f + Q' = E + R_e;$$

or, again,

$$R_c + R_a + R_d = R_e + (E - Q'). \quad . \quad . \quad . \quad . \quad \text{(II)}$$

The quantities designated by R are not given directly by experiment; they must be computed; which requires four new equations, in which R_c, R_a, R_d, R_e will be the unknown quanti-

ties. The equations of the expansion and the compression are easy to write, since the weight of fluid in action is constant. The fluid is enclosed in the cylinder, and cannot exchange heat except with the metal of the cylinder. When U_0 and U_1 represent the internal heat of the steam at the commencement and at the end of expansion, we shall have

$$U_0 - U_1 = T_d + R_d.$$

Similarly, the internal heat of the fluid at the commencement of compression was U_2. The heat T_c resulting from the work of compression is added to this, and this sum ought to preserve for the steam the heat denoted by U_3, and also to give R_c thermal units to the metal; whence

$$U_2 + T_c = U_3 + R_c.$$

In the periods of admission and exhaust, the problem is complicated by the fact that the steam, in coming into the cylinder, carries thither Q thermal units, and, in leaving the cylinder, it carries out $(C + c)$ thermal units to the condenser or the outer air. Whence, for the admission,

$$U_3 + Q = U_0 + T_a + R_a;$$

and for the exhaust,

$$U_1 + R_e + T_e = U_2 + (C + c).$$

These last equations can be written as below, and, adding those preceding, we have

$$\begin{aligned}
Q + Q' &= T + (C + c) + E; \quad \ldots \quad &(\mathrm{I})\\
R_c + R_a + R_d - R_e &= R = E - Q'; \quad \ldots \quad &(\mathrm{II})\\
R_a &= U_3 + Q - U_0 - T_a; \quad \ldots \quad &(\mathrm{III})\\
R_d &= U_0 - U_1 - T_d; \quad \ldots \quad &(\mathrm{IV})\\
R_e &= U_2 + (C + c) - U_1 - T_e; \quad \ldots \quad &(\mathrm{V})\\
R_c &= U_2 - U_3 + T_c. \quad \ldots \quad &(\mathrm{VI})
\end{aligned}$$

The quantity R_a does not represent solely the exchange of heat between the vapor and the metal. There is heat given out by the steam to compress that which, under low pressure,

filled the waste spaces at the end of the exhaust. This heat, not shown by the indicator, is an integral part of R_a. U_2 has been obtained on the hypothesis that, at the commencement of compression, there the steam is dry and saturated.

The object of the computations is to obtain the values of R_c, R_a, R_d, R_e, R_f, and to represent these values graphically. For this purpose the same scale is adopted for the exchange of heat as for the diagrams of pressure, and in the following manner: T_a represents a certain number of thermal units lost by the steam while the piston is generating the volume V_a cubic feet. In like manner, R_a represents the thermal units lost by the steam while the piston sweeps through the same volume. The value of T_a is represented on the diagram by a surface whose length, representing V_a, is the base. If the pressure during admission was constant and equal to p_a, this diagram would be a horizontal line at the height which is represented by p_a, and the area would be rectangular and equal to p_aV_a. If p_a is counted in pounds per square foot, then $p_aV_a = 772T_a$; whence $p_a = \frac{772T_a}{V_a}$.

Similarly, a height r_a can be calculated such that

$$r_aV_a = 772R_a,$$

whence

$$r_a = \frac{772R_a}{V_a};$$

also, in like manner,

$$r_d = \frac{772R_d}{V_d},$$

$$r_e = \frac{772R_e}{V_e},$$

$$r_c = \frac{772R_c}{V_c}.$$

If the exchanges are positive—that is to say, if it is the steam which furnishes heat to the metal—the ordinates r will be

carried above the axis in the forward stroke, and below in the backward stroke. If the exchanges are negative—that is to say, if it is the metal which furnishes heat to the steam—the ordinates r are carried below the axis for the forward, and above it for the backward stroke. In a trial, to be considered later, R_a has been found positive; R_d negative; R_e negative; and R_c positive. Then the diagram of exchanges is shown in Fig. 153.

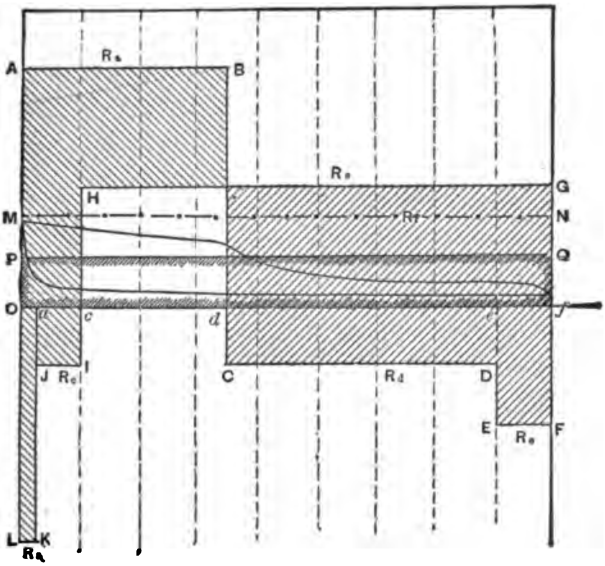
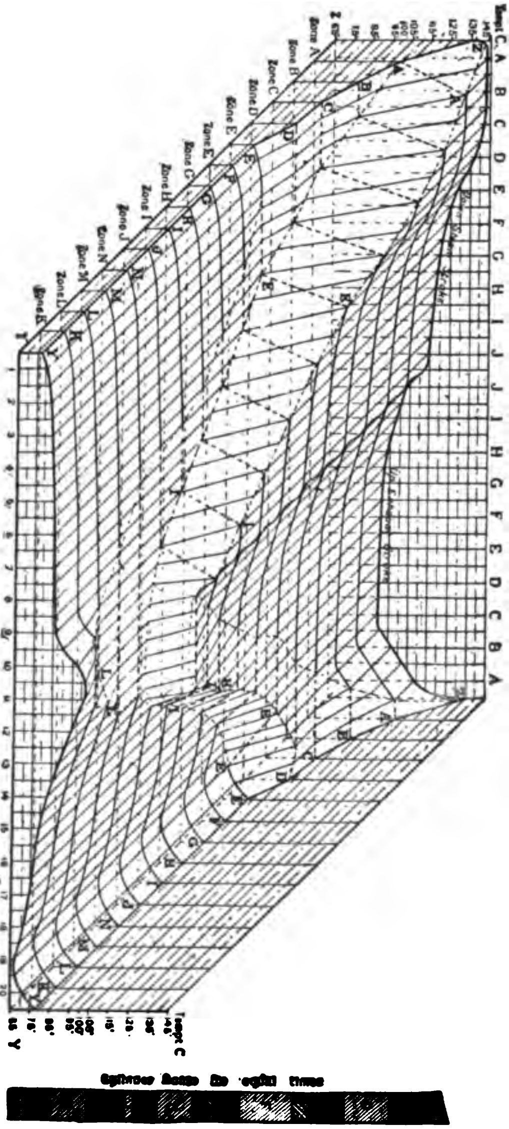


FIG. 153.—HEAT-EXCHANGES.

The area $aKLABda$ represents R_a;
" " $dCDed$ " R_d;
" " $eEFGHce$ " R_e;
" " $cIJac$ " R_c.

Positive exchange is represented by hatchings from left to right; negative transfer by hatchings from right to left. The difference of these two surfaces would be zero if there were no heat lost by external radiation, or received from a jacket. In the example there was a loss. On the diagram is a line MN, at a height such that the surface $OMNfO$ represents $R_f = R_c +$



THE HEAT TEMPERATURE OF CYLINDER WALLS

FIG. 159a.—HEAT-EXCHANGER.

$R_e + R_d$. This is the loss due to the action of the cylinder-walls. The straight line PQ is at such a height that, on the same scale, the surface, $OPQfO$, of which the contour is edged by hatchings, represents the positive work T_f of the steam.

We refer all the quantities to one pound of steam employed. We shall give an example of application in the chapter on engine-trials; which see. This method of waste is beautifully shown by Mr. Bard's relief-model of heat-exchanges in the cylinder, Fig. 153*a*, which was first illustrated in The London Engineer, Oct. 9, 1891.

Hirn and Hallauer have shown that, in ordinary cases, at least, in the computation of the efficiencies of steam-engines, it may be safely assumed: (1) that the weight of vapor remaining in the clearance-spaces may be neglected; (2) that this vapor, in compression, may be considered, if at all, as dry and saturated. It is only when the cylinder is so constructed as to hold precipitated water in its hollows that the presence of the liquid affects in this manner the economical working of the engine. See Chapter VIII for further illustration.

The rate at which the metal surface may condense the entering steam is probably, however, greatly modified by the extent to which water adheres to it and obstructs the entrance of heat, and by the rapidity and thoroughness with which water at any time precipitated is removed. Donkin has made visible such deposited water.

131. The Restriction of Cylinder-condensation may be effected, to a limited extent, by proper precautions.

This loss, as previously stated, is reduced by low ratios of expansion, by increasing back-pressures, by reducing initial pressures, by increasing speed of engine, and by special expedients, as steam-jacketing, superheating, and the division of the expansion between two or more cylinders, in "compound" or multiple-cylinder engines.

The proper methods of prevention of such wastes are, evidently, those reducing: (1) the heat-transferring power of the fluid; (2) the heat receiving and storing power of the surfaces in contact with it; (3) the time of exposure; and (4) the range of

temperature worked through. The methods actually practised or proposed are:

(1) Effectively drying the steam, as by superheating, by compression; by steam-jacketing, and by admixture of air or gas.

(2) Lining the cylinder with non-conductors, or bathing it with oil, or other non-volatile and slowly-conducting substance.

(3) Increasing the speed of engine.

(4) "Compounding."

Superheating is found to be most effective; but it is limited in the extent to which it may be carried; and, practically, up to the present time, it has been found undesirable to attempt much more than to thoroughly dry the steam before its entrance into the engine.

Wet steam has, thermodynamically, no other action in adiabatic expansion in the ideal engine, than dry steam, and the moisture does not affect efficiency. In the real engine, however, while, with low ratios of expansion little if any effect is noted, moisture may, at high ratio, produce sensible and perhaps in some cases serious losses of economy. Mr. Armstrong reports, in such an example, a waste of eight times the quantity of the water introduced with the steam.* This remarkable effect is attributed to the collection of entrained water within the engine-cylinder and its action in exaggerating initial condensation.

Steam-jacketing is frequently practised with compound engines, and sometimes with the simple engine when intended to work at high ratios of expansion.

The introduction of air, by reducing the conductivity of the fluid, has been found by Warsop, experimenting on locomotives,† and by others, to produce, in some cases, a gain of about ten per cent. This last is not a common practice; but the other methods are in common use, and are found to effect an economy which increases with decrease of efficiency in other respects; an economy which may probably average, in successful practice, about twenty per cent.

* Trans. A. S. M. E., 1897, vol. xviii, p. 1066.

† London Engineering; 1873.

Lining the cylinder with a non-conductor, if practicable, would considerably reduce this form of waste. Smeaton, a century ago, so lined his cylinder-heads, using wood for the purpose, and Emery has attempted, though without permanent success, to line the whole interior with glass or porcelain,* and the Author has reduced the heat-absorbing power of such surfaces, in experimental investigations, 40 and 60 per cent.† The free use of oil in the cylinder has been usually found to produce sensible, but costly, gain in efficiency. A well-polished internal surface, especially if bathed in oil, is hardly less effective in reducing wastes than is well-dried steam. Old and carefully handled engines are apt to show a decided advantage over new, and probably for this reason.

High speeds of engine, other things being equal, have been found to give very decided gains, as compared with low speeds; and, both for this reason and on account of the gain in power also to be thus secured, speeds have been increasing steadily, since the time of Watt, from his standard maximum velocity,

$$V = 128 \sqrt{s},$$

—where s is the stroke of piston in feet,—to several times that figure; to more than

$$V = 800 \sqrt{s}$$

in some cases, in locomotive engineering practice. Speeds of rotation have thus been brought up to 100, in even very large marine engines, and to 300 and upward in small stationary engines. At speeds exceeding about $N = \frac{600}{s}$, those wastes become unimportant. This expedient, as affecting a good type of simple engine, has been found quite as effective, on the average, as jacketing or moderate superheating in common practice, and even to give a close competition in

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* Trans. Am. Soc. Mech. Engrs.; 1881.

† Ibid.; 1889. See papers by Carpenter and Royse, also by the Author, in Trans. Am. Soc. C. E.; 1889–91.

many cases with the compound engine operated at low speeds. The following figures further illustrate gain by increasing speeds with a simple engine, as determined by trial (rated load 150 H.P.):

Revolutions per min....	20	30	40	60	80
Lbs. steam (at $r = 4$)...	30	28.5	28	27	26

"*Compounding*," or the use of two or more cylinders "in series," in which the ratio of expansion is restricted, in each, to a practically economical limit, is the now usual system, especially in marine-engine construction, is becoming daily more common with stationary engines, and is also coming into use in locomotives. By the adoption of this plan, steam-pressures and total ratios of expansion for maximum economy have been increased very greatly over those admissible with the single-cylinder engine. While the latter has been found, with steam at 60 pounds pressure, by gauge, to demand, ordinarily, 3 or 3½ pounds of coal, or 25 to 30 pounds of steam, per indicated horse-power per hour, the former, under similar circumstances, requires but 2 or 2¼ pounds of coal, 17 to 20 pounds of feed-water, per I. H. P. per hour; and the "triple-expansion," at 150 pounds, or ten atmospheres, takes 1½ to 1¾ pounds of coal, 14 to 18 pounds of steam; while the "quadruple-expansion" engine, at 12 to 15 atmospheres (180 to 225 lbs. per square inch) is said to demand only 13 to 15 pounds of feed-water, or 1¼ to 1½ pounds of good fuel, figures probably never yet reached by simple engines.

The ratios of expansion, which, with the simple engine, have not been usually successfully carried beyond 5 or 7, are thus increased to 8 or 10 in the "compound," to 12 or 15 with the "triple," and to 15 or 20, or even more, with the "quadruple-expansion" engine. The terminal pressure is usually between ½ and ⅔ atmosphere (7½ or 10 lbs. per square inch), absolute pressure, in the best forms of engine.

Compression of the exhaust as nearly to boiler-pressure as is possible, as already remarked, is decidedly advantageous—and especially with the non-condensing engine—not only as a means of filling the clearance and port spaces, and thus saving some steam, but also, and possibly in some cases to a still more

important extent, by transforming a certain amount of energy into heat and communicating this heat to the cooled surfaces of the cylinder; warming them up to approximately the temperature of the entering steam, thus checking initial condensation to an extent which may much more than compensate the, apparent, added waste of power in compression. With a "link-motion" valve-gear, an increase of the ratio of expansion is accompanied by increased compression, and thus the exaggeration, by increased expansion, of the evil here considered is partly checked by the coincident increase of compression. The locomotive is probably an illustration of marked gain occurring in this manner.

Practical limitations thus restrict the real engine to low efficiencies, as compared with those of the ideal, and the internal wastes, particularly, thus place a limit to the increasing thermodynamic conversion of heat with increasing expansion, as shown in the figure. The economy of the machine

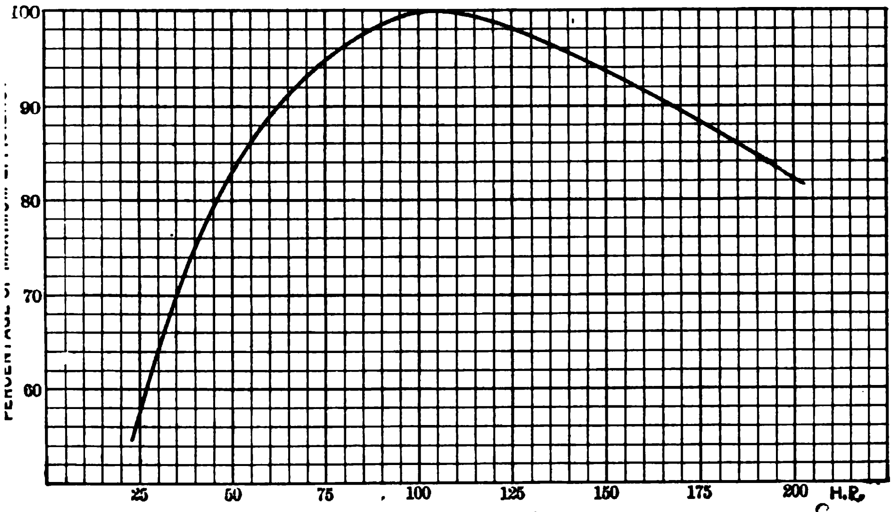

FIG. 153*b*.—EFFICIENCY OF ENGINES WITH VARYING LOADS.

falls off rapidly on either side its proper rating, and particularly with diminishing load. (Fig. 153*b* and p. 583.)

The subjects of compounding, jacketing, and superheating are of such importance as to demand their consideration in a separate chapter.

132. **The Friction of the Engine** itself, the "Internal Friction" of the machine, is usually a considerable quantity, and is a source of loss of energy by reduction of the efficiency of the engine as a machine. Since the efficiency of any train of mechanism, as a machine purely, is the ratio of the quantity of work delivered by it to whatever it may drive, or of work done by it upon the next element in order to the work which it receives, or the energy applied to its propulsion, the internal losses of mechanical energy become, to the engineer, subjects of real importance. As "Friction is thus the principal cause, and usually the only cause, of loss of energy and waste of work in machinery,"* "a given amount of energy being expended upon a driving-point in any machine, that amount will, in accordance with the principle of persistence of energy, be transmitted from piece to piece, from element to element, of the machine or train of mechanism, without diminution, if no permanent distortion takes place and no friction occurs between the several elements of the train, or between those parts and the frame or adjacent parts. Temporary distortion, within the limit of perfect elasticity, causes no waste of energy; permanent distortion causes a loss of energy equal to the total work performed in producing it; but permanent distortion is due to deficiency of strength, and to defective elasticity, and is never permitted, in well-designed machinery, properly operated. Hence the important principle:

"The only cause of lost work in mechanism which is to be anticipated in design, and calculated upon in deducing the theory of any special machine, is the friction necessarily consequent upon the relative motion of parts in contact and under pressure."†

The compound friction of lubricated surfaces, as it may

* Friction and Lost Work in Machinery and Mill-work; R. H. Thurston; N. Y., J. Wiley & Sons, 1885; p. 11; § 12.

† Ibid., pp. 11, 12.

be termed, or friction due to the action of surfaces of solids partly separated by a fluid, is observed in all cases in which the rubbing surfaces are lubricated. In such instances the solids are usually not completely separated by the liquid film interposed between them, but partly rub on each other, and are partly supported by the layer of lubricant which is retained in place by adhesion and by capillary action. The rubbing together of the two solids produces wear, the amount of which is indicated by the rate at which the lubricant becomes discolored and charged with abraded metal. The work of friction, both of solid and of liquid, is transformed into heat and is disposed of at the bearing heats, principally by radiation and conduction to adjacent parts, and partly by the heating of the lubricant. In all cases some abrasion is indicated by the change produced in the lubricant, and some heating is usually perceived in the bearing.

With very heavy pressures and slow speeds, the journal and bearing are forced into close contact, as is shown by their worn and often abraded wearing surfaces; while with very light pressures and high velocities the journal floats on the film of fluid which is continually interposed between it and the bearing. In this case the friction occurs between two fluid layers, one moving with each surface. There are thus evidently two limiting cases between which all examples of satisfactorily lubricated surfaces fall: the one limit is that of purely solid friction, which limit being passed, and sometimes before, abrasion ensues; the other limit is that at which the resistance is entirely that due to the friction of the film of fluid which separates the surfaces of the solids completely.

The laws governing the friction of lubricated surfaces are evidently neither those of solid friction nor those of fluid friction, but will approximate to the one or the other as the limits just described are approached. The value of the coefficient of friction varies with every change of velocity, of pressure, and of temperature, as well as with change of character of the surfaces in contact.

Where mixed friction is met with, it will usually be found

that its laws approximate to those of solid friction as the journal is run dry, and to those of fluid friction as it is the more effectively flooded with oil. Thus a journal or bearing surface fed with oil by an oil-cup, and where no oil-grooves are used to distribute the oil, will exhibit a total friction in some cases nearly proportional to the total pressure, the latter being varied; while similar surfaces flooded with oil, as by the oil-bath, offer a resistance sometimes nearly independent of the pressure, and but little, if appreciably any, greater with heavy than with light loads. A *perfectly* lubricated bearing should follow the laws of fluid friction, and its friction should be independent of the intensity of pressure produced by the load, varying as the square of the speed of rubbing. Such perfect lubrication has never yet been attained.

For perfect lubrication, assuming it practicable with complete separation of the surfaces, the laws of friction would become:

(1) The coefficient is inversely as the intensity of the pressure, and the resistance is independent of the load.

(2) The friction coefficient varies as the square of the speed.

(3) The resistance varies directly as the area of journal and bearing.

(4) The friction is reduced as temperature rises, and as the viscosity of the lubricant is thus decreased.

These laws will probably hold, even with the greases, which all become fluid when introduced between the rubbing surfaces.

It is found by experiment, as stated later, that the perfection of this form of lubrication depends upon the amount of fluid-pressure produced between the surfaces by forcing in the lubricant between them. This separation occurs to an important extent at high speed and less at low velocities. Hence, the friction of lubricated parts is often found to decrease at low speed with increase of velocity, while increasing at high speeds as velocity increases.

The limits of pressure for lubricated surfaces are determined by the nature of the materials composing them, and by their

smoothness and exactness of fit, as well as by the speed of rubbing, the character of the lubricant, and the methods of its application. A higher pressure is usually permissible on hard than on soft material; although when the soft materials, as for example common white alloys for bearings, are well sustained by a harder metal, the heaviest pressures allowed by the lubricant may be carried.

The more viscous the lubricating substance, and the stronger the capillary action taking it into the space between the journal and the bearing, the higher the pressure safely carried. With increase of speed the maximum pressure is lessened, and it is usual to take the intensity of pressure as inversely as the velocity of rubbing.

The magnitude of the waste of energy by friction is measured in horse-power by the expressions (British measure):

(1) Flat surfaces, $$HP = \frac{fPV}{33,000};$$

(2) Cylindrical surfaces, $$HP = \frac{fPRd}{127,000};$$

when f, P, and V are the coefficient of friction, the load and the speed of rubbing in feet, and R and d are the revolutions per minute and diameter of journal in inches.

The methods of reducing waste of energy by friction in mechanism are based upon very simple principles. It is evident that to make the work and power so lost a minimum it is necessary to adopt the following precautions:

(1) Make the coefficient of friction the least by proper choice of rubbing surfaces and by the best lubrication. To do this we should have at least one of the rubbing surfaces of a granular metal, and if possible both—that one which it is easier to replace being of the softer metal. The surfaces should not be subjected to a normal pressure beyond which the lubricating matter will be expelled. For slides, a much less pressure should be taken than for journals, as they have not as free a lubrication as well-arranged cylindrical journals; but this limit

is best determined by reference to the speed of rubbing and the nature of the lubricant.

(2) Make the space through which the friction is to act a minimum by reducing the diameters of all journals to the least compatible with safety under the stresses they are expected to sustain. The work done is independent of the length of the journal, except as it may modify pressures, and thus the coefficient of friction.

(3) Properly fitting the bearing surfaces, removing that portion of the bearing near the jaws, and transferring the bearing surface to the bottom, one sixth of the circumference of the journal may be thus removed. A journal well fitted cold is not necessarily a good fit after it becomes heated by friction, owing partly to the want of homogeneousness of the metal of the journal and bearing; a worn journal has less friction than when new. It is a question whether all journals should not be brought to a proper bearing and given a high polish before they are considered fit to perform their office. It is now usual carefully to grind all cylindrical journals, and to secure a very perfect fit in the bearing before setting the machinery at work.

(4) Giving the journals such forms and such size as will allow them to convey away the heat generated, either by radiation from their surfaces or by conduction through the mass of metal, to circulating water, to lubricating matter, or to adjacent masses.

(5) Securing an efficient system of supply of the lubricant.

Since lubrication has for its objects both the reduction of friction and the prevention of excessive development of heat, the engineer resorts to the expedient of interposing between the rubbing surfaces a substance having the lowest possible coefficient of friction and the greatest capacity for preventing or reducing the development of heat. It is evident that in order that any substance may be efficient as a lubricating material it must possess the following characteristics:

(1) Enough "body" or combined capillarity and viscosity to keep the surfaces between which it is interposed from coming in contact under maximum pressure.

(2) The greatest fluidity consistent with the preceding requirements, i.e., the least fluid-friction allowable.

(3) The lowest possible coefficient of friction under the conditions of actual use, i.e., the sum of the two components, solid and fluid friction, should be a minimum.

(4) A maximum capacity for receiving, transmitting, storing, and carrying away heat.

(5) Freedom from tendency to decompose or to change in composition by gumming or otherwise, on exposure to the air or while in use.

(6) Entire absence of acid or other properties liable to produce injury of materials or metals with which they may be brought in contact.

(7) A high temperature of vaporization and of decomposition, and a low temperature of solidification.

(8) Special adaptation to the conditions, as to speed and pressure of rubbing surfaces, under which the unguent is to be used.

(9) It must be free from grit and from all foreign matter.

Oils must be used with some caution when applied to journals upon which other lubricants have been employed. It sometimes happens that two oils are entirely incapable of working together, and this incompatibility may cause trouble when they are used together, or even successively. A minor good quality possessed by some lubricants in greater degree than others is that of being readily removed, and allowing the bearing surfaces to be easily cleansed when they have become soiled and gummed by alteration of the unguent, and by the gathering of dust and abraded metal upon them.

Oils should not be liable to decomposition by heat or wear, or to separation when mixed, either in use or by long standing, or by alteration of temperature. They should, if mixed, always have the same specified composition. Uniformity in this respect is as important as excellence of quality of the normal mixture, and the quality of the oil is usually of more importance than the quantity. The adhesiveness of the oil to the metal, and the ease of flow, with minimum fluid-friction, are

the essential characteristics of a good combination of materials in bearings and lubricant. Cast-iron is somewhat spongy in texture, and is therefore an exceptionally good metal for bearing surfaces, when of ample area.

Bearing surfaces are of bronze or other alloys, of cast-iron or other metal, or of wood, according to location, intensity of pressure, velocity of rubbing, and nature of the material of the journal. Ordnance bronze wears well under heavy pressures and at high speeds if not subjected to intense localized pressures by the springing or misfitting of parts; cast-iron has an advantage, if used under moderate pressures and in ample extent of surface, in its porosity and absorptive power and the persistence with which oil and grease adhere to it; wrought-iron and steel sustain heavy loads, if free from surface defects; "mild steel" is peculiarly valuable for journals, and hard steel ground to shape and well bedded in its bearing will safely carry pressures of enormous intensity; wood is only used in special cases. Too high a polish on the harder surfaces is objectionable where thin oils and heavy pressures are adopted, as the lubricant is difficult to feed between the metals in contact, or to keep there while in operation.

It is nearly always advisable to make the bearing of the softer metal, since its renewal is a matter of less difficulty and expense than that of the journal, and since the journal must usually have great strength. A hard bearing cuts the softer journal, and gives rise often to serious expense. It is from this consideration that bearings are often "babbitted" or lined with the soft white alloys.

The fitting of the surfaces in contact is as important a matter as the selection of the material of which they are composed. The theory of friction is based upon the assumption that all parts are accurately made to correct dimensions, and exactly fitted; and the conclusions derived are therefore invalidated by any departure from such assumed conditions. Precision and stability of form—stiffness of all loaded parts—are essential elements of successful working. Stability of form is dependent upon extent of surface exposed to wear: if this

area is ample, so that the two rubbing parts nowhere and at no time come into unrelieved metallic contact, no appreciable wear will occur, and their forms will be permanent.

Surfaces of similar area and form, even when well fitted, if of different materials will wear very differently. Thus the following table shows the comparative wear of axle-bearings. Thoroughly pure bronzes, like those fluxed with phosphorus, were reported as wearing very much less than ordinary compositions.

Bearing.	Composition.			Cost per 100 lbs.*	Miles run per lb.	Wear per 100 miles for four bearings.
	Copper.	Tin.	Antimony.			
Gun metal..............	83	17	..	$28 60	25,489	200 grs.†
" "	82	18	..	28 68	27,918	252 "
White-metal............	3	90	7	32 85	22,075	366 "
" "	5	85	10	32 27	24,857	284 "
Lead composition: lead, 84; antimony, 16.......	..	..	..	13 04	22,921	308 "
Gun-metal on brake-cars..	82	18	..	28 68	2,576	274 "

In many cases the excessive wear of a bearing is due to a misfit. The Hopkins bearing is a bronze bearing lined with a thin layer of lead, which, when new and unfitted, can accommodate itself to the distorted journal and permit gradual wear to a correct fit without danger of injury, such as occurs often with the common hard, unlined "brass." In the Defreest bearing a thin bronze bearing-piece is sustained by a strong iron backing-piece, and between them is a sheet-lead filling. Journals should be fitted without the use of emery or other gritty grinding material, which may adhere to its surface and thus produce injury.

Bearing surfaces of wood are, under the conditions already described as favorable to their use, exceedingly durable, and will carry enormous loads without abrasion. Thus lignum-vitæ will sustain pressures exceeding 1000 lbs. per square inch (70

* Including melting expenses, loss, etc. These figures are constantly varying.

† Seven thousand grains per pound.

kgs. per sq. cm.), where brass becomes rapidly abraded and destroyed under but little more than one fourth of that load, and will run continuously under 4000 lbs. (281 kgs. per sq. cm.) when bronze sets fast instantly. Camwood has been subjected to pressures exceeding 8000 lbs. per square inch (562 kgs. per sq. cm.), and has worked without injury; snakewood carries about as heavy a load as lignum-vitæ.

The bearing surfaces of watch-work are often made of ruby, agate, and other fine-grained and hard stones, and of gems.

A comparison made by the Author between surfaces of gun-bronze, of "Babbitt"-metal, and of other soft, white alloys, all working on steel, proved all to have substantially the same friction. In other words, the coefficient of friction was determined by the nature of the unguent and not by that of the rubbing surfaces, when the latter are in good order. The soft metals, however, heated more than the bronze, running at temperatures somewhat higher with equally free or even freer feed. To retain the temperature at 135° F. (57° C.), in some cases one half more oil—over 300 grammes, as against 200—was needed on the white metal than on the bronze. This probably does not, however, necessarily indicate a serious defect, but simply deficient conductivity. Lined journals may be expected to run normally warmer than unlined bronze of good quality. The following are the results of experiment with a "Babbitt"-metal, which was compared with bronze and a second white alloy:

	Bronzes.	White Metal.	
		No. 1.	No. 2.
Mean temperature, Fahr........	133°	152°	137°
Mean coefficient of friction......	0.010	0.013	0.010
Oil used per hour, ounces.... ..	7	17	12

These differences prove ordinary lubricated surfaces to have contact, since they give differences in the values of f where none could exist were the friction fluid-friction solely.

Riveting, in steam-boilers and bridge-work, or other constructions, is usually taken as having a coefficient, $f = 0.333$; but it should never be reckoned upon as an element of definite

value, although the enormous pressure produced by the shrinkage of heated rivets, while cooling, gives it some importance. The elastic limit of common iron is usually not far from 25,000 lbs. per square inch (1757.5 kgs. per sq. cm.), and one third this amount, above 8000 lbs. (562.4 kgs.) per unit of section of rivet, is a quantity of real value as an element of safety.

The friction of belts and of gearing has been often studied experimentally. Morin concluded its amount for belting to be proportional to the angle on the pulley subtended by the belt, to the logarithm of the ratio of tensions, and to be independent of the width of belt and of the linear measure of the arc embraced by it—i.e., independent of the area of contact. He obtained $f = 0.28$ to $f = 0.38$, the value varying with the condition of the belt.

Adopting the formula of Prony for the difference of tension on the two parts of the belt, the values of its coefficient, k, were obtained as in the table.

The maximum difference of tension allowable is

$$D = T_1 - T_2 = (k - 1)\, T_2.$$

The minimum tension allowable to prevent slip is taken as

$$T_2 = \frac{T_1 + T_2}{2} = \frac{1}{2}\frac{k+1}{k-1}D.$$

VALUE OF k IN PRONY'S FORMULA.

Proportion of Circumference in contact.	New Belt on Wooden Pulleys.	Ordinary on Wood.	Belts on Iron.	Wet Belts on Iron.	Rope on Wooden Axles.	
					Rough.	Smooth.
0.20	1.9	1.8	1.4	1.6	1.9	1.5
0.40	3.5	3.3	2.0	2.6	3.5	2.3
0.60	6.6	5.9	2.9	4.2	6.6	3.5
0.80	12.3	10.6	4.1	6.8	12.3	5.3
1.00	23.1	19.2	5.8	10.9	23.9	8.0
1.50	....	....	....	....	111.3	22.4
2.00	....	....	....	....	535.5	63.2
2.50	....	....	....	....	257.48	178.5

The maximum stress allowable on the leather was stated at about 350 lbs. per square inch of cross-section.

In the equations *

$$R = T_1 - T_2 = T_1(1 - e^{f\theta}) = T_2(e^{f\theta} - 1),$$

$$\frac{T_1 + T_2}{2R} = \frac{e^{f\theta} + 1}{2(e^{f\theta} - 1)},$$

f varies from 0.15 to 0.6, the former value being found only where the belt is actually wet with oil.

Reuleaux takes $f = 0.25$, and the experiments of Messrs. Towne and Briggs † indicate that this value is exceeded, under ordinary working conditions, more than 60 per cent.

Rubber belting has greater adhesion than leather, and values of f may be used exceeding very greatly those adopted for leather.

The angle $\theta = 2\pi n$, where n is the number of turns or part of turns taken by the belt about the pulley. Rankine gives ‡ the following values of the coefficient $2.7288f$ in the equation $e^{f\theta} = 10^{2.7288fn}$ which comes into use in the application of these formulas, as seen in Chapter II:

$f =$	0.15	0.25	0.42	0.56
$2.7288f =$	0.41	0.68	1.15	1.53

and, where $\theta = \pi$ and $n = \frac{1}{2}$, as is usual,

$T_1 \div T_2 =$	1.603	2.188	3.758	5.821
$T_1 \div R =$	2.66	1.84	1.36	1.21
$(T_1 + T_2) \div 2R =$	2.16	1.34	0.86	0.71

Usually we assume $T_2 = R$; $T_1 = 2R$; $(T_1 + T_2 + 2R = 1.5$ and f becomes 0.22.

* Friction and Lost Work; chapter II. § 31.

† Journal of the Franklin Institute; 1868.

‡ Machinery and Mill-work, p. 352.

Rankine* gives f for a wire-rope running on cast-iron at 0.15 and on gutta-percha at 0.25.

Rope-gearing has a value of $f = 0.25$ to $f = 0.8$, and the resistance to slipping is increased in proportion to the cosecant of the half-angle of the wedge-shaped groove of the carrying-wheel.†

The method of supply of oil should be carefully looked to, and a very free "feed," with a system of collection and reapplication of the oil leaving the bearing, will be found to give by far the greatest economy of power and cost. Experiments made for the Institution of Mechanical Engineers, in which oiling by a pad as in railway work, by a siphon lubricator or oil-cup, and by a bath, which keeps the surfaces flooded with oil, gave the following figures, showing an enormous advantage in the use of the last method:

METHODS OF OILING (RAPE-SEED OIL).

VELOCITY OF RUBBING, 157 FEET (46 M.) PER MINUTE.

	Actual Load.		Coefficient of Friction.	Comparative Friction.
	Kilogs. per sq. cm.	Lbs. per sq. in.		
Oil-bath..............	18.5	263	0.00139	1
Siphon lubricator.......	17.7	252	0.00980	7.06
Pad under journal......	19.1	272	0.00900	6.48

Conclusions.—*Specified qualities of lubricant* may, by the processes here described, be secured by test. If an unguent is desired for heavy pressures, or an oil for very light work, or for high or low speeds of rubbing under known pressures, the methods of study of the available lubricants which have been described will enable the engineer or the manufacturer to select that which is best suited to the specified purpose. He may go still further, and, by repeated mixing and test gradually improving the mixtures, may finally secure compounds having the best possible qualities for the various proposed

* Machinery and Mill-work, p. 352.

† American Machinist; November 1, 1884.

applications. The Author has in this manner sometimes produced lubricants for manufacturers which have been found peculiarly well suited for special lines of trade.

Studying the facts here stated, and the data acquired by many hundreds of other experiments, made on one or the other of these last-described machines for testing lubricants, we may recapitulate the facts and figures for ordinary use in machine-design and in estimating losses of power by friction as follows:

(1) The great cause of variation with well-cared-for journals, since they must work at ordinary temperatures, is alteration of pressure and variation in methods of supply; and it is seen that the higher pressures give the lowest percentages of loss of power by friction.

(2) The value of the coefficient is greatly modified by the state of the rubbing surfaces; a single scratch has its effect in wasting power. A good journal usually has its surface as smooth and as absolutely uniform as a mirror. Every well-kept journal acquires such a surface.

(3) For general purposes and for heavy work, as in the experiments of the Author, and at considerable speeds, the value of the coefficient varies nearly inversely as the square-root of the pressure, for pressures ranging from 50 to 500 lbs. per square inch.

(4) The coefficient for rest or starting may similarly be taken to vary nearly as the cube-root of the pressure. For closer estimates and other conditions, the tables just given can be referred to directly.

(5) The coefficient for the instant of coming to rest, under the special conditions here referred to, is nearly constant, and may be taken at 0.03.

(6) The resistance due to friction varies with velocity, decreasing with increasing velocity rapidly at very low speeds, as from 1 to 10 feet per second, and slowly as higher speeds are reached, until the law changes and increase at ordinary temperatures takes place, and at a low rate throughout the whole range of usual velocities of rubbing met with in machinery.

Its amount and the law vary with method of lubrication,

however. With oil-bath lubrication the value of f usually varies more nearly as the square-root of the velocity.

(7) With pressure and velocity varying, we may take the coefficient as varying as the fifth root of the velocity, divided by the square-root of the pressure for such work as is represented by the experiments of the Author.

(8) The effect of heating journals under conditions here illustrated is to *increase* the friction above 90° or 100° F., at a speed as low as 30 to 100 feet per minute, while at higher speeds and low pressures the opposite effect is produced, and the coefficient often *decreases* more nearly as the square-root of the rise of temperature.

(9) The temperature of minimum friction, under the conditions of the experiments here referred to, varies nearly as the cube-root of the velocity, for a pressure of about 200 lbs. per square inch.

(10) The endurance of any lubricant should be determined by actual wear upon a good journal under the pressures and velocities proposed for its use.

The economy with which it can be used will be dependent upon its natural method and rate of flow, and upon its capillary qualities, as well as upon its intrinsic wearing power and the method adopted in feeding it. Greases, therefore, are usually more economical in cost than oils, even if having less wearing capacity.

(11) The only method of learning the true value of a lubricant and its applicability in the arts is to place it under test, determining its friction-reducing power, and its other valuable qualities, not only at a standard pressure and velocity, and at ordinary temperatures, but measuring its friction and endurance as affected by changing temperatures, speeds, pressures, and methods of application, throughout the whole range of usual practice, and its wearing effect.

(12) The true value of an oil to the consumer is not proportional simply to its friction-reducing power and endurance, under the conditions of his work; but its value to him is measured by the difference in value of power expended, when using

the different lubricants, less the difference in total cost of oil or grease used; but for commercial purposes, no better method of grading prices seems practicable than that which makes their market value proportional to their endurance, divided by their coefficients of friction.

The consumer will usually find it economical to use that lubricant which is shown to be the best for his special case, with little regard to price, and often finds real economy in using the better material, gaining sufficient to repay excess in the total cost very many times over.

(13) To secure maximum economy, the journal should be subjected to a pressure the limit of which is determinable by either Rankine's or Thurston's formula;* the most efficient materials should be chosen for the rubbing surfaces; they should be reduced to the most perfect state of smoothness and perfection in form and fit; a lubricant should be chosen which is best adapted for use under the precise conditions assumed; the lubricant should be supplied precisely as needed, and by a method perfectly adapted to the special unguent chosen. The real problem is often, not what oil shall be used, but how to secure most effective lubrication.

(14) The semi-fluid lubricants, when equally good reducers of friction, are usually the most economical for heating journals, in consequence of their peculiar self-regulating flow, as the rubbing parts warm or cool while working. They are usually too viscous for economical use in ordinary work.

The loss by internal friction in the steam-engine includes the wastes at the journals of the shaft, crank-pin, and cross-head-pin or "wrist-pin," and of the valve-motion; of the sliding friction of cross-head and other guides; of the piston-rods and valve-stems in their "stuffing-boxes," and of the rubbing of pistons and valves on the surfaces over which they glide, and the resistance of air-pumps in condensing engines. Its total is ordinarily equivalent to from one pound on the square inch of piston, in very large engines in good order, to about four or

* Friction and Lost Work; § 127.

five pounds in small engines—of 25 to 50 horse-power—and becomes very much greater when the lubricants used are inefficient, the rubbing surfaces in bad shape, the stuffing-boxes too tightly packed, or the packing-rings set out too much. These figures ordinarily correspond to from five to as much as ten per cent of the total indicated power of the engine, in the best cases. and to from ten per cent upward, indefinitely, in worse cases.

Studying these losses in detail, it is found that the friction of the journals, when properly, uniformly, and effectively lubricated, is relatively less, though absolutely somewhat greater, as the pressures on them, and work transmitted across them, increase;* that the friction of guides follows the same law; that the work lost in the stuffing-boxes is probably independent of the work of the engine; and that the friction of the piston and of the valve may usually be taken as also independent of the engine-load, though probably always affected by the intensity of the steam-pressure.

Experiments made by the Author lead to the conclusion † that the method of variation of the internal friction of the steam-engine is not usually exactly that stated by early writers on the subject. It has been customary among engineers conversant with the operation of the steam-engine to take the "friction-card" obtained by applying the indicator to the unloaded engine as a measure of the friction of the engine at all times, whether loaded or unloaded; while it has been usual, in theory, to accept the formula of De Pambour, which is unquestionably accurate in form,

$$R = (1 + f) R_1 + R_0;$$

in which R is the total resistance, R_1 is that of the net work of the engine, its "useful" load, and R_0 is the work of friction of the parts of the machine itself, and f a coefficient of friction.

* Where this is not true, the deduction follows, inevitably, that the friction is that of solids, not, as it should be, "mediate," as Hirn calls it, and that the lubrication is not effective.

† Trans. Am. Soc. Mech. Engineers; 1886; vol. VIII.

This formula is based upon the very reasonable assumption that the total friction must be a minimum in the unloaded engine, and that the imposition of external work upon it must, by increasing the pressure on its running parts, add to the total by the amount of friction so arising. But whether this increase of waste energy amounts to so much as to become observable, or to be practically important in the operation of the engine; whether the engineer is right in theory, or correct in his practice, in usual cases, is not wholly certain. The friction of engine, as has been seen, consists of the resistances due to the motion of the various piston, valve, and other elements through stuffing-boxes and in guides, the friction of the piston-rings on the cylinder surface, the friction of eccentrics, and, often, of other parts which are independent of the magnitude of the load thrown upon the engine by the useful resistance, in addition to the friction of the journals transmitting the effort of the steam to the exterior resisting work, and of other parts directly and indirectly affected by its variation. It thus happens that the resistance due to the friction of the latter may be, and probably often is, but a small proportion of the whole friction of engine. The total friction of engine, as has been seen, in good engines of ordinary kinds, amounts to from 5 to 10 per cent of the total power developed when fully loaded; but the coefficient of friction of any one journal, if well lubricated, has often been found by the Author, under such pressures as are usual on the main journals of the steam-engine, to fall to a low figure, and the absorption of work and energy may thus be even a still lower proportion of the work of the steam as the speed of rubbing is less than that of the piston. The loss of power along the line of connection is probably always small. Again: the coefficient of friction, with really good lubrication, within the usual range of pressures on journals and guides, increases as pressures fall, and decreases when the pressures increase with variation of engine-power and load; and this compensation often occurs to such an extent that the total frictional resistance, on these parts even, varies slowly with variation of load; while the friction of the other portions of the engine re-

mains constant.* The resultant effect is often a practically constant friction of engine under all loads, the speed and steam-pressure being constant. In condensing engines this friction is subject to similar conditions; but the work of the air-pump should decrease with the reduction of the load.

Among the most excellent illustrations of thorough lubrication are those of Tower,† in which a near approach to perfect fluid-friction was attained, the total resistance thus becoming nearly constant at all pressures, and, nearly,

$$f = 20\,c\,\frac{\sqrt{v}}{p};$$

in which, as given by Kennedy,‡ c depends on the lubricant, and is about 0.0014 for sperm oil, 0.0015 for rapeseed, and 0.0018 for good mineral oils; v is the speed of rubbing in feet per minute; and p is the pressure per square inch. At $v = 250$ and $p = 310$, $f = c$, nearly.

It will be observed that the variation of fp, which is here a constant for a given velocity, is a gauge of the efficiency of the lubrication; since f is constant when the two solids are in actual contact.

The Efficiency of Machine, as distinguished from the efficiency of its thermodynamic operation, the efficiency of the mechanism, is measured by the ratio of the quantity of work done at the engine-shaft, to that shown at the piston by the indicator, and is less than unity as the lost work of friction reduces the former quantity. The value of this efficiency is, as a maximum, about 0.95 in the simplest and best constructions of non-condensing engine, and ranges from about 0.90 down to 80 or less with condensing engines; while 0.90 is a common value of the former, and 0.85 for the latter. Good marine engines should attain 0.88.

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* Friction and Lost Work; chapter VII.

† Trans. Brit. Inst. Mech. Engrs.; 1884.

‡ Mechanics of Machinery; p. 573.

Increasing the number of steam-cylinders, other things equal, increases the friction of the engine. For:

Let n = number of small cylinders;
d = diameter of each small cylinder;
D = " " " large cylinder;
l = length of stroke;
s = area rubbing surface of small cylinder;
S = " " " " large "

Then

$$s = n\pi dl;$$
$$S = \pi Dl = \sqrt{n}\pi dl;$$

and the friction increases as the square-root of the number of cylinders, where all the small cylinders are of equal size. This friction in a double engine would thus exceed by 40 per cent that of a simple engine.

133. **Investigations of Internal Engine-friction** were made, within a few years, to determine its nature, extent, and method of variation, and the conclusions reached have been sustained by still later experiment. Of these investigations, the first, made under the supervision of the Author, was conducted by Messrs. Aldrich and Mitchell,* with the following results:

Number of Card.	Revolutions.	Steam-pressure.	Brake H. P.	Indicator H. P.	Diff.	Friction per cent.
1	232	50	4.06	7.41	3.35	45
3	230	63	6.00	10.00	4.00	40
5	230	73	8.10	11.75	3.65	32
7	230	75	10.00	14.02	4.02	28
9	230	80	12.00	15.17	3.17	21
11	230	75	14.00	16.86	2.86	17
13	231	72	20.1	22.07	2.06	9
15	229	60	29.55	33.04	3.16	9.5
17	229	70	39.85	43.04	3.19	7.4
19	230	90	50.00	52.60	2.60	4.9

This engine was rated at 30 I. H. P., 8 inches in diameter of cylinder, 14 inches stroke of piston, having a rod 44 inches

* Trans. Am. Soc. Mech. Engrs.; 1886; vol. VIII; No. ccxxviii.

long between centres, a balanced valve with stroke of 2 to 4 inches, according to position or governor and eccentric, a fly-wheel 50 inches in diameter, weighing 2300 pounds, the steam and exhaust pipes having diameters of 2½ and 4 inches, respectively, and the whole machine weighing 2½ tons. The space occupied by the engine was 9 feet 4 inches in length, by 4 feet 8 inches in width, and 3 feet 10 inches in height.

Examining the above table of powers, it is seen that the difference between indicated and dynamometric power, the friction of the engine, varies somewhat, with varying steam-pressures and varying total power; but in such manner as to indicate the controlling cause to be irregular in action, and possibly to some extent due to errors of observation and to accident; and we are probably justified in taking it as approximately constant under all ordinary variations of load.

The repetition of the experiment upon an engine of another make, having a cylinder 9 inches in diameter and a stroke of piston of 12 inches, which would naturally give a somewhat increased percentage of friction, in consequence of the proportionally smaller stroke, at 20, 30, 50, and 65 horse-power, by brake, and running free, revolutions 300 per minute—a speed which may also have caused some increase in frictional resistance, not only in rubbing parts, but by increasing back-pressure—gave a friction of engine measuring from 2.66 horse-power unloaded, to 4 horse-power at 20 to 30 horse-power, 4.8 horse-power at 50, and 5.3 at 65 horse-power, the total friction increasing perceptibly, as assumed by De Pambour, but decreasing in percentage of load, from 16 to 7.5, between 20 and 65 horse-power. It is very nearly constant throughout the whole range of power that the engine would be worked under ordinary circumstances, and may be so taken without serious error. At their rated powers these two engines thus exhibit efficiencies of mechanism of about 94 and 90 per cent, respectively.

Another series of experiments was made by Messrs. Day and Riley during the year 1886, confirming the deductions already given. The engine taken for test was built for pur-

poses of experimental investigation. It was 12 inches stroke, and $6\frac{7}{8}$ inches in diameter.

The conclusion already reached is thus again confirmed. The following are the data obtained:

1 No. of Card.	2 Rev. per Minute.	3 Steam-pressure.	4 Brake Power. H. P.	5 Ind. H.P.P.I. per card.	6 Diff. Frict. H. P.	7 Mean F. Pres.	8 Frict. per cent.
1	282	19	0	2.26	2.26	3.70	100
3	286	66	7.61	10.95	3.33	5.25	30
5	285	71	13.10	15.99	2.61	4.25	18
7	284	74	18.55	20.73	2.65	4.18	12
9	279	65	23.61	25.95	2.33	3.73	9
11	280	72	29.03	32.22	3.19	5.15	10

These experiments lead to the discovery of the fact that the engine-friction varied, at constant load and speed, with variation of steam-pressure. In order to determine whether this hitherto unobserved fact were true, the following data were obtained:

No. of Card.	Rev.	Steam-pressure.	I. H. P.	Mean Pressure.	Mean F. Press.	Per cent. Frict.	
1	250	25	6.01	10.84	1.95	18	Ten pounds on the brake.
3	285	42	7.17	11.35	3.63	32	
5	271	58	6.81	11.28	3.16	28	
7	286	68	7.77	12.25	4.90	40	
9	296	82	7.87	12.00	4.68	39	
11	279	$66\frac{1}{2}$	1.995	3.22	3.22	100	No load on the brakes.
13	275	35	1.71	2.80	2.80	"	
15	272	25	1.876	3.11	3.11	"	
17	270	15	1.712	2.86	2.86	"	

In the first set of experiments, here numbered 1 to 9, inclusive, the weight on the brake-arm was kept constant at ten pounds; in the remaining experiments all weight was removed. In both cases, the same general effect is seen. As the steam-pressure rises, the speed being the same and the resistance the same, the friction of the engine increases; from 2 pounds, at 25 pounds' pressure in the steam-chest, to nearly five pounds per square inch of piston at the maximum, 82 pounds steam in the valve-chest. As the steam-pressure fell from this point to

15 pounds, in experiments 9 to 17, the load being thrown off entirely, and the speed being nearly constant, the mean pressure measuring the friction of engine falls again below 3 pounds per square inch of piston.

The accompanying figure illustrates graphically the method of variation of the internal resistance, in per cent of power developed, with variation of work done by the engine, as illustrated in the first series of trials.

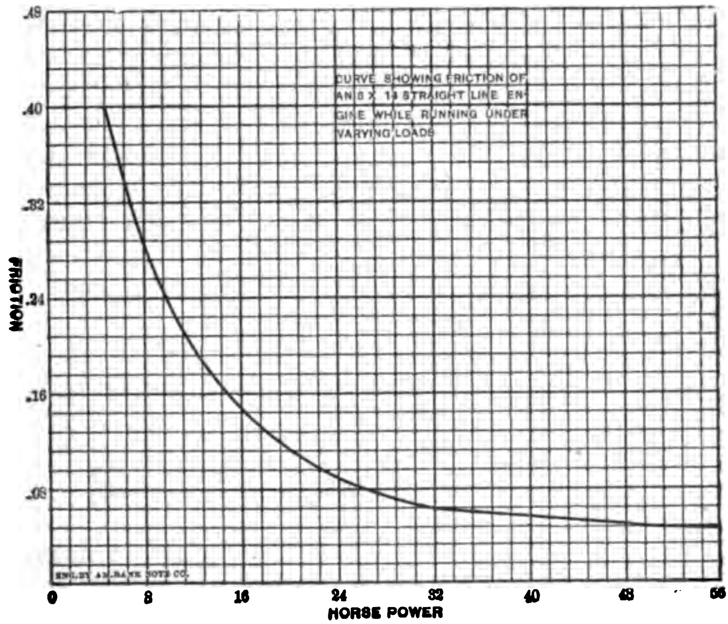

FIG. 154.—INTERNAL FRICTION OF ENGINE.

The curve is, evidently at least approximately hyperbolic.

Similar experiments conducted, for the Author, by Professor R. C. Carpenter, exhibited the same facts where the method of steam-distribution was changed from the "automatic" system of regulation and adjustment of the ratio of expansion to the "throttling" system.

A series of trials made to determine the effect of variation

of speed of engine showed a general tendency to increase of friction-resistance as the speed increased, and these and the experiments and data already obtained serve to give the law of variation with a very satisfactory degree of accuracy. The line most closely corresponding with the data which have been found most reliable has very exactly the equation

$$y = 0.008x;$$

and the internal friction of this engine in horse-power was about 0.8 per cent of the number of revolutions per minute.

Referring to the results obtained by the Author, Mr. D. K. Clark remarks: "The degree of nearness to uniformity of frictional resistance for various powers of the same engine, at the same speed, is probably dependent upon the degree of nearness by which the momentum of the reciprocating parts is balanced by the pressure of the steam." *

Earlier experiments have incidentally supplied some data relating to this form of waste of energy, thus:

A Porter-Allen engine, 16 inches diameter of cylinder and 30 inches stroke of piston, in trials by the American Institute in 1871 gave:

I. H. P.	27	56	84	109	142
Friction H. P.	9.1	9.5	8.5	8.7	12.7

A pair of Westinghouse single-acting engines 12 inches diameter and 11 inches stroke gave the following,† at 300 revolutions:

I. H. P., loaded	84	Friction H. P.	7
" light	10	" "	10

A "Buckeye" engine, 7×14 inches, at 280 revolutions, gave:

I. H. P., loaded	23.0	Friction H. P.	5.0
" light	5.1	" "	5.1

* The Steam-engine; vol. II. p. 619.

† Trans. Am. Soc. Mech. Engrs.; 1887.

MM. Hirn and Hallauer give the following for compounded engines,* condensing:

I. H. P., loaded.....	347	181	Friction..............	44	19
" reduced l'd..	185	137	"	40	25

Indicating, as would be anticipated, lessened waste energy with lessened load and correspondingly reduced air-pump work.

The experiments of M. Walther-Meunier on engines of a wide range of power show an average of efficiency of machine of 0.8815 for the compound and 0.9115 for the simple engine, the difference of 3 per cent being in favor of the latter. The former had the advantage, on the other hand, of 8 per cent in consumption of steam—a small gain, however.†

The internal friction of condensing engines has been the subject of an investigation by MM. Walther-Meunier and Ludwig,‡ a compound engine of some 300 indicated horse-power being used, with the following results:

(1) ENGINE WORKING COMPOUND.

I. H. P.	D. H. P.	Frict. H. P.	Efficiency.
288.45	248.97	39.48	0.863
222.73	188.68	34.05	0.847
136.07	108.28	27.79	0.795
(2) H. P. CYLINDER WITH CONDENSATION.			
153.12	128.38	24.74	0.839
108.96	88.19	20.77	0.809
55.19	37.94	17.25	0.689
(3) SAME WITHOUT CONDENSATION.			
145.87	128.38	17.49	0.880
103.93	88.19	15.74	0.848
51.34	37.94	13.40	0.738

* Alsatian Experiments; 1876.

† Congrès International de Mécanique appliquée; 1889; vol. II. p. 133.

‡ Bull. de la Soc. Ind. de Mulhouse; 1887; p. 140. Proc. Inst. C. E.; XC; 1886–7; part IV. p. 524.

With a range of work from about 150 to nearly 300 horse-power, the friction-waste was thus, as expressed by the formula of De Pambour,

$$P_f = P_o + 0.075P_e, \text{ nearly};$$

while, when the high-pressure engine only was at work, giving 55 to 150 H. P.,

$$P_f = P_o + 0.11P_e$$

with condenser in action, and

$$P_f = P_o + 0.06P_e$$

working, non-condensing, at about the same power, measured by indicator. The air-pump demanded 7.25 to 7.5 horse-power.

Earlier issues of the journal in which these data are recorded give, from various sources, the following figures:

Date.	Engine.	Builder.	Best H. P.	Max. Effic.
1864	Beam, simple.	G. A. Hirn.	115.00	90.8
1867	" Woolf.	Koechlin.	191.44	89.6
1876	Horizontal, Woolf.	Alsatian Soc.	174.46	89.1
1878	Corliss.	Berger-André.	144.82	91.5
1879	Horizontal, comp'd.	Weyher & Richmond.	60.00	87.5
1884	Collman.	Burghart Bros.	22.26	87.8
1884	Hor. portable.	Quiri & Co.	23.97	86.3
1885	" compound.	Alsatiah Soc.	59.26	89.1
1886	2 cyls. and condens.	Bitschweiler-Thaun.	248.97	86.3
	1 cyl. " "		128.38	83.9
	1 " " "		128.38	88.0

The data here collated show plainly the increase in *efficiency of machine* as the power demanded increases; but the last table also shows that, where equally well proportioned to their work, small engines may have practically equal efficiency, *as machines*, with large engines; and that horizontal and beam engines may be substantially equal in this respect. Single-cylinder engines but slightly excel good compound engines; and the triple-expansion engine with three equidistant cranks is still more satisfactory in its operation.

134. The Methods of Variation and of Distribution of Internal Friction of Engine are, so far as deducible from these data, and from those of other investigators, evidently as follows:

(1) The friction of the non-condensing engine, of the better class as here described, is sensibly constant, at any given speed, at all loads; and at different speeds, is independent of the magnitude of the load.

(2) The friction of such engines is variable with variation of speed of engine; increasing as speed increases, in some ratio as yet not fully determined, but probably differing with every engine, and, for the same engine, with every change of conditions of operation.

Generally, we may write

$$R = R_0 + f(R_1) + f(V).$$

(3) The friction of engines increases with increase of steam-pressure, in such cases, in a probably similarly variable manner with that observed with alteration of speed; neither method of variation being ordinarily capable of representation by any convenient algebraic expression.

(4) The total resistance measured at the piston of the engine is composed of two parts, the one sensibly constant at the working speed, the other variable with external load, and may be, for practical purposes, at least, represented by the expression

$$R = (1 + f)R_1 + R_0,$$

in which R is the total resistance, as shown on the indicator diagram, R_1 the resistance due to the external load—e.g., as measured by a Prony brake,—and R_0 the resistance of the unloaded engine.

Here $f = 0$ in the cases taken in § 133.

(5) In engines of this class, the internal friction varies directly with the speed, or sensibly so, other things being equal;

is directly proportional to the power exerted, and may be taken as a constant part thereof, whenever other conditions remain unchanged with varying speed.

(6) We usually find confirmation of the fact, well known to engineers of experience, that the operation of a well-cared-for engine will continuously, and for a long time, appreciably reduce the internal friction of the machine.

In Distribution, experiment shows the total friction of engine to be composed, in most cases, mainly of main shaft, piston, and valve-gear resistances, in the non-condensing engine, and of air-pump and load in condensing engines. Investigations made for the Author by Messrs. Carpenter and Preston give the following for a fast-running engine with unbalanced valve and "automatic" valve-gear; the total amounts to ten per cent of the rated power of the engine—20 I. H. P.

	Friction H. P.	Friction per cent.
Main shaft and eccentrics	0.867	42.4
Three-ported valve	0.560	27.4
Piston and rod	0.328	16.1
Cross-head and pin	0.174	8.5
Crank-pin	0.115	5.6
Total	2.044	100.0

The following distribution was found for a similar case with a balanced valve, the total being about 7½ per cent of the rated power:

	Friction H. P.	Friction per cent.
Main shaft, etc	0.867	56.9
Valve	0.038	2.6
Piston and rod	0.328	21.6
Cross-head and pin	0.174	11.5
Crank-pin	0.115	7.4
Total	1.522	100.0

The coefficient of friction can be deduced with certainty only for the main journals of the engine: since there is a variation in pressure of piston-rings, stuffing boxes, and in other quantities, which is, to a great extent, unknown.

If we call f the coefficient of friction, p the pressure on the bearings in pounds for engines light, and plus mean pressure on piston for engines loaded, c the circumference of the bearings in feet, n the number of revolutions per minute, $fpcn$ will thus equal the "lost work" of friction; which has been determined in the previous experiments, and is expressed as horse-power; this is indicated to foot-pounds by multiplying by 33,000.

Hence $$fpcn = 33{,}000 \text{ H. P.}$$

$$f = \frac{33{,}000 \text{ H. P.}}{pcn}.$$

The following shows the value of this coefficient for several engines, and the next table is a summary of results.

COEFFICIENT OF FRICTION FOR THE MAIN BEARINGS OF STEAM-ENGINES.

Engine.	F. H. P. due to Main Journals.	Weight on Journals in pounds.	Diameter of Journal in inches.	Coefficient of Friction, engine loaded.	Revolutions of Journal per minute.
(1) 6″ × 12″ "High Speed"	0.85	1500	3	.06	230
(2) 12″ × 18″ Automatic	3.70	2600	5	.05	190
(3) 7′ × 10″ Traction	0.68	500	$2\frac{1}{4}$	.08	200
(4) 21″ × 20″ Condensing	3.30	4000	$5\frac{1}{2}$	.04	206

The coefficient of friction of the common slide-valve is reported by Aspinall as from 6 to 10 per cent, usually averaging 9 on horizontal and 7 on vertical seats.

DISTRIBUTION OF FRICTION.

SUMMARY OF RESULTS.

Parts of Engine.	Percentage of Total Friction.				
	"High Speed," 6 x 12, Balanced Valve.	Same, 6 x 12, Unbalanced Valve.	7″ x 10″ Traction-Locomotive Valve-gear.	12″ x 18″ Automatic, Balanced Valve.	21″ x 20″ Condensing, Balanced Valve.
Main bearings	47.0	35.4	35.0	41.6	46.0
Piston and rod	32.9	25.0	21.0		
Crank-pin	6.8	5.1	13.0	49.1	21.8
Cross-head and wrist-pin	5.4	4.1			
Valve and rod	2.5	26.4	22.0	9.3	21.0
Eccentric-strap	5.3	4.0			
Link and eccentric			9.0		
Air-pump					12.0
Total	100.0	100.0	100.0	100.0	100.0

The friction-waste of a very small engine, tested by Professor Jacobus, as computed on the assumption of a constant coefficient of 8.5 per cent, is as below. The engine developed 0.944 D. H. P. at 100 revolutions per minute, with a mean pressure of 53 pounds; its size being $3\frac{1}{64}$ inches diameter and 5 inches stroke, with link-motion and unbalanced slide-valve.

FRICTION OF ENGINE.

	D. H. P.		D. H. P.
Valve	0.0240	Pins at cross-head	0.0068
Piston	0.0030	Guides	0.0079
Packing	0.0020	Crank-pin	0.0985
Eccentrics	0.0097	Shaft and wheel	0.0230

Total 0.1749

Actual by experiment .... 0.175 H. P.

Or 18.5 per cent. Efficiency of engine 0.815

The assumption of a coefficient of friction constant, at 10 per cent, gives the following, for 30 lbs. mean effective pressure:*

	H. P. 6 × 12 230 rev. 12 H. P.	7 × 10 200 rev. 15 H. P.	Remarks.
Shaft[a]......	0.93[b]	0.57[c]	[a]Including thrust of piston rod.
Crank-pin...	0.30	0.38	
Wrist-pin...	0.06	0.08	[b]Weight = 500 lbs.
Guides.....	0.13	0.12	[c] " = 1500 "
Valve......	0.17[d]	0.68	[d]Valve balanced.
Eccentrics..	0.08	0.48	[e]No packing on rod; using Sweet's metallic sleeve.
Piston......	0.16	0.13	
Packings....	0.00[e]	0.20	
Total.....	1.83	2.64	
Actual....	1.64	2.86	

Studying the data, it is seen that, in the engines here represented, the friction of the shaft and eccentrics is the principal item; that the friction of the valve and its stem is the next most serious item in the case in which it is tested under pressure unbalanced, but becomes only a fraction as great when well balanced, and is then comparatively unimportant; that the friction of the piston may be a heavy item, and that of the crank-pin is a very small proportion of the total. Since the sliding friction of the cross-head is known to be considerable, it is at once evident, on comparing that item with the last, that the friction of the cross-head-pin must be a very small, and probably an insignificant, part of the total. This is also to be inferred when the fact is considered that, although it is subject to the same pressure as the crank-pin, the extent of rubbing motion during a revolution of the engine is there very much less than on the latter.

* Stevens Indicator; Oct. 1890; p. 351.

The conclusions relative to the opportunity for, and the methods of, reducing this waste of energy are, evidently, (1) that it is advisable to secure a minimum shaft-friction, by careful selection of material and proportioning and finishing of journals; (2) to make piston-friction a minimum by securing the least possible pressure of rings and piston on the internal surface of the cylinder; (3) to adopt a good balanced valve—an essential desideratum, also, of all automatic regulation—and (4) especially to secure the most efficient possible lubrication. Similar general principles affect gas-engines.

In condensing engines, the wasted energy, in addition to the above, consists of that expended in taking the water from the condenser and expelling it from the system; the power required to move the air-pump valves and the bucket in the pump-barrel, the resistances of the circulating pump, when a surface condenser is employed, and the frictions of the pump mechanism. Of these quantities, the first, as a minimum, is approximately proportional to the quantity of steam to be condensed; the other quantities are nearly constant. The maximum mechanical efficiency of the locomotive is from 85 to 90 and sometimes to 95 per cent. The draw-bar pull is variable with the weight of engine relatively to train and with the head-resistance with varying loads and speeds. It is usually not far from 50 per cent of that due the indicated power developed and ranges from 40 to 70 per cent in common practice.

135. **The Conditions of Maximum Efficiency of Machine,** from what has preceded, are seen to be simply the conditions of minimum lost work by friction. Journals and all other rubbing parts must be of carefully adjusted size, well made, of proper material, and, above all, well lubricated. Piston-rings, if expanded by springs, should bear against the cylinder as lightly as possible, should be made of material giving minimum friction at the temperature of their operation, and under all the other peculiar physical conditions to which they are subjected. Stuffing-boxes, if used, should be deep, well filled, and lightly packed; and the whole system, including valve-gear and all connections, should be arranged to offer the least possible resistance. The machine, as a whole, should

be loaded to the maximum, consistent with economy of fuel and steam, and operated in such manner and at such speed as will give highest *total* efficiency.

The lubricating apparatus should, if possible, be so designed as to flood the journals constantly, and to utilize the lubricant fully, by a constant circulation. This system not only reduces the sliding friction of the machinery to a minimum, but also usually gives rise to minimum risk from failure of lubrication.

136. **The Conditions of Maximum Total Efficiency,** in the engine, are easily stated generally, but are not so easy of exact determination for a specified instance, or of complete realization in any case. In some directions, one element of efficiency is only promoted at the expense of another; and maximum total efficiency is always the resultant of compromises effected among conflicting conditions. Thus: increasing speed of engine usually diminishes exhaust-wastes, while increasing friction-losses; and, at the best velocity, any change of speed will increase aggregate loss, while diminishing some one or more of its elements. Increase of velocity giving rise to greater loss by friction than is compensated by decreased cylinder-condensation; a decrease in speed exaggerates total loss by producing waste at the exhaust in excess of the gain by decreased engine-friction. Similarly: a high ratio of expansion gives high thermodynamic efficiency; but it exaggerates condensation, and with considerable rapidity; the best ratio, from this point of view, is that at which this resultant efficiency is a maximum. It is this which limits the ratio of expansion practically allowable, often, in the condensing engine, a small fraction of that which the thermodynamic theory of the case would dictate.

The final test of total engine-efficiency, and of satisfactory design, construction, and operation, is the measure of the expenditure of steam or of fuel in the production of the required net, useful, work, the dynamometric power of the engine, as shown by a Prony brake or other apparatus. The test of *ultimate* value, to the purchaser and user, is a still different one: it is the money-cost of the power supplied, and of useful

work done, as measured by the total expense-account on the treasurer's books.

137. **Actual Efficiencies and Economy** of proposed steam-engines may be approximately computed, when operated under conditions similar to those of the experiments from which the available data are derived. As has been seen, all the expenditures of heat in the engine are now recognized; their magnitudes have been measured; the laws governing their variation with all the usual conditions have been, in some cases closely, in other instances roughly, determined; and it is practicable to make estimates that shall be, in many cases and for standard conditions and usual construction and methods of operation, fairly approximate, and which may also serve to guide the designer, the builder, and the user, in making estimates for proposed constructions.

The total expenditure of steam has been seen to be composed of: (1) that demanded for the thermodynamic cycle proposed for the engine; which can be computed with perfect accuracy; (2) that required to furnish the heat wasted by the engine otherwise than thermodynamically. This latter quantity is divided into two parts: (*a*) that needed to supply the heat wasted externally; (*b*) that wasted by internal transfer, by cylinder-condensation, without useful transformation. All these quantities are now easily computed, in most cases, with some degree of approximation; and the total probable heat and steam-supply may thus be obtained in the usual measures of heat and steam demanded per horse-power and per hour, and, when the efficiency of the boiler is known, in fuel, both per horse-power per hour, and as a total.

Examples of such computations have already been given for the ideal case (§ 117, pp. 462–3).

It is obvious that the computed expenditures for the ideal case must be increased in the proportion to which wastes occur, and that all the figures which have been thus tabulated must be increased from ten per cent upward to obtain probable values of weight demanded of steam and fuel in the actual case.

The following are selected illustrations of the ideal case for otherwise common practice, at the several pressures and ratio of expansion given; i.e., for the ideal case in which the steam

IDEAL EFFICIENCIES OF ENGINE.

Case No. (see pp. 462–3)	$p \div 144$.	r	E	Weight (per I. H. P. per hour), pounds.	
				Steam.	Fuel.
		CONDENSING.			
1	20	2	0.083	30.11	3.35
2	40	2.5	0.106	23.58	2.62
3	60	3.3	0.125	20.00	2.22
4	80	4.0	0.130	19.62	2.18
5	100	5.0	0.150	16.67	1.85
		NON-CONDENSING.			
6	60	2.5	0.074	33.78	3.38
7	80	3.33	0.091	27.78	2.78
8	100	5.0	0.105	23.81	2.38
9	120	5.0	0.115	21.74	2.17
10	160	5.0	0.127	18.90	1.89

is either worked in a non-conducting cylinder or in an otherwise perfect engine, the steam being kept in the dry and saturated state by adding heat during expansion in just the quantity needed to prevent its partial condensation in consequence of the conversion of its heat into work. Adding to the above computed quantities of steam and of fuel those demanded to supply the wastes invariably met with in greater or less amount in all actual engines, we may obtain figures of approximate, perhaps closely approximate, values, for every-day practice.

To determine the probable real efficiency of fluid, allowing for transfer without transformation, by internal wastes other than thermodynamic, assume the engines to be of moderate size and operated under familiar conditions, such as those which were met with in experiments conducted by the Author, in which the wastes were very exactly measured by the expression $c = \frac{a}{d} \sqrt{r} = 0.2 \sqrt{r}$ for the non-condensing unjacketed engine, and take the losses of the jacketed engine at a common

proportion, three fourths that amount, $c = 0.15\sqrt{r}$ for engines which we will take as of usual proportions, and will assume $d = 20$ inches diameter of cylinder. The speed of engine may be taken as about 500 feet per minute, that at which our data were secured, in these cases, $a = 4$, nearly (§ 130). Adding this proportion to the previously computed amounts for the ideal case, we obtain for the actual engine figures, assuming other losses too small to be here considered, which agree fairly with common experience.

Further, assume that it is practicable, in each case, to make the mechanical efficiency of the non-condensing machine 0.90 and the condensing engine 0.85, usual figures for the two classes. Then we obtain the following for indicated and for dynamometric power:

ACTUAL EFFICIENCIES OF ENGINE.

Case No.	$p + 14.4$	r	E	Steam.		Fuel.	
				I. H. P.	D. H. P.	I. H. P.	D. H. P.
1........	20	2	0.069	36.2	42.6	4.0	4.7
2........	40	2.5	0.085	29.2	34.4	3.2	3.8
3........	60	3.3	0.098	25.5	30.0	2.8	3.3
4........	80	4.0	0.100	25.0	29.2	2.8	3.2
5........	100	5.0	0.109	22.9	26.9	2.5	3.0
6........	60	2.5	0.050	44.9	50.0	4.5	5.0
7........	80	3.3	0.067	37.0	40.1	3.7	4.0
8........	100	5.0	0.073	34.2	38.0	3.4	3.8
9........	120	5.0	0.080	31.3	34.8	3.1	3.5
10........	160	5.0	0.087	28.7	32.0	2.9	3.2

Drier or superheated steam, higher piston-speed, larger powers of engine, efficient jacketing, will increase these efficiencies by reducing wastes; the opposite conditions will decrease them. Condensing engines are here found to promise about twenty per cent better performance than non-condensing: a promised fulfilled in good practice.

The differences between the steam-consumption figures of

the ideal and the real case represent those internal wastes which may be largely reduced by compounding; they amount to a nearly constant quantity, six pounds of steam for the condensing and ten pounds for the non-condensing engines.*

Similar computations, assuming, as before, that clearances may be neglected, and that the ideal case is first taken, then the corrections introduced for wastes, give the following results for an engine working steam at 500 pounds total absolute initial pressure, subject to 16 pounds back-pressure for the non-condensing and 5 pounds for the condensing machine, and taking the evaporation at 10 pounds for the former and 9 for the latter; the ratios of expansion ranging from 2 to 100. The condensation-waste is taken for the simple engine as the same as obtained in the Sandy Hook experiments, $c = 0.2\sqrt{r}$; i.e., corresponding to the simple engine of good construction and moderate speed, having about 20 inches diameter of cylinder. The feed-water is taken at the same temperature in all cases, 200° F.; since, at such pressures, a high temperature is advisable and is obtained by the use of heaters, in the one case taking heat from the exhaust-steam, in the other through jacket and receiver wastes. Buel's tables are here used; but Porter's or Peabody's will give similar results.

The data and results are as tabulated below:

HIGH (CONSTANT) PRESSURE, r, VARIABLE.

$v = 0.942$; $t_1 = 467°.42$ F.; $H = 1224.54$ B.T.U.; $H_1 = 815,650$ ft.-lbs.

IDEAL CASE, *Non-condensing.*

r	8	10	13	16	20	25	30
p_m	178.0	151.5	124.0	105.5	88.5	74.0	63.5
p_e	162.0	135.5	108.0	89.5	72.5	58.0	47.5
U	175,792	182,805	191,452	194,241	196,692	196,692	193,301
Efficiency	0.216	0.223	0.235	0.238	0.242	0.242	0.237
Steam	11.32	10.80	10.34	10.20	10.06	10.06	IC.24
Fuel	1.13	1.08	1.03	1.02	1.01	1.01	1.02

* The constancy of this waste, as thus computed, as already noted, accords singularly with the results of experiment.

Condensing.

r	20	30	40	50	60	80	100
p_m	88.5	63.5	50.0	41.5	35.5	28.0	23.0
p_e	83.5	58.5	45.0	36.5	30.5	23.0	18.0
U	226,535	238,066	344,170	247,561	248,240	249,596	244,170
Efficiency	0.278	0.292	0.299	0.304	0.304	0.306	0.299
Steam	8.75	8.32	8.11	7.99	7.98	7.93	8.11
Fuel	0.97	0.92	0.90	0.89	0.89	0.88	0.90

REAL CASE, *Non-condensing; Simple Engine.*

r	4.5	6	8	10	13	16
$1 + 0.2\sqrt{r}$	1.4242	1.4899	1.5658	1.6325	1.7211	1.800
Efficiency	0.1287	0.1356	0.1376	0.1365	0.1364	0.132
Steam	18.95	17.87	17.73	17.54	17.81	18.36
Fuel	1.89	1.79	1.77	1.75	1.78	1.84

Condensing, Simple Engine.

r	5	8	10	15	25	30
$1 + 0.2\sqrt{r}$	1.447	1.5658	1.6325	1.7746	2.000	2.095
Efficiency	0.1400	0.1486	0.1523	0.1481	0.1434	0.0139
Steam	17.40	16.54	15.90	16.47	16.94	17.43
Fuel	1.93	1.84	1.77	1.83	1.88	1.93

Studying the above figures, it is seen at a glance that, in such a case as is taken, the best work is done by the ideal engine, non-condensing, at about $r = 20$ and condensing at $r =$ about 80; while there is no great advantage, even in the ideal engine, in going beyond $r = 10$ expansions in the one or 20 in the other. Even these figures are reduced, in the case taken as actual, to $r = 6$ and $r = 10$.

By the adoption of the expedient of dividing the wastes by compounding the engine, these best ratios can be increased and the expenditure of steam and of fuel decreased very greatly, as seen elsewhere (Chapter VI, § 149), and it is evident from this study of the case and on comparison with cases of engines worked at lower pressures, that such high steam should not be used except in multiple-cylinder engines of three or four in series.

We neglect the effects of clearance and compression, in all these cases, assuming that, in all cases, they are made minima, the clearance being not only reduced to the least possible vol-

ume, but that the cushion-steam is expanded and compressed substantially in equal proportions, and that, for this reason, its action may be neglected. Hirn and Hallauer have shown that, in practice, the cushion-steam has not sensible effect, either theoretically or actually, in enhancing waste by cylinder-condensation; and many experiments conducted under the supervision of the Author have similarly shown the cushion-steam to be so absolutely dry, in even small engines, as to fully justify Hirn's conclusions.*

The Influence of Size of Engine may be very important as affecting wastes and the efficiencies of the engine. In all of the examples taken, it has been assumed that the engines were of fair size for factory engines, and of moderate speed of piston; at least, such that the rate of condensation found by experiment might be fairly assumed to apply to them. It will be interesting to endeavor to obtain some idea of the effect of variation of size of engine upon performance. That this is not necessarily serious, with even quite small engines, when proper precautions are taken to make the waste a minimum, is seen in the results of the trials of agricultural engines, where engines of ten and twenty horse power are reported giving as high efficiency as the average of fairly good engines of the same working pressures at sea, both simple and compound being compared. It is evident that the greater extent of surface exposed, per unit weight of working fluid subject to condensation, must, other circumstances being equal, give the larger engine the advantage.

But the heat-storing power of the unit of surface is less as the size of engine is less; since, if we follow Fourier, the rate of absorption varies, for a given temperature-head, nearly as the square-root of the total quantity of heat presented, and since, also, the water-flooded surface of the small engine is less effective, because of its reduced absorptive power, than the comparatively dry surface of the larger cylinder. Experience also

* In making such computations, for real cases, as are here illustrated, preliminary to designing engines, good average conditions are to be usually assumed; and when in doubt, less rather than more favorable conditions.

seems to indicate a less rapid rate of variation of internal waste than is indicated by the factor $\frac{1}{d}$, in $x = \frac{a'}{d}\sqrt{\frac{r}{N}}$, § 129.

To make this comparison, it is necessary to ascertain the waste per unit area of surface exposed, for one minute of exposure, and per unit range of temperature within the cylinder. The computations of Professor Marks* give for this quantity, assuming it for present purposes a constant, a value never far from $c = 0.02047$, which is here taken as the value affecting the cases assumed. Let the process of computation already illustrated be adopted, and let the data be as follows:

Data:

Engine, single-acting compound.

Clearance, 20 per cent.

Boiler pressure, 165 lbs. per sq. in., 23,660 per sq. ft.

Back pressure, 18 lbs. per in., 2592 per sq. ft.

Ratio of expansion in H. P. cylinder, 2.5.

Ratio of low- to high-pressure cylinder, 2.78 to 1.

Piston-speed, 600 feet per minute.

Initial volume, v_1, 2.8 feet; final, v_2, 7 feet; $p_2 = 8690$.

Results:

Weight of steam in low-pressure clearance, 0.554 lb.

Compression begins at 0.047; M. E. P., in H. P. cylinder, 6400 lbs.

Ditto in L. P. cylinder, 1940 lbs. per ft.

Weight of steam in L. P. cylinder, 1054 lbs.

Energy of steam per lb., 138,860 ft.-lbs.

Efficiency of the steam, $E = 0.1413$.

Water per H. P. per hour, lbs., 17.56.

Fuel at 10 lbs. per lb., 1.76.

Heat, at usual equivalent, per I. H. P. per hour, 19,766 B. T. U.

The above figures show what the ideal engine would do under the given conditions, and what would be the performance

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* Proportions of Steam-engine; 3d ed., p. 257.

of the real engine, irrespective of size, were there no wastes. With varying sizes, the volumes, v, worked at any given ratio of expansion, the stroke of piston being made variable with the diameter of the cylinder, will vary as the cubes of the diameters; while the surfaces, s, exposed will vary as the squares. The wastes occurring internally will thus vary as the quantity $s \div v$, or inversely as the diameter with cylinders of similar proportions. If the stroke be kept unchanged, the diameters varying, the wastes will vary as above, with the variations of surfaces and volumes. Ordinarily, short-stroke engines condense fifty per cent more rapidly than long-stroke.

In illustration, take three engines of the assumed type, having dimensions as below:

(1) 18″ and 30″ × 16″ stroke;
(2) 9″ and 15″ × 9″;
(3) 3″ and 5″ × 3″.

Taking the internal wastes, as already proposed, using the coefficient $c = 0.02047$, and computing the loss on the areas of the piston, the clearance, and port passages and interior of cylinder up to point of cut-off, we obtain the following results:

VARIATION OF EFFICIENCY WITH SIZE OF ENGINE.

Engine.	Area s.	I. H. P.	Fuel and Water per H. P.		Steam condensed per I. H. P.	Friction.
			I. H. P.	D. H. P.		
Ideal.			1.76; 17.6		0.0	
No. 1	10.16	220.7	1.3 ; 23	2.4; 34	5.4	5 p. c.
" 2	2.66	30.37	2.8 ; 27.9	3.1; 30.7	10.30	10 p. c.
" 3	0.294	1.132	4.8 ; 48.25	5.4; 45.2	30.7	15 p. c.

The enormous effect of this method of waste in small engines, and the very considerable influence of size upon its magnitude in the smaller classes of engines, are thus well exhibited. In the above instance, the interior wastes increase from 5.4 pounds to 10 and to 30 pounds per I. H. P., as size decreases, and the consumption of steam thus rises from 17.6

in the ideal case, to 28 and 48 pounds for the smaller engines. The modifying effects of the various expedients for reducing wastes and approximating more closely in real engines to the ideal case of pure thermodynamics will be illustrated in the chapter on compound engines; superheating and steam-jacketing; in which computations will be presented exemplifying those effects.

Computing the thermodynamic problem for the compound engines of the steamer City of Fall River from the data reported to the Author by Messrs. Adger and Sague, the observers, we may profitably compare the results with the actual performance.* This computation gives figures as below. The difference, 22 per cent, between the ideal and the real engine, being, in fact, probably, mainly the waste in one cylinder, as explained elsewhere, gives a measure of the extent to which cylinder-condensation affects the most wasteful of the two cylinders. The engine is of about 1600 I. H. P.

The steam was, in this case, dry; the engines large (44″ and 68″ diameters of cylinder; 8′ and 12′ stroke of piston) and the efficiency of boiler high. The engine had an efficiency of mechanism of 83 per cent, the paddles 80 per cent, 66 combined, and the whole machine was of excellent design. The lengths of trials ranged between eleven and twelve hours.

The following are the data and results:

p_1 = absolute pressure of admission = 11,808 lbs. per sq. ft.
p_2 = absolute pressure of release = 1363.68 " "
p_3 = mean absolute back-pressure = 704.16 " "
t_4 = absolute temperature of feed-water = 558°.36 Fahr.

The corresponding temperatures, densities, and latent heats are designated by the same subscripts:

$t_1 = 774°.50$ Fahr.; $t_2 = 652°.32$;
$L_1 = 131841.14$; $L_2 = 19000.39$;
$D_1 = .1909$; $D_2 = .02606$.

* Engine and Boiler Trials; R. H. Thurston; pp. 388–393.

From these data the following results were arrived at by considering the cylinders as non-conducting and the engine perfect: *

The ratio of expansion $r = 6.7167$.

Energy per cubic foot of steam admitted, $UD_1 = 27183.43$ foot-lbs.

Heat expended per cubic foot of steam admitted, $H_1D_1 = 163716.507$ foot-lbs.

Mean effective pressure, or energy per cubic foot swept through by piston,

$$\frac{UD_1}{r} = 4047.5 \text{ lbs. per sq. ft.}$$

Heat expended per cubic foot swept through by the piston, $\frac{H_1D_1}{r} = 24,377$ lbs. on square foot = pressure equivalent to heat expended.

Efficiency of steam $= \frac{UD_1}{H_1D_1} = \frac{U}{H_1} = .166.$

Net feed-water per cubic foot swept through by piston

$$= \frac{D_1}{r} = .0284.$$

Cubic feet to be swept through by piston for each indicated horse-power per hour $= \frac{1980000}{\text{M. E. P.}_1 = 4047.5} = 489.2$ cubic feet.

Feed-water per I. H. P. per hour $= 489.2 \times .0284 = 13.89$ lbs.

Actual feed-water $= 17.00$ lbs., nearly.

13.89

Difference, 3.11 lbs. = 22 per cent

* Steam-engine Trials; pp. 388, 389.

due to cylinder-condensation and leakage-waste and other wastes.

The Process of Computation of Total Efficiencies of Real Engines may be summarized in the following:

(1) Compute the work done per pound or per cubic foot of steam, under the assumed conditions of its use, by the thermodynamic formulas, and deduce weight of steam demanded per hour per horse-power, and the total for the specified total power.

(2) From the value of the mean effective pressure, p_e, thus computed, or from the quantity of steam and its known specific volume, determine the size of steam-cylinders needed to develop the demanded power, and then find the areas of surfaces subject to fluctuating temperature and wasting heat.

(3) Compute the *wastes* of heat and of steam, externally and internally, by conduction and radiation and by cylinder-condensation, thus:

(*a*) Take external wastes as 0.5 B. T. U. per square foot of well-lagged cylinder area and 2.5 B. T. U. for uncovered metal—as the heads—per Fahrenheit degree difference of temperature between these surfaces and surrounding bodies, per hour.

(*b*) Take internal wastes as measured by a fraction of the thermodynamic expenditure of heat and steam,

$$C = \frac{a}{d}\sqrt{\frac{r}{n}} = \frac{a}{d}\sqrt{rt};$$

where d, r, and t are the diameter of cylinder, the ratio of expansion in that cylinder, and the time of one revolution, or proportional to that of exposure to the exhaust; n, the revolutions per unit of time. For diameter in inches, and time-unit in seconds, $a = 4$, nearly, as obtained from the Sandy Hook experiments. $a = 3$ to $a = 3.5$ represent good values.

Or, take the waste as

$$H = C'A(T_1 - T_2)\sqrt{t};$$

where H is in B. T. U., A in square feet, T in Fahrenheit degrees, and t in seconds. $C = 20$, nearly. For H in pounds of steam wasted. $C' = 0.02$, nearly.

(*c*) Take Friction of Engine as from 0.06 to 0.12 the rated indicated power of the engine, accordingly as it is non-condensing or condensing.

Or, assume it as

$$F_1 = ad^2,$$

in h. p.; where a is about 0.04 for non-condensing and about 0.08 for condensing engines (d = inches).

(4) Sum up all wastes and expenditures, referring the total to both I.H.P. and D.H.P.

Proceed similarly with each of the given cases, tabulate computed quantities and plot curves exhibiting laws of variation of efficiency, with varying working conditions. (Notes, p. 991 *et seq.*)

A very extended account of the results of investigation of heat-wastes of engines will be found in the Appendix to § 149, page 995 *et seq.* Chap. VIII may also be consulted for illustration of the method of use of the "saturation-curve" in their measurement.

PROBABLE WATER CONSUMPTION. (Page 539.)

Class of Engine.	Gauge-reading.	Ratio of Indicated Load to Rated Capacity.						
		1/10	1/4	1/2	3/4	1	1¼	1½
		Lbs. of Water per Net or Delivered H. P. per Hour.						
Non-condensing.								
Throttling simple.... ...	80	Friction Load D. H. P. = O.	103	74.5	50.5	44.5	46.6	48.5
Automatic simple	80		77.0	47.5	39.6	35.6	36.6	37.8
" "	100		70.0	43.0	35.8	32.3	33.2	34.2
Corliss simple....	80		70.5	43.8	36.8	33.4	34.2	35.2
Automatic compound ..	100		59.5	37.7	31.8	28.8	29.4	30.1
" "	125		52.7	33.3	28.1	25.6	26.1	26.7
Condensing.								
Corliss simple	80	Less than Friction Load.	73.4	38.3	30.1	26.3	27.4	28.5
Automatic compound....	100		58.4	31.0	24.8	21.7	22.5	22.2
" " ...	125		53.4	28.5	22.7	20.0	20.7	21.4
Corliss compound.......	100		52.6	27.2	22.6	20.0	20.6	21.2
" "	125		47.4	25.6	19.9	18.3	18.8	19.3
Triple-expansion....	125		41.4	22.8	18.6	16.6	16.9	17.3
" "	150		39.2	21.6	17.5	15.7	16.0	16.5

CHAPTER VI.

THE COMPOUND OR MULTIPLE-CYLINDER ENGINE; STEAM-JACKETING AND SUPERHEATING.

138. The General Theory and the Construction of the multiple-cylinder engine are equally simple; and their correct forms may be readily and very exactly deduced from the principles and the facts already revealed by current practice and experience with the simple engine. As has been seen, the great source of avoidable wastes in the single cylinder is that alternate heating and cooling, and that consequent wasteful condensation and re-evaporation of steam, which is due to the exposure of the internal surfaces of the cylinder to the alternate heating action of entering steam and cooling effect of expansion and condensation. Any expedient which will reduce this waste by preventing that transfer of heat from the steam to the exhaust side of the engine without transformation in proper proportion, into work, will reduce this loss and increase the economical value of the machine. In "compound" engines, this is done by effecting a limited expansion and partial transformation of heat into work, submitting, so far as may be necessary to cylinder-condensation and re-evaporation, but then transferring the working steam, both the uncondensed and the re-evaporated, to a second cylinder in which the latter portion is enabled either to do some work or to balance its waste more or less fully. Any number of successive expansions may be thus practised; but experience indicates that not more than two is desirable at ordinary moderate pressures, three at from eight to ten atmospheres, or four with twelve to fifteen atmospheres pressure. Leakage is similarly prevented.

Experience, as well as the study of the distribution of

[Face page 584.]

FIG. 154*a*.—COMPOUND ENGINE AND GENERATOR.
(Direct-connected.)

wasted work in the machine, also indicates that a well-designed multiple-cylinder engine may exhibit higher efficiency of machine, i.e., less loss by friction, than ordinary simple engines arranged in pairs; thus giving still greater advantage when employed for marine work or wherever coupled engines are needed.

The multiple-cylinder engine is, therefore, any engine in which steam is used as the means of transformation of heat-energy into work, through a succession of expansions in cylinders placed "in series." In construction, this succession of steam-cylinders may be obtained, either by using structurally independent engines, or by making them parts of a single structure. The former system is sometimes seen, in stationary engine practice; the latter is usual in marine engines.

The question of adoption of the compound engine, or either form of multiple-cylinder engine, for the usual work of the locomotive or in any case in which the speed, pressure, or load, either or all, is expected to be variable, is complicated by the fact that, under such variable conditions, it is impracticable to find proportions of cylinders suitable, and permanently so. Whenever the load and speed may be expected to be reasonably constant, a suitable design may be produced. Hence the success of the marine engine in these forms and the less completely satisfactory results with other cases in which less uniform conditions are maintained, as in the locomotive. All the conditions affecting the choice and use of engines of differing type are those of practice, and quite apart from the thermodynamic problem.

In practice, the multiple-cylinder engine exhibits several advantages, and we may make a fairly-complete summary thus:

(1) Reduction of expansion in a single cylinder.

(2) Great restriction of internal waste.

(3) Ability to adopt large ratios of expansion, with light loads, without "wire-drawing."

(4) Reduced leakage in engine.

(5) Reduction of depreciation of boiler.

(6) Lighter blast; smoother draught; less waste, annoyance, and danger from sparks and cinder ejected from locomotives.

(7) Elevated limit of speed and power.

(8) Reduced loss by tender and fuel haulage.

(9) Greater uniformity of crank-moments.

(10) Larger efficiency of the machine.

139. **The Wastes of the Engine** are similar in kind, in all cases, to those of the simple engine.* Were it possible to construct a steam-engine of which the theory should be purely thermodynamic, an engine in which the only waste of energy should be that known as the necessary thermodynamic loss, its theory, as has been seen, would be most simple and most satisfactory. The efficiency of the engine and the quantities of heat, steam, and fuel demanded for its operation at a given power would be simple functions of the physical properties of the steam and of its ratio of expansion. The engineer, in constructing its theory, would only concern himself with the quantity of heat imported into the machine, the temperatures of the initial and terminal portions of the expansion-line, and the relation of initial to back pressures. The essential facts are the magnitudes of the pressures and volumes of the steam and the extent of adiabatic expansion, and it matters not whether the engine be one of a single cylinder or a multi-cylinder engine of indefinitely extended complexity. For this, the ideal case, the indicator-diagram represents precisely the amount of transformation of heat-energy into mechanical work, and the ratio of its measure in units of work to the mechanical equivalent of the total quantity of heat-energy supplied to the engine, while doing that work, is the measure of the efficiency of the engine; as it is of the thermodynamic efficiency of the working fluid. The thermodynamic efficiency, the dynamic efficiency of the machine, and the total efficiency of the engine are here identical.

* This portion of this chapter was presented, in part, at the Twentieth Meeting of the American Society of Mechanical Engineers. See Trans. 1889.

To ascertain how much heat, steam, and fuel are demanded by such an engine for the performance of work, it is only necessary to measure the quantity of work done by the steam upon the piston, as shown by the indicator, and to divide this quantity by the energy received by the engine from the boiler; the quotient is the efficiency of the engine. As the operation of the engine approaches more nearly the conditions of best effect, the magnitude of this measure of efficiency approaches a limit which is expressed by the quotient of the range of temperature worked through to the absolute temperature of the working fluid at entrance into the engine. The excess of the actual consumption of fuel, in the best engines, above the former figure measures the sum of all wastes in real engines due to imperfections other than of thermodynamic cycle. Thus, the best work of the Corliss compound mill-engine being taken as about sixteen pounds of steam per horse-power and per hour, where the thermodynamic efficiency is about twenty-five per cent, the ideal case demands about ten pounds, under similar conditions otherwise, and the wastes amount, in this case, therefore, to about six pounds per horse-power and per hour, or sixty per cent of the ideal consumption. This comparison is easily made by the method already presented, which enables the thermodynamic efficiency to be easily computed for any given case.

The wastes of the steam-engine have been shown to comprehend two principal classes: the external and the internal wastes; and these latter are of two distinct kinds. We may classify such losses thus:

(1) External wastes; consisting of those losses of untransformed heat which are produced by the conductivity and the radiating power of the materials of which the heated parts of the engine are composed. Two per cent should probably represent as large a percentage as is to be reasonably expected in good practice with engines of moderate or large size.

(2) Internal wastes; consisting of two parts:

(*a*) Thermodynamic, unavoidable, losses of heat rejected at the lower limit of temperature of the working fluid;

(*b*) Wastes by internal conduction and storage of heat, followed by later rejection with the exhaust-steam.

To these are to be added:

(3) Wastes of mechanical energy.

Of the internal losses, the first, (*a*), is, for any given set of initial and final temperatures of working fluid, a fixed quantity, and one which measures the defect of efficiency of the perfect engine working between the given temperatures. The second, (*b*), is a quantity of variable amount, capable of amelioration by one or all of several known expedients, and reducible from the enormous proportion observed in small and ill-designed or badly constructed engines to a very moderate amount in large engines of good type. The last item, (3), is one which is seldom large in good constructions, and may in some cases, by careful design, good construction, and skilful management, be brought down to less than five per cent in non-condensing and to perhaps ten per cent of the total energy in condensing engines of simple forms and high mean working pressures. The unavoidable thermodynamic waste is rarely less than seventy-five or eighty per cent of the total thermodynamic demand, and the internal wastes by conduction and storage with subsequent rejection, by cylinder or internal condensation, as it is customarily called, and by leakage, range from ten per cent, as a minimum, perhaps, to twenty-five or thirty per cent of the heat received from the boiler, in good engines, to fifty per cent, in many cases, and even to much more than the latter proportion in exceptional cases. It is this which has now been found to constitute, ordinarily, the great source of loss and inefficiency of the real, as distinguished from the ideal, engine. The proportions of stroke to diameter are usually better in the high-pressure cylinders of multiple-expansion engines than in simple engines, and condensing areas are thus reduced as well as rendered less active. (§ 130, p. 523. See Notes.)

The Flow of Energy in Engines is admirably illustrated in Figs. 154*a* and 154*b*, "Sankey diagrams," exhibiting the course of the current from the furnace through the boiler and engine, on the one hand, to the condenser, and, on the other hand, as

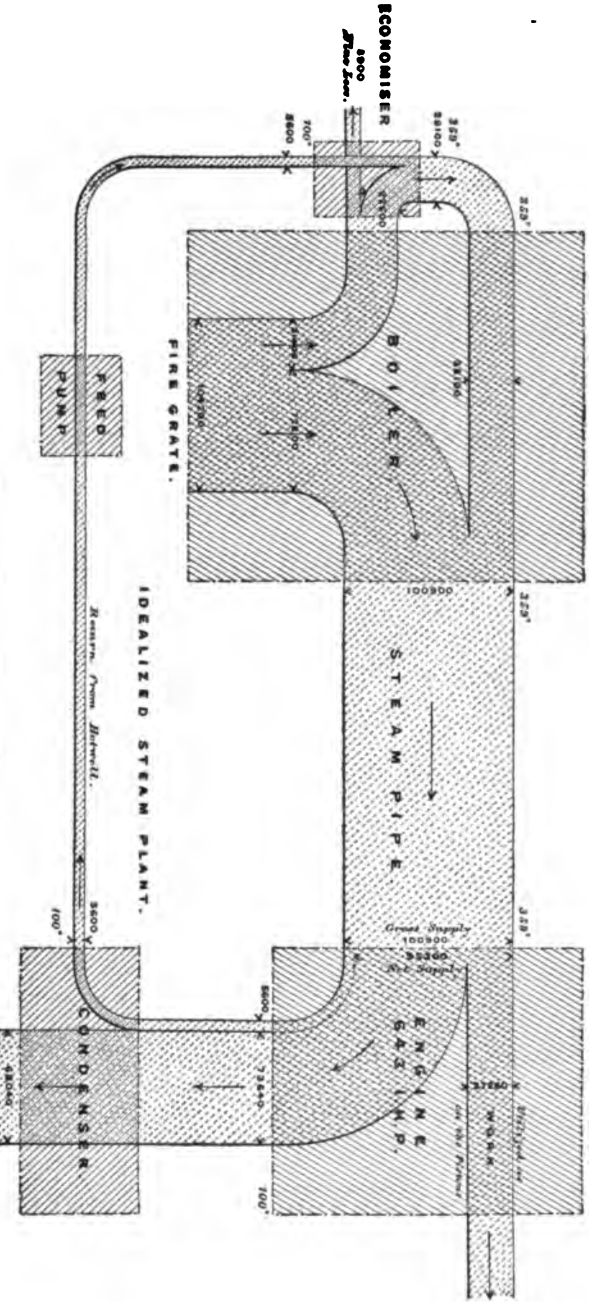

FIG. 154a.—FLOW OF ENERGY. (SANKEY.)

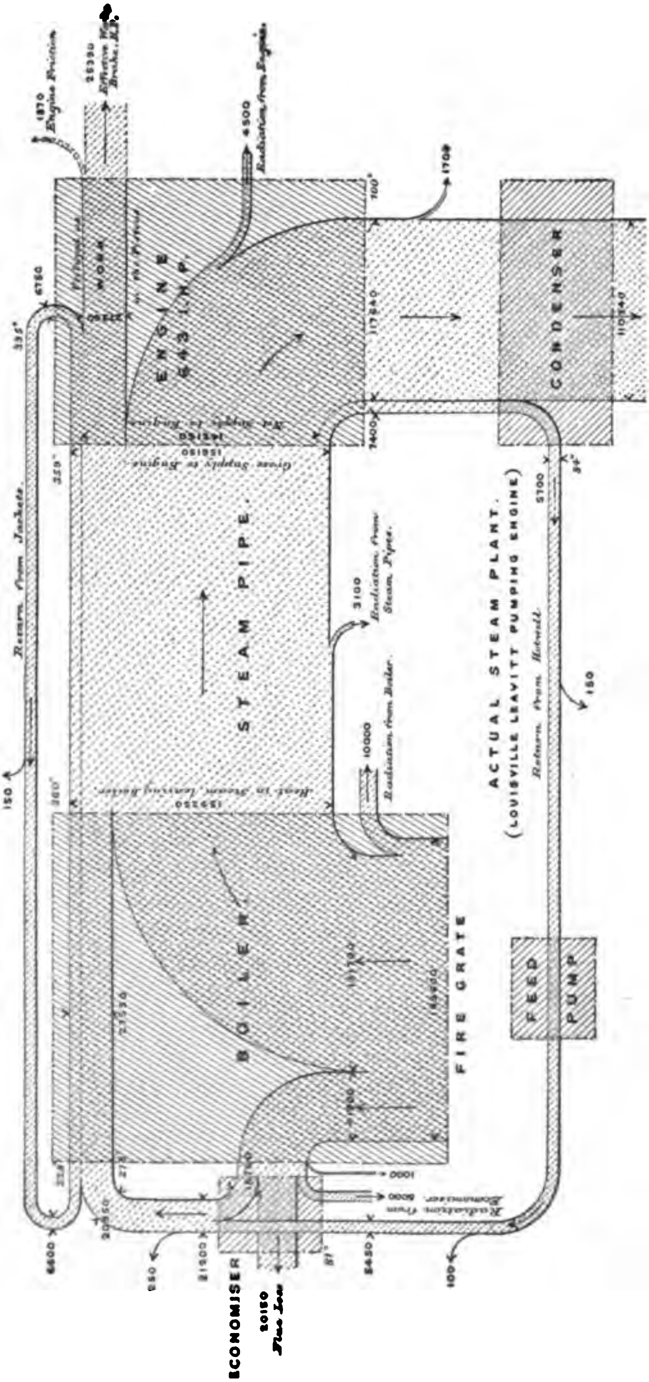

FIG. 154b.—FLOW OF ENERGY. (SANKEY.)

wasted at the chimney and by conduction and radiation *en route* to the condenser. Fig. 154*a* exhibits the case of the ideal engine to which the real case, Fig. 154*b*, would be reduced, could the machine be made purely thermodynamic and all extra thermodynamic wastes eliminated.

In comparing the simple with the compounded engine, in average practice, it will be found that the former excels, in the best types, in the small clearance practicable in a single cylinder; in the adaptability to that type of an effective expansion-gear, as illustrated in the Corliss type; in its giving a dynamic cycle which is represented by an indicator-diagram, of which the area is very nearly that of the ideal case; and, finally, in its lesser total area of exposed, radiating, and heat-wasting surfaces, exterior and interior.

On the other hand, the compounded type excels in the fact of its utilizing the wastes occurring in the full cycle, step by step, as they take place, more and more perfectly as the number of cylinders and of successive stages of expansion are increased, thus permitting an increase of the practically economical ratio of expansion. It also excels, in the marine type, in which two or three engines are found to be desirable in order to secure a good distribution of turning stresses and moments, by giving a more uniform pressure on the crank-pin and smoother and more nearly frictionless rotation of the shaft.

The advantage may lie on the one side or the other, in special cases; and it has usually been found practicable to attain substantially the same efficiency of working fluid in the one type as in the other by exercising sufficient care in provision against wastes. The simple engine must probably be largely dependent upon either jacketing or superheating for high efficiency; while the compounded types are more nearly independent of these expedients.

Comparing the ideal with the real engine, we may take in illustration the following from available data of engine-operation, computing the ideal case and comparing the results with those actually obtained :

DATA AND RESULTS.

Engines	Compound.	Standard.
Cases	I.	II.
p_1 (lbs. per sq. in.)	155	130
r	4	2
p_2	20	20
Temperature of feed-water	60° F.	60° F.
Efficiency of steam	0.1445	0.1197
" " furnace (Rankine's Eq.)	0.664	0.60

The fuel used on trial was reported as 4.1 and 4.7 pounds per D. H. P. per hour, respectively.

Coal per D. H. P. per hour	4.10 lbs.	4.7 lbs.
" " I. H. P. " " at 20 p. c. friction	3.28 "	3.6 "
" " " " " *ideal*	2.01 "	2.35 "
Wastes, extra-thermodynamic, per cent	30.9	37.6
Value of a ($c = a\sqrt{r}$)	0.22*	0.26

A gain of 15 per cent is made by reduction of waste, by compounding, in this instance, which represents an actual case in locomotive practice.

140. The Amelioration of Wastes thus becomes an important matter. The efficiency and economy of operation of the single cylinder, the "simple" engine, is at all times limited by internal waste, and the question which all engineers endeavor to solve is: In what manner may we best proceed to eliminate or ameliorate this loss? The three methods which have been found advantageous, and, in special cases, effective, have been seen to be:

(1) Superheating;
(2) Steam-jacketing;
(3) "Compounding."

Superheating is a well-known but not a common method. It is evident that, if the steam can be introduced into the engine at such a temperature that the cooling action of the metal of the cylinder will not cause its condensation initially, and the stroke may be performed without condensation in

* Closely corresponding with the Author's deduction for engines of quite nearly equal volume, and earlier reported.

consequence of doing work, no loss of heat from the cylinder can take place by re-evaporation ; and if no such loss occurs, the waste of heat at entrance, in turn, by initial cooling, will be reduced. Superheated steam, also, is a good non-conductor and a non-absorbent of heat, like the permanent gases. It is thus, also, less liable to this waste. But it is found, commonly, that superheating beyond a very moderate degree, perhaps 100° F., is inadvisable on account of risks of injury to engines and cost of repairs to superheater, which more than compensate its advantages. It has come to be regarded as an auxiliary in economizing, not as a complete remedy for interior wastes. This method of augmenting efficiency will be more fully discussed later.

Steam-jacketing is a common partial remedy for this waste. By surrounding the steam-cylinder with the steam-jacket, it is possible to produce, in part, the effect of superheating; that is, to secure drier steam in the engine throughout the stroke. The amount of re-evaporation, during the period succeeding cut-off and up to the closure of the exhaust-valve, and the quantity of heat of which the cylinder is thus robbed, measures closely the amount of initial condensation and waste and the weight of steam which must be supplied in excess of the thermodynamic demand to compensate that loss. The effect of the addition of a steam-jacket depends upon the conditions of operation of the engines, largely, and may be productive of marked advantage or, under unfavorable conditions, of no important useful effect. With steam initially dry, the jacket is probably usually helpful; but, with wet steam, or with superheated steam, it is of comparatively little value, even if not sometimes a positively wasteful adjunct. Steam-jacketing will be made the subject of a later article.

Of the several available methods of checking cylinder-wastes of heat, it is evident that only the plan of securing a non-conducting interior surface is purely economical in method. To superheat the entering steam is to reduce a great loss by submitting to a small one; and even permanent superheat, as in the conversion of the fluid into a gas, still leaves this loss only

ameliorated, not completely destroyed. In a single unjacketed cylinder, heat carried out by the exhaust is a pure waste; in the jacketed engine, this remains true, though in less degree, not only of the heat lost by initial condensation and later re-evaporation, but also of that heat which may have been employed in reducing its amount, either by drying the prime steam, or by the normal action of the jacket. In multi-cylinder engines, the heat employed in raising the temperature and reducing the initial condensation of the steam in the first cylinder is utilized in the second by there securing a better quality of steam, as well as by directly checking this waste. All the heat swept out of the last cylinder into the condenser is wasted. Utilization of the added heat, in either system, is obviously, at best, incomplete. With wet steam, the jacket may even exaggerate, rather than reduce, the loss; as it may, with considerable expansion, increase exhaust-wastes in greater amount than it decreases cylinder-condensation. The same remarks, to a less extent, apply to the systems of compounding where excessive expansion is adopted. Superheated, or at least dry, steam must be provided by the boiler to insure economy, either with or without these special constructions, and to enable the ratio of expansion to be economically increased.

"*Compounding*," or the use of the multiple-cylinder engine, in which the steam exhausted from one cylinder is again worked in a succeeding one, is now the most familiar of devices for extending the economical range of expansion and increasing the efficiency of the engine. The limit to useful increase of the ratio of expansion of steam in a single cylinder is found to be determined by the magnitude of the wastes incurred in the operation of an engine of which the working cylinder is a good conducting material. Any method of reducing this waste of heat internally will enable the efficiency of the engine to be increased by further profitable extension of the ratio of expansion.

141. **The Problems in Compounding** are now readily stated. Assuming it to be possible to divide the waste by

cylinder-condensation and leakage by two or more, it is evident that the limit to economical expansion and transformation of heat into work will be set correspondingly farther away. This is done by the multiple-cyclinder engine: the internal wastes are reduced approximately to those of one of its cylinders, and the gross percentage of waste is made less in the proportion of this division. The heat and steam rejected as waste by internal transfer without transformation from the first cylinder is utilized in the second nearly as effectively as if it were received directly from a boiler at the pressure of rejection from the first cylinder. Insomuch, therefore, as the pressure can be increased and the increase utilized by the addition of another cylinder, gain is secured.

Common experience shows that the best results are ordinarily obtained, in each class of multiple-cylinder engine, when, the engine being properly designed for its work, the terminal pressure for the system can be economically made something above the sum of back-pressure in the low-pressure cylinder, plus friction of engine. This total may be usually taken, probably, at about eight or ten pounds above a vacuum. The latter figure will be here assumed.

142. **The Three Fundamental Principles** are:

(1) Economical expansion in a single cylinder has a limit, due to increasing internal wastes; which limit is found at a comparatively low ratio of expansion.

(2) The method of expansion may be, for practical purposes, and such as are here in view, taken to be approximately hyperbolic; the best terminal pressure being something above that which corresponds to the sum of all useless resistances, and which may be here taken as, for example, about ten pounds per square inch above a vacuum.* The division of the initial tension by this terminal pressure will thus give an

* Mr. H. A. B. Cole finds the value of the index n, in ordinarily good engines of the triple-expansion type, to be approximately 1.2, varying but little with the range of expansion adopted. (Converting Compound into Triple-expansion Engines; Trans. Brit. Inst. N. A.; 1886.)

approximate measure of the *desirable* ratio of total expansion for the best existing engines.

(3) All steam entering any one cylinder will be rejected, *as steam*, into the succeeding cylinder, external wastes being neglected, and ultimately into the condenser; and the full amount of steam liquefied at entrance by absorption of heat by the interior surfaces of the cylinder will be re-evaporated later, and will pass into the condenser or into the next cylinder. Heat transferred in the one direction, in the one process, will be transferred in precisely equal amount in the opposite direction, in the other.

This last point is important, and is easily established: The cylinder, when in steady operation, is neither permanently heated nor permanently cooled; no progressive heating can go on, as it would, in that case, become heated above the temperature of the steam and become a super-heater; no progressive cooling can occur, since, in that case, the cylinder would become a condenser of indefinite capacity. It must, therefore, transfer to the next element of the system all the heat which it thus receives; assuming that external radiation and conduction may be neglected, and that the Rankine and Clausius phenomenon of liquefaction of steam by transformation of heat into work is ignored.* It also further follows that the introduction of one or of many cylinders between the terminal element and the boiler does not, through cylinder-condensation, affect the operation of the final cylinder, however great that condensation may be; provided the introduction of the added elements is effected by raising the steam-pressures commensurately, leaving to the final element of the series the same initial pressure as at first. The Rankine and Clausius phenomenon, it should however be noted, insignificant in amount and effect, in any one cylinder, with its customarily low ratio of expansion, produces a cumulative condensation in the series,

* This the Author would denominate Hirn's principle. See a paper by M. Dwelshauvers-Dery in the Bulletin de la Société Industrielle de Mulhouse, October 1888, on the theory of simple engines. A mathematical proof may be found in De Freminville's Cour de Machines à Vapeur; 1862; p. 121.

which, at high total ratios, has already been seen to be important, amounting to something between 15 and 20 per cent of the steam thermodynamically demanded. This condensation is not at all affected by the principle of "compounding," as the heat thus surrendered by the steam is transformed into work and thus taken out of the system instead of being temporarily stored.

The total waste by this form of loss is thus evidently measured, in the case of the multiple-cylinder engine, by the maximum waste in one cylinder. If all are equally subject to this loss, the rejected steam of re-evaporation from any one cylinder, as the high-pressure cylinder, supplies precisely what is needed to meet the waste by initial condensation in the next; and so on through the series. Thus the use of a series of cylinders, in this manner, divides the total waste for a single cylinder, approximately, at least, by the number of cylinders; and it is in this manner, largely, that the "compound" system gives its remarkable increase of efficiency.

The three principles which have been enunciated give a means of constructing a philosophy of the multiple-cylinder engine, which will meet all essential needs of the engineer. The first principle shows that, a limit existing to economical expansion in a single cylinder, the advisable number of cylinders in series may probably be determined, when that limit is ascertained for any case, either by experiment, by general experience, or by rational theory and computation. The second principle shows that we may find an approximate measure, at least, of the desirable total ratio of expansion for maximum efficiency, when the best terminal pressure for the chosen type of engine is settled upon. This total range of expansion is divided by the maximum admissible range for a single cylinder to determine the minimum desirable number of cylinders. Otherwise stated: The total ratio is a quantity which should approximately equal the admissible ratio for a single cylinder raised to a power denoted by the number of cylinders. Combining thus the two considerations referred to, we may obtain a determination, probably fairly approximate, of the proper mini-

mum number of cylinders in series. The third principle enables an estimate to be made of the total internal wastes of the series, and the probable expenditure of heat and of steam, and permits a solution of all problems of efficiency for the compound engine, of whatever type.

143. **The First Step in designing the "Compound" Engine** is the determination of the best ratio of expansion, under the assumed conditions of operation and for the given type of engine, for a single cylinder; then the best ratio of expansion for the series; this study being made largely from the financial standpoint. It is not the thermodynamic, nor the fluid, nor even the engine, efficiency, which must be finally allowed to fix the best ratio of expansion; but this must be the ratio of expansion at maximum commercial efficiency; that which will make the cost of operation at the desired power a minimum for the probable life of the system. The total ratio being settled upon, and that allowable, as a maximum, for the single cylinder, it becomes easy to determine the best number of cylinders in series. The first-mentioned ratio is that of maximum commercial efficiency, as just stated; but the second must be taken as that which gives highest efficiency of engine, the back-pressure in that cylinder and its friction, taken singly, being considered, together with its proper proportion of the friction of the engine as a whole.

Studying the method of distribution of wastes among the several cylinders of the multiple-cylinder engine, it will be observed that, since the pressures increase more rapidly than the temperatures, the range of temperature in the high-pressure cylinder is greatest; while, the same weight of steam passing through the whole series, the low-pressure cylinder presents the largest area of condensing surface in proportion to quantity of steam used.*

* In Professor Schröter's tests of the Augsburg triple-expansion engine, in 1889, the condensation in the cylinders ranged from an average of 14.4 per cent, in the small cylinder, to 33.7 and 51.9 per cent, in the intermediate and low-pressure cylinders, respectively; the total amounting to from 16 to 20 per cent of the whole steam-supply. Otherwise stated, these interior wastes

144. **The Extent of Economical Expansion** in a single cylinder will vary with the working range of temperature and pressure, and with the physical condition of the working fluid; but it may be taken, as determined by experience, as perhaps not above two and a half expansions for unjacketed engines with wet steam, or not over three or four for good practice with the better classes of engines. The total expansion-ratio thus becomes, for the several types of multiple-cylinder engines, as below:

MULTIPLE-CYLINDER ENGINES.

No. cyls.	1	2	3	4
r	2.5 to 3	6.25 to 9	16 to 27	40 to 81
p_1....	25 to 30 lbs.	60 to 100 lbs.	120 to 300 lbs.	350 to 800 lbs.

The result of this sharing of the wastes in the multiple-cylinder engine is that, in the triple-expansion engine, as an illustration, the total cylinder-condensation may be reduced from about 30 per cent, as in the parallel case of simple engine, to 10 or 12 per cent. This assumes good design and dry steam—i.e., steam containing less than three per cent water. In such case, the area of the combined indicator-diagram should approximate 80 per cent that of the ideal case. The compound engine should approximate 70 per cent. In common practice with 150 pounds steam, the temperature being equalized in the triple-expansion engine, the ratios of cylinder-volumes are about 1 : 2.5 : 7.5, or, equalizing work, not far from 1 : 2.8 : 7.1.

Thus a triple-expansion engine should do best work up to a pressure above 200 pounds, and the four-cylinder engine should be adopted from that point up to the highest pressures likely to be employed in the steam-engine; the common double-expansion compound serving its purpose well below the lowest figures assigned to the triple engine. Any type of engine may

amounted, each, to from $2\frac{1}{4}$ to 10 per cent of the total steam made; being, for example, in one trial, 2.6, 6.0, and 9.7 per cent in the three cylinders, respectively; the minima being 2.2, 5.4. and 7.3; and the maxima 2.9, 6.4, and 10.7; while the totals ranged from 16.1 to 20 per cent.

be made to overlap the range assigned it by suitably providing against wastes occurring within the engine; as by increased speed; by superheating; by any expedients giving higher effectiveness to the jackets, or by any other method of improvement. Any system which increases the efficiency of the simple engine will improve the efficiency of the compound, and will correspondingly increase the range of pressure through which it will give satisfactory gain as compared with the former.

145. **The Influence of Economical Expedients** recognized as useful in other forms of engine, as superheating, jacketing, and increasing speed of engine, may readily be perceived when the method of operation of the multiple-cylinder engine is understood in its relations to heat-transfer and heat-transformation. We may consider them in their order:

(1) *Superheating* the steam transferred from boiler to engine results in the supply of a fluid which may surrender to the metal of the working cylinder a certain portion of heat—measured by the product of its specific heat as a gas into the range of superheating and into its weight—without the production of initial liquefaction. If this quantity of heat is equal to or greater than the loss during expansion and exhaust, there will be no initial condensation; and the waste from the high-pressure cylinder will be nearly that due the passage of a gas through it under similar conditions of temperature and expansion; a comparatively small quantity, since any substance in the gaseous state possesses low conductivity and slight power of absorption and storage of heat. Should the superheating be in excess of this amount, the steam will not begin to condense until a later period, perhaps not at all; the only requirement to prevent liquefaction being now for heat to supply the amount required to keep the steam dry and saturated while expanding and doing work. If the superheating be less than the first-mentioned quantity, initial condensation will be reduced but not entirely prevented. It is probably never the fact that it is practicable to secure, safely and economically, so much superheating as is needed to keep the steam dry through-

out the stroke.* In any case, the quantity represented by the superheating will be a gauge of the amelioration of wastes by internal transfer of heat in every cylinder of the series. The steam leaving the high-pressure cylinder will be to that extent drier; and this will be true of the succeeding cylinder or cylinders.

Were there no other disappearance of heat than that due to cylinder-condensation, superheating at the first element of the series would give superheating at each of the others. In so far as condensation, such as was pointed out by Rankine and Clausius as the result of conversion of heat into work, takes effect, and so far as other wastes by transfer without transformation occur, to that extent will the gain, as observed in successive passages from cylinder to cylinder, be reduced; though the improvement of the working conditions above asserted will be none the less real. Each cylinder will have wetter steam than the preceding, in proportion as the condensation doing work and the losses by conduction and radiation increase, as a total, cylinder by cylinder. Superheating at the high-pressure cylinder will produce a favorable effect all through the series, including the low-pressure cylinder. Cylinder-condensation will, nevertheless, cumulatively increase throughout the series, in consequence of the fact that the wetter the steam entering any one cylinder the more the condensation, and the wetter that leaving it, both by this initial increase of humidity and heat-storing power and by the additional moisture coming from the Rankine and Clausius phenomenon, and from the loss by transfer to surrounding bodies. This last action will, however, be the less observable and the less important in its effect as the moisture of the entering steam and the magnitude of the waste by initial condensation become greater. The more nearly the total proportion of water in the mixture approaches one half, the more nearly does this phenomenon become a

* This subject is of greater importance in the case of the simple than in that of the multiple-cylinder engine (see Art. 168). Superheating is of greater proportional value for small than for large engines, and for unjacketed than for jacketed machines (see Art. 170).

vanishing quantity. It may probably be neglected entirely in the computation of efficiencies for a large proportion of the engines in use, without introducing sensible error, and very probably may be neglected in all cases without invalidating conclusions reached ignoring it. On the other hand, superheating is not likely ever to produce much effect upon this action.

(2) *Steam-jacketing*, the expedient devised by Watt for the purpose of reducing internal wastes, is a method of approximately "keeping the cylinder as hot as the steam which enters it," as Watt states it, in order that no such chilling of the entering steam may occur. Authorities disagree as to what extent and in what manner the jacket is advantageous in the multiple-cylinder engine. It is sometimes advised to jacket only the high-pressure cylinder; sometimes only the low-pressure, and sometimes the whole series, whether one, two or three cylinders, or more. The philosophy of the engine would indicate that, to secure maximum good effect, assuming the jacket on the whole desirable at all, the best system is the latter; and that, since the waste of the engine is most nearly measured by the losses of its most inefficient member, to omit the jacket from any one cylinder insures that the aggregate loss of heat in the whole engine will be increased by just the amount by which waste is increased in that one cylinder by such omission.

The question which actually arises in practice, for the designing engineer, is whether it will *pay* to jacket at all, or not. It can be readily seen that it is not as important, in a financial sense, that the multiple-cylinder engine be jacketed as it is to jacket a simple engine of similar total range of expansion. The value of the waste due to omission of the jacket is less, ordinarily, as the number of cylinders, in series, is the greater. It is also seen that those conditions which may make it unnecessary to jacket the simple cylinder make it still less important in the multiple-cylinder engine. As piston-speeds are increased, for example, the necessity of the jacket decreases, and the limit at which it will pay here to dispense with it is

sooner reached than in the single-cylinder engine. It is this principle which justifies the not uncommon practice of omitting jackets from engines which are driven up to 200 or 300 revolutions or to 1000 feet a minute, or more, of piston-speed; while pumping-engines, for example, in which the speed is low, must usually be jacketed, if high duty is demanded.

(3) *High engine-speed*, a device for reducing internal wastes, as well as decreasing cost of construction and weight, is evidently a matter of less serious importance as the number of cylinders is increased; yet it is equally evident that, to secure maximum efficiency, it is essential that the time of exposure to the action of the wasteful influences in each cylinder be made a minimum. At modern and customary speeds of piston and of rotation, the value of these several expedients for improving performance is much less than formerly; but all are to be adopted where it is hoped to secure such high efficiency as is coming to be demanded of the designing and the constructing engineer. So long as the advantages of further gain in this direction are safely attainable for the simple engine, they are still desirable, and may prove attainable, in the multiple-cylinder machine.

Non-conducting cylinders, such as were partly secured by Smeaton by the use of his wood-lined pistons and heads, and such as have since been sought by Emery and others; such as were shown to be needed by Watt, and later more conclusively by Rankine and his successors, would do away with the necessity of compounding on the ground of thermodynamic gain; but would leave the advantages of the multiple-cylinder engine, on the score of better division of stresses and work, unaffected.

Clearances are usually greater in the multiple-cylinder than in the simple engine; but it is also seen that the loss by clearance, and the rejected steam thus unutilized, in any one cylinder, goes to fill the clearances of the next; and thus the loss by this method of waste is divided approximately, also, by the number of cylinders, as in the case of other losses. It remains advisable to reduce the "dead-spaces" as much as is practi-

cable; but the importance of this matter is less than in the case of the simple engine.

Thus the adoption of the multiple-cylinder engine reduces wastes of every kind, except those coming of increased radiation from the exterior, where the total area is, commonly, increased, and the loss due to the friction of the engine when the number of cylinders is in excess. These are, however, minor wastes.

146. **The Number of Cylinders** to be introduced in series is finally settled by financial considerations. The fact that the loss by internal wastes is measured by that of one of the cylinders only indicates that, as a matter of economy of heat, simply, there is no natural limit to the number; except that the losses by external conduction and radiation may finally more than compensate the gain by further complication. This principle is easily shown, thus:

The work performed is proportional to the quantity $1 + \log r$, and the cost of that work is proportional to the quantity $1 + ar^{\frac{1}{mn}}$, since the expansion in one cylinder is the nth root of the total ratio of expansion for the series; m being taken as the index determined by the rate and method of variation of the cylinder-condensation with variation of the ratio of expansion, and which is not far from $m = 2$; and a is a constant coefficient, not far from 0.2. The cost of power, measured in terms of steam expended thermodynamically and by internal wastes, is a minimum when the quotient of the two expressions,

$$\frac{1 + \log r}{1 + ar^{\frac{1}{mn}}},$$

above is a maximum; this is a maximum when the denominator is a minimum; and this is a minimum when the value of n increases, without limit.

Assuming, in illustration, as the result of general experience in good practice, that, under the best customary conditions of operation, a good simple engine, working at high pressure,

condensing, and at the best ratio of expansion for maximum engine-efficiency, may be fairly expected to give as good a result as two pounds of fuel of satisfactory quality per horse-power and per hour. Under similarly favorable conditions, we may also, with equal likelihood, anticipate a probability that we may obtain better work with multiple-cylinder engines as below. Condensing engines are assumed.

Engine.	Consumption.	Gain, Total.	Gain, Difference.
Simple, one cylinder	2 lbs.	....	....
Compound (double-expansion)	1.6	20 p. c.	20 p. c.
Triple-expansion	1.4	30	10
Quadruple-expansion	1.2	40	10
Quintuple-expansion	1.0	50	10

The figures in the first three cases are based upon what is probably ample experience; the others are obtained by inference from the rate of progression thus established, and upon the principle, above enunciated, that the waste is reduced in proportion, approximately, to the number of cylinders in series. The probable first cost and running expense of adding one and another cylinder to any given type is easily ascertained by the engineer; and he can then, in such cases, readily determine whether the gain fairly to be anticipated is sufficient to compensate the cost of its acquirement, and to give a fair margin of profit. (See Appendix, Part II.)

Another important inference from what has preceded is that the question of use of one or another type of multiple-cylinder engine is not primarily settled by the magnitude of the steam-pressure to be adopted; although it may be taken as settled by experience and by the financial aspect of the question, as just indicated, that it will not usually pay to compound a machine working at very low pressures; nor to adopt a third cylinder until the pressure approaches some six or eight atmospheres; the advisability of adding cylinder after cylinder being, in part, determined by the rise in pressure, at the rate of perhaps not more than one cylinder for each four

or five atmospheres of pressure. Whatever the pressure, however, compounding will divide the total internal thermal loss, approximately, by the number of cylinders in series; but it does not at all follow that the efficiency of engine, or the commercial efficiency, will be reduced in similar ratio. On the contrary, as will be seen later, it will never pay to carry the complication as far as the study of the case from this point of view would dictate. The discrepancy will be found to be the greater as the real engine more closely approaches ideal perfection; the simple engine becoming the more desirable type as the efficiency of it, and of each of the several elements of the compound engine, becomes greater.

147. **As respects Size of Engine,** it is now easily seen that the gain by compounding is, so far as the considerations here studied are concerned, at least, likely to prove even more marked with small than with large engines. As the wastes are invariably, under similar working conditions, greater as size decreases, the desirability of reducing those losses would seem likely, ordinarily, to be also greater. In the case of the adaptation of this system to small engines, the effect of cylinder-condensation remains, in each cylinder, well marked, ordinarily, as is seen in the hitherto unnoticed effect observable where such small engines are constructed of the Wolff type, and the first effect of the cooling action of the metal upon the entering steam is shown by the sudden drop of pressure between the two cylinders, at the moment of opening communication; the fall being like that seen when exhaust occurs into the atmosphere from a high terminal expansion, and amounting, often, to several pounds.*

148. **Problems** relating to the relative efficiency of the various classes of multiple-cylinder engine may now be readily solved, the needed data being obtainable, by assuming the above enunciated principles to be applicable, and first computing the efficiency of the representative ideal engine, and then ascertain-

* This has been noticed and provided for by the designers of a familiar type of single-acting compound engine.

ing the probable wastes of heat, of power and of work, of the several cylinders, and of the engine as a whole. Obviously, the computation for the ideal engine is the same, whether the system is simple or complex. The wastes, however, vary with each type, and with every size and proportion of engine. If, as is now often possible, we may ascertain the approximate measure of waste for each cylinder and for each engine, whatever its type, it becomes perfectly practicable to determine the relative merits of each, and the probable efficiency and consumption of heat, of steam, and of fuel, also, if the efficiency of the boiler is given or can be computed. The difference of efficiency among the several types or examples indicates the relative standing of those various examples, and furnishes the basis for computation of all the efficiencies.

The following are illustrations of approximate solutions of such problems, as arising in common practice or as illustrated in the experiences of the engineer seeking to ascertain which of all available designs is the best for the special purposes in view:

The differences between the steam-consumption figures of the two tables given in the preceding chapter for the ideal and the actual efficiencies of simple engines have been seen to be the measure of those wastes which may be largely reduced by compounding; a nearly constant quantity, 6 pounds of steam for the condensing and 10 pounds for each form for the non-condensing engines. A two-cylinder compound engine should reduce these wastes to approximately 3 and 5 pounds, a triple-expansion to 2 and to 3.3 pounds. Case No. 5, in the last table, using 23 pounds of steam per hour per horse-power, would, as a compound engine, demand 20 pounds, as a triple-expansion 19 pounds, and as a quadruple-expansion engine about 18.2.

A familiar type of tandem compound high-speed engine is usually operated at a pressure of about 110 pounds by gauge, at a ratio of expansion of 9 and with cylinders having the ratio of 2.3 to 1. The following is the result of investigation

of this case, thermodynamically. It is first assumed that the engine is supplied with steam of variable pressure, next that the pressure is constant at the figure intended by its builders and the ratio of expansion varied. The deductions from these studies of efficiency are that both the boiler-pressure and the ratio of expansion assumed by the builders are very nearly ideally right for best economy with that form of engine. Further gain could be better secured, however, in this case, by an increase of the expansion than by that of the steam-pressure at the given ratio of expansion.

It is here assumed that the friction of engine is 10 per cent, the efficiency of machine being 90 per cent, and that jacket-wastes are 8 per cent, and external radiation 5 per cent; the net "efficiency of engine" thus becoming about 77 per cent the thermodynamic "efficiency of steam." The pressure in the valve-chest is taken as 0.97 that in the boiler.

It must be borne in mind, however, that the investigation represents the ideal, not the actual, case, and that the consumption of steam and fuel and the real efficiencies will be somewhat different; possibly varying from the computed figures 10 to 15 per cent, and correspondingly reducing the ratio of expansion and the pressure for best effect.

HIGH-SPEED ENGINE.

Variation of Pressure.

$$p_2 = 4; \quad r = R_1 \times R_2 = 9.$$

Boiler pressure..... p_0	50	75	100	110	120	140	160	180
Engine " ..... p_1	48.5	72.8	97.0	107	116	125	155	175
Receiver " ..... p_r	20.8	29.0	37.2	40.5	43.8	50.3	50.9	63.4
Mean total " ..... p_m	9.15	12.8	16.4	17.8	19.3	22.1	25.3	27.9
Mean eff. " ..... p_e	5.15	8.76	12.4	13.8	15.3	18.1	21.0	23.9
"Heat-pressure"... p_h	50.8	69.0	87.2	94.5	102	116	131	155
Effic. of Steam..... E_t	0.10	.127	.142	.146	.150	.156	.161	.163
Effic. of Engine.... E_e	0.08	.099	.111	.114	.117	.122	.126	.127
"Water-rate"...... W	17.3	13.8	12.5	12.0	11.8	11.2	10.9	10.8

Variation of Expansion-ratio.

$p_1 = 106.7.$

Expansion-ratio.... r	6	8	10	12	15	18	21
Receiver pressure .. p_r	42	41	39	39	37	35	34
Total mean " .. p_m	22.4	17.9	14.5	12.3	9.09	6.94	5.34
Mean effective " .. p_e	18.4	13.9	10.5	8.29	6.42	4.94	3.86
"Heat-pressure"... p_h	208	134	92	67	48	36	27
Effic. of Steam ..... E_t	0.09	.102	.113	.120	.129	.134	.138
" " Engine.... E_e	0.07	.080	.089	.094	.102	.105	.109
"Water-rate"...... W	14.2	12.3	11.1	10.5	9.65	9.35	9.0

The actual efficiencies will be reduced by the wastes to considerably smaller figures, as hereafter shown, and the water-rate thermodynamically computed will be increased, in such engines, ordinarily, by ten pounds, more or less, according to size and speed of engine, clearances, and other variable conditions affected by design, construction, and operation. With compound engines, the added quantity may be taken, for engines of considerable power, as about 6 pounds for compounds, 4 for triple-expansion, and 3 for quadruple-expansion.

The compound non-condensing engine is often employed, especially where it is difficult to secure a good and unfailing supply of condensing water. The following are the results of the investigation of this case, taking the total absolute pressure, and the back-pressure constant, as below, and assuming a variable ratio of expansion within the limits $r = 2$ and $r = 20$. The Rankine exact method and formulas are employed as before.

Let $p_1 = 180$ lbs. per sq. in., absolute; $p_2 = 16$; $r =$ variable.

Assume the available heat of the fuel at 10,000,000 ft.-lbs., and the evaporation to be 10 pounds of steam per pound of coal, as representing best practice, with a good feed-water heater and dry steam supplied at the steam-chest. Steam used, unity; $v_1 = 2.315$.

Then we obtain, in the manner already indicated:

NON-CONDENSING ENGINE.

Ideal Case.

$U = 404{,}330$; $p_1 = 180$; $p_2 = 16$; $t_4 = 140°$ F.; $T_4 = 600°$ F.; $v_1 = 2.31$[illegible] $h_4 = 83{,}459$.

r............	2	2.5	3.333	5	10	15
v_2............	4.63	5.79	7.72	11.58	23.15	30.87
p_2............	94.07	74.12	54.57	35.70	17.00	14.50
U_2..........	357,280	342,595	323,977	298,282	255,838	247,144
H_2..........	911,263	907,385	902,646	896,456	886,867	885,006
U'..........	47,407	62,071	80,650	106,176	148,515	157,140
h............	874,849	885,661	899,540	918,945	952,830	958,733
M. E. P......	71.1	74.5	72.6	63.7	44.5	34.8
h (rej.)......	827,442	823,590	818,890	812,861	804,315	801,593
Effic. St. p. c..	5.42	7.01	8.96	11.5	15.4	16.4
Fuel per H. P. per hour...	3.65	2.81	2.21	1.72	1.28	1.20
Steam per H. P. per hour.	36.5	28.1	22.1	17.2	12.8	12.0

Real Case.

Assume steam-wastes approximately constant at 6 lbs.; engine-friction to demand 3 lbs. steam in excess of that computed.

Indicated Power.

Fuel.........	4.25	3.41	2.81	2.32	1.88	1.80
Steam........	42.5	34.1	28.1	23.2	18.8	18.0

Dynamometric Power.

Fuel.........	4.52	3.71	3.11	2.62	2.18	2.10
Steam........	45.2	37.1	31.1	26.2	21.8	21.0

As another interesting case, assume a boiler-pressure, $p_1 = 250$, absolute, and back-pressures of 16 and 5 pounds, respectively, for the non-condensing and the condensing engine, feed-water temperatures 203° and 104° F., jacketed engines of such size and speed as to give internal wastes approximating $0.075 \sqrt{r}$, due to the action of the exhaust

period. Take Rankine's system of computation for the jacketed engine as the probably best approximation. Take the evaporation at 10 and 9 pounds for the two cases, respectively. Determine the variation of efficiency with varying expansion.

In this case, it will be seen that the variation of coal-consumption will differ from that of steam, in consequence of the fact that a part of the heat supplied the engine enters by way of the jacket, and, when condensed, this portion of the steam simply flows back to the boiler—if the drain-pipes are properly arranged—and does not enter into the measure of feed-water supply; though the heat which it conveys comes from the fuel, as really as does that transferred by the steam entering the cylinder. The fuel may thus be divided into two parts: that supplying heat to the entering steam; and that giving heat to the jacket. The measure of the heat supplied by the jacket may be obtained by deducting from the total computed heat-supply that required to furnish the steam entering the cylinder with its initial store. This gives us

$$h_j = H - (H_1 - h_4).$$

The weight of wa . and of steam worked in the cylinder is, per H. P. per hour,

$$W = 1{,}980{,}000 \div U';$$

where U' is the work performed by one pound of steam. The division of this quantity by the rate of evaporation gives the weight of fuel. It will be observed, on examining the tabulated results of such computations, that the minimum water-rate does not correspond, precisely, to the maximum efficiency; a consequence of the steady circulation of the jacket-steam and water. The minimum coal-consumption, on the other hand, does correspond exactly with the best efficiency; as it should. The following are the data and results of computation:

being due to the corresponding difference in total expansion, taking the one to work at a ratio of 2.5 for each cylinder, and the other at 2:

IDEAL MULTIPLE-CYLINDER ENGINE EFFICIENCIES.

Engine.	No. Cylinder.	E.	B. T. H. per I. H. P.	Water per I. H. P.	Coal per I. H. P.
Triple.....	1	.0811			
	2	.0730			
	3	.0779			
Total....	....	.231	11761	10.85	1.35
Quadruple.	1	.0637			
	2	.0598			
	3	.0580			
	4	.0598			
Total....	....	.2414	11577	10.68	1.34

The consumption of water and of fuel is thus seen to be ex- extremely low, as compared with the actual performance of the preceding cases of simple engines at lower pressures. Adding the prescribed allowances for internal wastes, we have:

EFFICIENCIES OF REAL ENGINES.

Engine.	Water per I. H. P.	Coal per I. H. P.
Ideal..	10.8	1.2
Simple..	17.3	1.9
Triple..	13.4	1.5
Quadruple..	13.1	1.4

Had these engines been unjacketed, assuming waste greater by one third in the actual and unchanged in the ideal case, we might probably have obtained the following:

UNJACKETED REAL ENGINES.

Engine.	Water per I. H. P.	Coal per I. H. P.
Ideal..	10.5	1.2
Simple..	19.4	2.2
Triple..	14.3	1.6
Quadruple..	13.8	1.5

The gain by increasing complication thus decreases as the number of cylinders increases. Reduced back pressure will considerably improve these figures.

Going into higher and unaccustomed pressures, it may be interesting to endeavor to compute the probable performance of a well-designed quintuple-expansion engine, working at a pressure of 500 pounds per square inch. The ratio of expansion is taken at $r = 2.3^5 = 64.4$, the back-pressure at five pounds. These results may be compared profitably with the case of the simple engine discussed in Chapter V, § 137, in which somewhat similar data are taken. Assume data thus:

QUINTUPLE-EXPANSION ENGINE.

Data:

$p_1 = 500 \times 144 = 71,000$ lbs. per sq. ft.
$p_3 = 5 \times 144 = 720.$
$r = 2.3^5 = 64.4.$

Results:

$p_2 = 862.2$ lbs. per sq. ft., 6 lbs. per sq. in.
Heat expended per lb., $H = 27,324$ ft. lbs. $= 1898$ B. T. U
$p_e = \frac{H}{V_2} = 4464$ lbs. per sq. ft., 31 lbs. per sq. in.
$p_h = 17,330$ lbs. per sq. ft., 120.3 lbs. per sq. in.
Efficiency of fluid, $E = \frac{p_e}{p_h} = 0.2576.$
B. T. U. per I. H. P. per hr. $= 10,189.$
Steam per I. H. P. per hr., at 1100 units per lb., $= 9.32$ lbs.
Coal per I. H. P. per hr., at 9 lbs. evap., $= 1.03$; say 1 lb.

For this case, therefore, the weights of steam and of fuel, for unity efficiency, would be approximately 2.4 pounds, and about 0.3 pound per horse-power per hour. Were the internal wastes to be taken as in the first part of this discussion, as indicated by experiments there referred to, we should have the following, assuming the losses to be reduced in proportion to the number of cylinders employed, and the efficiency of mechan-

ism to be 0.95 for the simple engine; 0.90, 0.90, 0.85, and 0.85 for the compounded engine in the five cases given, respectively:

EFFICIENCIES OF MULTIPLE-CYLINDER ENGINE.

Engine.	Water per I. H. P.	Fuel per I. H. P.	E. E.	Water per D.P.H.	Fuel per D. H. P.
	Pounds.	Pounds.		Pounds.	Pounds.
Ideal engine..........	9.32	1	1	9.32	1
Simple jacketed.......	20.5	2.2	95	21.4	2.4
Double-expansion.....	14.9	1.6	90	16.5	1.8
Triple-expansion......	13.0	1.4	90	15.0	1.7
Quadruple-expansion..	12.1	1.34	85	14.4	1.6
Quintuple-expansion..	11.6	1.24	85	13.6	1.5

The above is sufficient to give a fair idea, assuming the figures satisfactorily approximate for the conditions assumed, of the advances to be anticipated through the use of higher pressures and ratios of expansion, and with saturated steam. These figures may be further decreased by increasing boiler-efficiency and by superheating the steam. They have already been reduced about two pounds in best work to date.

150. **The General Results of Experience** and of experiment accord, very satisfactorily, in cases of good design and construction and of good management, with the deductions and computations which have now been presented.

Differences of type produce differences of performance, however, that sometimes modify the general conclusions which have been stated, to an observable extent. Thus the conclusions of Hallauer, after comparing the performance of the simple Corliss engine, with its efficient valve-gear and small clearance-spaces, with the ordinary Woolf compound, both working at about 5 atmospheres pressure, were that the one was substantially equal to the other; although the ratio of expansion of the latter was comparatively large, and both at their best working ratios.* This fact is probably quite as much due to the comparatively small port-spaces and clearances, and the separated steam and exhaust ports of the Corliss engine, as to any other cause.

* Trans. Soc. Indust. de Mulhouse; 1878.

A notable difference between the conditions dictating the design and construction of the locomotive and the marine engine is observed in these facts: the former must be proportioned and built to meet a great range of resistance and speed; as it must, on a level, haul at high velocity against low resistance; on a steep gradient, it must pull heavily at low speed. It may at one time haul light passenger trains, at another handle a heavy and slow merchandise traffic. The latter, on the other hand, has a steady load and practically constant speed, under ordinary conditions of operation. The locomotive is given large cylinders to meet the exigencies of heavy loads, and a link valve-gear to give high expansion and compression ratios under the opposite conditions. This is not as essential with the marine engine; with which, since the power demanded varies as the cube of the speed, the variation of velocity is usually moderate. These differences favor the use of the multiple-cylinder engine at sea more than on land, notwithstanding the fact that it is less affected than the older type by variations from the normal load. The necessity of proportioning the locomotive for its maximum pull and the comparatively constant liability to enormous variations of load and speed, its short periods of working and frequent stops, and its exposed cylinders and exaggerated wastes, are all conditions telling against this engine.

Experience at sea indicates that a good double-cylinder, compound, engine, with steam at 100 pounds by gauge (7 atmos., nearly) should not demand more than 2.2 pounds (1 kg.) of fuel of good quality per horse-power per hour; a triple-expansion engine 1.8 pounds (0.8 kg.); and a quadruple-expansion engine 1.5 pounds (0.7 kg.); the steam pressures and ratios of expansion adopted being appropriate to each.

The very considerable economy to be noted in such comparisons is not usually wholly attributable to differences in design and construction of engine. The greater steam-pressure and resultant higher ratio of expansion adopted with the later engines is generally, in part, the cause of the observed gain. But the simple engine could not be economically worked with

as high a ratio of expansion at such pressures as the compound engine, and the latter thus possesses a decided advantage; which advantage is, as is now known, due to its better arrangement for checking exhaust-wastes.

Trials of agricultural engines, made by Sir Frederick Bramwell and Mr. Anderson,* indicate that the efficiency of machine may be as high in compound as in simple engines, and give for the value of this factor from 0.75 to 0.94, the common values approximating 0.85, the steam consumed being about 33 pounds per dynamometric horse-power and per hour in the best simple engines, and 22 in the better class of small compound engines; the corresponding coal-consumption being nearly 3 and 2 pounds, respectively. The total friction of engine was thus about 15 per cent of the total power, or 3 H. P. on a 20-H. P. engine.

On the steamer Suez, the replacement of two-cylinder compound by quadruple-expansion engines was reported, "with the same kind of coal, the same revolutions, the same speed of ship, and the same propeller," to have reduced the fuel-consumption 34 per cent. The steam-pressure was raised, however, to above 150 pounds.†

An experience extending over three years, according to Mr. R. Wylie, with steamers having compound and triple-expansion engines gave a marked difference in favor of the latter, the former using nearly 14 tons a day, the latter 10¼; the former averaging 2.16 pounds of fuel per horse-power and per hour, the latter 1.41.‡

The quadruple-expansion engines of the steamship Singapore were reported, in 1890, to have demanded but 1.122 pounds of best navigation coals per hour, per I. H. P.

The compound pumping-engine designed by Mr. Corliss, in 1879, for the Pawtucket (R. I.) water-works, a small engine of but 15 and 30 inches diameter of cylinders and 30 inches stroke

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* Jour. Royal Agricult. Soc. of England; vol. xxii, 1887.

† London Engineer; Feb. 24, 1888; p. 162.

‡ Trans. Brit. Inst. M. E.; 1886.

of piston, was reported, in the year 1889, to have given, for the year, an average duty of 124,500,000 foot-pounds for 100 pounds of fuel consumed, on an evaporation of approximately 9 pounds of water per pound of fuel, or 13.7 pounds of steam and of feed-water, and 1.5 pounds of coal, per horse-power per hour for the whole year.* This extraordinary, probably unexampled, result is presumably due to the high steam-pressure (125 pounds by gauge); the choice of the most economical ratio of expansion (18) for that case; continuous steady work against a high head; unusually high speed for a pumping-engine (50 revolutions per minute), and remarkably good proportions and construction. In this engine, heads as well as sides of both engines are jacketed; but with apparently small practical advantage, either because of its speed, its employment of superheated steam or of an actual defect in jacketing.

An examination of records of trials of 60 engines in various parts of the world, and under a great variety of conditions, and for periods averaging about five months, gives an average gain of 18½ per cent, in comparing the compound locomotive with the simple engine.† Trials in the United States, on the E. Tennessee, Va., and Ga. Railway, resulted in the reporting of a gain of 1.6 pounds fuel per train-mile, or of 19 per cent, for standard engines, and of 4 pounds per mile, or 31 per cent, for 10-wheel engines by compounding.‡ Mr. Urquhart reports a gain of 18½ per cent in liquid fuel during the year 1890 and on a million of miles run.

The economy of the multiple-cylinder engine is thus seen to be mainly due to the cascade-like action of the machine, in its disposition of the heat-wastes in such manner that, with a given total range of expansion, the total internal waste is reduced approximately in proportion to the number in series; but it also is, in part, a consequence of the fact that the total condensing power is, or may be, less than that of the single

* Annual Report.

† Compound Locomotives; A. T. Woods; Jour. Assoc. Eng. Societies; May 1890.

‡ Railway Review; 1890.

cylinder that might displace it. Comparing the condensing power of a triple-expansion and of a compound engine, for example, with that of the corresponding simple engine, as measured by the product of range of temperature in each cylinder by its cooling surface, it will be found, as shown by M. Demoulin,* that the ratios of the sums of these products for each engine is not far from 65, 75, and 100, respectively, for usual practice; the reciprocals of which ratios, 1.3, 1.33, and 1, nearly, measure rather closely the commonly stated ratios of relative economy.

Assuming a steam-pressure of approximately 127 pounds per square inch by gauge, a ratio of expansion of 10 and a back-pressure of 4 pounds, M. Demoulin compares, in this respect, the simple, the two-cylinder compound, and the "triple-expansion" engines. These have diameters, respectively, of 1 metre, of $0^m.75$ and $1^m.5$, and of $0^m.61$, $0^m.96$, and $1^m.5$; and lengths of stroke, of $1^m.5$ for the first and 1 metre for the others.

Multiplying the ranges of temperature in each cylinder by the total areas of cylinder exposed to steam, their products are compared and the triple-expansion engine shown thus to possess an advantage of 15 per cent over the double and 34 per cent over the simple engine.†

The work of the compound engine illustrates a feature of the more economical types of that engine which is especially valuable when the load is not fixed and appropriate to the machine. Thus, in the figure, we have the method of variation of economy with varying ratios of expansion with three types of common forms of engine. It is seen that the efficiency of the compound is comparatively unaffected within any usual range of variation of load.

In the figure the upper curve represents the efficiency of the non-compound engine under variable loads. Many tests have

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* Machines à Vapeur; Paris, 1890; p. 6.

† Étude sur les Machines Compound à Triple Expansion; Paris, Baudry & Cie.; 1885.

determined the two corresponding curves for the compound engine, both with and without vacuum.

This peculiarity of the more economical type of engine makes it the more desirable where varying resistance is to be encountered.

As a general result of experience, it may be concluded that, for the average case, with good engines of the several classes:

(1) The volume of steam shown by the indicator, when superheated, or thoroughly dry, steam is used in well-jacketed compound engines, of moderate size, is nearly the same as

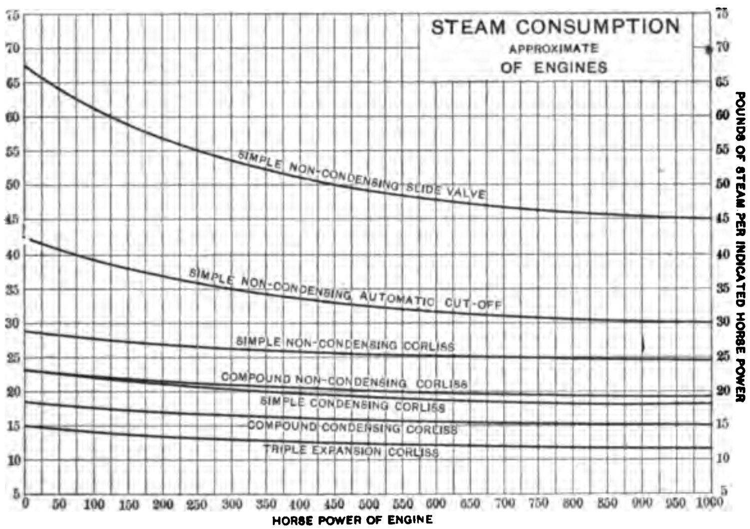

FIG. 155.—ECONOMY UNDER VARIOUS LOADS.

computed for a similar ideal engine, both at cut-off and at the end of stroke. The actual excess may be taken as not above fifteen per cent by weight at the first and ten per cent at the second point, if we follow Hirn, in such cases as were studied by him.

(2) Ordinary, nearly dry, steam—i.e., not containing five per cent moisture—worked in jacketed simple engines, may usually be expected to exhibit an excess at least one half greater than in the preceding cases, for good average practice.

(3) Moderately wet steam in any jacketed engine, or dry

steam in an unjacketed engine of any considerable size, may be expected to exhibit a waste of the kind here considered increasing rapidly with the ratio of expansion, and often double in amount that observed in the first case, above, in even good practice.

(4) Wet steam, in small and unjacketed engines, especially if worked at low speeds, may be expected to be condensed to such an extent as to give rise to expenditures of heat, steam, and fuel enormously in excess of, often several times greater than, those computed for the similar ideal case.

(5) The advantage of thus placing cylinders in series is less as wastes are less in the simple engines, as costs are less, and, in more detail, as the steam is drier, expansion less, speeds of engine higher, and as power demanded is greater; and the number in series is less for best effect, in all cases, as the performance of the actual engine approaches more nearly that computed for the ideal.

151. **The Balance of Forces** at the main shaft, in the multiple-cylinder engine, may often prove a matter of real consequence. Mr. John Elder, in 1866, stated that it was perfectly possible that a saving of 10 per cent and more of the indicated power might be wasted in an engine by avoidable friction at the shaft.* He ascribed much of the advantage of "compounding" to the division of the work of the engine and to the better consequent adjustment of pressures on the shaft and pins. A three-cylinder engine, with its cranks at angles of 120°, may be made to work with almost a balance of thrusts and pulls at the shaft. A double-cylinder compound engine, with cranks set opposite, is also thus advantageous; and, in both, the maximum pressures become a fraction of those in the simple engine.

The comparison of three similar British naval vessels, the Arethusa, the Octavia, and the Constance, fitted, respectively, with a pair of simple, trunk, engines, with cranks at 45°, a set of three single cylinders with cranks at 120°, and a three-cylinder "compound" engine, in 1865, running from Plymouth to

* Rankine's Life of Elder; 1871.

Funchal, resulted in giving, as the fuel-consumption, 3.64, 3.17, and 2.51 pounds per horse-power per hour; while the last two ships are reported to have shown a relative efficiency of mechanism of 100 to 127; or of 79 to 100.* This difference was slightly lessened as speeds and power increased. The last-described disposition of the engine also conduces to smoothness of motion and to regularity in crank-pin pressures and turning moments.

Variations of pressure on the running parts of the engine, due to extreme ranges of expansion, in the simple engine, may sometimes, and especially in marine engines, prove objectionable, and thus to constitute another argument in favor of the use of the multiple-cylinder engine. The steamers Polynesia and Circassian, of the Allan Line, were originally fitted out, the one with compound, the other with simple, engines. In all other respects they were alike. They were so designed that the same expansion could be adopted in both. The result was that the simple engine was badly shaken and injured, the machinery was removed, and engines similar to those of the Polynesia were put in, with thoroughly satisfactory results.†

The extent to which the stresses and strains due to high-pressure steam are relieved by "compounding" the engine may be readily seen by computing these quantities for parallel cases. It will be found that the simple engine is subject to double stress when expanding 10 to 12 times, as when working at a ratio of expansion of 3½ to 4, and must be correspondingly heavier and stronger. In multiple-cylinder engines, the total stresses may be made substantially equal in each, and the range of pressure reduced, and the strains as well, in similar proportion. A condensing triple-expansion engine, at ten atmospheres pressure (150 lbs.) by gauge, would be subject to about one fifth the stresses, on each piston and its connections, that would come upon the piston of its large cylinder, if all the work were done within it, or in a simple engine of the same size.

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* Rankine's Life of Elder; p. 44.

† King: Report on European Ships of War; 1877.

This reduction of loads is so considerable that it is actually possible, at high pressures, to save weight of engine by compounding. At very low pressure the simple engine has the advantage, both in weight and efficiency.

From the constructors' point of view, "compounding" the steam-engine often becomes, with the now usual boiler-pressures, a matter of vital importance; since it would be impracticable to successfully work the simple engine under those pressures, and with the enormous variations of pressure due to a high ratio of expansion. To do so would compel the adoption of such size and weight of parts, and such special proportions of journals, as would make the engine excessively heavy and costly, while at the same time causing great loss of engine-power through the friction of its own parts.

152. **Steam-jackets on Engines,** whether simple or other, have one and the same main purpose, in every case and on every type—the reduction of internal wastes due to initial condensation. In the older Worthington direct-acting type, and perhaps in other pumping-engines, the use of the jacket may bring an incidental advantage of some practical value, enabling, as it does in this case, the stroke to be completed at a higher ratio of expansion than it could otherwise be, a result of the higher terminal pressures produced by it, and of prevention of water in the cylinders.

A special reason for the use of the jacket on engines liable, as is the Cornish pumping-engine, or to a certain extent in marine engines, for example, to be stopped occasionally for intervals of greater or less length and to be started up again at a moment's notice, is that the cylinder can be kept heated, the engine "warmed up," however long the stop, and thus kept in condition for immediate starting, without danger or delay. The jacket also, incidentally, is useful in keeping the bore of the cylinder unstrained, if properly constructed. This is considered so important that, in some cases, the "liner" is inserted only after the engine is set up in place.

As is well known, the use of the steam-jacket was original with Watt, who remarks, in a letter to Professor Jardine, that,

after his experiments on the Newcomen model, his next, and an easy, step was "to inquire what was the cause of the great consumption of fuel. This, too, was readily suggested: viz., the waste of fuel which was necessary to bring the whole cylinder, piston, and adjacent parts, from the coldness of water to the heat of the steam no fewer than 15 or 20 times a minute." *

He invented, first the separate condenser, then the steam-jacket, in order "to keep the steam-cylinder as hot as the steam which entered it." The cause of the great internal waste detected by Watt is now well known and has been described as cylinder, or internal, or initial condensation.

Combes, in papers presented to and published by the Académie des Sciences, was probably the first to introduce into the theory of the steam-engine the consideration of that phenomenon, discovered by Watt, to check the wasteful effects of which the latter invented the steam-jacket.† That author gradually gave shape to his ideas, as time went on, publishing them in 1845,‡ and, later, in 1853–67.§ He says in his first paper, just mentioned: "The utility of the jacket, or rather that of heating the cylinders of steam-engines from the outside, . . . is rendered unquestionable, both by direct experiment and by detailed observation of the phenomena characterizing the action of steam in the cylinder, and the logical discussion of these observations." "Jackets have not for their main result the maintenance of the temperature of the steam during expansion; their use consists in the prevention of refrigeration of the walls of the cylinder while in communication with the condenser:" probably the first exact statement of this effect ever printed.‖

Mr. Gill, as early as 1844, says: "If the cylinder be supplied with dry steam, and no heat is dissipated by radiation, there will still be a loss of heat in the cylinder occasioned by the sudden expansion of the steam when the communication with

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* History of the Steam-engine; Thurston; p. 88.

† Comptes Rendus; 1843.

‡ Traité d'exploration des Mines.

§ Principes de la Théorie Mécanique de la Chaleur.

‖ Memoirs of 1843; p. 245.

the condenser is opened. . . . As the heat for evaporation is furnished by the hot metal of the cylinder, piston, etc., such heat must be returned to them by the condensation of steam during the succeeding stroke, such condensation and evaporation going on until an equilibrium is established." He suggests superheating as the best remedy.*

Hirn published his *Mémoire sur l'Utilité des Enveloppes à Vapeur* in 1855.† This memorable paper gives us the first analysis of experiments showing the quantitative measures of the thermal action of the walls of the steam-cylinder. He concludes:

"(1) There is a capital difference between the thermal phenomena characterizing two types of engine: In the simple engine, the cylinder-walls always yield heat to the steam during expansion; though the amount is less when the jacket is working than when shut off. In the Wolff engine, the surfaces of the cylinder take heat from the steam, even during the expansion, and lose it again during the exhaust."

"(2) With the simple engine the walls of the cylinder give to the steam the same amount of heat with as without the jacket; but, in the former case, the heat is given up during the expansion, and thus, without cost, adds considerably to the amount of work done; while, without the jacket, this heat is all lost by being thrown into the condenser without doing any work, uselessly evaporating the condensed water, mainly after the exhaust-valve has opened."

As explained by many recent writers, the benefit of the jacket comes of the facts that it not only reduces initial condensation but insures that a part, at least, of such heat-waste as does take place shall occur through condensation within the jacket, where it does no additional harm, instead of in the cylinder, where it would produce, indirectly, wastes out of all proportion to its own amount. It is by allowing the surfaces of the cylinder exposed to the entering steam to become as hot, approximately, as the steam itself, and nearly or quite dry, so as

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* Improvements of the steam-engine; Weale's paper; Jan. 1844.

† Bulletin de la Société Industrielle de Mulhouse; t. XXVII. pp. 105–206.

to largely check, if not to prevent, initial condensation, that the steam-jacket gives its economic advantage.

As has been well stated by Holmes: "A jacket operates in two ways, in keeping the temperature of the cylinder-walls constant: first, by keeping the working steam comparatively dry, it reduces the power of the sides of receiving heat from, and of giving it out to, the former, and thus deprives the sides of the power of taking up the extremes of temperature which would otherwise be possible; and, second, whatever differences of temperature would actually occur are further greatly reduced by the flow of the heat from the jacket-steam to the inner walls of the cylinder. It is only the heat supplied in the latter process which costs the jacket-steam anything. The great gain due to the rendering of the working steam non-conducting and non-radiating costs nothing whatever; seeing that it is an indirect effect of keeping the sides hot. Thus, the steam-jacket, though for half the time warming the exhaust, has proven in the majority of cases to be an undoubted source of economy." *

The operation of the jacket may thus be defined to be that of improving the working fluid, converting a defective into an efficient, changing a heat-absorbing into a non-absorbing material, a wet into a dry vapor, or into a gas, more or less completely.

Thus the quantity of heat and steam lost in the jacket is not, as often assumed and stated, precisely the equivalent of the amount which would, without it, be wasted inside the cylinder. The real effect of the jacket is to present a comparatively hot and dry internal surface to the entering working steam, and thus to prevent any condensation of that steam at its admission, and corresponding re-evaporation during exhaust. The transfer of heat by internal conduction is thus made to take effect between dry surfaces and through a comparatively dry medium with the result of greatly reducing the quantity so transferred and, to the extent of that reduction, adding to

* The Steam-engine; 1887; p. 451.

the efficiency of the engine. The jacket wastes, if it is one of high efficiency, only the quantity of heat needed to preserve the working steam in the "dry and saturated" condition.

The jacket thus acts usefully in two distinct ways: (1) by preventing exchange of heat between the steam and the cylinder-walls, by keeping the steam more nearly gaseous; (2) by reduction of the range of temperature occurring within those metallic masses, and of their tendency to initiate and continue the waste.

Throughout the whole cycle of the engine, however, the jacket is either transferring heat through the sides of the cylinder to the steam, or is compensating a previous loss by storing heat in the metal composing the inner layers of cylinder, piston, and heads; constantly draining heat into the engine from the boiler, and all the time wasting it; either by transfer without transformation, or by transformation within a smaller range of temperature than the maximum. It is a wasteful device for preventing or ameliorating a greater waste.

When this latter is a larger loss than that due to the jacket itself, a gain occurs; when the internal wastes are otherwise reduced to the magnitude of minimum jacket-waste, that accessory has no value; whenever, as by superheating, or other device or combination of expedients, the interior wastes are made less than the normal waste due the jacket itself, the latter can have no useful effect; and finally, an inefficient, or an exceptionally wasteful, jacket may possibly prove absolutely hurtful. This has been observed, for example, in some reported cases of locomotive performance, and in cases which, perhaps, the heat wasted from it during the terminal portion of the expansion-period and during the exhaust-stage is more than the equivalent of the earlier gain by reduced initial condensation and during expansion. This last effect may be a consequence of excessive wetness of steam, causing the presence of water in its mass up to and beyond the termination of the expansion-line.

The action of an effective jacket, notwithstanding its production of a drain of heat into the cylinder, results in greatly

accelerating the re-evaporation, and in its completion at so early a period in the stroke as to accomplish two results: (1) the conversion of the water from this condensation into effective working fluid; (2) the drying and warming of the walls of the cylinder so completely, before the succeeding admission, as to make the heat-absorption and the consequent initial condensation minima. The net result is usually a gain by reduction of interior wastes; and the exterior losses, although exaggerated by the increased areas of surface exposed, remain insignificant when the cylinder is properly clothed.

153. **The Action of the Jacket, in Detail,** is probably not complicated; but it is obscure because of the facts that it is so far out of reach of the investigator that the variations of temperature and in heat-storage and transfer affect variable quantities of metal and fluid which the engineer cannot easily measure, and are subject to intricate and uncertain physical changes of condition and quality of the mixture of steam and water, or possibly, at times, of dry and superheated steam, similarly difficult of determination.

We will examine several typical cases (see § 122, Fig. 140, p. 473):

(1) *Jacket and cylinder receive gaseous steam;* i.e., the fluid is highly superheated and behaves like a gas.

In this case the action of the jacket tends to keep the inner walls of the cylinder up to its own temperature. Assume this possible. The gaseous steam enters the cylinder at maximum temperature, expands, doing work, constantly losing both heat and temperature, down to a minimum, at exhaust, and is finally discharged, it may be assumed, dry but saturated. Each entering charge finds the inner surface of the cylinder slightly cooler than itself, before expansion begins, but absorbs its heat continually, once expansion has begun, up to the close of the exhaust-period. This heat is partly utilized by conversion into work, but within a reduced range of temperature and with reduced efficiency, and is in part discharged as pure waste. But the total quantity so absorbed will be small, since the fluid has small specific heat, large specific volume, and insensible conductivity.

Precisely what the internal waste would be under such conditions is not precisely known; but experience with gas-engines and with superheated steam would indicate that it would not usually be ten per cent in large engines, and probably not be less than five for what might be taken as fair examples.

We may perhaps call eight per cent the normal waste due to the action of the jacket, and the minimum to be anticipated with the best possible jacketing. The gain by the use of a jacket is approximately the difference between this and the waste of the same cylinder without the jacket. Experience indicates this to be, usually, in such cases, a very small quantity, and often inappreciable.

(2) *Jackets and cylinders receive dry steam.* In this case, the jacket readily keeps the external surface of the cylinder-walls at maximum temperature, that of the steam itself, and due its pressure. The slightest reduction of temperature at once produces condensation in the jacket, and the temperature of the cylinder-surface next the jacket is restored by absorption and storage of the latent heat of the jacket-steam so condensed. This process of transfer by condensation is known to be one of such great rapidity that we are justified in assuming that the surface of the cylinder which is exposed to jacket-steam is kept up fully to the temperature of the latter throughout the whole cycle.

Consider the four phases of the engine-cycle: (1) induction; (2) expansion; (3) exhaust; (4) compression. During the first, the steam has the same temperature and pressure on both interior and exterior of the cylinder-walls; during the second period, differences of temperature and pressure on the two surfaces are observed, progressively increasing to the end of the expansion and the establishment of the back-pressure; during the exhaust, this difference remains nearly constant and a maximum; while the compression-period sees this difference once more reduced, we will assume, to zero. Thus both "prime" steam and jacket-steam at first unite in restoring to the metal heat lost during the preceding cycle, and none passes from the jacket into the interior of the cylinder. Jacket-heat

flows into the engine throughout the remainder of the cycle, and is partly converted into work, partly transferred and wasted as heat; and the proportion of these two quantities, the partial waste by inefficient transformation and the pure waste, is determined both by the extent to which expansion is carried and by the quality of the working fluid.*

If the steam be dry or nearly so, at the close of the first period, and if the second, the expansion-period, is sufficiently prolonged, the action of the jacket and the heat-storing property of the metal of the cylinder promptly results in superheating the expanding steam and so checking further waste of heat from jacket, and from cylinder-walls, during the terminal period of expansion and during the exhaust, and thus allows the jacket to raise the temperature of the cylinder promptly and fully to that of the entering steam. This being accomplished, initial condensation is, in turn, reduced to an unimportant quantity; the total waste is mainly jacket-waste, and is a minimum.

On the other hand, if the amount of water produced, either by initial condensation or by the work of expansion, or both, is so great that it cannot be all re-evaporated early in the stroke, and if the cooling of the cylinder-walls is thus continued, the jacket-waste becomes increased, the waste which it is intended to check may remain serious, and the result may be a considerable net loss and but little or no advantage from the jacket.

This must be the result, probably, to a greater or less extent, whenever the drying of the cylinder and steam is not nearly or quite completed at the opening of the exhaust-valve, as when the jacket is defective or the steam too wet. It would seem possible that intermediate conditions might prove to be those of best jacket-action.

The process is here, probably, one in which the first effect of the jacket, during expansion, is to dry the steam which contains, always, if not superheated, suspended within its mass, more or less of the water of initial condensation; next the

* The resistance to transfer of heat from the metal into a gas is 30 or 40 times as great as to water.

checking of condensation due to the work of expansion, and finally the superheating of the steam, if the earlier stages are completed early enough, and existing conditions permit. The first portion of this process gives a gain of work by adding steam to that existing, as such, at the beginning of expansion; the latter portion by giving the steam larger work-power. The whole operation is a waste of a smaller, to gain by reducing the waste of a larger, quantity of heat-energy.

During the exhaust-period there is a pure waste of heat with a compensating gain by drying and heating the interior surfaces of the cylinder preparatory to the entrance of the next charge of steam. Compression has a similar effect, as a result of the conversion of the work of compression into heat.

During the engine-cycle, the metal is first drenched by the water of condensation, which gives it heat from the entering steam, then cooled by evaporation and lowering of temperature during expansion, and then it is dried off, and is finally warmed up, more or less nearly to the temperature of the prime steam, by the combined action of the jacket and compression.

(3) *Wet steam is supplied.* In this case, the jacket, on its side, acts precisely as before. The water in the steam in the jacket drains out or is trapped off, and is returned to the boiler, leaving the steam practically dry, as before. But the interior of the engine is placed under quite different conditions:

In addition to the heat demanded of the jacket to keep the working steam dry, and to first dry off and then warm up the interior surfaces of the cylinder, a quantity of heat, which, within limits, will be larger as the steam is initially wetter, and which may be often very great, is drawn from the metal and from the jacket, throughout substantially the whole cycle, to evaporate all or a part of the entrained water, and to *then*, if possible, dry off the metal and to heat it up again to the maximum temperature. Not only is this amount of heat increased with increase in quantity of water entering with the steam; but the proportion of heat drained off wastefully in the terminal portion of the expansion, and throughout the exhaust-period,

is continually increased as the quantity of water to be evaporated is greater; so that it may readily be believed that the interior of the cylinder, drenched and flooded with water at the opening of the steam-valve, may continue to act as a waste-producing boiler quite through the cycle; thus causing an enormous loss during the exhaust-period, when, the difference of temperature being a maximum, the heat which the jacket is capable of thus wasting becomes itself a maximum and both absolutely and relatively very large.

If the water of initial condensation is not, in any instance, all re-evaporated during the expansion-period, it will be reconverted into steam during the exhaust-period.

It is thus obvious that the quality of the boiler-steam is a vitally important matter; and it may be easily seen that dry steam is an essential element of successful action of the jacket. It may perhaps even be possible, under specially unfavorable conditions in this respect, that a jacket may do more harm by loss of heat during this wasteful period than it can save by its legitimate action earlier in the cycle. It is as unquestionably the fact that dry steam is essential to the best action of the jacket, as that superheated steam, as shown later, may render the jacket unnecessary and useless.

It is uncertainty as to the condition of the steam supplied, and the probability that it may have been both wet and variable in its humidity, that makes it difficult to secure safe and reliable deductions from many experiments hitherto made on jacketed engines. It is impossible to base on data obtained in such cases any useful computations.

M. Hirn concludes, from observations made by him on engines with and without jackets, that the action of the walls of the cylinder can only affect the working mixture of steam and water either in actual contact or in close juxtaposition with them. This conclusion is confirmed by the computations and experiments of Cotterill and of Dixwell, and of many other authorities.

Under ordinarily favorable circumstances, and in ordinary practice, as M. Dwelshauvers-Dery remarks: "If the jacket be

applied to a single cylinder, it gives up little heat, although the effect produced is very considerable; for the larger part of the heat given up by the walls, and employed in useful work during expansion, is that already imparted by the steam to the metal during admission. In a compound engine, on the other hand, the heat given up by the steam in the jacket increases the work performed during expansion."*

We find, thus, that the jacket may produce economy by simply preventing external losses from the working barrel, giving absolutely no heat to the steam, but simply preventing its losing as much as it otherwise would, at the critical instants, by transfer to the metal of the cylinder. It is easy to see that the use of the jacket is ordinarily advantageous by preventing transfer of heat to the metal of the cylinder during admission, and that the function of the jacket is usually substantially completed at the close of this period, and, consequently, that the engine of large diameter and small stroke, a given volume being assumed, and with jacketed heads, has, ideally at least, an advantage. In general, the greater the area of wetted surfaces, and the wetter those surfaces, the greater is the waste and the more is a jacket needed; but, possibly, also, the nearer may be the limit beyond which the jacket ceases to be advantageous.

154. **Jacket-wastes and Cylinder-wastes,** in the sense in which the latter term has come to be understood, must evidently be carefully distinguished. In an engine without the jacket, it is obvious that the latter form of loss has no limit, up to that set by the complete raising of the whole mass of metal exposed to prime steam up to the temperature of the latter, with subsequent equally complete rejection and waste of this store of energy, down to the temperature of exhaust and back-pressure; except as the limit is determined by conductivity of metal and fluid and by restriction of the period of action. Experience proves, however, that high speed of engine, by reducing the time allowed for alternate absorption and rejection of

* Lond. Eng'g; Dec. 13, 1889; p. 692.

heat by the metal, and by making the quantity of steam passed through the engine greater relatively to this waste, may, in large engines, especially, reduce it, as a percentage of heat supplied, to a comparatively small amount.

Jacket-wastes, on the other hand, are determined by the mean difference of temperature between jacket and cylinder and by the quality of the working fluid. In the same engine, they may be great with large expansion and small with late cut-off; or large with wet steam and insignificant with effective superheating. But they can never become zero; nor can a jacketed engine ever be entirely free from waste internally by complete suppression of these two forms; both will always have sensible value, and probably considerable magnitude.

The economy of steam-jacketing is evidently the difference between the total intrinsic cylinder-wastes without the jacket and those wastes with it, reduced by the amount of the jacket-waste proper. Since no heat can pass from jacket to cylinder-steam during the steam-stroke, up to the point of cut-off, and since all heat supplied later is either partly or wholly wasted, it is obvious that the net loss is a minimum, and the gain by the use of the jacket is a maximum, when, later, it dries off and brings the temperature of the interior of the cylinder up to that of initial steam with most promptness, completeness, and certainty.

The total jacket-waste is easily determined, and is, for many cases, well known, being obtained simply by measuring the water draining from the jacket, and deducting from the total heat which it represents that wasted externally by conduction and radiation, a quantity of small amount and easy of approximate computation, if not determinable by direct experiment.

It is obvious that a steam-jacket will be useful or injurious, more or less, accordingly as it wastes less or more heat—by the drain constantly going on, into, and through the engine, to the condenser or the atmosphere—than it saves by reducing the normal internal wastes of the unjacketed engine. It may, at one or another period, in the cycle of the engine, thus effect a net saving or a net loss; or it may produce no sensible effect;

and the total net result may be either a positive, a negative, or a doubtful gain. Any case in which, through the use in it of exhaust steam or steam of too low pressure, or in consequence of malconstruction or misuse, the jacket, on the whole, acts as a refrigerator, will give a negative and wasteful net result.

Could a perfectly efficient jacket be made, in the sense of being capable of instantly and fully supplying any demand, however sudden or great, for heat needed at the beginning of the stroke, on the interior of the engine, and could the steam be supplied perfectly dry initially, the vapor would remain perfectly dry throughout the stroke; none would be condensed at the beginning, to be re-evaporated later, at the expense of heat from the jacket; and the cost would be only that of the comparatively small normal heat-waste of a dry gas; while a saving would be effected of substantially all the initial condensation that would otherwise have occurred, and at insignificant expense.

Under such conditions, the more readily the jacket surrenders heat, the less the amount it is called upon to yield, and to waste. This was first seen and proved by Hirn.

The weight of steam condensed in the jackets is a very variable quantity. It obviously may be taken as a measure of the efficiency of jacket-action; but it may nevertheless be the fact that highest efficiency of jacket-action may not insure maximum efficiency of engine, as it may, especially with wet steam, induce excessive wastes during the exhaust-period. The amount of this condensation is variable between very wide limits. The Pawtucket Pumping-engine gives but five per cent. In Professor Unwin's report on the Worthington "High-duty" Engine he gives the jacket-water as 15 to 20 per cent of the total;* in the Lawrence and Lynn engines of Mr. Leavitt's design, it amounted to about 16 per cent;† in Donkin & Co.'s engine at the Eichburg paper-mill it was 10 per cent,‡ and

* Lond. Eng'g; Dec. 7, 1888; p. 566.

† Eng'g and Mining Jour.; Nov. 25, 1871.

‡ Zeitschrift des Vereins Deutscher Ing.; Apr. 1869.

about the same on the London Gas Works,* on expenditures for these several engines of 17, 14.4, 16.8, 22.2, and 25 pounds of water per horse-power per hour.

The minimum jacket-drainage reported by investigators is below ten per cent and its lowest value may be perhaps safely assumed at about five per cent; which may be taken as the jacket-waste proper. With perfectly dry steam, it has been known to be less than five per cent. By the expenditure of five to fifteen per cent in this direction, therefore, a reduction of cylinder-condensation from twenty to forty per cent down to perhaps ten or less may be sometimes effected; and this net gain of ten to twenty-five per cent then constitutes the advantage of jacketing in such cases as the above.

With the introduction of other methods of reduction of the second form of loss, the relative value of the jacket, and the return for its expenditure and waste become less, and, with high engine-speed and compounding, or superheating, the gain may become insignificant; a deduction amply confirmed by experience.

In the development of the thermodynamic theory of the steam-engine (1859), Rankine assumes "that the steam in the cylinder, while expanding, receives just enough of heat from the steam in the jacket to prevent any appreciable part of it from condensing, without superheating it." This assumption is founded on the fact that dry steam is a bad conductor of heat as compared with liquid water, or with cloudy steam, and that after cloudy steam has received enough of heat to make it dry, or nearly dry, it will receive additional heat very slowly. The assumption is justified by the fact that its results are confirmed by experiment.† Rankine's assumption, as is now well understood, involves the further assumption that the jacket is preliminarily effective in preventing initial condensation. His theory of the jacketed engine thus becomes the theory of a dry, saturated-steam engine.

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* Lond. Eng'g; Feb. 1, 1878.

† Steam-engine; § 287, p. 396.

155. **Computations of Efficiency** of jacketed engines and of jacket-waste may be made which are fairly approximate for good examples of actual practice. From what has preceded, it is seen that the ideal engine with non-conducting cylinder, free, as it is, from internal wastes, must have higher efficiency than the ideal jacketed engine which is subject to pure jacket-waste, but not to the second method of internal loss; while the real engine, with its combined jacket- and cylinder-wastes, reduced by the jacket, as the latter are, to a minimum amount, is more wasteful than either of the preceding, but is more efficient than the same real engine would be without a jacket. In the ideal cases, jacketing results in loss; in actual cases, it commonly produces gain. Could we approximate in real engines to the ideal conditions, we might lose, rather than gain, by the action of the jacket; should the jacket actually waste during exhaust more than it saves on the steam-stroke, it might also, in inefficient engines, even produce loss. It gives maximum gain under intermediate conditions and when its own waste is a minimum, while its activity in reducing other loss is a maximum.

The following results of computation illustrate these deductions. The methods and formulas adopted are the same as those previously presented. In all cases, the *real*, not the apparent, ratio of expansion, is assumed, and no allowance is made for compression.

COMPARISON OF THE EFFICIENCY OF IDEAL JACKETED AND UNJACKETED CYLINDERS.

The approximate formulas are here used, having been proved sufficiently accurate for present purposes.

ASSUMPTIONS: Ideal non-condensing engines.

DATA:

$p_1 = 60, 80, 100, 120$ lbs. per sq. in. (absolute).

$\frac{1}{r} = 0.15;\ 0.2;\ 0.25;\ 0.3;\ 0.4;\ 0.5.$

$p_3 = 18$ lbs. per sq. in. (absolute).

$T_4 = 110°$ F.

RESULTS:

(*a*) *Pressures—Unjacketed Non-conducting Cylinders.*

	$\frac{1}{r}$	0.15	0.2	0.25	0.3	0.4	0.5
	$\frac{p_m}{p_1}$	.407	.496	.572	.639	.748	.833
$p_1 =$ 60:	p_m	24.42	29.76	34.32	38.34	44.88	49.98
	p_e	6.42	11.76	16.32	20.34	26.88	31.98
$p_1 =$ 80:	p_m	32.56	39.68	45.76	51.12	59.84	66.64
	p_e	14.56	21.68	27.76	33.12	41.84	48.64
$p_1 =$ 100:	p_m	40.70	49.60	57.20	63.90	74.80	83.30
	p_e	22.70	31.60	39.20	45.90	56.80	65.30
$p_1 =$ 120:	p_m	48.84	59.52	68.64	76.68	89.76	99.96
	p_e	30.84	41.50	50.64	58.68	71.76	81.96

$$H_1D_1 = 13\tfrac{1}{3}p_1 + 4{,}000 \qquad p_1 = \text{lbs. per sq. in.}$$

$$\frac{H_1D_1}{144} = \frac{13\frac{1}{3}p_1}{144} + 27.77$$

p_1 per sq. in.	$\frac{H_1D_1}{144}$
60	827.8
80	1094.4
100	1361.1
120	1627.8

$$UD_1 = rp_e \qquad p_e = \text{lbs. per sq. in.}$$

$$\frac{UD_1}{144} = \frac{rp_e}{144}$$

$\frac{1}{r}$	0.15	0.2	0.25	0.3	0.4	0.5
$\frac{UD}{144}$ for $p_1 =$ 60	42.8	58.8	65.28	67.8	67.2	63.96
$\frac{UD}{144}$ for $p_1 =$ 80	97	108.4	111.04	110.4	104.6	97.28
$\frac{UD}{144}$ for $p_1 =$ 100	151.1	158	156.8	150.3	142	130.6
$\frac{UD}{144}$ for $p_1 =$ 120	205.6	207.6	202.56	195.6	179.4	163.9

(*b*) *Pressures—Jacketed Cylinders.*

$\frac{1}{r}$	0.15	0.2	0.25	0.3	0.4	0.5
$\frac{p_m}{p_1}$	.417	.505	.582	.648	.756	.840
$p_1 = 60$: p_m	25.02	30.30	34.92	38.88	45.36	50.40
p_e	7.02	12.30	16.92	20.88	27.36	32.40
$p_1 = 80$: p_m	33.36	40.40	46.56	51.84	60.48	67.20
p_e	15.36	22.40	28.56	33.84	42.48	49.20
$p_1 = 100$: p_m	41.70	50.50	58.20	64.80	75.60	84.00
p_e	23.70	32.50	40.20	46.80	57.60	66.00
$p_1 = 120$: p_m	50.04	60.60	69.84	77.76	90.72	100.80
p_e	32.04	42.60	51.84	59.76	72.72	82.80

$$p_h = \frac{15.5p_1}{r}$$

$\frac{1}{r}$	0.15	0.2	0.25	0.3	0.4	0.5
p for $p_1 = 60$	139.5	186	232.5	279	372	465
p_h for $p_1 = 80$	186	248	310	372	496	620
p for $p_1 = 100$	232.5	310	387.5	465	620	775
p_h for $p_1 = 120$	279	372	465	558	744	930

(*c*) *Efficiencies.*

(*a*) For the unjacketed cylinders $E_1 = \frac{UD_1}{H_1D_1}$.

(*b*) For the jacketed cylinders $E_2 = \frac{p_e}{p_h}$.

$\frac{1}{r}$	0.15	0.2	0.25	0.3	0.4	0.5
$p_1 = 60$: $E_1 = \frac{U}{H}$	$\frac{42.8}{827.8} = .052$	$\frac{58.8}{827.8} = .071$	$\frac{65.28}{827.8} = .079$	$\frac{67.8}{827.8} = .082$	$\frac{67.2}{827.8} = .081$	$\frac{63.96}{827.8} = .077$
$E_2 = \frac{p_e}{p_h}$	$\frac{7.02}{139.5} = .05$	$\frac{12.3}{186} = .066$	$\frac{16.92}{232.5} = .073$	$\frac{20.88}{279} = .075$	$\frac{27.36}{372} = .074$	$\frac{32.4}{465} = .07$
$p_1 = 80$: $E_1 = \frac{U}{H}$	$\frac{97}{1094.4} = .09$	$\frac{108.4}{1094.4} = .099$	$\frac{111.04}{1094.4} = .101$	$\frac{110.4}{1094.4} = .10$	$\frac{104.6}{1094.4} = .096$	$\frac{97.28}{1094.4} = .09$
$E_2 = \frac{p_e}{p_h}$	$\frac{15.36}{186} = .083$	$\frac{22.4}{248} = .09$	$\frac{28.56}{310} = .092$	$\frac{33.84}{372} = .091$	$\frac{42.48}{496} = .086$	$\frac{49.2}{620} = .08$
$p_1 = 100$: $E_1 = \frac{U}{H}$	$\frac{151.1}{1361} = .111$	$\frac{158}{1361} = .116$	$\frac{156.8}{1361} = .115$	$\frac{150.3}{1361} = .11$	$\frac{142}{1361} = .105$	$\frac{130.6}{1361} = .095$
$E_2 = \frac{p_e}{p_h}$	$\frac{23.7}{232.5} = .102$	$\frac{32.5}{310} = .105$	$\frac{40.2}{387.5} = .103$	$\frac{46.8}{465} = .10$	$\frac{57.6}{620} = .093$	$\frac{66}{775} = .085$
$p_1 = 120$: $E_1 = \frac{U}{H}$	$\frac{205.6}{1627.8} = .126$	$\frac{207.6}{1627.8} = .128$	$\frac{202.56}{1627.8} = .124$	$\frac{195.6}{1627.8} = .12$	$\frac{179.4}{1627.8} = .11$	$\frac{163.92}{1627.8} = .10$
$E_2 = \frac{p_e}{p_h}$	$\frac{32.04}{279} = .115$	$\frac{42.6}{372} = .114$	$\frac{51.84}{465} = .111$	$\frac{59.76}{558} = .107$	$\frac{72.72}{744} = .098$	$\frac{82.8}{930} = .09$

(*a*) *For Maximum Efficiency of Fluid.*

p_1	60	80	100	120
$\frac{1}{r}$	0.3	0.25	0.2	0.2
E_1	.082	.101	.116	.128
E_2	.075	.092	.105	.114
$\frac{E_2}{E_1}$	.914	.911	.905	.899

It will be observed that maximum efficiency of fluid increases as p_1 increases, and the value of r for maximum efficiency also increases as p_1 increases; but the value of $\frac{E_2}{E_1}$ decreases as p_1 increases—that is, the loss due to the jacket increases, in these ideal cases, with increase of initial pressure.

(*e*) *Fuel-consumption.* Assume an effective evaporative power of 9 to 1. Also the available heat per lb. of coal = 6,700,000 ft. lbs.

$$\frac{60 \times 33,000}{E \times 6,700,000} = \frac{0.295}{E} = \text{lbs. of coal per H. P. per hour.}$$

IDEAL ENGINE. FUEL-CONSUMPTION.

	p_1...	60	80	100	120
Lbs. coal per H. P. per hr.—	Unjacketed.	3.6	2.85	2.55	2.3
" " " " " "	Jacketed....	3.9	3.2	2.8	2.5

The fact that the steam-jacket, as employed on the steam-engine, of whatever form and arrangement, is intrinsically a wasteful element, and that its use only gives, in certain cases, an economical advantage by its repression of wastes of larger magnitude, is also shown by the following illustrations, computed with and without jacket for various ratios of expansion. The results, as given in the following tables and as illustrated in the curves plotted from them, show clearly that the jacketed engine is always more wasteful than the ideal unjacketed engine.*

* Journal Franklin Institute; April 1891. "On a Maximum Efficiency of Steam-jacket;" R. H. Thurston.

Making the computations by the methods already employed and tabulating the results, we have, for $p_1 = 115$ lbs. absolute, $\tau_1 = 799°$ F., and $p_2 = 4$:

EFFICIENCIES OF WORKING FLUID.

Steam-engine, Jacketed and Unjacketed.

Cut-off.	Ratio Exp.	Eff. without.	Eff. with Jacket.
0.05	20.00	0.2073	0.1930
.10	10.00	.1934	.1808
.15	6.66	.1795	.1665
.25	4.00	.1566	.1442
.35	2.85	.1358	.1302
.45	2.22	.1237	.1209
.55	1.82	.1119	.1087
.75	1.33	.0898	.0812
1.00	1.00	.0707	.0707

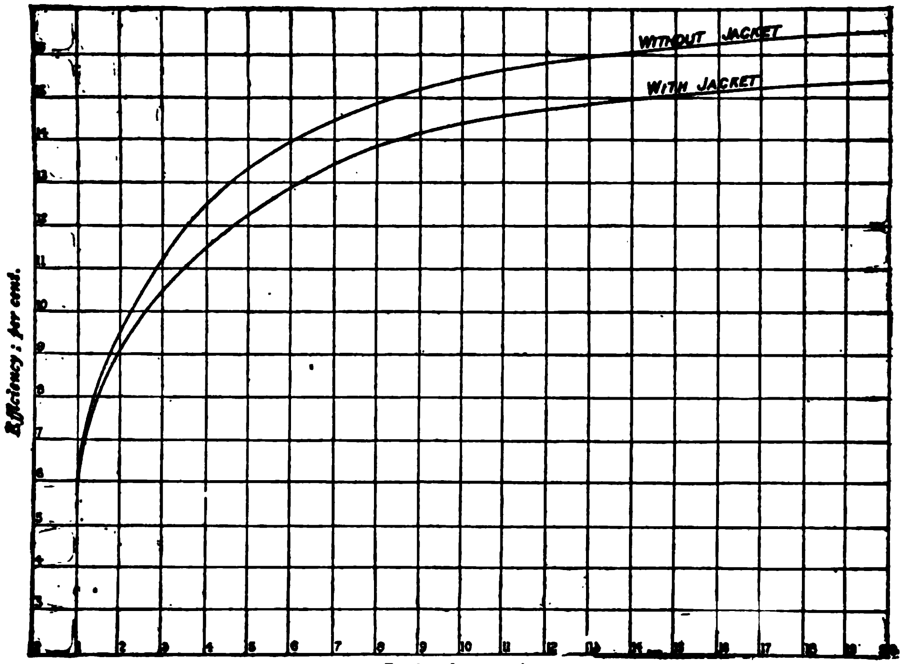

FIG. 156.—EFFECT OF JACKET.

An examination of the tables, of the curves still better, will show clearly the wasteful influence of the steam-jacket, as an element considered by itself. Within the useful range of practice, from about five or six to fifteen or twenty expansions, under the assumed conditions of initial pressure and cut-off, it it is seen that the loss by its application is fairly constant at something over one per cent, in these cases; rapidly falling to zero as the ratio of expansion falls from the lower figures to unity. The consumption of steam, in pounds per horse-power per hour, may be computed very approximately by dividing 2.3 by the computed efficiencies. The cases assumed are for condensing engines, and the evaporation always taken at nine pounds of steam per pound of fuel, the fuel expenditure may be gauged by dividing the weight of steam computed by 9. This gives, for example, about 12.06 and 12.95 pounds for the unjacketed and for the jacketed engine, respectively, at a ratio of 20, in steam demanded; and of about 1.33 and 1.44 pounds of fuel. For a ratio of expansion of 4, the figures become about 16 and 17.3, respectively, for the steam and 1.75 and 1.85 pounds of fuel. At full stroke, the figures become 35 pounds of steam and of feed-water, and 4 pounds of fuel per horse-power and per hour, for both engines.

The consumption of fuel by the ideal jacketed engine is thus found to exceed that of the ideal unjacketed engine. To determine what such engines would actually demand, we must know their size, speed, and such other data as will enable us to estimate the probable cylinder-wastes. Assuming that they are of such size and character as to give wastes for the unjacketed and jacketed engines, respectively, of $0.2\sqrt{r}$ and of $0.1\sqrt{r}$, they would consume:

Actual Consumption of Fuel.

p_1	60	80	100	120
(*a*) Unjacketed	4.8	4.0	3.2	2.8
(*b*) Jacketed	4.2	3.5	3.0	2.5

The ratio of expansion would usually be larger at these higher pressures and the actual gain by the jacket greater.

But the assumption made in these computations—that the steam is kept by the steam-jacket just dry and saturated during expansion—is probably never true except with initially dry and perhaps superheated steam. The fact is, more likely, that waste usually goes on during the whole exhaust-period, and that the total jacket-waste is thus seldom less than, and may often even exceed, ten per cent. A maximum efficiency of jacket is always found in practice between full stroke, where the cylinder-waste is a minimum, and extreme expansion, where jacket-waste is a maximum, extending through the exhaust-period. Fortunately, this maximum rises as pressure increases, precisely as required for best results.

It is obvious that, in the computation of probable efficiency, and of steam-consumption, in the case of the engines efficiently jacketed, in the manner here assumed, the volume of steam at the opening of the exhaust-valve, measures the amount used and requires no correction. A table will be found in the Appendix, computed by Mr. Buel, exhibiting, concisely, the nomenclature, data, formulas, and results for this case.

The following table * represents the results of computations of probable efficiency and performance, on the assumption that the initial pressure is 250 pounds per square inch, absolute, the internal wastes as found in experimental work already referred to, and measured by the expression $1 + 0.075 \sqrt{r}$, the engine being a jacketed tandem-compound engine, and these wastes assumed to be those due a single jacketed cylinder of moderate size and under usual conditions of operation.† Back-pressures are taken as 5 pounds condensing, and 16 non-condensing, feed-temperatures as 104° F. and 203° F., respectively, and the evaporations as 9 and 10 respectively. Rankine's assumption as to effectiveness of jacket is accepted, the wastes above referred to being taken as those of the exhaust-period.

* Trans. Am. Soc. M. E.; ccccli; vol. XII; 1891.

† Mon. Haton de la Goupillière coincides with Sinigaglia; who says that this function was first proposed by the Author, and subsequently confirmed by direct experiment at Sandy Hook and elsewhere.—Cours des Machines; vol. II.

COMPOUND STEAM-ENGINE—JACKETED.

NON-CONDENSING.

IDEAL CASE.

r	v_2	$v_3 = rv_2$	p_2	p_3	$v_3(p_3-p_2)$	U_1	U_2	$U_1 - U_2$	$U' = U_1 - U_3 + v_3(p_3 - p_2)$	H_3	(Feed = 203) h_4	$H = U_1 - U_3 + H_3 - h_4$
5	1.84	9.2	6494	2304	38548	420280	312633	107647	146195	899854	132360	875141
8	"	14.72	3931	"	23949	"	283189	137091	161040	892976	"	897647
10	"	18.40	3102	"	14683	"	269523	150757	165440	889960	"	908357
15	"	27.60	1987	"	− 8749	"	245228	175052	166303	884604	"	927296
20	"	36.80	1483	"	−30213	"	228319	191961	161748	881073	"	940694

CONDENSING.

r	v_2	$v_3 = rv_2$	p_2	p_3	$v_3(p_3-p_2)$	U_1	U_2	$U_1 - U_2$	$U' = U_1 - U_3 + v_3(p_3 - p_2)$	H_3	(Feed = 203) h_4	$H = U_1 - U_3 + H_3 - h_4$
5	18.4	9.20	64.94	720	53121	420280	312633	107647	160768	899854	55612	951889
10	"	18.40	31.02	"	43829	"	269523	150757	194586	889960	"	985105
20	"	36.8	1483	"	28078	"	228319	191961	220039	881093	"	1017442
30	"	55.2	961.	"	13303	"	204958	215322	228625	876434	"	1036144
40	"	73.16	709.9	"	− 743	"	189023	231257	230514	873373	"	1049018
50	"	92.0	559.9	"	−14729	"	176665	243625	228896	871067	"	1059080

NON-CONDENSING.

IDEAL CASE.							REAL CASE.			
$H_1 - h_4$	Q Heat supplied by Jacket per lb. steam. $H - (H_1 - h_4)$	Efficiency $= \frac{U'}{H}$	S Lbs. H_2O worked in cyl. per I. H. P. per hr. $\frac{1980000}{U'}$	M (10) Coal to evap. S $\frac{S}{10 \& 9}$	N Coal to supply heat supplied Jacket. $\frac{QM}{H_1 - h_4}$	Total fuel, $M + N$	$1 + 0.0075\sqrt{r}$	Efficiency.	Water per I. H. P. per hour.	Coal per I. H. P. per hour.
797251	77890	.1671	13.54	1.354	.1354	1.489	1.168	.1430	15.815	1.739
"	100396	.1794	12.29	1.229	.1598	1.389	1.212	.1480	14.895	1.683
"	111106	.1821	11.96	1.196	.1674	1.363	1.237	.1472	14.795	1.686
"	130045	.1793	11.91	1.191	.1906	1.382	1.290	.1392	15.364	1.783
"	143443	.1720	12.24	1.224	.2203	1.444	1.335	.1290	16.340	1.928

CONDENSING.

$H_1 - h_4$	Q	Efficiency	S	M	N	Total fuel	$1 + 0.0075\sqrt{r}$	Efficiency.	Water per I. H. P. per hour.	Coal per I. H. P. per hour.
873999	77890	.1689	12.315	1.368	.122	1.490	1.168	.1446	14.384	1.740
"	111106	.1975	10.170	1.130	.142	1.272	1.237	.1597	12.580	1.573
"	143443	.2163	8.995	.999	.172	1.171	1.335	.1620	12.008	1.563
"	162145	.2207	8.660	.962	.1732	1.135	1.411	.1564	12.360	1.601
"	175019	.2197	8.590	.954	.189	1.143	1.474	.1491	12.662	1.685
"	185081	.2161	8.650	.961	.208	1.169	1.520	.1410	13.235	1.789

p_1 = 36,000 lbs. per sq. ft. = 250 lbs. per sq. in.

It will be seen that the efficiencies range from 16.7 to 18.2 per cent in the case of the non-condensing, and from 16.9 to 22 per cent for the condensing engine, the maximum being found at a ratio of expansion, in the first case, of about 10, and in the second of about 30. Beyond these ratios the terminal pressure falls below the back-pressure, and a waste follows, instead of gain, by further expansion.

These results are still better exhibited by the curves (Figs. 157 and 158) plotted from the numerical values; the ideal case, in both sets, being represented by dotted lines, and the real engine giving the widely different curves in full line. The great difference between the condensing and the non-condensing engine, for the ideal case, is well shown, not only as to consumption of fuel at similar ratios of expansion, but also as affected by changing values of that ratio. The gain by expansion in the former case continues far beyond that at which the latter finds a limit; while the point of maximum effect is far more sharply defined with the non-condensing engine. Variation from the best ratio for the latter causes much more marked loss than with the condensing engine. The numerical values obtained are presumably those which we should obtain if we were to find a way of building engines with working cylinders having non-conducting inner surfaces. The points of maximum efficiency and those for minimum consumption of steam and of fuel are coincident in these cases, and also that for minimum supply of feed-water. As will be seen presently, this last is not the case for jacketed engines, in either the ideal or the real case, in consequence of the fact that a part of the working fluid circulates continuously between jacket and boiler and makes no demand upon the source of supply for replenishment.

The efficiencies of the real engine range from 13 to about 15 per cent, and from 14 to 16 per cent, in the two engines, respectively; while the best results are now given at a ratio of expansion of not far from 8 and 20 in the two cases, respectively. The water-consumption has increased from 12 to 14.8 pounds, and from 8 to 12 pounds, and the fuel account has risen from 1.36 to 1.68 and from 1.13 to 1.55 pounds per horse-

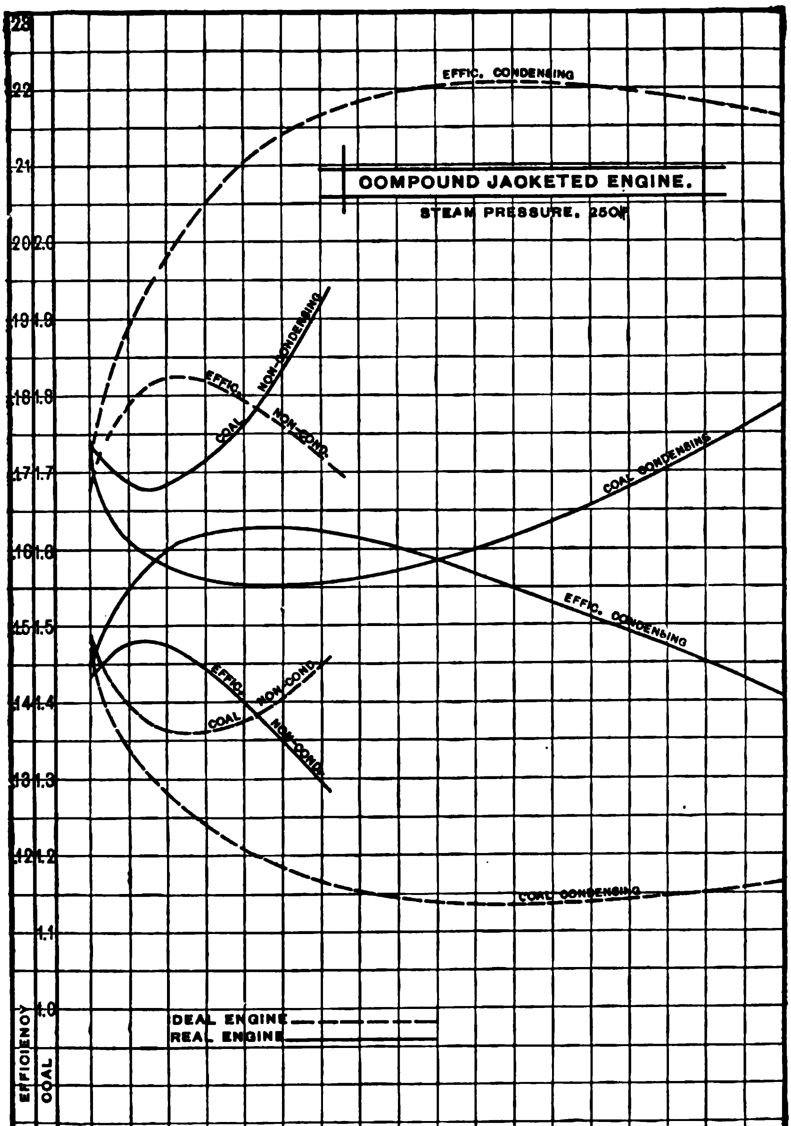

FIG. 157.—ENGINE EFFICIENCIES.

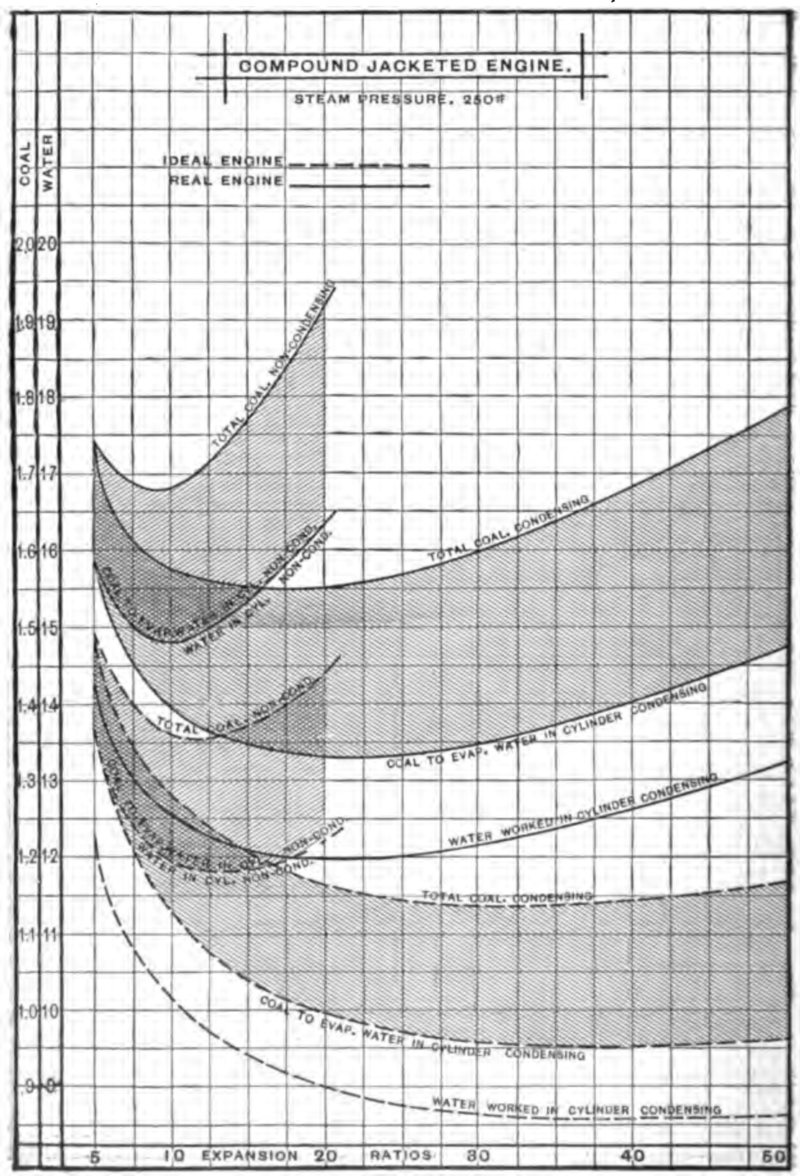

FIG. 158.—FUEL AND WATER CONSUMPTION.

power and per hour for non-condensing and for condensing engines. These changes are best seen on the curves; the lower sets being those for the real engine, and the differences being best exhibited by the shaded areas separating the pairs on the second plate.

It will be seen that the effect of this introduction of wastes in the ideal, as in the real, engine is to greatly reduce that ratio of expansion, which gives maximum efficiency, and to make variation from that ratio of maximum efficiency more seriously productive of loss; while at the same time making the differences between the several cases less than in the ideal engine. The following are the values of the ratios for maximum efficiency and for minimum steam and water consumption for the cases taken:

COMPOUND JACKETED ENGINE.

$p_1 = 250$; $p_2 = 5$ and 16; r variable.

Case.	Non-condensing.		Condensing.	
	Ideal.	Real.	Ideal.	Real.
r for maximum efficiency	11	8.5	32	17
Water-rate	12	14.75	8.5	12
Fuel-rate	1.35	1.68	1.1	1.55

The real measure of the useful power of an engine is the dynamometric power at the point at which the engine delivers its energy to the machinery of transmission. A well-built non-condensing engine should have an efficiency of machine as high as 92.5 per cent. An equally well-built condensing engine should approximate 90 per cent efficiency of machine. Taking these values, the last table becomes modified thus:

COMPOUND JACKETED ENGINE.

(Data as above.)

Case.	Non-condensing.		Condensing.	
	Ideal.	Real.	Ideal.	Real.
r for maximum efficiency	11	8.5	32	17
Water per D. H. P.	13	16	9.5	13.5
Fuel per D. H. P.	1.5	1.8	1.2	1.7

156. Limitations of Jacket-action have been noted, in many cases; and, while the precise methods of operation of their various causes have not always been fully revealed, we are perfectly familiar with their general action and effects. It has been found that the use of superheated steam, the compounding of the steam-engine, or the increase of speed of piston and of rotation—in fact, any circumstance independently promoting economy—reduces the value of the jacket, and sets a limit beyond which it would presumably have no useful effect. That this limit is sometimes reached is unquestionable. Hirn first detected such limitation in the application of superheated steam to a compound engine. Later experience has very often illustrated the fact that the jacket may be of little service, especially on compounded high-speed engines; and it is sufficiently obvious that any conditions which tend to make the net jacket-waste and the net cylinder-waste equal, either by exaggerating the former or by reducing the latter, will tend to bring about this result; as will any defect in the design, construction, or operation of the jacket which renders it inefficient in its working.

Precisely where the limit is reached in any class of engines is not easy to determine. A clue to the solution of such problems is found in the measurement of the condensation in the jacket; the quantity of water trapped off being a measure of the total heat-supply to the cylinder, dry steam being received from the boiler.

Mon. Dwelshauvers-Dery, analyzing the data supplied by test of the Whitworth Laboratory experimental engine, obtains the following:

Let Q = heat supplied by the boiler, directly;
Q_1 = that supplied by the jacket;
T = total indicated work;
E = rejected heat externally;
$C + c$ = that sent to the condenser.

Then

$$Q + Q_1 = T + E + (C + c);$$

$$\frac{T}{Q + Q_1} + \frac{E_1}{Q + Q_1} + \frac{C + c}{Q + Q_1} = 1.$$

Referring to six tests in three of which the jackets were all in use, and in three of which they were on the reservoirs, only, and were shut off on the cylinders, the following table is obtained: *

HEAT-DISTRIBUTION.

Number of trial		44	33	56	41	35	40
Heat of the direct steam,	$\frac{Q}{Q+Q_1}$	0.749	0.789	0.816	0.869	0.893	0.904
Heat of the jacket,	$\frac{Q_1}{Q+Q_1}$	0.251	0.211	0.184	0.131	0.107	0.096
Work,	$\frac{T}{Q+Q_1}$	0.153	0.165	0.171	0.125	0.135	0.136
Radiated waste,	$\frac{E}{Q+Q_1}$	0.127	0.091	0.060	0.040	0.053	0.045
Heat utilized by jacket,	$\frac{Q-E}{Q+Q_1}$	0.124	0.120	0.124	0.091	0.054	0.051
Heat rejected,	$\frac{C+c}{Q+Q_1}$	0.720	0.744	0.769	0.835	0.812	0.819

Thus, deducting the quantities of heat wasted by external radiation, the jackets supply an almost perfectly uniform quantity of heat, the figures being 12.4, 12, and 12.4 per cent in the first three trials; the cause of the greater variation in the last three cases is indeterminate from the available data.

Mon. Dwelshauvers concludes from his somewhat extended observations, and experimental researches, that, other things equal, and under usual working conditions, the jacket has little value at a low ratio of expansion; and that, to enable it to be of much service, that ratio must exceed at least 5. He has observed an economy, in his own trials, of 15 per cent at a ratio of 5, of 3 to 4 per cent at ratios approximating 3.3. In compound engines, when the expansion in the high-pressure cylinder is small, he sees little advantage in the use of the jacket; while he considers it indispensable on the large cylinder. Heat wasted in the smaller cylinder may be utilized

* Correspondence.

in the larger; but waste from the latter cannot be compensated.

Could the conditions assumed for the ideal case, as illustrated in the cases of jacketed engine and dry saturated steam, elsewhere computed, be actually secured, the exhaust delivering dry steam to the condenser, it is probable that the waste of heat from the cylinder during that period would be slight and the efficiency of the engine in actual operation thus made to approximate a maximum. Were the jacket made so effective —as, for example, in the case of Donkin's gas-heated jacket— as to superheat the steam exhausted; or were it so ineffective, as is probably usual, as to permit the exhaust-steam to be sent to the condenser wet, it is probable that the resultant, total, efficiency would be less. This consideration justifies an apothegm of Dwelshauvers-Dery: the waste by the cylinder-walls is measured by the heat demanded to evaporate the water in the exhaust-steam at the end of the expansion-period.

In all cases where the cylinders are provided with steam-jackets, if practicable, steam should be introduced into the cylinder-heads; and non-conducting coverings should be applied to the heads as well as to the cylinders, proper. The jacket-steam should not be allowed to become either stagnant or charged with air; and it should not pass into the cylinders. The jacket should be neither too greatly nor too slightly energetic; its action should be sufficient to insure dryness of the surfaces of the cylinder at the close of the exhaust, so as to prevent initial condensation; but it should not superheat the steam during expansion or exhaust.* Such efficiency of the jacket must apparently be secured, first, by proper construction of the jacket and cylinder and, secondly, and especially, by insuring reasonably dry or slightly superheated steam.

It would seem, from all that has preceded, that where a high ratio of expansion is proposed in any one cylinder, and when the steam supplied it is initially dry, or fairly dry, the steam-jacket may be confidently expected to give an unmis-

* See Ledieu: Machines à Feu; 1882; p. 714.

takable and very desirable economy, even from the final commercial point of view from which all costs, direct and incidental, are noted; but when the expansion is restricted, the range of temperature in the cylinder slight, the steam superheated, or, on the other hand, when it is so wet that the jacket connot completely dry and thoroughly reheat the metal of the cylinder before it is again exposed to the entering steam, the value of the steam-jacket may become questionable, or null, or even negative.

A good jacket covers all active condensing areas, permits neither water nor air to remain in it, returns all water of condensation immediately to the boiler, and is itself well covered by non-conductors.

In 1886, a "Research Committee" was appointed by the British Institution of Mechanical Engineers, to investigate the action of the steam-jacket.* A very extensive collection of data pertaining to the efficiency of the jacket was secured, and from these the following figures were collated and results deduced: †

The first case is a single-cylinder non-condensing Corliss engine, 21.65×43.31 inches, the body only jacketed. The jackets were supplied by a small pipe from the main steam-pipe and were automatically drained.

The second case is a single-cylinder condensing engine (Corliss), cylinder dimensions as before, body only jacketed; experiments carried on at the same place, in the same manner.

The third case is a horizontal compound condensing tandem engine, the body of the cylinders only being jacketed. The whole steam-supply to the cylinders passed through the jackets, which were drained by trap; and when not in use the jackets were open to the air.

The last trials were carried on at a constant boiler-pressure

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* Proceedings, 1890.

† Journal Franklin Institute; Apr. 1891. "On a Maximum Efficiency of Steam-jacket;" R. H. Thurston.

of 42 pounds above the atmosphere and a piston-speed of 196 feet per minute.

VARYING JACKET-EFFICIENCIES.

	Ratio of Exp.	Eff. of Jacket. Per Cent.	
$p_1 = 110$	6.2	21.33	Non-condensing.
	6	20.61	
	5	15.79	
$p_1 = 80$	5.5	13.6	
	5	12	
	4	9.63	
	3	9.38	
$p_1 = 50$	4	7.64	
	3	7	
	2	3.93	
$p_1 = 110$	12	23.43	
	11	23.82	
	10	23.78	
	9	23.2	
	8	21.5	
	7	20.32	
$p_1 = 80$	10	21.95	
	9	22.14	
	8	19.23	
	7	16.66	
	6	12.34	
	10	9.25	
	9	8.62	

	Ratio of Exp.	Eff. of Jacket. Per Cent.
$p_1 = 65$	8	7.38
	7	5.94
	6	4.67
	5	3.6
	4	1.64
$p_1 = 50$	10	7.45
	9	6.28
	8	5.13
	7	3.84
	6	3.08
	5	2.03
$p_1 = 42$	9	19.23
	8	20.45
	7	22.55
$p_1 = 42$	9	26.34
	8	26.34
	7	26.24
	6	26.5
$p_1 = 42$	9	29
	8	29.35
	7	30
	6	31.62

In the first case, that of the simple non-condensing Corliss engine, the heads unjacketed, we find, taking the first example, and plotting the data, that the use of the jacket reduced the cylinder-wastes from about 25 per cent of the ideal consumption of steam and feed-water to about half that proportion, for ratios of expansion approximating 6; from one third to about one tenth, at a ratio of 5; and apparently from 20 to 10 per cent at 4.4. The same general effect is observed

throughout, with some discrepancies which may be due either to varying action of the jacket or to slight errors of observation, or to both combined, the latter being the probable fact.

In this first case, also, it will be observed that the jacket gives best results, with 110 pounds of steam, when the ratio of expansion approximates 6. When the steam-pressure falls to approximately 80 pounds, the best work of the jacket occurs at a ratio not far from 4.75; while, at the pressure of 50 pounds, the value of the jacket increases through the whole range of the experiments; and not only so, but the curve assumes a rectilinear form, indicative of probable improvement indefinitely in the direction of increasing expansion. The highest efficiencies, however, either with or without the jacket are found, in this case, at the lowest ratios adopted, and indicate a maximum value at about 3.25. The ratios of expansion for maximum efficiency of fluid, in the other cases, are seen to be, for 110 pounds, about 5, and for 80 pounds, about 3.5.

Similarly studying the performance of the condensing engine, we find that the best work done, whether jacketed or not, is at a ratio of expansion of 10 (at a steam-pressure of 110 pounds), but that the jacketed engine reduces the internal wastes from 50 per cent at highest ratios, and from one fourth at the lowest ratios, in the case of the unjacketed engine, to 5 per cent, and, in some cases, probably to within the magnitude of the errors of observation. At a pressure of 90 pounds, the best ratio seems to be, for this engine, under the given conditions of operation, about 6.5 when unjacketed, and 8.5 jacketed; while the lower pressures still further reduce both the efficiencies and the savings effected by the jacket. The best work of the jacket, as an economizer of heat, is done, at high pressure, at a ratio of expansion of 12 or more. In all cases it seems to be the fact, with these engines, at least, that the jacket is useful beyond the ratios of maximum efficiency of fluid.

The compound engine is operated at altogether too low a pressure to bring out the best effect of compounding; but it exhibits the same general effects which have been noted in the

cases of working of simple engines. The effect of the jacket is less pronounced than in the simple engine, and the efficiencies of fluid vary less with variation of the ratios of expansion. It gives its best result at ratios of expansion ranging from 7.5 to 10.5, the variations of value being very much more observable in the last case, in which both jackets are in use than in either of the others, at least in that case in which only the high-pressure jacket was employed. By far the best work was done by the engine when both jackets were in use.

The discovery of a maximum efficiency of jacket throws some light upon the causes of the conflicting, and sometimes apparently irreconcilable, results of trials of engines with and without jackets, and with jackets variously constructed. The discovery may prove of value to the designer, as aiding him in securing the best proportions and arrangement of his engine.

157. **Jackets on Multiple-cylinder Engines** have given varying results; but the general action of this accessory in these machines may be readily traced. It is substantially the same as in the simple engine; but its effects are, in some respects, characteristically different.

In each cylinder, and throughout the series, the jacket is a source from which heat continually drains into the working fluid, adding constantly to its stock of heat-energy. Where the conversion of heat into work is considerable, or where the wastes of heat or of steam are large, the effect may be simply to reduce the rapidity of condensation during the expansion-period, as well as to check initial condensation and to waste heat during the exhaust-period; but where the jacket is effective and these losses are less, the result is not only to raise the temperature and pressure of the steam, during expansion, but also to make the steam entering each successive cylinder drier and drier, as compared with the case in which the jacket is not used.

This may even result, in some cases, perhaps, in not only preventing initial condensation in the series, after the first cylinder, but in giving dry, or even appreciably superheated,

steam throughout; the jacket supplying steam mainly to do work, and not to waste.

In computing probable expenditures of heat, steam, and fuel, for the compound engine, as affected by the jacket, it is seen that the method of treatment is precisely the same as in the cases already illustrated. The ideal case is first computed; next the wastes are added, each cylinder being treated separately, and the wastes of the system taken as equal to the loss of the most wasteful cylinder in the series. These wastes are computed for each cylinder as for the jacketed simple engine, and are ordinarily, perhaps, one half to two thirds as great with as without the jacket. The advantage of the jacket will be thus found to be less as the number of cylinders is increased.

The philosophy of the multiple-cylinder engine, as already outlined, would obviously indicate that, to secure maximum good effect, assuming the jacket on the whole desirable at all, the best system is to jacket all; and that, since the waste of the engine is measured by the waste of its most wasteful member, to omit the jacket from any one cylinder insures that the aggregate loss of heat in the whole engine will be increased by just the amount by which waste is increased in that one cylinder by such omission.

The resulting effect, in detail, is evidently the following: Assume the intermediate cylinder to be unjacketed. That cylinder, being exposed to a wider range of heating and cooling action as it alternately takes steam and exhausts it, is subject to a greater waste by internal condensation than either of the others; it thus discharges into the next cylinder a nearly equal quantity of heat and steam, but it does less work than it would have otherwise done, and to that extent produces decreased efficiency. Assume the high-pressure cylinder unjacketed, it demands more steam from the boiler, as it condenses a larger proportion of that entering by this process of initial liquefaction; it is thus itself more wasteful and furthermore transmits to the succeeding cylinders a larger quantity, and therefore a more uneconomical apportionment, of steam than it would otherwise have released. In proportion as its own efficiency

is thus reduced, it reduces the economical working of the whole; and, in proportion as the steam rejected from it is a less economical storehouse of heat for use in the other cylinders, they are in turn rendered less efficient. The low-pressure cylinder being left unjacketed, it becomes more wasteful in proportion to the increased initial condensation thus permitted, and the whole system is again, to that extent, given impaired efficiency.

With high speeds of engine, however, and especially if compounded, or when using superheated steam, these wastes become too small to be sensibly affected by the jacket; and on "high-speed engines," and all fast-running "compounds," its addition is of such doubtful utility, on the whole, that it is usually omitted. This is the case on small electric-light engines and on the enormous triple-expansion engines of trans-atlantic steamers alike.

158. Jacketing and Superheating have been already seen to be, in a way, incompatible. Both are methods of reducing interior wastes, and either being adopted, the desirability of the other is reduced. There is, however, this difference: superheating may, and sometimes does, entirely suppress initial condensation; while jacketing cannot do this. Yet, even with highly superheated steam, there must atways be some interior waste by storage in the metal and subsequent transfer. Both methods of economizing give wastes; and both produce gain, but in different degree; and, of the two, superheating is probably capable of approximating most closely to the ideal case; and even moderate superheating reduces the total of these losses below minimum jacket-waste, and thus renders the jacket useless.

159. Jackets on High-speed Engines have comparatively little value for the same reason as in the case of superheating. The cylinder-wastes are reduced, by high speed of engine, to so small a quantity that this excess over jacket-wastes is so small as to be of comparatively little importance. Where, as is now often the fact, these engines are compounded, the still

further reduction of wastes and the addition of the jacket may, in such cases, prove entirely useless, practically.

Thus, the Author has been furnished with the following record of test of a high-speed compound engine of small size, working with and without the jackets in operation. The small size of the engine makes the "water-rate"—i.e., the water used per horse-power per hour—rather large for an engine of this class, as well as giving a large amount of engine-friction. The jacket evidently has no practical value on such an engine.

	Jacket off.	Jacket on.
Boiler-pressure	90	90
Speed	343	343
Brake load	209	209
Duration of test, hours	10	10
Water used per hour	1722	1686
Vacuum	None	None
Initial pressure	88	86
Terminal pressure	11	11
Ratio of expansion	396	389
High-pressure M. E. P.	44.77	43.97
Low-pressure M. E. P.	15.34	15.04
Indicated horse-power	66.81	65.57
Brake horse-power	54.99	54.99
Loss or friction	11.82	10.58
Percentage of loss	17.7	16.13
Indicated water-rate	25.8	25.7
Brake water-rate, lbs., per h. p. per hour	31.3	30.66

In another case, the water-rate is brought down to 20 or 21 pounds by the use of the condenser, and it is found that the jacket saves 1 or 2 pounds on these figures. In still another instance, we have the following:

The engine was a "tandem compound," with cylinders 8×12 and 13×12, running at 300 revolutions per minute; steam at about 95 pounds. Each test was for one hour.

(1) First hour, steam-jacket on high-pressure cylinder only:
I. H. P., 64.46; water per h. p. per hour, 19.57 lbs.

Second hour, with steam-jacket on both cylinders:
I. H. P., 66.03; water per h. p. per hour, 18.88 lbs.

Third hour, with steam-jacket on both cylinders:
I. H. P., 65.00; water per h. p. per hour, 18.44 lbs.

(2) Same engine, same conditions, same boiler-pressure.

First hour, with steam-jackets on:
I. H. P., 71.49; water per h. p. per hour, 19.59 lbs.

Second hour, without steam-jackets:
I. H. P., 77.57; water per h. p. per hour, 19.71 lbs.

In other cases, however, jackets have done excellent work on engines making 200 revolutions and over. A Committee of the British Institute of Mechanical Engineers report a gain of a third at 220 revolutions, and Professor Alden has gained fifteen per cent, similarly, at 290 revolutions. Small engines give highest gains.

160. The Temperatures and Pressures of steam in the jacket must undoubtedly have an important influence on its economic value. Where, as is usual, the same or a slightly greater pressure is carried in the jacket as in the engine during the induction-period, the drain of heat into the metal must be little or none, after the first effect of the initial condensation is passed, and up to the point of cut-off. By raising the pressure in the jacket, as is sometimes done, by the use of an auxiliary boiler, or by throttling the steam entering the cylinder, a difference of temperatures, a temperature-head, may be obtained which will cause a more prompt restoration of the chilled cylinder to the normal maximum temperature, and will produce a drain of heat into the cylinder throughout the induction-phase. an increase of drying and in work-effect during expansion, and, also, a greater waste during the exhaust-stage.

Evidently this may be productive of either increase or diminution of efficiency of fluid, accordingly as the gains or the losses shall be found to be augmented more or less, relatively,

and as the net result is thus rendered more or less favorable. Similarly, a reduced pressure in the jacket, as, for example, when the boiler-steam passes through it on its way to the steam-chest, will modify the final effect either the one way or the other. The conclusion from general experience, thus far, would seem to be that, within the usual limits of these variations, as hitherto practised, the jacket works better with higher pressures and less satisfactorily at lower, assuming the engine receives dry steam. To jacket with exhaust-steam, as has been actually sometimes done, would seem to be an absurdity.

Some time prior to 1855, Mr. D. K. Clark proposed supplying the steam-jacket from an independent boiler at a pressure exceeding that of the main boiler, and this was done by Mr. Spencer and others.*

Experimental data relating to the economy obtainable by the use, in the jacket, of steam of considerably higher temperature and pressure than that in the cylinder are rare. Mr. Guzzi, of Milan, in 1886–7, employed, thus, steam of about 180 pounds pressure where the working steam was only 55.† The engine was of but 26 horse-power; the weight of feed-water demanded was 19.5 pounds per horse-power and per hour, as compared with 23.5 with boiler-steam in the jackets.

161. The Quality of the Steam and Condition of the Surfaces must have an important effect upon steam-jacket action. While superheating steam previously to its entrance into the engine may so reduce interior wastes as to render the jacket unnecessary, it is also unquestionable that very wet steam may exaggerate wastes to such extent as to make the jacket comparatively impotent in effecting the result to accomplish which it is employed. Also, should either the conductivity or the heat-capacity of the metal, or the transmitting power of its surface, be varied, the need of a jacket and its effectiveness, if applied, will both be modified. Low conductivity and small specific heat are the characteristics desirable in the material of

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* The Engineer; Lond.; Feb. 17, 1860; p. 106.

† Revue Universelle des Mines; Sept. 1887.

a cylinder without jacket; while large conductivity and small heat-capacity are the ideal conditions where a jacket is employed.

We may therefore conclude that dryness of steam is important in order that it shall produce minimum tendency to waste of heat by storage in the metal of the cylinder, and that a thin "liner" is desirable with a jacket; while any expedient which will reduce the absorbing and storing power of the interior surfaces of the cylinder-walls will prove useful. To secure the first result, liners are now customarily made of comparatively thin steel. Experience shows that the polishing of the inner surfaces of the cylinder, coating them with non-conductors, or bathing them in oil—a somewhat expensive process—will produce the last-mentioned effect.* It is probably practicable to find methods of securing a gain by suitable treatment of the heads of the cylinder and the sides of the piston, and the working of the engine effects the polishing of the cylinder, proper, which is perhaps next best to giving these parts non-conductivity.

The effective action of the jacket cannot even begin until the process of evaporation of moisture, by its heat-supply, has ceased. Hence the efficiency attainable by its action depends upon the early cessation of that evaporation during the induction or the expansion period, and the prompt conversion of the steam into the superheated, or at least dry, state.

Incrustations probably often exist on the jacket side of the cylinder-wall, from deposits from oil or remaining from inefficient cleaning of the casting in the foundry, which seriously reduce, in such cases, the effective action of the jacket.

Where, as usually in multiple-cylinder engines, the range of expansion is considerable, and the Rankine and Clausius form of cylinder-condensation results in liquefaction to the extent of

* The Author has devised a system of treatment of interior surfaces with first acid, then with drying oil or other suitable substance having low conductivity and specific heat per unit of volume, which method the experiments of Professor Carpenter and of Mr. Chamberlain show to be capable of reducing their wasting action more than forty per cent. See Trans. A. S. C. E.; 1890.

ten or fifteen per cent, the intermediate receiver has an office to perform as a separator and drying-chamber, as well as in the adjustment of pressures; and it receives the products of this condensation increased by all that due to the cooling through external conduction and radiation of the cylinder from which it takes the exhaust.

162. **Jacketing the Piston** is sometimes practised, notwithstanding the practical difficulties attending it, and is stated to have been first successfully attempted by M. Normand, of Havre, in France, and, later, by Mr. Davidson in Great Britain. Where the system of piston-jacket drainage can be made certain and effective in its operation, this will probably prove advantageous — as much so as jacketing the heads, a comparatively common, and always desirable, arrangement.

The cylinder-heads and piston are the parts most affected by the variations of temperature which cause those wastes to check which jackets are introduced. They are subject to as wide a range of variation of temperature as are the clearance and port spaces. They are usually comparatively rough, and therefore peculiarly active transmitters of heat. Again: at the point of cut-off their surfaces usually constitute by far the larger portion of all areas exposed to pressure and temperature changes. The advisability of jacketing both heads and of admitting steam to the interior of the piston is thus sufficiently obvious; the only objection and drawback being the difficulty of supplying steam and securing thorough drainage. These conclusions would seem to be also justified by experience, so far as it goes.

Where the heads of the cylinders, but not the piston, are jacketed, it is obvious that reduced clearances should give improved performance; since the heat in the head would act to a certain extent effectively in drying the surfaces of the piston when close together, and the more so as they the more closely approach each other. It would seem, on all accounts, that if any portion of the cylinder be jacketed, it should be the heads and steam-passages.

163. Proportions of Engines having Jackets.—Assuming the jacket to be of good design and construction and properly managed, it has been seen that its activity and its efficiency are largely determined by the proportions of the engine and the quality of the steam entering the working cylinder. Of the latter, enough has already been said. The relation of diameter to length of stroke evidently determines what proportion of heat-wasting surface exists in cylinder-heads and sides of the piston.

Evidently, to secure best effect, these proportions should vary with the type of engine. Jacketing on the sides and not on the heads is best where diameter of cylinder is small and the stroke long; if the heads, and especially if heads and piston, be jacketed, on the other hand, the reverse proportions, larger diameter and shorter stroke, give better effect.

Clark concludes, after a comparison of jacketed and unjacketed cylinders, both on long- and on short-stroke simple engines, that the evidence so gathered "proves what has long been acknowledged—the economical advantage of superheating the steam; and, more remarkably, the striking disadvantage of short-stroke, as versus long-stroke, cylinders. . . . The relatively large absorbing surfaces of the covers and the piston of short-stroke engines are disturbing influences which affect the operation of the steam in the cylinder to a greater extent, proportionally, than in long-stroke cylinders."* He adds later: "Large second cylinders proportionally to first cylinders, in the ratio of 4 or 4½ to 1, may be employed with economy when the cylinders are thoroughly steam-jacketed; but they are unfavorable for economy when the cylinders are only partly, or not at all, jacketed."†

Mr. Druitt Halpin, computing the quantities affecting cylinder-wastes in a standard triple-expansion engine, obtained the following:

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* Steam-engine; vol. 1. p. 577.
† Ibid., p. 581.

S. S. Para.	High.	Interm.	Low.
Diameter of cylinders, inches	19	35	53
Stroke, inches	33	33	33
Steam pressure per gauge, lbs	146		
" temperature, maximum, Fahr.	364°		
" " in cylinders	384°	266°	199°
Diff. " " "	16°	98°	165°
Volume of cylinders, cu. in.	9,356	31,750	72,804
Area of surface of cylinders, sq. in.	1,970	3,629	5,497
Ratio $\frac{\text{volume}}{\text{jacket-surface}}$	4.75	8.75	13.25
" $\frac{\text{temperature-range}}{\text{volume} \div \text{surface}}$	3.4	11.2	12.5

The last-stated quantities are taken as fairly measuring the relative liability to waste and advisability of jacketing.*

164. **The Defects of Steam-jacketing** usually result in faulty drainage. In some cases, no effective drainage or circulation of steam through them is possible; in other cases, this circulation is secured by carrying the steam through the jacket into the cylinder, with all its burden of water of condensation; a process, often probably, of exaggeration, rather than of reduction, of wastes. Provision for exit of air is often not well attended to, and spaces are sometimes found in the interior which form basins holding standing pools of water indefinitely. As remarked by an engineer familiar with such difficulties: "To secure the advantages of the steam-jacket, it is not sufficient to merely place around the cylinder a casing that *may* contain steam. Care must be taken that this jacket always *does* contain steam. Few but those who have actually tried it fully appreciate how soon a jacket may be rendered ineffective by the accummulation of air or of water." †

A sensible proportion of the water in the jacket may be due to external radiation. Experiments on the engine at

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* Proc. Inst. M. E., 1887; p. 59.

† Lond. Eng'g; Aug. 3, 1877; p. 88.

University College, London, showed that, in that special case, 80 per cent of this water, or 0.471 out of 0.587 pound per minute, was thus produced. Only 20 per cent, 0.111 pound, or 0.6 pound per indicated horse-power per hour, was due to true jacket-action.*

The failure to remove the sand of the cores, thoroughly, from the jacket-space and from the surface of the enclosed cylinder or "barrel" may sometimes produce such reduction of heat-transmission to the working steam as to reduce the efficiency of the jacket or possibly, in some cases, reverse its action as an economical device.

What may, perhaps, be termed the effect of a negative jacket-action is illustrated by Clark's experience on locomotives. The office of the jacket is to supply heat to the cylinder to keep up its temperature during all the fluctuations of pressure of the working fluid within it, and thus partly to ameliorate the wasteful action of the heat-conducting material of which it is composed. In locomotive practice, the cylinders of outside connected engines are exposed to the refrigerating influence of the air-currents sweeping past them, while *en route*, and thus to the precisely opposite action. Clark says:

"The action of the steam in the outside cylinder is broadly distinguished from that of the steam in the inside cylinder." †

165. **Experimental Results** are not wholly satisfactory, despite the fact that they are numerous and varied.‡

Professor Schröter, experimenting in his own laboratory at Munich on a simple engine of the Sulzer type, 280^{mm} in diameter and of 650^{mm} stroke (11 inches by 25⅝), determined the effect of its jacket at varying ratios of expansion, the steam passing through the jacket on its way to the cylinder.

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* Lond. Eng'g; Oct. 2, 1885.

† Railway Machinery, 1851; pp. 82–84. On the Behavior of Steam; Proc. Inst. C. E.; No. 1910; vol. LXXII; 1882–3.

‡ See Authorities on the Steam-Jacket; R. H. Thurston; Trans. A. S. M. E.; Nov. 1890.

One head was also jacketed. The points of cut-off were as below, and the gains by the jacket are stated therewith: *

	53 Rev.					39 Rev.				
Cut-off....	0.1	0.2	0.3	0.4	0.5	0.1	0.2	0.3	0.4	0.5
Gain pr. ct..	15.7	12.25	8.96	4.57	(?)	18.85	16.80	14.00	8.72	6.05

These figures indicate a gain, at high ratios of expansion, of 15 to nearly 20 per cent, the largest amounts being given at the lowest speeds, and that gain progressively decreasing with reduction of values of the ratio of expansion. An increase of speed of about 30 per cent gives an economy of about 20 per cent at shortest cut-offs, and of nearer 50 per cent at low expansions, where the expenditure of steam in ratio to power is greater, while the percentages of total waste are less.

In the case of the best work which the Author has yet (1891) seen reported, Professor Schröter obtained from a triple-expansion engine of 200 horse-power, steam at 156 pounds pressure, the remarkable figure of 12.2 pounds of dry steam entering the engine per I. H. P. per hour.† This corresponds to a duty of a trifle over 162 million ft.-lbs. per 100 lbs. coal at an evaporation of 10 to 1, or of 146 million at 9 to 1. The efficiency of machine was 88 per cent nearly. The jackets condensed a large percentage of the steam, thus proving their effective working. The total, about 20 per cent, was distributed thus: In the first cylinder-jacket, 2.2 per cent; second and intermediate receiver, 6.4 per cent; in third and receiver, 10.7 per cent; the drain of heat into cylinders being greater as their mean working pressures and temperatures fell.

M. Schneider, of Creusot, made numerous experiments, extending over a period of six months, upon a Corliss engine, at the Creusot works, the results of which were reported by M. Delafond in the following year.‡ In these experiments careful

* Correspondence.

† Zeits. des Ver. Deutscher Ingenieure, vol. xxxiv.; Lond. Eng., Dec. 5, 1890, p. 669.

‡ "Essais effectués sur une machine Corliss;" Annales des Mines, September, October, 1884.

examination was made of the disputed useful effect of the steam-jacket, with what M. Delafond considers satisfactory results. The jacket covered the cylindrical portion of the engine only.

The results of these elaborate and carefully conducted investigations, so far as they relate to the steam-jacket, are the following:

"The jacket reduces the expenditure the more, at equal ratios of expansion, as the pressure is higher; its effect, important at 7.75 atmospheres, with condensation, becomes very slight at 2.5.

"The economy due to the jacket is the less at the same pressure as the effective power is the greater; i.e., as the expansion is less.

"It is found advantageous to employ in the jacket steam of higher temperature than that in the engine cylinder."

The gain by the jacket in these experiments was usually not far from 15 or 20 per cent under ordinary conditions of operation.

Major English found that even jacketing the steam-pipe in engines tested by him sometimes increased their efficiency 5 per cent and over, so sensitive is the expansive steam-engine to variations of quality of steam.*

One of the most satisfactory of recent determinations of the value of the steam-jacket on compounded engines is that of Professor Osborne Reynolds, of Owens College, Manchester, employing the triple-expansion engines of the Whitworth Laboratory.† (See Frontispiece and page 499.)

The three independent engines combined in the compound machine were of the following dimensions:

	Cylinder. Diam.	Cylinder. Stroke.
No. 1	5 inches	10 inches
" 2	8 "	10 "
" 3	12 "	15 "
Air-pump on No. 3	9 "	4½ "
Feed-pump	1½ "	2 "

* Trans. Inst. M. E., 1887.

† Proc. Brit. Inst. C. E., Dec. 10, 1889.

All were jacketed, sides and head; steam was carried at 200 pounds per square inch, and boiler pressure was maintained in all the jackets.

The results were the following, with and without the jackets in use:

	With Jackets.	Without.
Coal, per horse-power per hour..	1.33 to 1.50	1.62 to 1.81
Water........................	12.68 " 14.10	15.90 " 17.30

The effect of radiation was determined, and found somewhat considerable. Deducting this waste, the figures stand:

	With Jackets.	Without.
Coal................	1.21 to 1.30	1.54 to 1.77
Water...............	11.90 " 12.30	15.10 " 16.60

This is a most satisfactory approximation to the ideal engine and to minimum wastes.

In this remarkably economical engine the loss by shutting off the jackets was from 25 to over 35 per cent in fuel-consumption; or from 25 to 30 per cent in water-expenditure.

Of the total heat received, exclusive of radiation, 19.4 per cent was converted into work with jackets in action, and but 15 without; a difference of over 23 per cent of the first quantity, or 29 of the latter. The ideal engine, under similar thermodynamic conditions, would have utilized 23 per cent.

The effect of the jacket on the high-pressure cylinder, where the difference of temperature between jacket-steam and initial was small, was found to be slight as affecting cylinder-condensation. In No. 2, the effect, with a difference of temperatures, in this respect, of 80° Fahr., that condensation was reduced from 30 to 5 per cent; while in No. 3, with a difference of 180° Fahr., such condensation was sensibly zero and the "saturation expansion-curve," assumed by Rankine to be attainable by this use of the jacket, was, perhaps, for the first time produced.

The following are data and results, reported by Mr. Buel,

as obtained in trials of an engine corresponding to the ideal case summarized in the Appendix:*

NON-CONDENSING ENGINE, WITH STEAM-JACKET AND SATURATED STEAM.

Number of Experiment	Diameter of Cylinder, inches.	Stroke. Inches.	Clearance, per cent of piston-displacement.	Absolute-Initial Pressure, pounds per square inch.	Apparent Cut-off, fraction of stroke.	Piston-speed, feet per minute.	Effective Horse-power.	Steam hourly, per effective horse-power. Pounds.
1				118.8	.13	450	152.6	25.7
2				118	.16	450	170.9	25.4
3				92.4	.23	435	153.1	23.5
4	22	43.5	3.7	92.9	.3	439	185.8	24.1
5				118.5	.58	455	211.6	24.8
6				49.9	.1	439	134.7	50.4

The influence of size of engine on the necessity and efficiency of the jacket is well shown in the experience of makers of slow-moving multiple-cylinder engines, who sometimes find that it affects the economical operation of their small engines appreciably, while insensible in its action on large machines. Experiments by Messrs. Beare and Donkin on large British locomotives, 1895–6, at 40 to 50 miles an hour, showed, with and without jackets, coal consumptions of 2.73 and 3.07 pounds per H.P. hour, and 24.49 and 26.7 pounds steam, saving nearly 10 per cent by jacketing. The cylinders were 19 by 26 inches, and developed about 500 horse-power.

166. Conclusion relative to Jacketing.—From what has preceded, it is sufficiently obvious that if jackets are used, as is advisable, at least in the case of slowly running engines, care should be taken to meet the following essential conditions of efficient and economical working:

(1) The jacket should be provided with ample supply-pipes and with effective traps or other drainage arrangements, and for air as well as water. If the jacket can be made to drain back to the boiler, that plan should always be adopted.

(2) They should be kept supplied with steam at a pressure

* Am. Machinist; Sept. 1888.

fully equal to that in the boiler. It is probably wise to jacket all the cylinders of a multiple-cylinder engine.

(3) All surfaces exposed to full-pressure steam should be jacketed, if practicable.

(4) The jacket itself should be very carefully and thoroughly lagged, and so made secure against serious external waste of heat.

(5) Provision for safe expansion and contraction should be very carefully made.

(6) It should be seen that the jacket-steam has everywhere complete contact with the inner or working cylinder, and that all water precipitated therefrom may promptly and completely drain away.

(7) The walls of the cylinder or "liner" should be as thin as practicable, and yet safe; all core-spaces should be free and clear; all core-sand thoroughly removed; no pockets should exist in which water may gather; and all fits and joints should be made with extreme care.

A jacket through which the steam entering the cylinder should pass would have a great advantage in efficiency of heat-transfer; but unless the entrained water and condensed steam could be completely removed, it would cause counterbalancing and probably greater losses, as compared with the usual arrangement, by carrying that water into the engine to exaggerate wastes.

In any case, whenever the jacket-waste, as measured by the condensation therein, exceeds the amount by which the internal waste is reduced by its action, the jacket is useless, or even a disadvantage.

The character of the steam, as has been seen, has a great influence on the activity and economical value of the jacket, and the resultant effect is due to quality of steam quite as much as quality of design and construction of jacket. The main points are:

(1) If the steam is so wet that it and the cylinder-walls cannot be dried before the end of the expansion-period, especially if the jacket is thus rendered active during the exhaust-

period, it may waste more than it saves, and thus may have even a negative value.

(2) If so dry or so far superheated that the cylinder-waste would be, without the jacket, no greater than the normal jacket-waste with the same steam, the value of the jacket will be zero.

(3) If, in the latter case, cylinder-wastes without jacket are, as is usual, greater than the normal jacket-waste for the same engine, with the same steam, the net value of the jacket will be a positive quantity proportional to the magnitude of this difference.

In compound or multiple-cylinder engines, as a rule, the temperature-head driving heat from the jacket into the cylinder increases as the pressures successively decrease, in the series of cylinders, and the activity of its action is thus similarly increased. It may sometimes prove that the plan of securing higher pressures in the high-pressure engine jacket and graded lower pressures on the others, each jacket being kept at a higher temperature than the steam entering its own cylinder, may prove advantageous.

The jacket may prove of great value with slow-moving engines and high ratios of expansion, but it is certainly not usually so with high speeds of rotation or small expansion. Since the active useful period of the jacket is mainly during the early part of expansion only, no drain into the cylinder being possible during the induction-period, and its action at the end of expansion and during exhaust involving waste, the value of the jacket becomes the more questionable as that active useful period is the less.

In all cases, and under all conditions, the use of a steam-jacket is "a violation of a fundamental law of maximum efficiency of heat-engines, which requires that they should receive all their heat at the maximum and give it out at the minimum temperature, and not, as in the case of an engine with a steam-jacket, at temperatures between these, and at times when the heat imparted lessens efficiency, which it evidently must do at and near the end of the stroke." It is a

necessary evil, justified only by the conditions affecting the use and the construction of the engine. The advantage to be derived thus varies according to circumstances, and the jacket may not only sometimes be useless, but wasteful. The necessity for a careful study of the conditions of use, of care in its application, and of exact determination of its value, is evident.

167. **Superheated Steam as a Working Fluid** can probably never be used in the ordinary steam-engine, and even superheating, in its legitimate function of reducing liability to interior wastes, is employed comparatively infrequently. It is not used as a working substance for the reason that, in order that it may retain the gaseous state throughout the expansion-period, it must be superheated initially to a higher temperature than is found ordinarily safe or practicable; or, otherwise, a way, as yet undiscovered, must be found to so modify the engine itself as to permit its safe use and at the same time to prevent those wastes of heat which now so promptly convert the steam, at entrance, from the superheated to the saturated or wet condition.

Could it be employed as a working fluid, however, its physical characteristics would be more nearly those of the gases; it would insure, possibly, a similarly high thermodynamic efficiency, and would possess the characteristic advantage of high tension, at the same time with high temperatures, initially, and would thus permit the use of a comparatively small volume of engine, and thus the attainment of that high efficiency of mechanism which is now the distinguishing excellence of the steam-engine. That this may some time be accomplished may be perfectly possible. In such event the engine would combine the high thermodynamic efficiency of the gas-engine with the high efficiency of machine of the contemporary steam-engine. According to Hirn, steam becomes steam-gas, and so remains, when the superheating exceeds about 9° C. (16° F.). Siemens found this margin to be 18° F. at the boiling-point under atmospheric pressure. For steam-gas $pv = 85.5\,T$.

168. **The Steam-engine using Superheated Steam** is simply an engine in which the working fluid, by previous

superheating, is rendered a more satisfactory working substance, with a certain nearly unaltered range of temperature and expansion; one in which the expansion is rendered more nearly adiabatic, and the conditions of maximum thermodynamic efficiency are more nearly attained. The condensation of steam at entrance is reduced in amount; but, ordinarily, at least, the fluid is still more or less wet at the point of cut-off, and continues nearly at the saturation-point throughout its whole expansion-period. Superheating is thus, as commonly practised, simply a method of economizing by reduction of interior waste.

The higher the temperature of superheating is carried above that of saturation, within usually practicable limits, the more complete is this improvement of working quality of the steam, the less the waste, and the higher the efficiency of the working fluid. Could this elevation of temperature be carried far enough, the steam might surrender all heat demanded from it to raise the walls of the cylinder up to, or above, the temperature of saturation, without itself becoming condensed, and it might thus eliminate that kind of waste entirely, substituting for it the comparatively small cylinder-wastes of a gaseous working substance. Could the "adheating" be carried still further, the working fluid would be a gas of high tension, but of low temperature as compared with the gases worked in the other forms of heat-engine.

Superheating the steam, as has already been stated (§ 145), thus results in the supply of a fluid which may surrender a certain portion of heat, measured by the product of its specific heat as a gas into the range of superheating and into its weight, to the metal of the working cylinder without the production of initial condensation. If this quantity is equal to or greater than the loss of heat during expansion and exhaust, there will be no initial condensation, and the waste from the high-pressure cylinder will be nearly that due to the passage of a gas through it under similar conditions of temperature and expansion,—a comparatively small quantity, since any substance in the gaseous state possesses low conductivity and slight

power of absorption and storage of heat. Should the superheating be in excess of this amount, the steam will not begin to condense until a later period, perhaps not at all, the only demand being now for heat to supply the amount required to keep the steam dry and saturated while expanding and doing work. If the superheating be less than the first-mentioned quantity, initial condensation will be reduced, but not entirely prevented. It is probably never the fact, in practice, that it is possible to secure, safely and economically, so much superheating as is needed to keep the steam dry throughout the stroke.*

In any case of use in the multiple-cylinder engine, the quantity of heat represented by the superheating will be a gauge of the amelioration of wastes by internal transfer of heat in every cylinder of the series. The steam leaving the high-pressure cylinder will be to that extent drier than it would otherwise be; and this will be true of the succeeding cylinder or cylinders. Were there no other disappearance of heat than that due to cylinder-condensation, superheating at the first of the series would give superheating at each of the others. In so far as condensation doing work, such as was pointed out by Rankine and Clausius, takes effect, and so far as other wastes by transfer without transformation occur, to that extent will the gain, as observed in successive passages from cylinder to cylinder, be reduced; though the improvement of the working conditions will be none the less real. Each cylinder will have wetter steam than the preceding, in proportion as the condensation doing work and the losses by conduction and radiation increase, as a total, cylinder by cylinder. Superheating at the high-pressure cylinder will produce a favorable effect all through the series, including the low-pressure cylinder. Cylinder-condensation will, nevertheless, cumulatively increase through the series, in consequence of the fact that the wetter the steam entering any one cylinder the more the condensation and the wetter that leaving it, both by this

* In one case reported to the writer an initial superheating of 500° F. was required to give 50° F. superheating at exhaust; 100° F. has usually been considered a practical maximum superheat.

initial increase of humidity and by the additional moisture coming from the Rankine and Clausius phenomenon, and from the loss by transfer to surrounding bodies. This last action will, however, be the less observable and the less important in its effect as the moisture of the entering steam and the magnitude of the waste by initial condensation become greater.

The compound engine offers peculiar facilities for superheating effectively, since the steam may be *reheated* between the cylinders, and thus kept comparatively dry with lower maximum temperature than in the simple engine; or, otherwise stated, with the same range of temperature, the working substance is a more perfect fluid for its purpose. This has been effectively practised by Cowper in Great Britain, and by Corliss and by Leavitt in the United States. The best work on record has since been reported where this expedient has been adopted, sensible superheating being secured.

In some instances, as in the Worthington "high-duty" engine, the "reheating" is obtained by the use of "prime" steam from the boiler in a "re-heater" constructed like a surface-condenser, the water of condensation flowing back to the boiler at nearly the temperature of the latter.

Reported experiments by Mr. Barrus on engines using superheated steam lead him to question its production of an economy in usual cases, even of effective drying, exceeding about ten per cent. His data show the effect of a small range, as from 15° to 25° Fahrenheit, to be slight; while superheating 60° to 80° reduces the cylinder-wastes one half or two thirds: results fairly to have been anticipated in view of general experience and the economics of the case. For such cases as the latter he obtains as the internal waste

$$W = a\sqrt{r};$$

$a = .07$, nearly, for the best cases, when for saturated steam $a = 0.10$ or $a = 0.15$. For the former class we find $a = 0.9$ to $a = 0.12$ where, with saturated steam, $a = 0.12$ or $a = 0.15$. This gain by superheating is thus made not far from one half

the total internal wastes, or, in common cases, about ten per cent net on total expenditures of steam.

169. **The Limit in Superheating** is, to-day, considered to be practically somewhere inside of the temperature 500° F. (260° C.), or within a range of not much above 100° F. (56° C.) above the now usual maximum temperature of saturation. If this amount of adheating can be secured, steadily and with certainty, no serious difficulties are anticipated; but at higher points on the scale the burning out of superheaters and the difficulties of cylinder-lubrication are such as are likely to intimidate both engineer and owner.

The desirable limit of superheating is determined, for the purposes now in view, by the amount of initial condensation to which the steam is liable if supplied in the saturated, or the wet, state. Assuming, for example, that each pound of wet steam entering the engine, bringing with it 1200 thermal units from the fuel, is subject to loss of 25 per cent of its latent heat by cylinder-condensation, storing about 250 B. T. U. in the metal of the engine: since the specific heat of gaseous steam is, according to Regnault, 0.4805, it is seen that the amount of superheating required in order that it may surrender this quantity of heat without condensation on admission must be approximately

$$\frac{250}{0.4805} = 521° \text{ Fahr.};$$

which is far beyond the practically advisable limit as fixed by experience to date.

Fortunately, however, this is not necessary, and very much less adheating is amply sufficient to accomplish the purpose in view, and a small addition by superheating, as by jacketing, suffices to greatly reduce or even suppress initial condensation. All that is necessary, in this case, is to supply an excess sufficient to meet the demand due to interior wastes of a fluid of the character of that actually at the moment worked in the engine-cylinder. The drying and the superheating of the steam continually improve the working of the engine in

two distinct ways: (1) giving a better working substance, and thus initially reducing interior wastes; (2) at the same time meeting more completely the demand for heat to bring up the temperature of the metal to that of the prime steam before the entrance of the latter into the cylinder; thus, each process conspiring with the other, the final effect is large economy with small expenditure.

It is found that in engines of moderate size—as 200 or 300 I. H. P.—superheating 80° F. to 100° F. will sometimes check all sensible condensation. This indicates that superheated steam is in such cases productive of cylinder-waste to the extent of not more than about

$$\frac{100 \times 0.4805}{1000} = 0.048,$$

or less than 5 per cent; initial condensation being entirely prevented. Against this saving by the reduction of waste perhaps by about $25 - 5 = 20$ per cent, must be charged the cost of superheating. This, when the extra heat is obtained at the chimney-flue, will be only the financial charge for first cost and maintenance of superheaters, and by simple extension of heating surface, and will be only its proportion of the cost of steam-production, in other cases; or

$$\frac{1000 + 48}{1000} - 1 \doteq 0.048,$$

to give a gross gain of about 25 per cent in steam by the expenditure of 5 per cent additional fuel, or a net gain of 20 per cent; a not infrequently reported case.

The experiments of Mr. G. B. Dixwell show that the amount of superheating required to prevent cylinder-condensation is, as is readily seen must be the fact, variable with the ratio of expansion, with the quantity of steam used and the proportion of surfaces exposed; these varying with the point of cut-off. He found that in a small engine, steam

entering the engine at 550° F., the temperature retained at two-thirds stroke without cut-off was 500°; while cutting off at one-third, the temperature dropped to 274°. Mr. Dixwell found the higher temperature perfectly safe, even at low ratios of expansion, and considered the comparatively high absorbing and radiating power of the vapor of water an important element in producing its economic effects.* The gain obtained by the reduction of cylinder-condensation, amounting to 69 per cent, was computed at 55 per cent, only about 20 per cent of its true value being expended in its extinction; while the gain in power was at the same time 16 per cent. The fact that a temperature of superheat, safe at high ratios of expansion, might not be safe at low ratios, was very clearly exhibited.†

The extent, however, to which superheating is required to check a known amount of cylinder-condensation, as already seen, cannot as yet be computed; but it will be something intermediate between that giving heat-storage in the fluid equivalent to that observed in the metal and zero. The precise location of this minimum is presumably determined by the size and physical characteristics of the engine. Recorded data have led the Author to assume that less than one half this maximum will often suffice. It may be computed thus:

Let l = latent heat of the saturated steam;
m = that fraction initially condensed;
C_p = 0.48, its specific heat at constant pressure;
t' = range of superheating required;
a = coefficient.

Then

$$t' = \frac{aml}{C_p} = 2aml, \text{ nearly.} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If $a = 0.25$, as above assumed,

$$t' = ml \div 2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

* Hirn had already set this maximum safe temperature at 230° C. (446° F.).
† On Cylinder-condensation; Trans. Society of Arts; Boston, 1875.

For example, let

$$p_1 = 90 \text{ lbs. absolute;}$$
$$l = 890 \text{ B. T. U.;}$$
$$m = 0.25; \quad a = 0.5;$$

then

$$t' = \tfrac{1}{2}ml = .125 \times 890 = 111^\circ \text{ F., nearly.}$$

Thus, roughly speaking, superheating about five degrees for each one per cent initial condensation is considered sufficient.

A large engine, working with dry steam and at moderate speed, should not waste over about thirty per cent by this process; in which case the superheating demanded would be computed as about

$$t' = 0.15 \times 890 = 134^\circ \text{ Fahr., nearly.}$$

When the available range, t, of superheating is given, the condensation may be, on the above assumptions, reduced by the quantity

$$m' = 2t' \div l.$$

Thus, when $t' = 100^\circ$ F. and $m = 0.25$,

$$m' = 2t' \div l = 200 \div 890 = 0.22,$$

and the cylinder-condensation may be reduced to something like

$$m - m' = 0.25 - 0.22 = 0.03;$$

or, in the case of the larger engine, completely with a surplus to extend the period of pre-condensation in the forward stroke, in the first case, or to $0.15 - 0.22 = -.07$, superheat. It is to be remembered, however, that, even with complete suppression of condensation by superheating the steam, heat-waste still goes on, to some extent, by storage and transfer, as before.

A good illustration of the computation of efficiency and steam-consumption in the ideal case, in which it is assumed that, to suppress initial condensation, the superheating must give a surplus of heat precisely equivalent to the anticipated or actual waste of the same engine using saturated steam, will be found in the Appendix, with all its nomenclature, formulas, data, and results. The following is a comparison of computed

with actual results obtained by test of engines presumed comparable:

GAIN BY SUPERHEATED STEAM IN NON-CONDENSING ENGINES WITH UNJACKETED CYLINDERS.

Computed Results.					Results from Experiments of U. S. N. Board, 1877.				
Point of Cut-off.	Pounds of Steam hourly per effective horse-power.				Point of Cut-off.	Pounds of Steam hourly per indicated horse-power.			
	Saturated Steam.	Superheated Steam.	Difference.	Per cent of difference.		Saturated Steam.	Superheated Steam.	Difference.	Per cent of difference.
F. S.*	40.7	38.8	1.9	4.9					
3/4	39.3	36.6	2.7	7.4	.69	48.2	35.2	13	37
1/2	30.3	26.6	3.7	13.9	.46	42.2	31.7	10.5	33.1
1/3	28.6	23.7	4.9	20.7					
1/4	28.	22.1	5.9	26.7	.25	45.3	35.8	9.5	26.6
1/6	30.2	22.	8.2	37.3					
1/8	34.9	24.2	10.7	44.2					
1/12	42.2	26.1	16.1	61.7					

* "F. S." = full stroke.

The only special assumption made in the ideal case is that, as in the ideal jacketed engine, condensation is prevented and the steam is dry and saturated at the end of the expansion-period.

SUPERHEATED-STEAM CONDENSING ENGINES WITH UNJACKETED CYLINDERS.

Theoretical Results.					Practical Results.				
Point of Cut-off.	Pounds of Steam hourly per effective horse-power.				Point of Cut-off.	Pounds of Steam or Coal hourly per indicated horse-power.			
	Saturated Steam.	Superheated Steam.	Difference.	Per cent of difference.		Saturated Steam.	Superheated Steam.	Difference.	Per cent of difference.
F. S.	35.9	33.2	2.7	8.1	.65	3.71	2.99	.72	24.1
3/4	34.7	31.2	3.5	11.2	.60	3.07	2.74	.33	12.1
1/2	26.6	22.4	4.2	18.8	.58	31.4	26.1	5.3	20.3
1/3	24.5	19.3	4.6	23.8	.50	32.7	25.1	7.6	30.3
1/4	23.4	17.3	6.1	35.3	.45	3.38	2.91	.47	16.2
1/6	23.4	15.9	7.5	47.2	.35	2.73	2.33	.40	17.2
1/8	24.6	15.7	8.9	56.7	.32	30.6	28.4	2.2	7.8
1/12	24.8	13.9	10.9	78.4					

The next table is given for the case of condensing engines by Mr. Buel; and the following cases are from Bourne:*

GAIN BY USE OF SUPERHEATED STEAM IN MARINE ENGINES.

Vessel.	Total Coal—Pounds.			
	Saturated Steam.	Super-heated Steam.	Difference.	Per cent of difference.
Alhambra, Southampton to Lisbon and return	405,440	275,520	129,920	47.2
Colombo, Southampton to Alexandria and return	2,900,800	2,287,280	613,520	26.8
Norman, Southampton to Cape of Good Hope and return	1,554,560	1,189,440	365,120	30.7
Ceylon, Southampton to Alexandria and return	3,364,480	2,291,520	1,072,960	46.8

Since these dates, however, the increasing pressures, advances in general efficiency and especially high temperatures and wide range of expansion, which have become common, have greatly reduced the margin for gain by superheating. The S. S. "Inchdune," 1900, with steam at 18 atmospheres, temperature, superheated, 469° F. (243° C.), feed-water entering the boiler at 209° F. (98° C.), flue-gases at stack at 319° F. (160° C.), worked in a quadruple-expansion engine, is reported to have obtained as low a consumption as 0.97 pound of fuel per horse-power-hour.

It is evident that, as long since observed by Professor Hirsch, a most serious obstacle to the employment of superheated steam exists in the difficulty of regulating the quantity of added temperature. It is also obvious that, to secure every desired favorable condition, a method must be found of apportioning the degree of superheating to the varying demands of the engine, as determined by variation of the ratio of expansion, from time to time, and by the quality of steam entering the superheater.

170. **Experience and Testimony** derived from many experiments prove the value of moderate superheating. Mons. Hirn reports, as the results of trials in which he was aided by

* Treatise on the Steam-engine, by John Bourne; 1859.

Messrs. Dwelshauvers-Dery, Grossteste, and Hallauer, the following figures, checked by Cotterill:

SUPERHEATING.

Extent.		p_1	r	Per cent of waste.
Steam superheated..........	157° F.	61	4	7.8
" "	0°	54	4	15.6
" "	95°	56	7	12.4
" "	0°	55	7	21.8

The engines built, in 1832, for H.M.S. Dee demanded 3.9 pounds of coal per I. H. P. per hour with saturated steam, but only 2.74 pounds at a temperature exceeding that of saturation by 188°, the pressure being but 9 pounds. The Ceylon in 1860 gained over 25 per cent by superheating about 100° F.; the Alhambra gained over 25 per cent; the Nepaul about 50 per cent.*

The following table, compiled by Mr. Dixwell from the experiments of Isherwood, Emery, and Loring, shows well the advantages of superheating steam within the safe limit and at moderate pressure. It thus appears, as remarked by Mr. Dix-

Name of Steamer.	Kind of Engine.	Kind of Steam used.	Boiler-pressure above Atmosphere. Lbs. per sq. in.	Actual Cut-off.	Pounds of Coal consumed per net horse-power per hour.
Michigan....	Simple	Saturated	21	.29	4.5
Mackinaw...	"	"	35	.43	3.49
Eutaw.....	"	"	27	.54	3.84
Dexter.....	"	"	67	.29	3.4
Dallas......	"	"	32	.31	3.8
Bache......	Compound Jacketed	"	80	.20	2.66
Rush........	"	"	69	.16	2.71
Georgeanna.	Simple	Superheated	33	.31	2.58
Adelaide....	"	"	34	.39	2.45
Mackinaw...	"	"	39	.29	2.48
Eutaw......	"	"	28	.54	2.99

* Proc. Brit. Inst. C. E.; vol. XIX. p. 473.

well, that the Georgeana, Adelaide, Mackinaw, and Eutaw, working with superheated steam at moderate pressures and without jackets, surpassed the performances of jacketed compound engines working with much higher pressures and much greater expansion. The table shows best results to date.

Conclusions relative to superheating may evidently be arrived at, and without question, favorable to the moderate use of superheating. It is certain that, as long since pointed out by Hirn, this method is more thorough in its reduction of cylinder-wastes than jacketing, or even, if it can be carried sufficiently far with safety in the simple engine, than "compounding." It gives dry steam initially, and throughout the expansion-period, and is not productive of loss during the exhaust-period, a phase in the engine-cycle during which wastes by the jacket are especially active where it has not left the steam and the walls of the cylinder dry at the end of expansion. The jacket keeps these surfaces approximately at the boiler temperature, even during this last most wasteful part of the whole revolution; while superheated steam produces its effects just when and where they are needed. Superheating is less effective at high than at low pressures.

Where the superheating is effected by the saving of heat which would otherwise have passed up the chimney, as is often, perhaps usually, the case, the gain at the engine is a real gain. When, however, the superheater simply produces dry and superheated steam where it would otherwise have been wet, and by the application of heat that might otherwise have been employed in the boiler in the production of saturated steam, the apparent gain must be reduced by this expenditure and the net and real saving is correspondingly lessened. This net saving is to be measured in fuel, rather than steam, consumption. A net gain amounting to from 50 to 75 per cent the apparent saving has been attained in practice, in such cases, by a reduction of cylinder-wastes to a very small quantity, as to five per cent, or even less.

There exists, for every engine, a set of conditions, and especially a quality of steam, which make the jacket most

Schmidt Engines.	No. of Test.	Performance in H. P.		Boiler Pressure in Lbs.	Temperature of the Steam in Degrees Fa	
		Electrical.	Indicated		Saturated.	Sup
						At Boiler
Simple horizontal single-acting two-cylinder non-condensing	1		129.4	138.4	359.6	[illegible]
	2		103.1	144.1	362 5	[illegible]
Horizontal tandem compound condensing	3		124.6	166.2	372.8	
	4		117.9	167.6	372 9	
	5		106.6	165.7	372.8	
	6		104 9	165.6	372.8	[illegible]
	7		103.6	161.5	370 9	[illegible]
Compound with high-pressure cylinder horizontal, low-pressure cylinder vertical	8	109.8	118.9	160.2	369.5	[illegible]
	9	101.1	110 2	160.3	370.0	[illegible]
	10	65.39	71.95	108.4	342.5	[illegible]
	11	65.38	72.12	108.8	342.5	654
Do. do. do	12	60.70	68.73	153.6	366.8	[illegible]
Vertical tandem compound condensing	13	62.23	75.32	169.2	374.4	[illegible]
	14	61.74	71.38	167.8	373.8	[illegible]
Compound with high-pressure cylinder vertical, low-pressure cylinder horizontal	15	85 67	110.15	124.1	351.9	
	16	79.08	102.21	130.1	354.9	
	17	86.13	109.79	131.4	355.9	
Simple horizontal double-acting	18		64.97	91.7	332.2	531
Compound horizontal	19	31.96	34.13	113.7	345.9	568
Two-cylinder single-acting	20	39.45	46.45	142 2	361.4	651
Vertical two-cylinder single-acting	21	37.96	39.45	128.3	354.0	
	22	39.20	40.90	127.1	352.4	
Non-condensing	23	40.90	47.82	125.3	352.0	
Horizontal compound	24		26.97	103.9	339.8	611
Simple horizontal	25	20.12	21.45	106.6	341.8	619
" "	26	18.32	22.36	125.1	552.4	667
" "	27	18.36	22.53	123.1	352.0	690
17 H. P. Schmidt motor	28	18.40				700
	29	18.30				592
	30	18.88				460
	31	18.64			374	without
	32	8.12				646
	33	10.38				676
	34	16.20				680
	35	18.34				700
Vertical high-speed	36		16.57	105.2	339.8	572
Simple single-acting	37	3.54	4.47	114.5	345.9	
	38	3.75	4.85	112.3	345.0	
Horizontal cross-compound double-acting	39		184.2	140	359.6	745
	40		124.8	140	359.6	without
Single-acting twin tandem compound condensing	41		257.6	157.3	369.2	728

* Trans. A. S. M E.,

team Consumption in Lbs. per Hour.		Coal Consumption in Lbs. per Hour.		Test reported by
E. H. P.	I. H. P.	E. H. P.	I. H. P.	
	16.87		2.59	"Mitt. aus der Praxis des Dampfk.- u. Dampfmasch.—Betr.," 1898, p. 226.
	18.43		2.68	
	10.07		1.43	"Zeitschr. des Bayrischen Dampfkessel Revisionsvereines," 1897, Nos. 11 and 12.
	9.76		1.43	
	9.50		1.55	
	9.99		1.56	
	10.14		1.54	
11.45	10.57	1.96	1.81	"Zeitschrift des Vereines deutscher Ingenieure," Nov. 28, 1896.
11.70	10.74	1.87	1.71	
12.55	11.41	2.01	1.83	
11.93	10.82	2.02	1.83	
10.96	10.42	1.72	1.52	*Ibid.*, Dec 5, 1896.
12.31	10.17	1.55	1.28	*Ibid.*, Jan. 5, 1895.
12.58	10.88	1.60	1.38	
16.1	12.5	2.56	1.99	Gritzner in Durlach.
14.6	11.3	2.71	2.12	
14.6	11.5	2.77	2.17	
	21.7		3.55	"Zeitschrift des Vereines deutscher Ingenieure," Dec. 5, 1896.
	18.5		2.01	
23.9	20.3	2.57	2.18	Gritzner, Elektr. Ausst., Karlsruhe.
17.9	17.2	1.97	1.89	"Zeitschr. des Verbandes der Dampfk. Ueberwachungsvereine," 1894, No. 16.
17.2	16.5	2.22	2.13	
17.8	15.3			"Mitt. aus der Prax. d. Dampfk.- und Dampfmasch.—Betr.," 1895, p. 3.
	23.5		4.02	"Zeitschrift des Vereines deutscher Ingenieure," Dec. 5, 1896.
	21.9		2.10	
18.3	15.0	2.88	2.87	
18.4	15.0	3.22	2.88	
18.3				Prof. Wm. Ripper, in Sheffield; see "Zeitschrift des Vereines deutscher Ingenieure," 1897, p. 1406.
21.7				
35.7				
40.7				
26.8				
23.2				
19.2				
18.3				
	19.4		2.79	"Zeitschrift des Vereines deutscher Ingenieure," Dec. 5, 1896.
26.5	21.0	4.41	3.49	"Zeitschr. d. Verb. d. Dampfk. Ueberwachungsvereine," 1894, No. 17.
26.5	20.5	4.21	3.25	
	10.4		1.31	Prof. Ewing, Cambridge University.
	17.2		2.10	Portion of steam passed through reheating receiver before reaching throttle.
	8.97			Prof. Lewicki, Dresden.

ive. With sufficiently superheated steam, the jacket is eeded at all; it would add nothing to the efficiency of the ɩe; with wet steam it might be possible that the loss from ıcket during the terminal portion of the expansion-period, throughout the exhaust, might exceed the gain in the :r part of the active period of jacket-action, and during ompression. With intermediate conditions, a maximum by the jacket-action might be observed. This maximum be expected to be found when the steam is at least fairly ,nd the ratio of expansion considerable.

'nce the surfaces become dry, they can yield but little to the enclosed vapor, and the jacket can then promptly ; them up to approximate the temperature of the entering n. This action is that desired of the jacket, in fact.

'he steam-turbine gains only thermodynamically by superng, not being subject to initial condensation; but gains :r by accelerated discharge.

71. **Compression and Clearances** have rather definite ions; nevertheless they are not related by purely kinematic :iple; even if the usual treatment, by such a process, have really important bearing. Were there no exchange of to be anticipated, between the working fluid and the walls e cylinder, the proper treatment would be to secure com- : expansion and full compression to initial pressure. But only does this transfer occur and thus modify the case, :he purely dynamic exigencies of operation may enter as ›rtant factors in determining these relations.

'he "clearance" in the steam-engine is the small space nec- :ily left between the piston and head, at the end of stroke, vade danger of their being brought into actual contact, ugh wear, accident, or carelessness in adjustment of length king up wear on the connecting-rod "brasses," or in other ings "in series" with it. The "dead-spaces" include this :ance and the port-spaces; which latter are often large. total varies from below 2 per cent up to 6, 8, or even 10 cent of the volume of the cylinder. Since these spaces t be filled with steam at every stroke, they constitute a

source of waste; except they are filled from the back-pressure steam by compression.

Thus the waste due to clearance may be reduced and in some cases made zero by suitable compression. Where expansion is incomplete, it will be found that, dynamically, the best result is secured when the compression is somewhat in excess of the expansion-ratio, and, under usual conditions, not far from 50 per cent higher.* The thermal effect in reduction of internal wastes is sufficiently important, however, to make it advisable to aim at compressing, in most cases, probably, well up toward boiler-pressure, regardless of this aspect of the problem. The dynamic loss, in engines with large clearance, as 6 to 10 per cent, may be as much as 10 and 15 per cent without compression, and but one third these figures with best adjustment.

Zeuner's principle, affecting the action of the clearance and port spaces, is the following:

In any case, complete compression, if practised, annuls the wasteful effect of those spaces with complete expansion.

Complete expansion occurs when the pressure at its end is equal to the back-pressure; complete compression is that which carries the final pressure of compression up to the initial pressure of admission. Assuming that the law of compression is the same as that of expansion, and also assuming the law of Mariotte:

Let v_1 = the volume of steam entering at the initial pressure p_1;
v = the volume of the dead-space;
p_0 = the back-pressure.

The expansion will be complete when the pressure at the end of expansion is equal to p_0, which requires that the volume at that point shall be greater than at the beginning of

* See Cotterill; p. 258.

sion in the proportion $\frac{p_1}{p_0}$. In a cylinder having no
nce, the work per stroke of piston is, in such case,

$$U_1 = p_1v_1 + p_1v_1 \log\frac{p_1}{p_0} - \left(\frac{p_1}{p_0}v_1\right)p_0 = p_1v_1 \log\frac{p_1}{p_0}.$$

hen there exists a dead-space, v, the initial volume of
, v_1, first fills a portion, $v - v\frac{p_0}{p_1}$, of this space, and then
the piston through a volume, $v_1 - v + v\frac{p_0}{p_1}$, during
sion. The work at full pressure is

$$p_1v_1 - p_1v + p_0v.$$

ıe total volume of steam at the end of the admission is

$$v_1 + v\frac{p_0}{p_1};$$

the work of expansion is measured by

$$\log\frac{p_1}{p_0}(p_1v_1 + p_0v).$$

he volume of steam at the commencement of the exhaust

$$\frac{p_1}{p_0}\left(v_1 + v\frac{p_0}{p_1}\right) = v_1\frac{p_1}{p_0} + v;$$

olume at the beginning of the compression is, in order that
ll be complete, evidently $v\frac{p_1}{p_0}$.

he work of the back-pressure is then

$$p_0\left\{v_1\frac{p_1}{p_0} + v - v\frac{p_1}{p_0}\right\} = p_1v_1 + p_0v - p_1v;$$

and the work of the compression will be

$$p_0 v \log \frac{p_1}{p_0}.$$

The net amount of work done is thus, finally,

$$U_2 = p_1 v_1 + p_1 v + p_0 v + (p_1 v_1 + p_0 v) \log \frac{p_1}{p_0}$$
$$- p_1 v_1 - p_1 v - p_0 v - p_0 v \log \frac{p_1}{p_0};$$

$$U_2 = p_1 v_1 \log \frac{p_1}{p_0} = U_1.$$

It is perfectly obvious, however, that the action of the cylinder-walls may completely invalidate all conclusions drawn from this purely kinematic principle.

Where it is necessary to take cognizance of the clearance and its effect, it is obvious that r, the true ratio of expansion, has the relation to the apparent ratio, r', as practically measured on the guides at the instant of seating of the cut-off valve, for example,

$$\frac{r}{r'} = \frac{1 + c}{1 + cr'};$$

$$r = \frac{r' + cr'}{1 + cr'};$$

where c is the clearance-ratio.

The volumes and weight of vapor, as shown by the indicator, will be greater, in the case of an engine with clearance, in the proportion $1 + cr'$, where c is the clearance-fraction. It is obvious that the volume traversed by the piston to do the same work must be, with clearance, greater in the proportion

ınd either the size of cylinder or speed of piston corre-
ıdingly increased. The efficiency will also be diminished slightly less ratio than that of increased steam used, unless compression be adopted; in which case no such loss occurs.
\ comparison of the weight of steam demanded in the m-engine at various ratios of expansion, as affected by ·ance, and neglecting the influence of compression, is easily e.

.et r = ratio of expansion (nominal);
c = ratio of clearance to stroke;
s = stroke of piston; A = its area;
w = specific weight of steam.

Then the ratio of weight of steam to work done will be

$$m = \frac{As(1 + c)w}{As(p_1 - p_2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

he engine without expansion,

$$m' = \frac{As\left(\frac{1}{r} + c\right)w}{As(p_1 - p_2)n}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

he case of expanding steam; p_1 and p_2 being the pressures .e beginning of expansion and during exhaust, measured a vacuum, and n is the ratio of the mean effective pres- to the difference $p_1 - p_2$, the initial effective pressure.
`he ratio of these two quantities, (1) and (2), is

$$r'' = \frac{\frac{1}{r} + c}{n(1 + c)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

`he following tables give values of n and of r'', as calcu- ı by Professor Schröter.*

---

* Bayerisches Industrie- und Gewerbeblatt; 1881; Heft VI.

VALUES OF n.

$r =$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03
$p_1 = 2$	100	90.4	82.0	74.0	66.7	60.0	53.5	47.4	—	—	—	—	—	—	—	—	—
	—	90.9	82.8	75.3	68.5	62.1	56.2	50.6	45.5	—	—	—	—	—	—	—	—
3	—	90.4	81.9	73.9	66.5	59.7	53.0	46.6	40.2	—	—	—	—	—	—	—	—
	—	90.9	82.8	75.3	68.3	61.8	55.7	50.0	44.4	39.7	—	—	—	—	—	—	—
4	—	90.4	81.9	73.8	66.4	59.5	52.8	46.3	39.7	—	—	—	—	—	—	—	—
	—	90.9	82.8	75.2	68.2	61.6	55.5	49.6	43.9	38.8	38.4	37.9	—	—	—	—	—
5	—	90.4	81.9	73.8	66.4	59.4	52.7	46.1	39.4	32.4	—	—	—	—	—	—	—
	—	90.9	82.8	75.2	68.2	61.5	55.4	49.4	43.7	38.3	37.9	37.3	37.0	36.5	—	—	—
6	—	90.4	81.9	73.7	66.3	59.4	52.6	46.0	39.2	31.9	31.1	—	—	—	—	—	—
	—	90.9	82.8	75.1	68.1	61.5	55.3	49.3	43.4	38.0	37.4	37.0	36.5	36.1	35.7	35.4	—
8	—	90.4	81.9	73.7	66.3	59.3	52.6	45.9	39.0	31.5	30.6	29.8	28.9	—	—	—	—
	—	90.9	82.7	75.1	68.1	61.4	55.2	49.2	43.2	37.6	37.0	36.4	36.0	35.5	35.1	34.7	34.4
10	—	90.4	81.8	73.7	66.3	59.2	52.5	45.8	38.8	31.2	30.4	29.5	28.5	27.6	26.6	—	—
	—	90.9	82.7	75.1	68.0	61.3	55.1	49.1	43.1	37.3	36.8	26.2	35.7	35.3	34.7	34.3	34.0

VALUES OF r''.

RATIO OF STEAM DEMANDED WITH EXPANSION TO THAT REQUIRED WITHOUT.

$r =$	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03
$p_1 = 2$	1.0	0.995	0.976	0.946	0.899	0.833	0.747	0.633	—	—	—	—	—	—	—	—	—
	—	—	0.977	0.948	0.904	0.844	0.763	0.658	0.523	—	—	—	—	—	—	—	—
3	—	—	0.977	0.947	0.902	0.838	0.754	0.643	0.497	—	—	—	—	—	—	—	—
	—	—	0.978	0.949	0.906	0.848	0.769	0.667	0.536	0.360	—	—	—	—	—	—	—
4	—	—	0.977	0.948	0.903	0.840	0.757	0.648	0.503	—	—	—	—	—	—	—	—
	—	—	0.978	0.950	0.907	0.850	0.772	0.672	0.542	0.368	0.347	0.327	—	—	—	—	—
5	—	—	0.977	0.948	0.904	0.841	0.759	0.650	0.507	0.309	—	—	—	—	—	—	—
	—	—	0.978	0.950	0.908	0.851	0.774	0.674	0.545	0.373	0.352	0.332	0.309	0.287	—	—	—
6	—	—	0.977	0.949	0.905	0.842	0.760	0.652	0.510	0.313	0.289	—	—	—	—	—	—
	—	—	0.978	0.951	0.909	0.852	0.775	0.676	0.548	0.376	0.356	0.335	0.313	0.290	0.267	0.242	—
8	—	—	0.977	0.949	0.905	0.843	0.761	0.654	0.513	0.317	0.294	0.268	0.242	—	—	—	—
	—	—	0.979	0.951	0.909	0.853	0.777	0.678	0.551	0.380	0.360	0.340	0.317	0.295	0.271	0.247	0.221
10	—	—	0.978	0.949	0.905	0.844	0.762	0.656	0.515	0.320	0.296	0.271	0.245	0.217	0.188	—	—
	—	—	0.979	0.951	0.910	0.854	0.778	0.679	0.552	0.383	0.362	0.342	0.320	0.297	0.274	0.250	0.224

The main value of high compression, as is seen in some types of engine, certainly, is not to secure that nice adjustment which would prevent a slight waste of power due to maladjustment of the ratios of expansion and compression, but to secure a smooth-running engine by "cushioning," in such manner as to take up, by the spring thus produced, that impact and jar, that "pound," otherwise liable to occur with annoying, if not dangerous, consequences, every time the crank swings past the centre. In high-speed engines the designer carefully adjusts the volume of clearance to be adopted, with this end in view, making the "dead-space" comparatively large to insure that the work of compression shall furnish the needed means of absorption of the energy of retardation.

This heat-supply is less effective and economical, however, than is similar surplus heat, in the form of superheat, direct from the boiler. Dwelshavers has found that, in some cases at least, such gain by compression is offset by more than commensurate loss. (Revue de Mecanique, 1897.)

From this point of view no computation is required, or is yet possible, that shall exactly determine the magnitude of these effects. It is, however, obvious that compression to boiler-pressure might be desirable, and that the volume of dead-spaces should be then such as makes the work of compression approximately equal to so much of the stored energy of the reciprocating parts as is required to be absorbed.*

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* Leloutre remarks: "I can easily demonstrate, by an immense number of diagrams and of calorimetric observations made on a large scale, that the law of Mariotte is radically false in its application to the steam-engine. This law is expressed by the equation $\frac{P_n}{P_m} = \frac{V_m}{V_n}$. Rankine was the first, I think, to propose the expression $\frac{P_n}{P_m} = \left(\frac{V_m}{V_n}\right)^{1.11}$. More recently MM. Hirn and Cazin, in the courses of thoroughly scientific investigations, have found the value, for superheated steam, $\frac{p_n}{p_m} = \left(\frac{V_m}{V_n}\right)^{1.33}$. But in the application of these

It is evident, further, that compression is a necessary and effective adjunct to all other methods of economizing; hough the magnitude of the dead-spaces and the waste by arance is a matter of less importance with the multiple-inder engines.

So essential is the use of compression to insure smooth :ion in high-speed engines with their large inertia-effects that ɜir usually large clearances are sometimes even purposely aggerated to obtain ample cushioning. In such makes of gine the clearances are carefully proportioned with this purse in view. Thus Messrs. H. Westinghouse and Rites introce a "clearance-chamber of carefully determined proportions tween the two cylinders of the single-acting compound en-ıe, which space is constantly open to the small cylinder, in der that the initial and compression pressures may be made ual. The action of the engine is that characteristic of the oolf or receiverless engine, and the result of this arrange-ent is that the compression in the small cylinder is made dependent of the load, but variable with the steam-pressure, is compression always beginning when the low-pressure :pansion begins, producing the distribution shown in ig. 159, the diagram being that used in designing the ıgine.

In this diagram three variations of load are shown, respec-vely, by the heavy, light, and light-dotted lines, the compression

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st formulas to our industrial motors they will be found even more incorrect an the law of Mariotte. Through numberless researches I have reached the llowing conclusion: There *is* no fixed law of expansion in these engines; or, ther, the general law, if one can be established, varies in its effects from one roke of the piston to another. . . . I have already demonstrated, in a report ı the superheated-steam-engine of Mons. Hirn, that the succession of press-es during the expansion is represented very exactly by the general formula $\frac{x}{n} = \left(\frac{V_m}{V_n}\right)^a$, in which the index α is generally less than 1, and, consequently, at the machine has slightly more power than the constructors consider them-lves able to guarantee." (Bulletin de la Société Industrielle de Mulhouse: 73.)

of each commencing at *c*, *b*, and *a*, respectively, but following the same curve, and terminating in each case at the same initial pressure, *M*. In like manner, with the steam-pressure raised to *N*, we get the heavy-dotted diagram, in which cut-off having taken place earlier, compression would commence earlier at *d*, but terminating at the new initial pressure, *N*. Whatever be the exhaust-pressure at the commencement of compression in the small cylinder, due to changes of load or of boiler-pressure, it is automatically compensated by shifting the point of com-

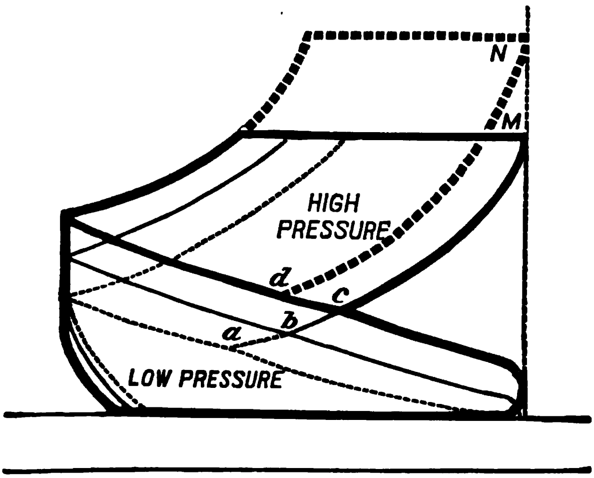


FIG. 159.—FULL COMPRESSION.

pression itself to such a position as will insure final pressure equal to that of the admitted steam. Expansion in the large cylinder should commence coincidently with compression in the small cylinder.

This result is arrived at by the simple combination of correct valve-travel and proportion, with a specific and constant clearance-volume in the small cylinder.

In this case, also, the clearance and compression are adjusted to compensate that loss of pressure between the cylinders due to cylinder-condensation in the initial stage in the low-pressure engine.

Where the two pistons are secured on the same rod, as in st tandem compound engines, the smooth running of the ine is facilitated by the aid given in the cushioning of the ιm in the high-pressure cylinder, when, as in condensing ines, large compression in the low-pressure cylinder becomes cult.

Compression was not used by Mr. Corliss in his engines, ιtever their speed. Mr. Henthorn advises, for Corliss en- :s, a compression not to exceed the terminal pressure on the ansion line for condensing engines, and an excess over this ;sure of about five pounds for non-condensing engines.*

The loss of work by the clearance and the cushion-steam is lily computed as a purely dynamic quantity; but the real by clearance and the thermodynamic effect of the com- ;sion are not, as yet, capable of computation with accu- r.

If the pressure and volume of the steam at exhaust are p_2, the back-pressure p_3, and the volume of the clearance-space the pressure and volume of the cushion-steam at the begin- g and end of compression, and the ratio of compression, re- ctively, p_3v_3, p_4v_4, and r_c, the work of compression is, very rly,

$$U_c = p_3v_3 \log_e r_c$$

$$= p_3v_3 \log_e \frac{p_4}{p_3};$$

erbolic expansion being assumed. The work of expansion he cushion-steam is

$$U_c' = p_4v_4 \log_e \frac{p_4}{p_3}$$

$$= p_3v_3 \log_e \frac{p_2}{p_3}.$$

The Corliss Engine; Henthorn and Thurber; N. Y., E. P. Watson, 1891.

The difference in work lost by incomplete expansion of the cushion-steam is

$$U_c - U_c' = p_3 v_3 \left(\log_e \frac{p_4}{p_3} - \log_e \frac{p_4}{p_2}\right).$$

When, to insure best thermal action or effective cushioning, the compression is made complete and $p_4 = p_1$,

$$U_c - U_c' = p_3 v_3 \left(\log_e \frac{p_1}{p_3} - \log_e \frac{p_1}{p_2}\right);$$

$$= p_3 v_3 (\log_e r_c - \log_e r).$$

With complete expansion, $r_c = r$; with clearance reduced to zero, $v_4 = 0$; in either case $U_c - U_c' = 0$.

The effect of clearance in producing a difference between the real and the apparent ratio of expansion is exhibited by the following diagram and tables which were prepared by Mr. Buel.*

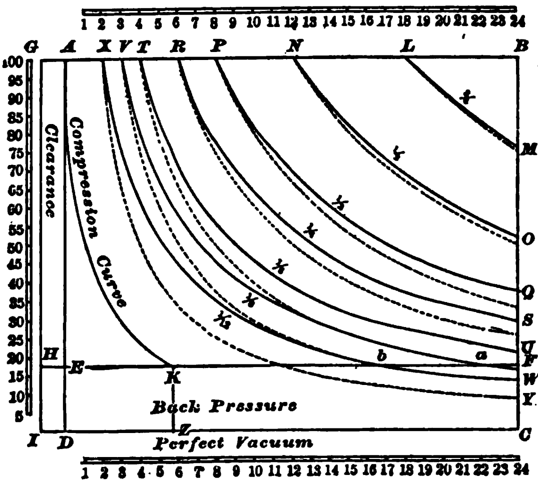


FIG. 160.—EXPANSION-CURVES.

A clearance of 5 per cent is assumed. For the effect of clearance on the cut-off and ratio of expansion, see Appendix.

* Am. Machinist; Apr. 14, 1888, p. 2.

?ig. 160 is a diagram showing the expansion of steam in erbolic curves, at the points oi cut-off noted, the initial sure being 100 pounds per square inch:

1) In a cylinder with 5 per cent clearance (curves in full ;).

2) In a cylinder with no clearance-spaces (curves in broken ;).

[n the following table the numbers in column 4 are mean ;sures, corrected for back-pressure, for stroke plus clearance, the numbers in column 5 are the mean effective pressures olumn 4, corrected. Compression to initial pressure reduces mean effective pressure, but the steam in the clearance-:e is saved. This case is illustrated in the diagram by the re *KA*, the clearance being *AG*.

:ETICAL RESULTS OF USING STEAM EXPANSIVELY—CORRECTED FOR BACK PRESSURE, CLEARANCE AND CUSHION.

	Mean pressure, clearance space included.		No Cushion.						Cushioning to fill clearance spaces with steam of initial pressure.				
	Total.	Effective.	Mean effective pressure.	Relative mean pressure.	Area of Cylinder.	Steam used.	Relative steam used.	Percentage of saving.	Mean effective pressure.	Relative mean pressure.	Area of cylinder.	Steam used.	Percentage of saving.
	3	4	5	6	7	8	9	10	11	12	13	14	15
	100.0	82.5	82.5	1.000	1.00	1.050	1.000		77.9	1.000	1.00	1.000	
2	97.0	79.5	78.5	.952	1.05	.840	.800	20.0	73.9	.949	1.05	.788	21.2
4	86.2	68.7	67.1	.818	1.28	.677	.645	35.5	62.5	.802	1.25	.625	37.5
5	73.3	55.8	53.6	.650	1.54	.590	.562	43.8	49.0	.629	1.59	.530	47.0
6	64.4	46.9	44.2	.536	1.87	.561	.534	46.6	39.6	.508	1.97	.493	50.7
6	53.2	35.7	32.5	.394	2.54	.550	.524	47.6	27.9	.358	2.79	.465	53.5
7	46.5	29.0	25.5	.309	3.24	.567	.540	46.0	20.9	.268	3.73	.466	53.6
7	38.8	21.3	17.4	.211	4.74	.682	.602	39.8	12.8	.164	6.10	.508	49.2

Another method of treatment is the following: The quan-r of steam, q, entering the cushion-spaces is the difference :ween that required to fill them at boiler-pressure, p_1, and ıt compressed into them, reduced to the same pressure; i.e., v_c is the "dead-space,"

$$q = v_c - v_1 .$$

But, assuming hyperbolic expansion,

$$p_1v_1 = p_cv_c = (v_c + x)p_0;$$

$$v_1 = (v_c + x)\frac{p_0}{p_1} = \frac{p_cv_c}{p_1};$$

$$q = v_c\left(1 - \frac{p_c}{p_1}\right) = v_c\left(1 - \frac{p_0}{p_1}\right) - x\frac{p_0}{p_1};$$

which value becomes 0 when the compression is complete, and $v_c = v_1$.

The total steam admitted up to the point of cut-off is

$$V_1 = q + v_1 = v_c\left(1 - \frac{p_c}{p_1}\right) + v_1;$$

$$= v_c\left(1 + \frac{p_0}{p_1}\right) - x\frac{p_c}{p_0} + v_1.$$

The larger the ratio of expansion and the greater the volume v_c, the more serious is the loss due to incomplete expansion of the cushion-steam and, the clearance being given, the useful effect of increasing pressure becomes less and less as the pressure rises.

The greater the back-pressure, the less the ratio needed or desirable, either to effect complete compression or to annul the waste by cooling. Non-condensing engines are given insignificant ratios of compression as compared with those required for complete compression in condensing engines. Other things equal, the higher the initial pressure, the less should be the clearance. Large port and clearance spaces increase the cost of the engine, as they decrease the net useful work of the machine, both by actual reduction of the indicated work and by increasing the waste-work due to friction.

EXPANSION OF STEAM.

Initial pressure, p_1.	Mean total pressure, p_m.	Quantity of steam.	Per cent saving.
100	100	1000	—
"	96.4	780	22.0
"	84.7	590	41.0
"	70.0	477	52.3
"	59.7	420	58.0
"	46.5	358	64.2
"	38.5	325	67.5
"	29.0	288	71.2

:, allowing $17\frac{1}{2}$ lbs. back-pressure:

	p_e		
100	82.5	1000	—
"	78.9	780	22
"	67.2	615	38.5
"	52.5	523	47.7
"	42.2	488	51.2
"	29.0	473	52.7
"	21.0	490	51.0
"	11.5	596	40.0

The Binary-vapor System is a method of what may ed "compounding" engines with transfer of heat and transfer of working fluid from the high- to the low- element of the series. The general principles are the main, the same as in the usual form of multiple- engine; but with important differences of result due ical differences of physical conditions of environment peration.

le the principle of Carnot, asserting that, thermodynami- l working substances have the same value of efficiency when working through the same range of temperature batic expansion, in the ideal engine, it happens to be : that it is often practically impossible to obtain the onditions of maximum efficiency in all cases. Some re more liable to loss of heat in actual working,

through internal and external conduction and radiation, than others; and the pressures of the various possible working substances at any temperatures vary enormously; vapors of ether and chloroform, for example, having much higher pressures than steam.

A defect in the action of steam, as commonly used, is that, at high temperature, it has, if saturated, dangerously and even uncontrollably high pressures; while, at low temperatures, its pressure falls below that of the atmosphere and compels the use of an expensive and cumbersome system of condensation if we seek to transform low-temperature heat into work. The binary-vapor system is one in which this latter difficulty is sought to be remedied by using a volatile fluid as the condensing medium, so that the latter may be vaporized at a good working pressure by the condensation of the former and may then, in turn, be used in a supplementary engine, transforming a new and sometimes large quantity of thermal into dynamic energy. Thus a kind of "compounding" results in the substitution of a second engine, "in series" with the first, for a condensing apparatus. This added machine must necessarily also be made a surface-condensing engine in order that its always costly and sometimes dangerous working fluid may be saved and used over and over again. By the use of such a system, the gain due to decreased cylinder-condensation and increased range of expansion, combined, may prove to be considerable, when compared with the economy of the ordinary steam-engine.

The following, adopting Rankine's methods, is the theory of this case: *

Let p_1 be the absolute pressure of the steam at its admission;

v_1, the volume of 1 lb. of it when admitted;

rv_1, the volume to which it expands.

Let H_1 denote the available heat expended, in foot-lbs. per lb. of steam;

U, the energy exerted on the piston by 1 lb. of steam.

* Rankine; p. 145.

the *heat rejected* by each lb. of steam, and given up her, is

$$H_2 = H_1 - U. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

nd what *volume* will be filled with ether-vapor, the ex- e of heat *per cubic foot of ether-vapor*, at the pressure nich it is evaporated, p_1'—temperature lower than the :ure at which the steam is condensed:

$$L' + Jc'D'(T' - T'''), \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$\frac{dp'}{dT'}$ is the latent heat of evaporation of one cubic foot of ether-vapor under the given pressure;

9.1 foot-lbs. per degree Fahrenheit, is the specific heat of liquid ether;

: weight of one cubic foot of ether-vapor;

e temperature at which the ether is evaporated, and t at which it is condensed.

iitial volume of the ether evaporated, per lb. of steam ed, is

$$u' = \frac{H_2}{L' + Jc'D'(T' - T''')}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

p'' denote the intended final pressure of the ether- ind p''' its mean back-pressure; about 5 lbs. on the nch. Then by means of the formulæ for steam, already ubstituting the constants which apply to ether, we may

ratio of expansion, r', and the final volume, $r'u'$, of the 'aporated per lb. of steam; the energy exerted by that T', and the ratio

$$\frac{r'u'}{rv_1}$$

of the volume of the ether-cylinder to that of the steam- . In practice, those cylinders are usually of equal the ether-cylinder somewhat larger.

The heat per lb. of steam, abstracted by the cold water in the ether-condenser, is

$$H_1 - U - U'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

The mean effective pressures in the steam and ether cylinders are

$$\frac{U}{rv_1} \text{ and } \frac{U'}{ru'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

But the amount of energy obtained by the addition of the ether-engine to the steam-engine might be obtained by continuing the expansion of the steam.

The following are *means*, computed from results given in the report of M. Gouin, on the performance of the steam and ether engines of the Brésil, designed by Du Trembly about 1844:

	Pressures in Pounds on the Square Inch.		
	In Boiler or Evaporator.	Back-pressure.	Mean Effective.
Steam............	43.2	7.6	11.6
Ether.............	31.2	5.3	7.1

Total M. E. P. reduced to the area of *one piston*, the areas and strokes of the pistons having been the same........................ 18.7

It appears that the proportions of the power obtained in the cylinders, respectively, were:

Steam.......................... $\frac{11.6}{18.7} = .62$

Ether.......................... $\frac{7.1}{18.7} = .38$

The gain of power by the addition of the ether-engine is not so great as this shows; because, had the steam-cylinder been used alone, the back-pressure would have been in all probability about 4.6 instead of 7.6; so that the mean effective pressure in the steam-cylinder would have been 14.6 instead of

and the proportion of the power of the steam-engine to)f the binary engine would have been

$$\frac{14.6}{18.7} = .77,$$

ıg

$$1.00 - .77 = .23$$

: power of the binary engine, as the gain due to the ether- e.

ıe consumption of fuel was either 2.8 or 2.44 lbs. of coal ıdicated horse-power per hour, according as certain ex- ents made under peculiarly adverse circumstances were led or excluded. Rankine adds:

Γhe binary engine is not more economical than steam- es designed with due regard to economy of fuel; but by ddition of an ether-engine, a wasteful steam-engine may nverted into an economical binary engine"—a conclu- which is sufficiently obvious from the fact that such s are considered rather high for the ordinary compound -engine.

binary-vapor engine, tested by Mr. Haswell, in which the ary fluid was carbon disulphide, gave the following re- n a trial in which the operation of the engine was con- l five hours, which, as that period involved the cleaning fire, was held to afford time for a test.*

ıe reported data are as below:

ressure, steam—boiler............	75.8 pounds
" " shell.............	15.3 "
" vapor—engine...........	76 "
" " mean, by indicator..	31.35 "
Vater evaporated.................	5.71 cubic feet
:evolutions per minute............	100
'acuum........................	9.85 pounds
oal consumed.....................	600 "
Iorse-power indicated.............	86.64

* rans. Am. Soc. C. E.; 1887; also Steam-engine and Boiler Trials; p. 454.

From which it appears that steam at a pressure of 75.8 pounds per square inch passed through the automatic regulating valve to the shell surrounding the generator at the reduced pressure of 15.3 pounds, due to a temperature of 250.4 degrees, produced a vapor in the generator of 76 pounds.

The consumption of coal was thus reported as 1.385 pounds per indicated horse-power per hour.

These results confirm the indications of thermodynamic science, that substantially as good work may be done with other vapors as with steam; but the steam-engine has actually given as good economical results as those here reported, and has many practical points of superiority. This trial was, however, too short to be taken as fully satisfactory. A trial of a sulphurous acid combination by Professor Josse (1899) at Berlin showed a reduction from 8.6 kg. to 5.5 kg. steam per I. H. P., as shown on page 704.

The data and results obtained by Mr. Barrus, by three tests of a Campbell *ammonia-engine* and boiler, as reported to the Campbell Engine Co., April 1887, were as follow:

DIMENSIONS OF BOILER AND ENGINE.

Boiler—One horizontal-return tubular, set in brick-work.

Diameter of shell	42 in.
Length of shell	10 ft.
Inside diameter of tubes	1.75 in.
Area of water-heating surface	369.3 sq. ft.
Area of steam-heating surface	308.3 "
Area of grate-surface	9.17 "
Collective area for draught through 67 tubes	1.12 "
Ratio of water-heating surface to grate-surface	40.3 to 1
Ratio of steam-heating surface to grate-surface	33.6 to 1
Height of smoke-stack above grate	30 ft.

Engine—Porter-Allen automatic cut-off, single cylinder.

Diameter of cylinder	11.5 in.
Stroke of piston	20 "

DATA AND RESULTS OF TESTS.

e	1887,	March 8,	March 9,	April 16.
ation of test	hrs.	8	10	7.45
:entage of ashes, etc.	per cent		9.9	8.2
l per hour per sq. ft. of ·ate	lbs.	19.09	15.27	16.07
er-pressure above at-osphere		100	95.5	86.6
ıp. of feed-liquid en-:ring boiler	deg. F.		167.6	167
ıp. of gases entering ack	"		390	394
uum in feed-well	inches		11.5	11
'olutions of engine per ıinute	revolu.	205.2	204.5	201.5
icated horse-power de-eloped by engine	H. P.	61.80	57.53	54
portion of stroke com-leted at cut-off			.189	.211
portion of stroke com-leted at release			.773	.791
portion of return stroke ncompleted at compres-on			.307	.342
l consumed per indi-ated horse-power per our	lbs.	2.832	2.433	2.729

Professor Josse, of Berlin, employs SO_2 as the secondary
l and adopts a modified triple-expansion engine.

The outcome of these experiments is exhibited in the
ompanying table, and it is seen that this binary-vapor
ine, or waste-heat engine, has broken the world's record
thermal efficiency. It has, in one trial, produced the
se-power-hour on 8.36 pounds of steam, equivalent; under

the circumstances of the test, to about 10,725 B.T.U.* The steam-engine rejects about one-half of all the heat sent to it from the boiler and this heat is nearly as well utilized in the secondary engine as is the steam in the primary. Thermodynamically the engine is a success.

The next question to be determined before it can be confidently affirmed that the plan is practicable and likely to become permanently employed is that of commercial efficiency. Is it certain that power thus produced will, in the long run, cost less than if produced by the usual system and the single-fluid engine?

This is a question which can only be satisfactorily answered after experience shall have shown what are the practical objections to the use of the secondary fluid. Should it prove dangerous, liable to cause interruptions of power-supply, costly in maintenance or even seriously inconvenient in operation, it may prove that the investment of capital in this type of engine is less profitable, on the whole, than if put into a steam-engine, and the final question is: Will the cost of power and the influence of the action of the engine on the dividends payable by the enterprise with which it is associated be more satisfactory, the life of the machine and its replacement being considered, than with the equivalent steam-engine? It is this question which remains to be settled; but it is reported that so far as experience has yet been had with the "waste-heat engine," it has proved in these respects satisfactory. It costs little if any more, if, for example, built as a three-cylinder engine, in the one form than in the other. It has given no trouble in operation and exhibits no evidence of more rapid deterioration than the steam-engine. It can be overhauled and repaired, wherever the secondary fluid is not encountered, with as little trouble as the steam-engine, and it is found that the vaporizer and its contents seem little likely to

* It is to be remembered that, in this case, the first figure is not a true measure of efficiency.

TESTS COMBINED STEAM WASTE-HEAT-ENGINE (AVERAGE VALUE).—JOSSE.*

Arrangement during the test.	Steam-engine working triple expansion. Superheated Steam.	Steam-engine working triple expansion. Superheated Steam.	Steam-engine working triple expansion. Superheated Steam. With High Vacuum.	Steam-engine working triple expansion. Superheated Steam.	Steam-engine working triple expansion. Dry Saturated.	Steam-engine working comp'd. Superheated Steam.	Steam-engine working comp'd. Superheated Steam. With Small Quantity of Circulating Water.	Steam-engine working comp'd. Superheated Steam. At Half Load.
Speed r. p. m	136.3	143.5	137.4	148	149	148	137	146
Volts at dynamo	189.5	210	191	231	230	235.8	210	230
Amperes from dynamo	529	584	414.5	610	609.5	531	470	272
Steam-engine. Temperatures — Boiler steam—admission	309°	310°	306°	310°	189.5°	326°	332°	330
Steam-engine. Steam pressures — Inlet pressure—High-pressure cylinder—lbs. per square inch above atmosphere	156.5	158	156.5	156.5	156.5	163.7	156.7	156.5
Steam-engine. Steam pressures — Per cent vacuum in condenser	80.5	70	85	68.5	68.2	69.1	70.2	75.2
Steam-engine. Indicated output — horse-power — High-pressure cylinder	55.2	68.6	40.9	76.5	59.2	86.7	73.9	50.8
Steam-engine. Indicated output — horse-power — Intermediate cylinder	35.7	43.2	31.3	42.5	47.3			
Steam-engine. Indicated output — horse-power — Low-pressure cylinder	34.3	42.4	29.4	42.9	49.8	53.8	43.7	31.6
Steam-engine. Indicated output — horse-power — Complete engine	125.2	154.2	101.6	161.0	156.3	140.5	117.6	82.4
Steam-engine. Steam consumption in lbs. per hour — Condensation from the vaporizer	1405	1881	1465	2133	2562	1885	1612	1197
Steam-engine. Steam consumption in lbs. per hour — " " steam-jackets								
Steam-engine. Steam consumption in lbs. per hour — Total condensation	1405	1881	1465	2133	2562	1885	1612	1197
Steam-engine. Steam consumption in lbs. per hour — Per H.-P. indicated	11.2	12.2	14.4	13.2	16.4	13.4	13.7	14.5
Waste-heat engine. Temperatures — SO_2 vapor. Admission to cylinder	56.5°	66.5°	50.4°	68.4°	67.5°	68.6°	67.9°	64.4°
Waste-heat engine. Temperatures — SO_2 liquid. Outlet from condenser	18.8°	19.8°	18.0°	20.0°	21.1°	19.4°	24.0°	18.4°
Waste-heat engine. Temperatures — Circulating water—inlet	9.9°	9.9°	10.1°	10.1°	10.1°	10.1°	10.0°	10.0°
Waste-heat engine. Temperatures — " " discharge	15.7°	16.9	15.7°	17.46°	18.44	17.4°	24.8°	17.2°
Waste-heat engine. SO_2 pressures (above atmospheric) in lbs. per sq. in. — in vaporizer	128	172	110.9	186.3	180.6	183.4	180.6	164.9
Waste-heat engine. SO_2 pressures (above atmospheric) in lbs. per sq. in. — in condenser	33.5	34.8	31.2	36.2	38	34.8	46.2	33.1
Waste-heat engine. Output—H.P — Indicated	42.8	56.8	31.0	61.3	66.0	55.6	40.8	30.6
Waste-heat engine. Output—H.P — In per cent of steam-engine	34.2	37.0	30.5	37.9	42.1	39.5	34.6	37.2
Waste-heat engine. Consumption of waste steam per I.H.P. per hour, lbs. — Condensation from the steam-jacket and receiver	32.8	33.2	47.3	34.8	38.9	33.9	39.5	39.1
Combined engine. Output—H.P. — Indicated	168	211	132.6	223.2	222.3	196.1	158.4	113
Combined engine. Output—H.P. — Effective	143.8	177.2	114.2	202.9	101	180.2	141.9	89.7
Combined engine. Output—H.P. — Electrical	134	164.5	106.3	189.1	187.2	167.5	132	83.7
Combined engine. Total steam consumption per hour, lbs	1405	1881	1465	2133	2562	1885	1612	1197
Combined engine. Steam consumption per I.H.P. per hour, lbs	8.36	8.6	11.05	9.55	11.5	9.6	10.22	10.6
Combined engine. Quantity of circulating water in gallons — Total per hour	11660	13090	... .	12468	13310	12056	5236	8140
Combined engine. Quantity of circulating water in gallons — Per I.H.P. per hour	69.5	61.6	.....	55.8	59.9	61.5	33.0	71.9
Combined engine. Mechanical efficiency — Electrical output ÷ I.H.P.	80.1	78	80.4	84.5	84.2	85.5	83.5	74
Combined engine. Mechanical efficiency — B.H.P.† ÷ I.H.P.	86.2	83.8	87.5	90.8	90.5	92	89.8	79.5

* From the "Mittheilungen aus dem Maschinen-Laboratorium der Kgl. Tech. Hochschule zu Berlin." † Dynamo efficiency assumed 93 per cent.

cause difficulty or expense. There is no sediment or incrustation, and there need be no objectionable leakage, as asserted by the investigator of the operation and general behavior of this machine. Such doubt as may remain respecting this aspect of the subject can only be removed by time.

CHAPTER VII.

THE MAXIMUM EFFICIENCIES OF THE STEAM-ENGINE.

173. **The Mathematical Theory of Efficiencies** has been comparatively little studied. The thermodynamic theory, and the efficiency of the ideal engine free from all other than thermodynamic wastes, has been fully developed by Clausius and Rankine and their successors; but neglect of experimental and mathematical investigation of the physics of the case, and consequent ignoring of the practically important conditions distinguishing the real from the ideal case, has often led to serious misconceptions, and to enormous losses of money, in the attempt to realize in practice the advantages indicated as attainable by the pure thermodynamic treatment. In the establishment of a correct and practically applicable theory of efficiencies, it is not only essential that the physical, as well as the purely thermodynamic, conditions of working should be taken into the account; but, also, that the several efficiencies should be very carefully distinguished, and that the finance of practical operation should be no less carefully studied. The latter division of the subject, in fact, includes, and depends upon, all the preceding, and, to the user of the engine, presents the controlling considerations and the essential problem.

174. **The Several Efficiencies of the Steam-engine.***— In the design of the steam-engine the engineer has frequent occasion to solve certain problems relating to its economical performance, to determine what proportions of engine and boiler are best adapted to give maximum economy of fuel or of money under certain conditions precisely defined in advance.

* Trans. Am. Soc. M. E.; 1882.

Such problems may usually be solved by the determination of the ratio of expansion producing maximum economy under the given conditions.

Several problems of this character may be classed together, all of which relate to one or another of the "Several Efficiencies of the Steam-engine," as the Author has called them.

These are:

(1) *Thermodynamic Efficiency of Fluid.*—This is measured by the ratio of work done by the working substance to the mechanical equivalent of the heat expended on it to do that work. In the perfect engine-cycle this efficiency is measured by the quantity $\frac{\tau_1 - \tau_2}{\tau_1}$; the range of temperature worked through, divided by the maximum, initial, absolute temperature of the fluid entering the cylinder of the engine.

To obtain a measure of the thermodynamic efficiency of the working substance, as has already been seen, it is only necessary to measure the work done, as by the measurement of the indicator-diagram, and compare its amount with the mechanical equivalent of the heat expended in its performance. In the case of the steam-engine, this requires the determination of the volume of steam and its weight, at the point of cut-off, the determination, by computation or from the tables, of the quantity of heat required in its production from the feed-water, and, finally, the division of the work shown in the diagram by this quantity. This is substantially the method adopted by Rankine, in the first construction of the thermodynamic theory of the heat-engines.

In real engines great losses occur by incomplete expansion and by direct transfer of heat from induction to exhaust without production of work.

(2) *Actual Efficiency of Working Substance.*—This is here considered to be that observed in the actual operation of the engine as the ratio of heat conveyed into the engine by the working fluid, and acting purely thermodynamically, to the total heat entering the system.

Various working fluids have different values in this respect.

Thus a gas has little conducting or radiating power, can surrender but little heat and can absorb but little, in its contact with the parts of the machine in which it is employed, while a saturated vapor like steam may take it up with comparative freedom when in contact with hotter substances, and can reject it with enormous rapidity if brought in juxtaposition with a cold body. The latter is a less efficient vehicle of heat for thermodynamic purposes than the former, and, in this respect, a much less satisfactory working substance. The "actual efficiency of the working substance" is lower with saturated than with superheated steam, and with steam than with gas. It varies with every known working substance.

(3) *Efficiency of the Machine.*—This is measured by the ratio of the quantity of work yielded to the "machinery of transmission" to that done upon the piston by the working fluid.

This is the ratio of the "dynamometric power" to the indicated power, and is less as the waste in engine-friction is greater.

(4) *Efficiency of the Engine.*—In some cases the product of the total efficiency of the fluid by the efficiency of the machine is called the Efficiency of the Engine or Efficiency of the System. It measures the ratio of the work performed by the engine, externally, to the work-equivalent of the heat supplied it.

(5) *The Efficiency of the Furnace* is the ratio of quantity of heat transferred to the working substance to that developed by combustion of the fuel.

(6) *The Efficiency of Combustion*, or ratio of heat produced by combustion to that latent in the fuel.

(7) *The Total Efficiency of the Apparatus, or of Plant*, as the Author would term it, is the product of these several partial efficiencies, and is the fraction of the total calorific power of the fuel which is delivered to the machinery of transmission as mechanical energy. It is a maximum when each of its factors is a maximum.

(8) *The Efficiency of Capital*, or the Commercial Efficiency of Steam Machinery, is measured by the amount of capital re-

quired, or the total running expenses, per unit of time, for a given power required and obtained; i.e., it determines how small a sum will provide a given amount of power, and *what size of engine must be selected for the given work*, a problem first enunciated by Rankine.*

Each of the above efficiencies is made a maximum by a set of conditions the determination of which constitutes an important problem in the science of engineering. Each must be solved, and in a certain definite order, in the application of steam-power to any given case. The determination of the efficiency of fluid is included in the problem relating to efficiency of engine, and this and all other efficiencies are included in the last,—the efficiency of capital,—which cannot be exactly determined unless they are first ascertained.

(9) In addition to the above, another problem may present itself to the user of power, although seldom to the designer, or to any one proposing to purchase a steam-engine; viz., the determination of the maximum economy of a given plant; i.e., how the most work may be obtained for the unit of cost from a given engine already constructed. This is entirely a different problem from the preceding; its solution leads to very different results, and does not usually, if ever, determine maximum commercial efficiency. This problem relates to what may be called the "*Maximum Commercial Efficiency of a Given Plant.*"

(10) It may, finally, be necessary to determine still another question: "*What is the Maximum Amount of power that can be profitably obtained from a Given Plant?*" This is a more commonly familiar problem than the last, and in most cases of more direct and practical importance.

The solution of all these problems in the case of the real engine and for the purposes of the designing engineer, of the builder, or of the proprietor, is complicated by the presence among the data to be introduced of the varying thermal internal wastes. As has already been stated, however, and as will

* Trans. Royal Society of Edinburgh; 1851; vol. XXI. Rankine's Miscellaneous Papers; No. XVI. p. 295. Shipbuilding, Appendix; p. 292.

be again shown later, the engineer is always able to say, in advance, how these variations of wastes will affect the problem, and can say in advance, with some degree of approximation, what will be the probable size of the engine, and the slight uncertainty arising from a first approximation based on data obtained in this manner becomes insensible with a second approximation obtained by repeating the process of computation or graphical construction, as presently described and illustrated.

175. **Maximum Thermodynamic Efficiency**, or the efficiency of the working fluid operating under purely thermodynamic conditions, is, as has been seen, entirely independent of the nature of the fluid selected, and is dependent simply on the limits of temperature adopted and the character of the cycle employed. With the cycle of maximum efficiency, as the Carnot cycle, the measure is invariably $\frac{T_1 - T_2}{T_1}$; with other methods of operation this efficiency is measured by the ratio of work done by the fluid, and of heat thermodynamically transformed in its performance, to the quantity of heat supplied from the source during the same period; this period being that of a cycle or of some stated number of complete cycles. The processes by which this ratio is calculated have been already given and examples presented, illustrating their use and practical application.

176. **Estimates of Heat, Steam, and Fuel** are easily made. Were it possible to utilize all heat stored in the steam supplied to the engine, under the usual conditions of practice, there would be demanded but about 2¼ pounds (about one kilogram) of feed-water, or of dry steam, per horse-power per hour. The horse-power is the equivalent of 1,980,000 foot-pounds (or in metric H. P. 270,000 kilogram-metres per hour, equal to 2565.5 B. T. U. per hour or 43 units, nearly, per minute (metric: 637 calories per hour, or 10.6 per minute). Assuming the total available heat to be 1150 B. T. U. per pound as a maximum, the steam of ordinary pressure demanded in a perfect engine of efficiency unity would thus be between 2.2

and 2.25 pounds (one kilog., nearly) per horse-power per hour. Dividing the quantity 2.2 pounds (1 kilog.) by the thermodynamic efficiency of fluid will give the weight of steam demanded at that efficiency, and, assuming a maximum practically attainable evaporation of 9 or 10 to 1, the weight of coal required is obtained by dividing this weight of steam by 9 or 10 for condensing or for non-condensing engines, respectively.

Thus: a steam-engine receiving steam at a pressure 100 pounds above vacuum, and condensing it at a temperature corresponding to 4 pounds, the ratio of expansion being 5, has a thermodynamic efficiency of 0.15, nearly; it would demand about 15 pounds of feed-water per horse-power per hour, and about 1.7 pounds of coal.

A non-condensing engine similarly operated would have an efficiency of fluid 0.10, nearly, would use about 22 pounds of steam and 2.2 pounds of fuel, the engine being, as before, thermodynamically perfect. If, in the latter case, the steam-pressure were ten atmospheres, this efficiency would become 0.125, nearly, and the steam and fuel consumption 18 pounds and 1.8 pounds, respectively.

With larger ratios of expansion the efficiencies would be increased and the expenditure of steam and fuel correspondingly reduced.

The Gain by Expansion, in an engine free from the wastes which characterize the steam-engine as actually used or for an ideally perfect case, is seen in the table on p. 711, which assumes hyperbolic expansion.

Thus it is found that the gross or "absolute" work done by a pound of steam, or, as assumed in the table, by that giving 100 units of power, at full stroke, increases enormously with its use expansively, doubling at one-third stroke; and becoming three times the initial amount at $r = 8$, and four times at $r = 20$. But, as will be seen when studying the losses of the actual engine, these gains are rarely even approximately realized. The extent and the nature and effect of the losses in real engines have been already fully indicated.

GAIN BY EXPANSION.

Point of Cut-off.	Number of Expansions. r	Work and Power.
Full stroke	1	100
$\frac{1}{2}$ "	2	169.3
$\frac{1}{3}$ "	3	209.8
$\frac{1}{4}$ "	4	238.6
$\frac{1}{5}$ "	5	260.9
$\frac{1}{6}$ "	6	279.1
$\frac{1}{8}$ "	8	307.9
$\frac{1}{9}$ "	9	319.7
$\frac{1}{10}$ "	10	330.2
$\frac{1}{12}$ "	12	348.4
$\frac{1}{14}$ "	14	363.9
$\frac{1}{16}$ "	16	377.2
$\frac{1}{18}$ "	18	389.0
$\frac{1}{20}$ "	20	399.5

The values in the last column of the table are evidently proportional to the quantity,

$$p_m = \frac{p_1(1 + \log_e r)}{r}.$$

The true, net, work of the engine would be proportional to

$$p_m = \frac{p_1(1 + \log_e r)}{r} - p_b;$$

where p_b is the mean back-pressure.

Were it possible to expand steam in a non-conducting cylinder, the adiabatic curve would differ slightly from the hyperbola, and the relative work of the steam would correspondingly differ, giving figures as follow, for ideal non-condensing engines ($p_1 = 120$; $p_2 = 1.5$):

WORK OF ADIABATIC EXPANSION.

Point of Cut-off	1	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
Value of U per pound	1.000	1.285	1.459	1.667	1.905	2.278	2.854
Steam per H.P. per hour	31.3	23.8	21.4	18.8	16.4	15.4	11.0
Mean pressure	120	115	107	96	91	61	22

177. The Actual Efficiency of Working Substances has been seen to be very greatly less than the thermodynamic efficiency, in any real engines; the difference being mainly due to wastes of heat by internal storage, conduction, and radiation. As shown by experimental investigations, such as have been already described, the magnitudes of these wastes vary with the area of the confining walls of the working cylinder, and the differences of temperatures produced, and probably nearly as the square roots of the times of exposure of the working fluid to refrigerating influences.

It is becoming practicable to determine, approximately, the amount of waste to be anticipated when the size of engine and the conditions of its operation are known. This quantity being added to that demanded by the thermodynamic action of the engine, the total weight of steam required is obtained, and the quotient of the work done, or its heat-equivalent, by the work-equivalent, or the total heat supplied, as just indicated, is the measure of the actual efficiency of working substance in the real, as distinguished from the ideal, engine.

Thus: in the cases considered in the preceding section, the ideal condensing engine has a thermodynamic efficiency of about 0.15, and requires about 14.7 pounds of steam, or 1.47 of coal, per hour and per horse-power; but its exhaust-wastes, due to internal conduction and loss, may amount to one third of all steam entering the engine, fifty per cent of the thermodynamic requirement, or to nearly ten pounds of steam and one pound of coal, making the total 25 pounds of steam and 2.5 of fuel, nearly; which are very common figures for good engines of moderate size. Similarly, the non-condensing engine, requiring, thermodynamically, 18 pounds of steam, or 1.8 pounds of fuel, if subject to similar losses, would actually demand 32 and 3.2 pounds. The actual efficiency thus becomes, for the condensing engine 0.10, and for the non-condensing engine 0.065, instead of 0.15 and 0.10, as for the ideal case.

Relative Actual Efficiency is the efficiency actually attained, as compared with the computed ideal efficiency. It is here

$\frac{10}{15} = 0.667$ for the one, and $\frac{.065}{.100} = 0.65$, for the second of these two examples.

178. Estimating Consumption of heat, of steam, and of fuel, for the actual case, becomes a very simple matter, approximations such as may be based upon the researches already described being accepted. The engineer may desire either to estimate the probable total absolute weight of steam condensed in the cylinder; or he may, for purposes to be presently detailed at some length, find it desirable to estimate this waste as percentage or as a function of the ratio of expansion, simply, where all other conditions are constant, and the expansion-ratio is the only variable; thus making two cases.

The weight of steam condensed may be estimated as a function of range of temperature, or pressure, of area of internal surfaces, and of time of exposure, or speed of engine. It may also be reckoned as a fraction of the thermodynamic consumption of steam, and in terms of the ratio of expansion.

The Relative Actual Efficiency of the working fluid is thus from 0.90 to 0.75 for these cases.

The quantity of heat, of steam, or of fuel, being estimated thermodynamically, as already indicated in the preceding section and the last chapter, the quotient of the quantities so obtained by a known relative actual efficiency of a working substance gives the amount of heat, of steam, or of fuel, to be actually consumed.

Thus, if the efficiency, calculated from the thermodynamic conditions, be 0.15; the heat demanded being $\frac{42.75}{0.15} = 285$ British thermal units per horse-power per minute or $\frac{2545}{0.15} = 16,100$ per hour; the steam called for amounting to $\frac{2.2}{0.15} = 14.7$ pounds per hour; and the fuel amounting to $\frac{0.22}{0.15} = 1.5$ pounds,—the product of these quantities by the reciprocal of the

relative actual efficiency, $\frac{1}{0.90} = 1.111$, gives for the real demand per indicated horse-power 18,817 thermal units; 15.1 pounds of steam; and 1.67 pounds of coal—figures often attained by modern engines.

The net efficiency of the fluid is thus found to be, for this case, the "*indicated power*" being considered,

$$E = 0.15 \times 0.90 = 0.135.$$

It should be remembered that this efficiency of the fluid employed as the medium of energy-transformation is determined both by the physical properties of the substance and by the conditions of its employment in the engine.

179. **The Efficiency of the Engine, as a Machine,** and below unity, as has been seen, is less as the friction of its moving parts is greater. It has been further seen that this friction may probably be usually taken as sensibly constant for all loads, and, for any given, or for the rated, load, as a determinable fraction of the resistance or power. In its absolute amount, it may be taken as equal to the product of a nearly constant *friction-pressure*, as it may be termed, into the area and speed of piston; and the work of friction is the product of that intensity of pressure, p_f, into the volume ASN traversed by the piston in the given time. This pressure being taken as p_f, we have

$$U_f = p_f AS$$

as the work of friction per stroke of piston, and the efficiency of the machine as

$$E_m = \frac{p_m - p_f}{p_m} = 1 - \frac{p_f}{p_m}.$$

This efficiency usually varies from $E_m = 0.80$, in small engines, to above $E_m = 0.90$, in large engines of the best construction. The smaller values are the more common.

The total efficiency of the engine is the continued product

of the thermodynamic efficiency, the relative actual efficiency, and the efficiency of the machine. For the case last considered, this becomes

$$E_e = E_t \times E_r \times E_f = 0.15 \times 0.90 \times 0.95 = 0.129.$$

For a more common case, in which these values are much less,

$$E_e = 0.08 \times 0.75 \times 0.90 = 0.054.$$

and only about one eighteenth the energy supplied by the steam-boiler is here converted into useful work, such as is measured by the absorbing dynamometer and known as the "dynamometric power," the D. H. P., as often symbolized when given in horse-power.

The Actual Demand of the engine, as measured in heat, steam, and fuel, is thus known to be often much greater than the quantity computed for the ideal engine, and is, as already seen, readily estimated by multiplying the values for the ideal case by the reciprocal of total, final, efficiency. Thus, for the last example, we have

Heat per horse-power per hour $\dfrac{2566}{0.054} = 49,370;$

Steam " " " " $\dfrac{2.2}{0.054} = 40.4;$

Coal " " " " $\dfrac{0.22}{0.054} = 4.04.$

And, for the case next preceding,

Heat, $\dfrac{2545}{0.129} = 18,170$ B. T. U.;

Steam, $\dfrac{2.2}{0.129} = 17.054$ lbs.;

Fuel, $\dfrac{0.22}{0.129} = 1.71$ lbs.

180. Thermal Lines and "Curves of Efficiency," as the Author has called the latter, may be now studied for the case of the actual engine.*

It has been shown that friction and—often to a vastly greater extent—cylinder-condensation, due to expansion of a heated fluid in a working cylinder made of a material of high conducting power, modify the methods of expansion and of expenditure of heat so greatly that the ratio of expansion for maximum efficiency, in unjacketed engines, is small, although its value would otherwise be, often, several times greater than it actually is. It was also shown that these modifying conditions very differently affect different kinds of steam-engine and different engines and also individual engines, at various pressures and piston-speeds. It has become evident that in no case, in steam-engines as to-day constructed, can the expansion-line or the curve of mean pressures for varying ratios of expansion be such as would be obtained in a non-conducting cylinder. Steam must always be more or less condensed at the beginning, and must always carry away heat by its re-evaporization at the end of the stroke. The steam-jacket checks the first operation, but accelerates the last, and, with wet steam, may possibly even increase the evil that it is designed to prevent.

The actual expansion-line is not only modified in position and in form by the conductivity of the cylinder, but, also, although perhaps less seriously, by the quantity of water contained in the mass of fluid at the instant of closing the expansion-valve.

The expansion-curve may be often closely represented by a regular curve of the hyperbolic class, $p_1v_1^n = pv^n$, the exponent n varying with the proportions of steam and water in the mixture at the commencement of the expansion, which is assumed to take place in a non-conducting cylinder. Table

* On the Ratio of Expansion at Maximum Efficiency in Steam-engines; Trans. Am. Soc. Mech. Engrs., 1881; Jour. Franklin Institute, May 1881. On the Behavior of Steam in the Steam-engine, and on Curves of Efficiency; Jour. Franklin Institute, Feb. 1882.

III, appended, gives the values of the ratio of mean pressure to initial pressure, $\frac{p_m}{p_1}$, for various mixtures from steam 1.00, water 0, to steam 0.50, water 0.50, assuming the formula to be practically accurate within that range. With these are given the adiabatics for superheated steam, $n = 1.333$. Table III also gives the values of $\frac{p_m}{p_1}$ for steam-expansion in a jacketed metal cylinder, in which it is kept just dry and saturated by heat from the jacketed sides and ends; the values for wet air compressed in air-compressors, in which n is frequently found to be 1.2; and for peculiar cases in actual steam-engines in which leakage or re-evaporation, or both, raise the terminal pressures greatly, giving $n = 0.50$, $n = 0.75$. Table IV, similarly, gives the ratios $\frac{p_1}{p_2}$.

It is, as yet, impossible to predict which of these curves will be found, in any case, and the engineer is compelled to rely entirely upon the "indicator" for information of this character. The greatest possible variety of curves are found to occur in such cases,* but they approach the adiabatic more nearly, as the steam is drier and as the speed of piston is increased, rarely departing far from the common hyperbola in good engines. Perfectly dry or superheated steam, in fast-running engines, gives a curve most closely approaching the adiabatic; but the deviation is more marked as the speed of engine is decreased, and as the amount of moisture in the steam, initially, increases. The limit may be taken as $pv = p_1v_1$, on the one side, and to $pv^{\frac{1}{2}} = p_1v_1^{\frac{1}{2}}$ on the other; the latter being the rare case sometimes met with of an unjacketed engine working at a piston-speed below 50 feet per minute (under 15 metres), and with a

* An indicator-diagram lying before the Author gives $n = 1.001$ at the beginning of the stroke, $n = 0.94$ at the middle, and $n = 0.89$ at the end. The compression-line starts with $n = 1.52$ and varies thus, $n = 1.29$, $n = 90.6$ to the end, where $n = 0.77$, showing that the mean temperature of the surfaces in contact with steam is above that of the vapor during the first half of the period of compression, and below that of the fluid during the second half.

high ratio of expansion; while the former is a very usual limiting value with well-constructed jacketed engines at good speed Where the steam contains muc water, the expansion-line i actual engines often, especially if leakage of the steam-valve occurs, lies entirely above the curve of Mariotte, the value of n being less than unity. In other cases, the line may fall under the hyperbola at the beginning, but rise far above it toward the end, of the expansion, giving a curve more nearly parabolic in

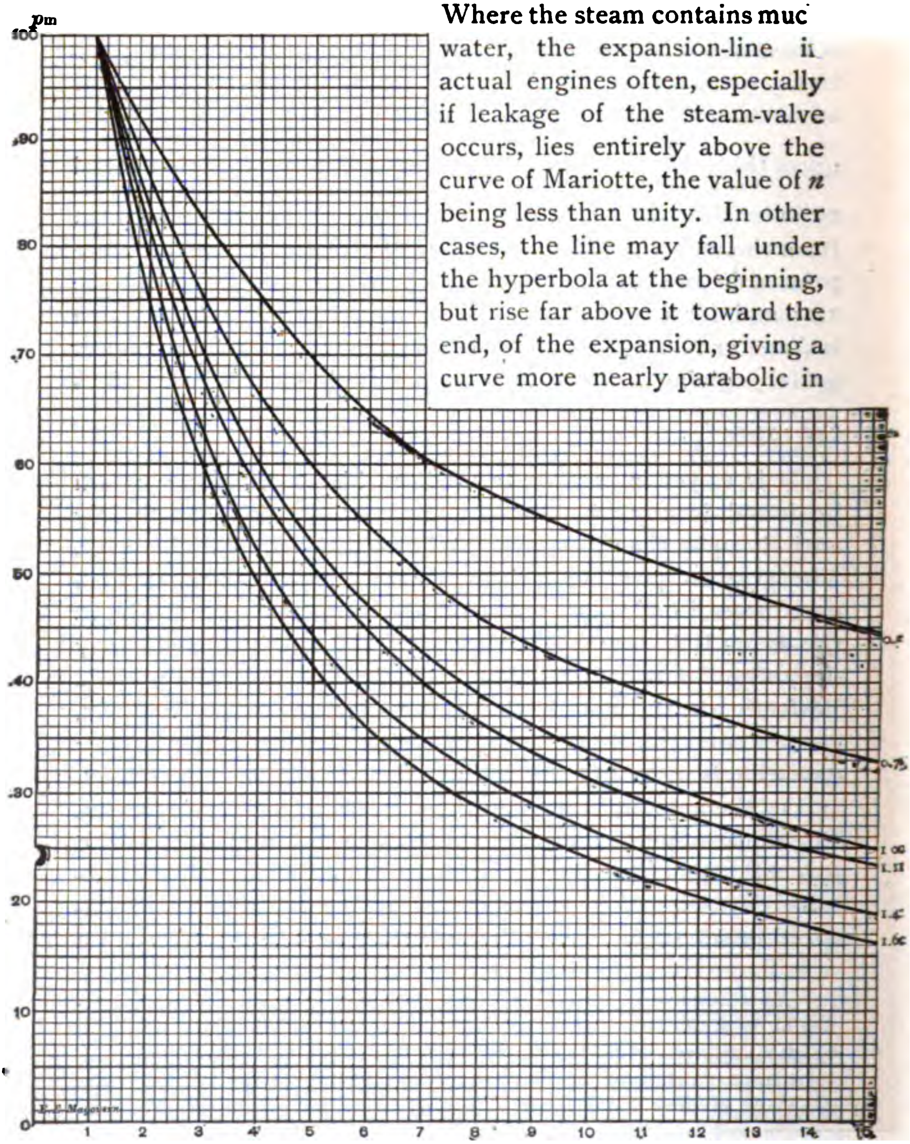
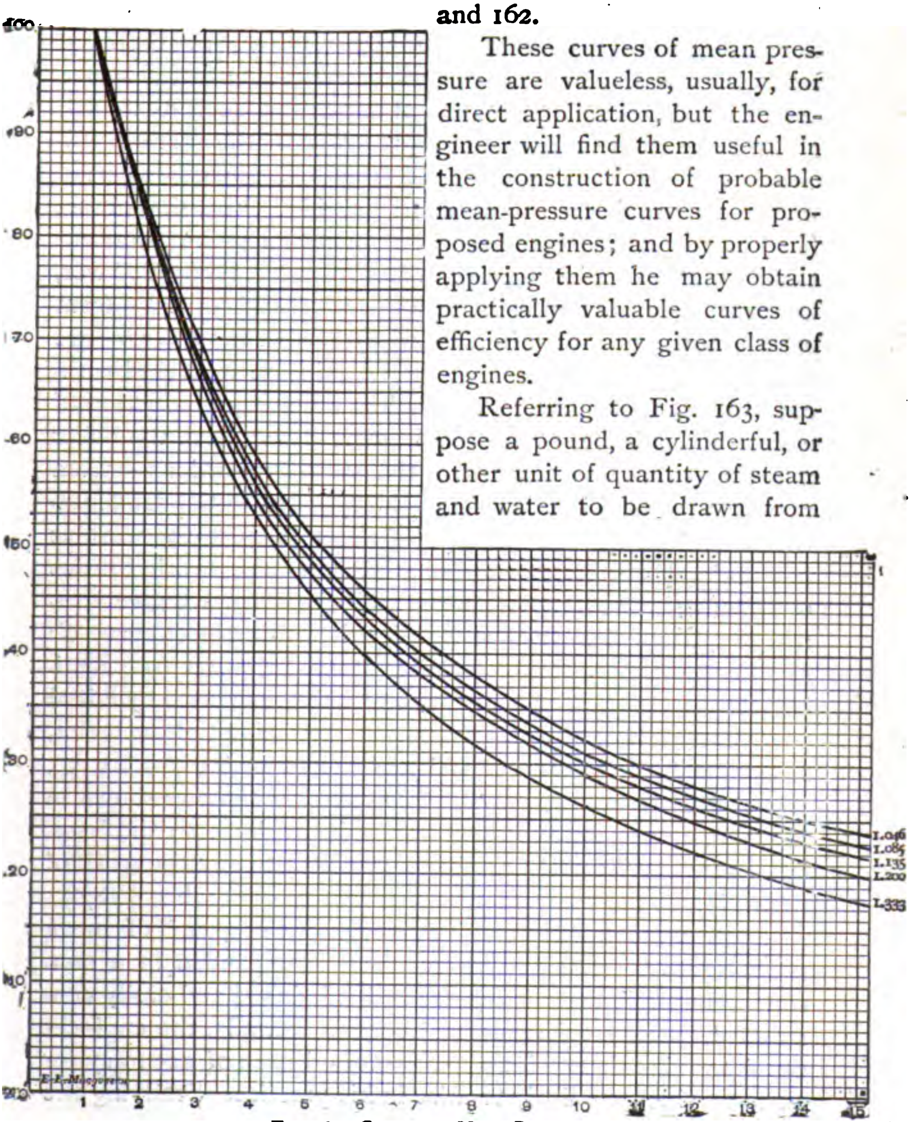


FIG. 161.—CURVES OF MEAN PRESSURE.

appearance, and also with a mean value of n less than unity. The values of $\frac{p_m}{p_1}$ given in the tables are plotted in Figs. 161 and 162.



These curves of mean pressure are valueless, usually, for direct application, but the engineer will find them useful in the construction of probable mean-pressure curves for proposed engines; and by properly applying them he may obtain practically valuable curves of efficiency for any given class of engines.

Referring to Fig. 163, suppose a pound, a cylinderful, or other unit of quantity of steam and water to be drawn from

FIG. 162.—CURVES OF MEAN PRESSURE.

the boiler, carrying 10 per cent its total weight of water, 90 per cent being saturated steam, and to have a pressure which may be called 1.00. When separated from the boiler and carried into the cylinder it will retain the pressure 1.00 and, worked at full stroke, will do the work 1.00. If supplied with additional heat until completely dry, the work becomes 1.11 at full stroke and, if worked at different ratios of expansion, such steam will give a series of mean pressures represented by the curve of efficiency, A_1, Fig. 163, as obtained from the expansion-curves whose equation is $pv^{1.135}$ = constant, provided expansion occurs in a non-conducting cylinder where no condensation can occur except such as is due to performance of work. Expanded wet, as drawn from the boiler, the mean pressures of curve B—from $pv^{1.125}$ = constant, which is deduced by Zeuner for $x = 90$—are proportional to the work done by the mixture if worked without change of proportion other than occurs by production of work. If, again, the same weight were drawn from the boiler at the pressure assumed and in the same proportions—steam 90, water 10—and if, on entering the cylinder, initial condensation should double the quantity of water present, the work at full stroke would be .90 and the mixture would, at other ratios of expansion, the proportion remaining unchanged, give relative quantities of work measured by the ordinates of curve C: $pv^{1.116}$ = constant. It now contains steam 81, water 19. Similarly, the proportion of water present being increased by initial condensation from the original amount carried out of the boiler, so as to reduce the work of unity of weight to .80, .70, .60, .50, etc., at full stroke, the curves of efficiency become as shown in Fig. 163, curves D, E, F, etc., successively, down to the base-line where condensation has become complete and the work of expansion of the water may be neglected. (See, also, § 187.)

Such are the curves of efficiency, of work, and of mean pressures to be obtained where steam is expanded in a non-conducting cylinder. They are easily deduced and easily constructed, and, by reference to Zeuner's formula, the engineer can determine them with a satisfactory degree of accuracy for all cases which are likely to arise in his practice. Studying the

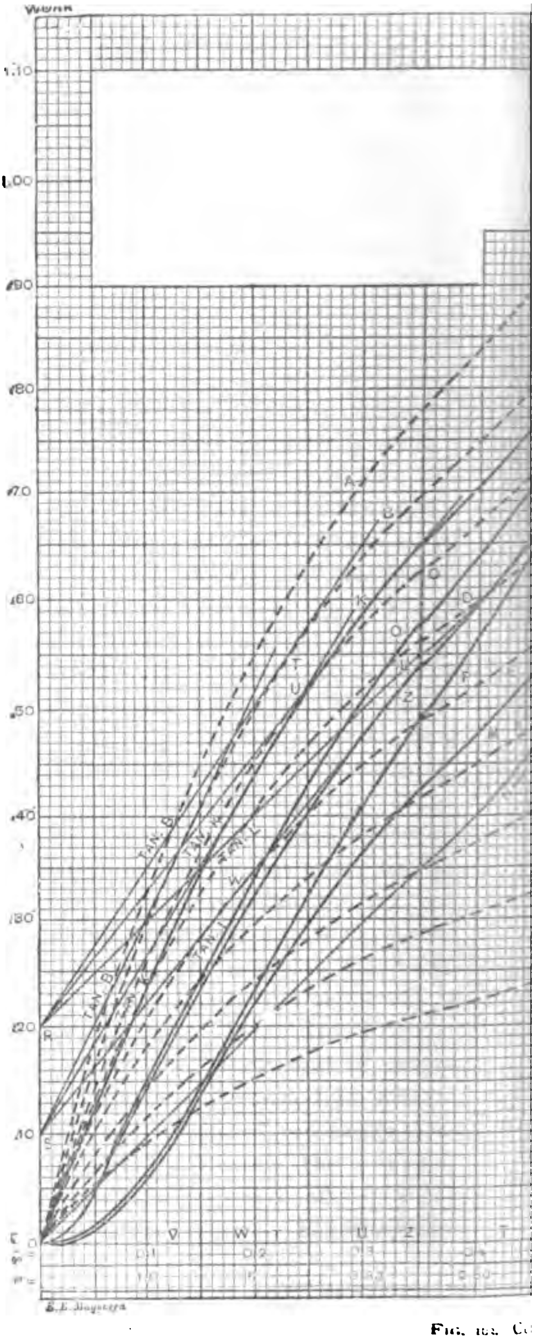
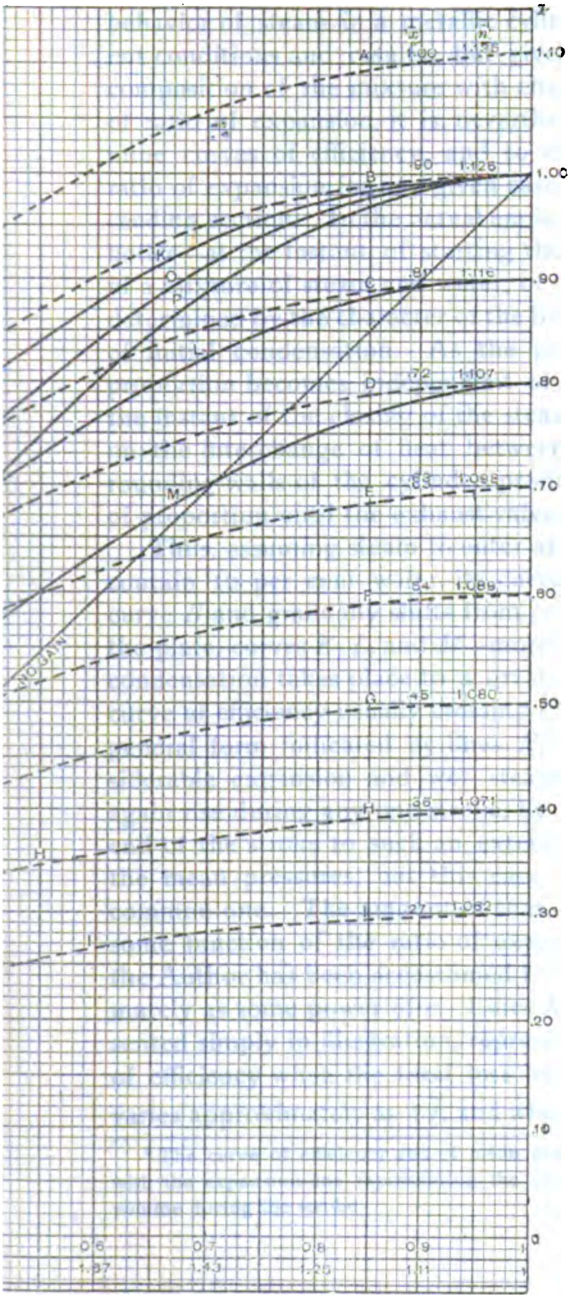

WORK
1.10
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
A
B
K
T
U
O
Z
F
R
S
TAN. B
TAN. W
TAN. T
TAN. K
V
W
U
Z

FIG. [illegible] C


1.10
1.00
.90
.80
.70
.60
.50
.40
.30
.20
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0
A
B
C
D
E
F
G
H
I
K
L
M
O
P
NO GAIN
1.90
1.135
.90
1.125
.81
1.116
.72
1.107
.63
1.098
.54
1.089
.45
1.080
.36
1.071
.27
1.062
0.6
0.7
0.8
0.9
1.67
1.43
1.25
1.11
1

behavior of steam in a metallic cylinder, we find vitally different conditions and results; but given the law of variation of composition of the mixture with change of point of cut-off, or of ratio of expansion, it is, nevertheless, practicable to determine curves of efficiency, and to deduce values of the best ratio of expansion for any given case, as illustrated in the succeeding section. In the actual engine, steam entering from the boiler—at the instant of starting the piston forward—consists of a mixture of steam and water, of which the proportions are determined by the character of the boiler-steam and the amount of initial condensation. As the piston moves forward, this proportion becomes independent of all external conditions at the instant of the closing of the steam-valve. From this point on, the interchange of heat between the steam and the surrounding walls of the cylinder produces a continuous change of proportion until the exhaust-valve opens.

Thus, assuming steam to enter at a pressure of 1.00, and to contain 10 per cent water, its curve of efficiency* starts on curve B and gradually shifts from curve to curve—as seen on the plate, curves K, L, and M—more or less rapidly, as cylinder-condensation takes place to a greater or less extent, the real curve of efficiency usually crossing C, D, E, etc., and taking the general form indicated by lines K_1 L, O, and P. With considerable expansion and wet steam, the expansion-line may again rise during any one stroke, by re-evaporation, toward the end of the stroke to such an extent as to somewhat increase the mean pressures, but this case is, apparently, not a very common one. The amount of that condensation is, evidently, some function of the ratio of expansion in every engine, and the Author has been accustomed to take it as varying approximately as some power of r. Lines K, L, and M, which are presented simply in illustration, represent, respectively, the curves of efficiency when the total loss by cylinder-condensation, h_c, varies approximately as $\sqrt{r}$, and when $h_c = 0.1 \sqrt{r}$, $h_c = 0.2 \sqrt{r}$,

* The curve of efficiency and of mean pressures must not be confounded with the expansion-line representing the varying relations of pressure and volume during the stroke.

$h_c = 0.3 \sqrt{r}$, nearly; values in per cent of total steam demanded not uncommon in engineering practice. The abscissas of the curves are, as before, measures of weights of steam used. If, in any case, condensation were so to vary that no gain should be derived from expansion—and such cases are, within a limited range of expansion, sometimes nearly approximated to—the curve of efficiency would become a straight line, N, the "line of constant efficiency," Fig. 163. The curves O and P are obtained by altering the vertical scales of L and M, so as to give them a common initial point with B and K at $p = 100$, and thus enabling the reader to compare the differences of form of the several lines, and of the two kinds of curve more satisfactorily. It will be seen, later, on comparing the second of the two kinds of curve with those derived from experiment on working engines, and to be presented later, that the curve of efficiency here obtained by induction is of precisely the same character as that given by direct experiment.

Referring once more to the set of curves of efficiency, Fig. 163, we may deduce the same conclusions from graphical construction, and obtain results far more easily and rapidly. Selecting values of $\frac{p_b}{p_1}$ such as are often obtained with non-condensing and with condensing engines, respectively,— $\frac{p_b}{p_1} = .20$; $\frac{p_b}{p_1} = .10$,—we may determine ratios for maximum efficiency of engine thus: From the points .20 and .10 on the axis of ordinates on the scale measuring total work per stroke, draw lines tangent to the several curves, as RT, RV, SV, SW, etc., etc. The points of tangency being found, the values of their abscissas measure the quantities of steam to be used per stroke to give maximum engine efficiency, since the ordinate of any point divided by the abscissa is a measure of the ratio of work done to steam expended in doing it, and, for the assumed back-pressures, the *net* amount of work per unit's weight of steam is a maximum at the points just identified.*

* This principle was pointed out by Rankine. See his Miscellaneous Papers, p. 295, and Shipbuilding, Appendix.

On making the construction it will be found that these maxima are found for very nearly the same values of abscissa and, therefore, for the same ratio of expansion, nearly, whatever the dryness-fraction of the steam used in the non-conducting cylinder. But, drawing tangents *RK*, *RY*, *SX*, *SZ*, etc., to curves *K*, *C*, and *M*, to determine the best ratios for the metallic steam-cylinder, values are formed for r far removed from those just obtained for the non-conducting cylinder, and also differing among themselves greatly with the proportion of water present. In the cases shown on the plate, the ratio for the non-condensing engine is decreased to two thirds, and for the condensing engines to less than half that found best for the non-conducting cylinder. It is to be remembered that the quantity of steam used per stroke, although in direct proportion to the distances "followed" by the steam up to point of cut-off in the non-conducting cylinder, may be in widely different proportion with the metal cylinder. In the latter it varies from nearly an equal proportion at full stroke to, often, a double proportion at high ratios of expansion.

181. The Ratio of Expansion at Maximum Efficiency.—In all heat-engines the method of transformation of heat-energy into useful mechanical work has been seen to be the following: *

A certain mass of the working fluid is heated from a temperature which is usually not far from that of the atmosphere up to some higher temperature. This is accompanied by a definite increase of volume, or of pressure, or of both, and in the case of liquids by a change of physical state after passing a certain point which is variable, but definite for each pressure; this latter temperature is the boiling point, and the change is that known as vaporization. Evaporation being complete, the mass is expanded in the working cylinder of the engine until it has attained a certain larger volume, v_2, the magnitude of which is r times that of the initial volume, v_1, with which expansion began. We thus have the "*ratio of*

* See Journal Franklin Institute; May 1881.

expansion," $r = \frac{v_2}{v_1}$. When expansion is complete, the whole volume, v_2, of steam or gas at the pressure p_2 is rejected from the cylinder into a condenser or into the atmosphere, and the piston which it has impelled through the total volume, v_2, returns to the starting-point, resisted by the "*back*-pressure," p_2, of the condenser or of the atmosphere. During the latter operation all heat which has not been transformed into work is rejected, and an additional amount is expended, which is equivalent to the work done by the piston upon the fluid during its expulsion. This operation is that which has already been more than once described.

This process is thus graphically represented: In Fig. 164, the fluid, initially in the state measured by the pressure *aE* or *a'E'* and volume *Oa* or *Oa'*, is heated, sometimes at constant volume, as *Oa*, and sometimes with compression, as from *Oa'* to a higher temperature, the pressure and volume varying as shown by *EA* or by *E'A*. Heated next at constant pressure or at constant temperature, the mass expands, doing work, to *B* or to *B'*. At this point, v_1, p_1, the supply of heat ceases and the fluid expands "adiabatically," transforming into mechanical energy all the heat demanded as equivalent to the work measured by the area *bBcC*, and drawing upon its own stock of heat to supply this demand. At the end of this stage the fluid has a lower temperature and a pressure and a volume, *cC*, *Oc* (p_2, v_2,) determined by that temperature and the value of $r = \frac{v_2}{v_1}$, and which are indicated by the location of the point *C*. Rejecting heat at constant volume, v_2, pressure falls to *D*, p_2, and then rejection of heat continuing at constant pressure, p_2, the volume is reduced to that with which it started.

The *total* or *gross* work done is, in gas-engines, measured by the area *ABCcaA*, in steam and vapor engines by this area increased by a very considerable amount—the measure of internal, of molecular, work which cannot appear on the indicator-diagram.

The net work done is measured by the area included in the

indicator-diagram $ABCDEA$. This work is the equivalent of all heat transformed into mechanical work or energy. *The efficiency of the fluid* is the ratio of *net* work done to total heat received by the fluid, and is a maximum when the area $ABCDE$ is a maximum, assuming the *ratio of expansion* alone to vary. It is evident that this maximum is determined, therefore, by the conditions which make the area $bBcC$ a maximum, which conditions are very simple in the hot-air engine, and are easily expressed, while in the steam and in vapor engines they are very difficult of determination and expression in consequence of their extreme variability. But the efficiency of the fluid is but one factor in the determination of the ratio of expansion for maximum economy. The heat in the fluid is compelled to do its work, not simply through that fluid as a transmitting mechanism, but also through a machine which, as an apparatus intended to imprison and direct so subtle and elusive a form of energy as heat, is extremely imperfect, and which has the additional and very serious defect of being itself cumbersome and difficult to start and to keep in motion without considerable loss of power within itself.

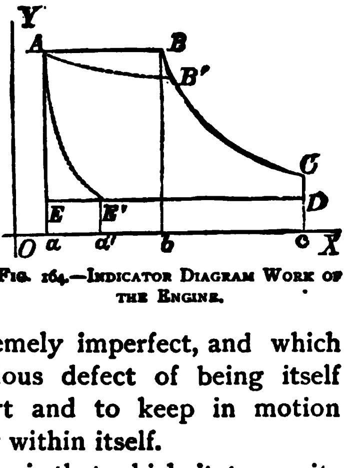

FIG. 164.—INDICATOR DIAGRAM WORK OF THE ENGINE.

The useful work of the machine is that which it transmits beyond its own boundaries to other mechanisms, and this is a maximum at that ratio of expansion which gives energy to the machinery of transmission beyond the engine at least cost in heat expended. This *efficiency of the system* is therefore the product of the factors, *total efficiency of the fluid* and *actual efficiency of the engine* considered as a piece of mechanism.

Taking first the purely ideal case in which the mechanism is assumed to be perfect and the ratio of expansion the only variable element, we may by examining Fig. 165 see at once what should be the value of that ratio.

It is obvious that the ratio of expansion simply determines

how far the transformation of stored heat-energy existing at B shall be continued by transformation into work during the expansion of the working fluid. It is equally obvious that this expansion should continue until the gain of work by further expansion is more than balanced by losses avoidable by termination of that process.

Where the only loss is due to a fixed back-pressure, $FD = p_2$, it is seen that, were expansion to cease at C, the work which would have been done had the expansion-line BC extended to the right beyond C, is lost, and that the counterwork of

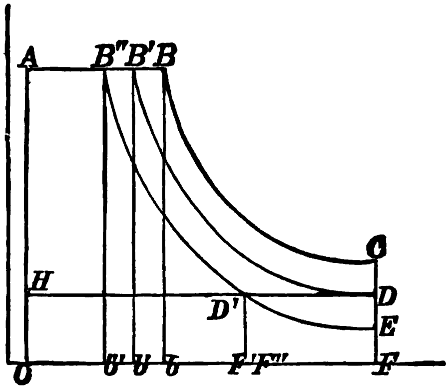

FIG. 165.—ENGINE CYCLES.

back-pressure beyond that point is gained; but the former exceeds the latter, and the net result is a loss by incomplete expansion. On the other hand, were the ratio of expansion increased so that the expansion-line becomes $B'' E$, the back-pressure line is reached at D'; and, beyond this point, we note a gain of work done usefully, which is measured by the area $D'EFF'D'$, while a loss accrues by back-pressure measured by $D'DFF'D'$. We thus again meet with a net loss which is represented by $D'DED'$, and expansion has evidently been carried too far. Making the value of $r = \frac{v_2}{v_1}$ such that expansion reaches the back-pressure line at D and p_2 becomes equal to p_2, we meet with neither kind of loss, and it follows that

expansion should in this ideal case be continued until the expansion-line meets the back-pressure line.

This may be readily shown by other methods: It was shown, nearly two generations ago, by Sadi Carnot, that maximum efficiency of *fluid* is attained when expanding between the widest possible limits of temperature. It is now well known, and it is shown by every elementary treatise on physics, or mechanics, or thermodynamics, and on heat-engines, that the efficiency of the *fluid* in any heat-engine is measured by the expression $\frac{T_1 - T_2}{T_1}$, in which T_1 and T_2 are the temperatures of reception and rejection of heat measured from the "absolute" zero. But this maximum range of temperature corresponds to the maximum attainable range of pressure, and, the upper limit being fixed, this range is determined by the value of r and is a maximum when $p_2 = p_3$ and expansion continues to the back-pressure line. A general analytical demonstration is obtained in the following manner: *Problem:* Given p_1, v_1, v_2, p_3, to find the value of the *ratio of expansion*, r, which will make the net work done a maximum for the Ideal Case.

This work, *ABCDE*, figure, is measured by

$$W_u = p_1 v_1 + \int_{v_1}^{v_2} p dv - p_3 v_2; \quad . \quad . \quad . \quad . \quad (1)$$

and is a maximum when the variable part $\int_{v_1}^{v_2} p dv - p_3 v_2$ is a maximum.

The method of variation of p with variation of v is determined by various conditions which do not affect the analysis. Let this relation be such that we may write, as experiment indicates that we may with practically close approximation,

$$p_1 v_1^n = p_2 v_2^n = \text{const.}; \quad \frac{v_2}{v_1} = r.$$

Thus we have

$$W_n = p_1 v_1 + \int_{v_1}^{v_2} p dv - p_2 v_2;$$

$$= p_1 v_1 + \frac{p_1 v_1 - p_2 v_2}{n - 1} - p_3 v_2; \quad . \quad . \quad . \quad (2)$$

or, for hyperbolic expansion, where $n = 1$,

$$W_n = p_1 v_1 (1 + \log_e r) - p_3 v_2 . \quad . \quad . \quad . \quad . \quad (3)$$

Determining the maximum for the first and usual case, we get

$$\frac{dW_n}{dr} = d\left(p_1 v_1 + \frac{p_1 v_1 - p_1 v_1 r^{1-n}}{n - 1} - p_3 r v_1\right) \cdot \frac{1}{dr} = 0;$$

whence

$$r = \left(\frac{p_1 v_1}{p_2 v_1}\right)^{\frac{1}{n}} = \left(\frac{p_1 v_1}{p_3 v_1}\right)^{\frac{1}{n}} = \frac{v_2}{v_1} . \quad . \quad . \quad . \quad . \quad (4)$$

Hence

$$p_2 = p_3,$$

and the ratio of expansion for maximum efficiency of fluid is that which makes the terminal direct pressure equal to the pressure resisting the motion of the piston, and irrespective of the method of variation of p with v, or of the value of n.

This analysis must be modified when the expansion-line is taken as an equilateral hyperbola; in which case we have $n = 1$ and $p_1 v_1 = p_2 v_2$. This case is often assumed in the theory of gas and air engines, as it is in those cases that of isothermal expansion; but it is probably rarely observed in actual practice, and perhaps never occurs in steam and vapor engines. In simple computations of work, however, the assumption does not lead to serious error, and, so expanding the working fluid, the energy exerted by it, up to the point of cut-off, is equal to the lost work due to back-pressure; the net work done is measured by the total area under the expansion-line of the

indicator-diagram, and the efficiency is proportional to the hyperbolic logarithm of r.

Thus we have

$$W_n = p_1 v_1 (1 + \log_e r) - p_3 r v_1;$$

$$\frac{dW_n}{dr} = 0 = \frac{p_1 v_1}{r} - p_3 v_1;$$

$$r = \frac{p_1}{p_3} = \frac{v_2}{v_1} = \frac{p_1}{p_2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

whence we again find

$$p_2 = p_3.$$

The following are values of n for various cases commonly taken in these discussions:

VALUES OF n IN pv^n = CONSTANT.

Air, isothermal expansion	1.0
" adiabatic "	1.4
" wet and adiabatic	1.2
Gases generally, isothermal	1.0
" " adiabatic	1.4
" in explosive gas-engines	1.6
Steam, dry and saturated	1.046
" adiabatic	1.135
Steam, 0.76; water, 0.24	1.111
" superheated	1.333
Steam and water generally	$1.035 + \frac{x}{10}$

But in all *real* engines we have a resistance to the motion produced by the expanding fluid, which is composed of two parts: an actual back-pressure on the piston, $p_b = p_3$, as in the *ideal* case above, and a resistance due to friction of engine, including pumps and all attachments. It is evident that, as this latter resistance, p_f, like the back-pressure, p_b, is a constant

source of lost work, we must terminate the expansion as soon as it produces a greater loss of power or of work than is gained by further expansion. In fact: given a certain value for the sum of these resistances, $p_b + p_f$, we may consider the whole as back-pressure, if we choose; and it is a matter of indifference, so far as the determination of the ratio of expansion is concerned, what are their individual magnitudes.

To determine $p_b + p_f$, the sum of resistances due to back-pressure, p_b, and to the frictional and other resistances—as of pumps, etc.—denoted by p_f, take an indicator-card from the engine unloaded. Its mean pressure measures the friction, p_f, of the unloaded engine, and this, sometimes, probably, increased by a fraction of the pressure added by the load, is the value of p_f. Or, still better, determine the indicated and the dynamometric power of the engine simultaneously; their difference is lost work, and the value of p_f, corresponding to that work, is that required.

Hence, for actual engines, where no other cause of loss exists of any appreciable magnitude, we may write

$$W_u = p_1v_1 + \frac{p_1v_1 - p_2v_2}{n - 1} - (p_b + p_f)v_2; \quad . \quad . \quad . \quad (6)$$

and, by the process already outlined, we obtain a maximum and deduce

$$p_2 = p_b + p_f.$$

Hence: ***Where the lost energy and work is that due to back-pressure and to friction of engine, the ratio of expansion should be such as to carry the expansion-line down to the mean-pressure line of the engine-diagram taken without load.***

The useful work is, as before, the gross work done during expansion; and, thus adjusted, the net useful work and the efficiency are nearly proportional to $\log_e r$. This conclusion is obviously true, whatever the value of n or the character of the expansion-line.

Thus, as stated by Rankine, "the greatest useful work is obtained by making the expansion cease when the forward-

pressure is just equal to the back-pressure, added to a pressure equivalent to the friction of the engine." *

For all actual steam and other engines still further and still greater modification is necessary, since in such engines the departure from the ideal conditions first assumed is so great as, in most cases, to lead to radically different ratios of expansion. Even in the gas-engines, the action of the working fluid, as assumed above, is very greatly modified by such variations from the ideal conditions as are here referred to.

For any given engine, there is always a certain ratio of expansion appropriate to every steam-pressure, and which gives, on the whole, the most economical performance. Every engine must therefore be most carefully proportioned to the usual conditions of its operation.

The best ratio of expansion, kinematically, when the expansion-curve is defined by the expression $p^m v^n =$ const., is

$$r_e = \left(\frac{p_1}{p_2}\right)^{\frac{m}{n}};$$

and, for engine-efficiency, friction being considered,

$$r_e' = \left(\frac{p_1}{p_2 + f}\right)^{\frac{m}{n}}.$$

The defining equation usually takes the form $p^m v^{m+1} =$ const.; when we have

$$r_e = \left(\frac{p_1}{p_2}\right)^{\frac{m}{m+1}}; \quad r_e' = \left(\frac{p_1}{p_2 + f}\right)^{\frac{m}{m+1}}.$$

It may evidently be concluded from what has preceded:

(1) That the *work done* in a non-conducting cylinder, the fluid expanding adiabatically, varies so little with the proportion of water present that this variation may be neglected by the engineer, and he may assume the performance of work to

* Life of John Elder; 1871; p. 16.

be such as would come of hyperbolic expansion; while the heat thus expended may be computed, as in the thermodynamic case, from the quantity of work, when the latter is known.

(2) That, in cylinders of *metal*, the work done at any given point of cut-off is nearly the same as in the non-conducting cylinders; but that the quantity of heat and of steam expended in doing it are increased, and usually very greatly increased, by cylinder-condensation, if ordinary nearly dry steam is used, or by other methods of storage and transfer of heat to the exhaust, and consequent waste, if superheated steam or other gaseous working fluid is employed.

(3) That the ratio of expansion at maximum efficiency of fluid would be but slightly changed by ordinary variations in the proportion of water entrained by the steam, if it were worked in a non-conducting cylinder, and the value of that ratio, r_e, is very nearly $\frac{p_1}{p_b}$, the quotient of initial pressure by the sum of the cylinder back-pressure and other wasteful resistances.

(4) That the ratio of expansion at maximum efficiency of fluid, when steam expands in a metallic cylinder, is affected by the introduction of water entrained by the steam; and this difference is increased and usually is made a serious one by the occurrence of cylinder-condensation, or other method of transfer of heat to the exhaust. This ratio becomes, in this case, much less, usually, than $r_e = \frac{p_1}{p_b}$.

(5) That the quantity of fluid used per stroke, in the non-conducting cylinder, is in direct and exact proportion with the volume of the cylinder open to the supply-pipe at the instant of closing the expansion-valve, and is measured by $\frac{1}{r}$, the reciprocal of the ratio of expansion.

(6) That the volume of steam worked per stroke, in the metal cylinder, is *not* in direct proportion to volume of cylinder open to steam at the point of cut-off; but that it is often very

greatly in excess of the latter quantity, and is in greater excess, as the ratio of expansion is increased, indefinitely.

(7) That the ratio of expansion is not a gauge of the volume of steam demanded from the boiler, and paid for by the proprietor of the apparatus, when the metal cylinder is employed; but that the volume of steam used, and quantity of heat demanded, must always exceed the proportion $\frac{1}{r}$ in real engines.

(8) The Curve of Variation of Efficiency—of which the abscissas measure varying quantities of steam used in a given steam-cylinder, while the ordinates are proportional to the quantities of work done by those amounts of steam—is a curve of entirely different character and form, and often widely different in location, with the actual engine, from the curve of adiabatic mean pressures, or other curve of mean pressures exhibiting the work done by various quantities of steam expanding in a non-conducting vessel.

(9) That no predetermination of the efficiency of any proposed engine, whether of fluid, of machine, or of capital, can be made unless the elements of the true curve of efficiency can be obtained for the assumed case.

(10) That the most certain and the most satisfactory solution of any problem of efficiency will be that obtained by first securing the data for the curve of efficiency, from actual engines, operated in the manner proposed for the case taken.

(11) That, having obtained, by experiment upon any engine, the true "Curve of Efficiency," as defined by the Author, the efficiency of fluid, of engine, and of capital expended to do a given amount of work, and the quantity of work to be obtained most cheaply from a given engine, may all be obtained for any given set of conditions; and the ratio of expansion at maximum efficiency, of fluid, of engine, and of capital, and the ratio of expansion which, with a given "plant," gives most work for a dollar of total expense of operation, may all be determined with a degree of exactness only limited by the magnitude of the errors of observation.

To construct the theory of cases of non-adiabatic expansion, the Author has taken the following method:* We may take two distinct cases: (1) That in which, as when the cylinder is unjacketed and unprotected against radiation and the ratio of expansion small, so little re-evaporation occurs that it may be neglected; (2) That in which, as in most cases familiar to the engineer, and especially in jacketed cylinders with considerable expansion, nearly all condensation occurs before the point of cut-off is reached, and re-evaporation takes place throughout the remainder of the stroke.

Case 1.—It has been seen that the form of the adiabatic expansion-line may be obtained from approximate expressions of the form $pv^n = p_1v_1^n$; $p_2 = p_1r^{-n}$.

Since loss of pressure occurs in the metallic cylinder by a transfer of heat, taking place by initial condensation and later re-evaporation, and since the amount of this loss is determined, in any given cylinder, by the magnitude of the ratio of expansion, we may write

$$p = p_1\left(\frac{v_1}{v}\right)^n [1 - f(r)].$$

The values as well as the form of this function of r, $f(r)$ above, are not yet exactly ascertained. The Author has found that for the ordinary values of the ratio of expansion we may assume, as an approximation, $f(r) = ar^m$; m being taken constant.

In this expression a, for any engine, has a value which is determined by the condition of the steam at entrance into the cylinder, and is connected with the exponent n by some definite, though as yet unascertained, relation. The value of m is dependent upon the character of the engine and the method of its operation, so far as they determine the variation of the proportions of steam and water during expansion. Given the values of n and of a, m becomes determinable. We have

* On the Behavior of Steam in the Steam-engine, etc. Trans. N. Y. Acad. Sci., 1882; Jour. Franklin Inst., Feb. 1882.

$$\frac{p_2}{p_1} = r^{-n} - ar^{m-n}; \quad m = \frac{\log\left[\frac{1}{a}\left(r^{-n} - \frac{p_2}{p_1}\right)\right]}{\log r} + n,$$

where p_2 is the terminal pressure, a quantity always known when either it or r is obtained by experiment.

The equation for the expansion-line, the working substance being enclosed in a metallic cylinder, is then

$$p = p_1\left(\frac{v_1}{v}\right)^n\left[1 - a\left(\frac{v}{v_1}\right)^m\right].$$

The work done by expansion is

$$\int_{v_1}^{v_2} pdv = p_1v_1^n\int_{v_1}^{v_2}\left[1 - a\left(\frac{v}{v_1}\right)^m\right]v^{-n}dv.$$

The net work is

$$W_n = p_1'v_1 + \int_{v_1}^{v_2} pdv - p_bv_2,$$

in which p_b is the back-pressure plus friction and useless resistance.

The terminal pressure is given above. Making $r = 1$, we obtain from that equation $p_2 = p_1(1 - a) = p_1'$, showing that p_1 is not the initial cylinder-pressure, p_1', but the pressure which the same weight of steam would have given if working at the same volume and without condensation in the same cylinder; p_1 exceeds p_1' in the ratio $1 : 1 - a$; which ratio measures the relative working values of the same mass of steam with and without cylinder-condensation.*

Integrating the expression for net work done during expansion,

$$W_e = \int_{v_1}^{v_2} pdv = p_1v_1^n\int_{v_1}^{v_2} v^{-n}\left[1 - a\left(\frac{v}{v_1}\right)^m\right]dv,$$

* If x is the "dryness-fraction" of the steam when worked to the end of stroke, it having been dry when drawn from the boiler, $p_1' = p_1x$; $x_1 = \frac{p_1'}{x}$.

we obtain

$$W_e = \frac{p_1v_2}{1-n}r^{2-n} - \frac{p_1v_2}{1-n}r^{-1} - \frac{ap_1v_2}{m-n+1}r^{m-n} + \frac{ap_1v_2}{m-n+1}r^{-1} - p_bv_2$$
$$= \frac{p_1v_1}{1-n}r^{1-n} - \frac{p_1v_1}{1-n} - \frac{ap_1v_1}{m-n+1}r^{m-n+1} + \frac{ap_1v_1}{m-n+1} - p_bv_1r,$$

while the total useful work per stroke is $W_u = W_e + p_1'v_1$.

In this analysis the work-effect of re-evaporation is neglected as unimportant.

The equation of these curves of efficiency for adiabatic expansion is

$$y = p_1\frac{n - r^{1-n}}{n - 1}x.$$

The equation for the present case is

$$y = \left[\frac{r^{3-n} - 1}{1 - n} - \frac{ar^{m-n+1} - a}{m - n + 1} + 1\right]p_1x, \text{ nearly.}$$

The mean pressure is then

$$p_m = \frac{p_1r^{2-n} - p_1r^{-1}}{1 - n} - \frac{ap_1r^{m-n} - ap_1r^{-1}}{m - n + 1} + p_1'r^{-1},$$

and the mean effective pressure is

$$p_e = \frac{p_1r^{2-n} - p_1r^{-1}}{1 - n} - \frac{ap_1r^{m-n} - ap_1r^{-1}}{m - n + 1} + p_1'r^{-1} - p_b.$$

The mean effective pressure and the work of the engine are maxima, r varying and the back-pressure, p_b, being fixed, when

$$p_2 = p_1r^{-n}[1 - ar^m] = p_b,^*$$

* In fact, however, re-evaporation—the effect of which is not in such cases usually found to be important in increasing efficiency—usually prevents the fall of terminal pressure to the value $p_2 = p_b$.

provided, as assumed, re-evaporation may be neglected. Then

$$r^{-n} - ar^{m-n} = \frac{p_b}{p_1}.$$

The Ratio of Expansion for Maximum Efficiency of Fluid is, however, that which makes $\frac{W_n(1 - ar^m)}{p_1 v_1}$ a maximum. The "cut-off," or fraction of stroke completed at the instant of closing the steam-valve, is $\frac{1}{r} = c$, and its value for maximum work is that which gives $c^n - ac^{n-m} = \frac{p_b}{p_1}$.

The following cases, illustrating the results of this method of treatment, as applied to several selected examples, such as are met with in ordinary practice, are given as exhibiting a very usual range of values of the quantities involved in the preceding equations:

	Character of Engine.	p_1	p_b	a	m	n	r_e
I.	Non-condensing engine	100	20	0.2	1.5	1.115	4.5
II.	Condensing, unjacketed	40	5	0.2	0.5	1.115	2.5
III.	" compound, jacketed	60	6	0.1	1.1	1.125	6.0
IV.	" " "	100	5	0.1	0.0	1.135	10.0

In the first three of the above cases, the steam is taken from the boiler nearly dry; in the last, it is so far superheated that it expands as practically dry steam, cylinder-condensation being negligible.

Case 2.—The second assumed case is probably that usually met with in practice, initial condensation ceasing with the closing of the expansion-valve, and re-evaporation occurs throughout substantially the whole period of expansion. Then, taking $b = 1 - a$, b thus measures the proportion of actual work done at full stroke to that which the same steam, without cylinder-condensation, would do; while r^q is a factor proportional to the wastes at other ratios of expansion. We may write, for the net power delivered:

$$W_n = br^q p_1 v_1 \frac{n - r^{1-n}}{n - 1} - p_b r v_1.$$

Here p_1v_1 measures, as before, the work obtainable from the same weight of *dry* steam, up to the given point of cut-off, when working at the same ratio of expansion, and when, therefore, $bp_1v_1 = p_1'v_1 = (1 - a)p_1v_1$, as taken in the first case. The above expression, r varying, becomes a maximum when

$$r^{q-1} - \frac{q+1-n}{qn} r^{q-n} = \frac{n-1}{bqn}\frac{p_b}{p_1},$$

The mean effective pressure is

$$p_e = br^{q-1}p_1\frac{n - r^{1-n}}{n-1} - p_b.$$

and the equation of the curve of efficiency is, for this case of non-adiabatic expansion,

$$y = b^2r^{2q-2}\frac{n - r^{1-n}}{n-1}p_1x.$$

For the case of nearly hyperbolic expansion, which is a common one for this class of engines,

$$W_n = bp_1v_1(1 + \log_e r)r - p_brv_1,$$

nearly; which is a maximum when

$$\left[\left(q(1 + \log_e r)\right) + 1\right]r^{q-1} = \frac{p_b}{bp_1}.$$

The mean effective pressure is $p_e = bp_1(1 + \log_e r)r^{q-1} - p_b$.

The value of q varies from 0, nearly, to $-.5$; being greatest with most efficient engines and always negative.

The ratios of expansion for maximum efficiency are those which satisfy the above equations.

The following are corresponding values of a, b, and n:

a	0.00	.10	.20	.30
b	1.00	.90	.80	.70
n	1.135	1.125	1.115	1.105

The consumption of steam and cost of power in these cases is measured by the volume actually introduced at the initial pressure, as with the non-conducting cylinder.

The values of a and b are very widely variable, as has already been seen (Chap. V), with variation of working conditions, size and construction of engine; the engineer easily obtains a fairly approximate figure for either, taking that found by experience to be usually characteristic of similar engines of nearly the size of that which his judgment commonly leads him to anticipate will be approximately that of the engine to be designed. Where the commercial and other problems to be here discussed relate to an engine actually in use, these quantities may sometimes be directly determined.

Professor Marks has solved this problem, incorporating in his expressions for efficiency the Rankine function of condensation-waste.* These expressions thus become somewhat complicated, and graphical methods are commonly preferred by the engineer, in solving all problems of this class.

182. The Efficiency of Capital is the final and the most vitally important of the problems of maximum efficiency. It determines, when solved, the best ratio of expansion, all things considered. But since the quantity of work to be performed and the power of the engine are the magnitudes usually given, and since the size of engine needed to do a given amount of work varies, other conditions being the same, with the extent to which expansion is carried, the solution of the problem giving the ratio of expansion at maximum commercial efficiency, or efficiency of capital, is, really, the determination of the proper size of engine for the case taken.

The solution of this problem evidently involves a study of all the conditions affecting either first cost or expenses of operation, immediate or remote, direct or indirect, during the life of the apparatus. Of these items of cost, some are constant for the case assumed; some vary with the size of engine; and

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* Steam-engine; 3d ed., p. 191.

others are variable with the size of boiler and quantity of steam demanded.*

In Case 8, § 174, making the sum of both items of variable annual expense—those variable with size of engine and those variable with quantity of steam demanded—a minimum, the sum of these items and of all invariable expenses, i.e., of the total running expense, becomes a minimum, and the problem is solved when the ratio of that sum to the quantity of work is thus made a minimum. A knowledge of these conditions and of all other expenses, constant as well as variable, is also essential to the treatment of Case 9.†

Since economy of fuel and steam demands the use of a large engine, working steam with considerable expansion, and gives reduced size and weight of boiler, it is evident that the first of the two problems, Case 8, § 174, is to be solved by determining what proportion of engine and boiler will be cheapest when summed up at the end of the life of the plant; this is settled when the ratio of expansion at maximum commercial efficiency is known, since the mean pressure is thus fixed, and the best size of engine and boiler is thus settled. The work will then be done less economically either by a larger engine and a smaller boiler, or by a smaller engine supplied with more steam by larger boilers.

The last enunciated problem, Case 9, is solved by determining what degree of expansion and resulting mean pressure and work will give the power, from an engine and boiler already installed, at least total cost per horse-power. The first of these problems contains, as elements, all items of cost variable with change of proportions of engines and boilers capable of doing the same given quantity of work; the second considers every item of expense, while the amount of power is the variable quantity. Both problems require the study of all the costs of steam-power, the determination of the way in which each is

* The Several Efficiencies of the Steam-engine; R. H. Thurston.

† First treated, so far as the writer is aware, by Messrs. Wolff and Denton. Trans. Am. Society Mech. Engrs., 1881; American Engineer, 1881.

related to total expense, and the manner in which each varies with variation of the variable quantities in either case. The first of these is the designer's problem, the second the owner's or the user's, as the Author has customarily designated them.

If we have given a certain annual invariable expense of operation, certain additional expenses variable with size of engine, and therefore with the ratio of expansion adopted, and certain other additional expenses variable with quantity of steam demanded and with size of boiler needed, and thus also dependent upon the ratio of expansion at which that steam is used, we may call the two latter quantities, respectively, $f'(r)$ and $f''(r)$, while the constant part may be called C. Then the total annual expense is $f'(r)+f''(r)+C$, which is a minimum when the variable part, $f'(r)+f''(r)=f(r)$ is a minimum, and this is a minimum when its ratio to work done, $F(r)$, is a minimum, i.e., when $\frac{f(r)}{F(r)}$ is a minimum, or $d\frac{f(r)}{F(r)} \div dr = 0$. The value of r which satisfies this condition determines the required mean pressure, and gives Maximum Commercial Efficiency.

The determination of the value of r which makes $\frac{f(r)+C}{F(r)}$ a minimum gives the solution of Case 9.

Case 10 is solved by determining at what ratio of expansion the cost of power becomes equal to the market value of the power, less a stated paying profit.

The Annual Cost of Steam Power thus consists:

(1) Of certain expenses which are invariable, whether the work is done by a large engine with high ratio of expansion and small boilers, or with a smaller engine working at a low ratio of expansion and with necessarily larger boilers. These expenses are, usually: rent of building or interest on cost; taxes, repairs, etc., etc., on structure and cost of location; the "engineer's" salary, and sometimes all, sometimes part, of the fireman's or "stoker's" wages; also sundry minor expenses, or a part of each of other expenses, which as a whole are variable.

(2) The interest on first cost of engine, in place; the cost

of maintenance and repairs; and a sum which measures the depreciation in value of the machine due to its natural wear, or to its decreasing value in presence of changes that finally compel the substitution for it of an improved engine. Oil, waste, and other engineer's stores fall under this head. All these items are variable with size of engine.

(3) The expenses of supplying the engine with steam. These are:

(*a*) The cost, on fuel account, of the steam supplied; and which includes also the cost of steam condensed *en route* to the engine, and that wasted by "cylinder-condensation" and by leakage, as well as that actually utilized. This total quantity of steam greatly exceeds that actually used in the production of power by simple transformation of heat energy.

This item varies with the efficiency of engine, and determines the size of boiler demanded.

(*b*) The interest on cost of boilers in place, and their appurtenances; rent of boiler-room, or interest on its cost; depreciation, taxes, repairs and insurance, wholly chargeable to boilers.

This item is variable with size of boiler.

(*c*) Cost of attendance in excess of the costs included in the constant quantity of item (1) and variable with size of boiler or quantity of steam demanded.

The pay of the engineer in charge is usually not chargeable to either engine or boiler alone; his position is one of supervision over the whole apparatus, and a good engineer usually keeps the closest watch over the boilers. With small engines, the engineer is also the fireman. With large engines, the number of additional firemen may be taken as proportional to the quantity of steam demanded; and, with very large marine engines, a similar remark may apply to engine-room attendance.

In working up this account, it will be most convenient to refer all costs to volumes of cylinder, and to so express variable quantities that they may enter the equations in terms of the ratio of expansion, which ratio is to be taken, as hereafter shown, as an independent variable upon which all other variable quantities are made dependent. We will enter all con-

stant quantities as so many dollars of *annual* expense; the total, invariable expense being denoted by A, which includes all such expenses, whether chargeable to engines or boilers, or both. The first cost of an engine varies according to no definite rule, and differs greatly with type of engine, kind of valve-gear, character of work, and value of material and labor, both at the manufactory and at the place of installation. With standard forms of engine, however, it is found that the cost may be reckoned, for ordinary variations of size, as approximately proportional to volume of steam-cylinder; and prices may be fixed on that basis. The cost of transportation, other things being equal, may often be similarly estimated; as may expenditures for repairs, engineer's supplies, etc.; although these items are less exactly determinable.

For present purposes, it may be assumed that interest on cost of engine in place, depreciation, repairs, and all other expenses varying with size of engine, may be reckoned per cubic foot of cylinder.

The cost of steam supplied to the engine, exclusive of the constant quantity entered in (1) may be reckoned as a certain number of dollars per pound, or per cubic foot of steam worked in the cylinder.

The weight of steam supplied for the performance of work —when the weight per cubic foot of steam at the given pressure, p, is w; and its total volume is $v_1 = v_2 \div r$, where r is the "real" ratio of expansion—is $wv_1 = \frac{wv_2}{r}$; its cost per cubic foot of steam-cylinder is $\frac{cwv_1}{v_2} = \frac{cw}{r}$, and its total cost per year is $2Rcwv_1 = 2Rc\frac{wv_2}{r}$, where R is the number of revolutions made by the engine per annum.

To this weight is to be added steam wasted by cylinder-condensation, by leakage, and by conduction and radiation from engine and boiler. This may be allowed for by multiplying the last item by a factor greater than unity, determined as elsewhere shown.

183. Theory of Efficiencies of the Ideal Engine.—When the cylinder-condensation and other wastes, and their variation with variation of the ratio of expansion, may be neglected, the "Equation of Ideal Steam-engine Efficiencies" may be written:

$$V = \frac{1}{E'''} = \frac{Arv_1 + Bv_1}{2RW_n} = \frac{A + Br^{-1}}{2R\left(p_1\frac{nr^{-1} - r^{-n}}{n-1} - p_b\right)}.$$

Where V may be called the counter-efficiency, and E''' is the ratio of work done to variable costs, and therefore, in the sense here adopted, the efficiency. This quantity becomes a minimum, and the best ratio of expansion and the corresponding mean pressure are obtained when, r being made the independent variable,

$$Ap_1\frac{n - nr^{1-n}}{n-1} - B(p_1r^{-n} - p_b) = 0; \; r^{-n} - M\frac{n - nr^{1-n}}{n-1} = \frac{p_b}{p_1}.$$

Here r has become r_e'''. A is the total annual charge per cubic foot of cylinder on engine account, B is the annual cost of steam per cubic foot *filled* each stroke, and is measured by $2Rwc$, when R is the number of revolutions of engine *per annum*, w the weight of a cubic foot of steam at the pressure p_1, and c its cost per pound, including all running expenses, in the boiler-room, and $M = \frac{A}{B}$.

More explicitly: since this problem demands minimum cost of a known power, and the ratio of expansion at Maximum Commercial Efficiency, we have

$$p_1v_1\frac{n - r^{1-n}}{n-1} - p_brv_1 = \text{Constant} = W.$$

The variable cost will be, as before,

$$P = Arv_1 + Bv_1,$$

which is to be made a minimum. But from the equation of condition, just given,

$$v_1 = \frac{W}{p_1 \frac{n - r^{1-n}}{n - 1} - p_b r}.$$

Thence

$$u = \frac{M + r^{-1}}{nr^{-1} - r^{-n} - \frac{p_b}{p_1}(n - 1)};$$

and the minimum is found, as above, when $\frac{du}{dr} = 0$; i.e., when

$$r^{-n} - M\frac{n - nr^{1-n}}{n - 1} = \frac{p_b}{p_1}.$$

The construction of this equation shows that, under the assumed conditions, this ratio for maximum commercial economy is not dependent simply on the size of engine or ratio of expansion; but in the real engine small cylinders have a higher value of p_b than large engines, are more subject to wastes, internally and externally, and have greater friction. They therefore require to be worked, under similar external conditions, with less expansion than large engines.

Thus the solution of the problem determining the ratio of expansion r_e''' and the mean pressure at "Maximum Commercial Efficiency, or Efficiency of Capital," Case 8, fixes the size of that engine which, doing the required work, will do it at least cost. The sum of all variable expenses being here made a minimum, the total running expense, which includes all invariable charges, also becomes the least possible, and the prescribed work is done at least total annual cost.

To find the ratio of expansion at which any given engine, already constructed and in place, Case 9, will give the largest amount of work for the unit of running expense, i.e., to determine the "Ratio of Expansion, r_e^{iv}, at Maximum Efficiency, of a Given Plant," we may use the same general equation. In

this case, the size of the engine being fixed, the whole annual "cost of engine" becomes constant, and we write the equation in precisely the same form as before,

$$V' = \frac{1}{E^{iv}} = \frac{A'rv_1 + Bv_1}{2RW_n}.$$

but making the symbol A' cover *all* annual expenses of the engine-room, estimated per cubic foot of cylinder, and including all *constant* charges of attendance in the boiler-room as well; while B now only includes those costs which are still variable with the steam-supply; V' thus measures the ratio of total annual expenses of operation to work done. We now obtain, by the same process as before, such a ratio of expansion that

$$r^{-n} - N\frac{n - nr^{1-n}}{n - 1} = \frac{p_b}{p_1}.$$

when N is a modification of M, such that it represents the ratio of the total expenses classed with engine-cost to the "cost of full steam," as already taken, and r has become r_e^{iv}.

Again: making A and M or N equal zero in the general equation, and making p_b the sum of useless resistances expressed as the intensity of pressure on the piston,

$$r^{-n} = \frac{p_b}{p_1}$$

and $r = r_e''$, the ratio of expansion at "Maximum Efficiency of Engine."

Similarly, if p_s is the actual back-pressure in the steam-cylinder,

$$r^{-n} = \frac{p_s}{p_1},$$

and we have the ratio of expansion at "maximum efficiency of fluid," $r = r_e'$.

To solve this problem, therefore, we are to determine the costs of steam, assuming the engine to work at full stroke, in-

cluding all incidentals dependent upon its quantity; make this the scale of measurement; find the total costs of engine in the same manner and on the same scale; ascertain the total constant annual or hourly expenses; introduce these quantities into our general equation, or our graphical construction, and solve for the required ratio of expansion. This determined, we are to find what size of engine, working at this ratio, will give the demanded power, and the problem is completely solved.

Should the size so determined be far different from that assumed in the estimates of costs and losses, a second approximation, based upon the new estimates of these quantities, will give a satisfactory solution.

In each of these several cases the expression obtained is derived, it will be noted, by making r the independent variable, and determined by the magnitude of the ratio of the two cost-items, and is the result, under the given conditions, independent of the actual size of the engine. Thus we determine, in each case, the ratio of efficiency which is correct, under the assumed conditions, for all engines of the class upon which our estimates are based. We thus are able now to *tabulate* the proper size of engine for assumed quantities of work, and the powers at which each engine, once set at work, will operate with maximum efficiency, commercial or other. Finally, comparing costs, it can be determined in any known case just when a change of engine will be financially advisable.

But this simple method of treatment cannot be applied where cylinder-condensation becomes a serious item; in fact, therefore, it is comparatively valueless for very many cases in engineering practice.

184. Rankine's Diagram of Efficiency.—For the ideal case, or any fair physical approximation, Rankine's graphical treatment of the problem here studied is conveniently applicable, and by its use the engineer may easily solve such problems by a simple construction on his drawing-board.

In illustration: Suppose an engine, of one cubic foot capacity, to be in operation, expanding steam adiabatically, its

cylinder and piston being impervious to heat, and the engine having an adjustable expansion-gear. When following full-stroke it uses one cubic foot of steam per stroke, at initial pressure; when "cutting off" at half-stroke, one half cubic

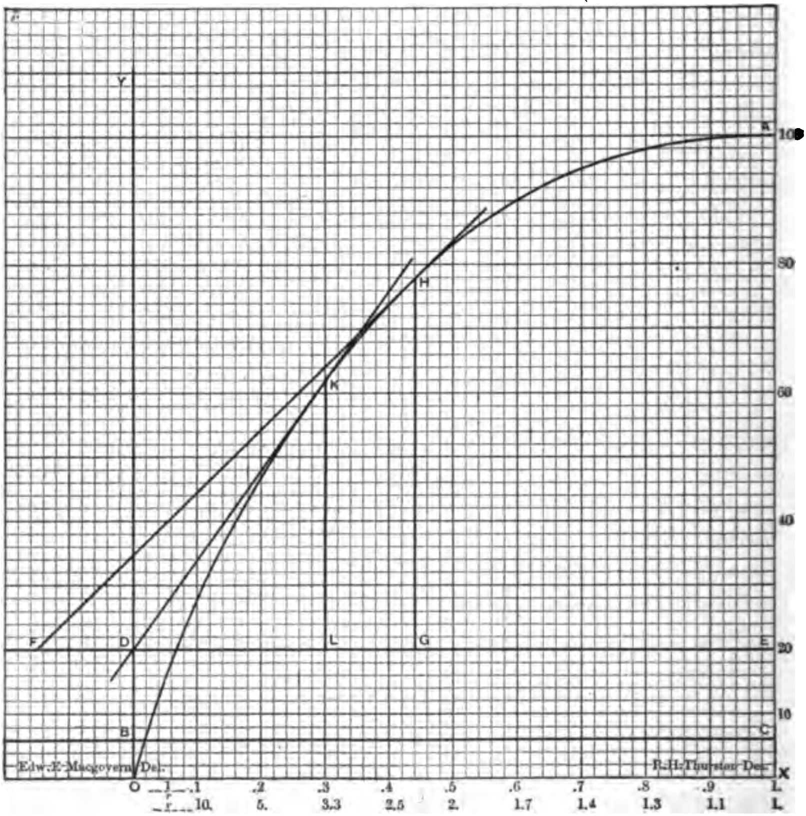


FIG. 166.—RANKINE'S EFFICIENCY-DIAGRAM.

foot, and at a cut-off of one quarter, one fourth of a foot, are used, the quantity used always being inversely as the ratio of expansion. To determine the best ratio of expansion: Construct a curve, *OA*, Fig. 166, of which the abscissas are proportional to the amount of steam used, while the ordinates are

proportional to the mean absolute pressure for that degree of expansion, and the "total work" of the steam so measured off. Drawing a line, *BC*, parallel to the base, and at a height proportional to the back-pressure in the engine-cylinder, the ordinate from any point in the curve down to this line will measure the corresponding "mean-effective pressure" shown by the indicator for that degree of expansion, and will be proportional to the "indicated power" of the engine. Again: Drawing a line, *DE*, at the height measuring the sum of all useless resistances, the "net" or "dynamometric" power of the engine, as transmitted to the machinery of transmission, is measured by ordinates between the curve and this line. Finally, extending this second line toward the left, and measuring off upon it a distance proportional to the cost of operation so far as it depends upon the plant, and measured on the same scale as that used in laying off *DG* on the base-line in terms of cost of steam, the sum of the two costs, as *GF*, measures the total expense of obtaining the power; while the height of ordinate *GH*, measured from the last drawn line, is proportional to the net amount of power obtained. For any one amount of constant expense, as determined by the location of the point, *F*, the line *FH*, drawn tangent to the curve, touches the latter at a point marking the ratio of expansion at maximum commercial economy, or if drawn from the axis *OY*, as *DK*, it identifies the ratio for maximum "efficiency of engine."

To solve this problem of maximum efficiency: Draw the mean-pressure curve *OA*, making the base-line, *OX*, a measure of all costs, "at full steam," variable with quantity of steam demanded by the engine, and the ordinates proportional to the mean pressure, corresponding to the cut-off. Draw a line parallel to the base, as *BC* or *DE*, at a height corresponding to the back-pressure plus useless resistances of engine.* Take *DF* equal to the unvarying costs, independent of steam-supply, on the same scale on which *DE* measures costs of full

* I.e., back-pressure plus mean-effective pressure as found on the "friction-diagram."

steam. Draw a tangent, FH, to the curve OHA and let fall a perpendicular from H to the base-line.

The point thus identified on OX will indicate the proper ratio of expansion for highest total commercial efficiency.

This simple and beautiful construction is correct and exact, when cylinder-condensation and other wastes of the real engine, as leakage, may be neglected. For other cases this construction may lead to widely inaccurate results. It is obvious that any accurate and reliable method must take account of *all* losses of heat, and must thus distinguish between efficient and inefficient classes of heat-engines.

185. Theory of Efficiencies of Real Engines.—The direct process of analytical treatment of this general problem for real engines, adopted by the Author, is the following:

Let it be known what style of engine is to be adopted, for any case, and what kind of boilers and attachments are to be used in supplying steam. Let the costs of attendance and all other expenses be ascertainable. Then, to adopt Rankine's terms, ascertain A, the annual variable "cost of engine" of the selected type, per cubic foot of steam-cylinder, and B, the annual variable "cost of boiler," per cubic foot of steam-cylinder supplied without expansion and without allowance for cylinder-condensation or leakage; ascertain all other costs, invariable with change of size of either engine or boiler within the range of the problem, and call their total C.

The "cost of engine" will be, as before, $Av_2 = Arv_1$; the "cost of boiler" will be Bv_1 and the constant charges C. Make $\frac{A}{B} = M$.

The work done per stroke may be called W_n, and work per annum becomes $2RW_n$.

The ratio of the total of annual variable costs of power to work done by the engine is

$$u = \frac{Arv_1 + Bv_1}{2RW_n} = \frac{Av_2 + Bv_2r^{-1}}{2RW_n},$$

which is a minimum when $\frac{M + r^{-1}}{W_n}$ is a minimum.

The value of W_n may here be obtained by multiplying the value of W'_n for adiabatic expansion, such as would be obtained in a non-conducting cylinder, by a factor variable with the ratio of expansion, as already shown, which shall measure the ratio of actual work done in the metallic cylinder to that performed with adiabatic expansion. Thus:

Let b represent the proportion of steam present in the working cylinder when $r = 1$, as reduced by the cylinder-condensation; let r^q represent the rate of variation of losses with increase of ratio of expansion; and let n be the index for the expansion-line of the mixture.

Then we shall have: $W_n = bp_1v_2\dfrac{nr^{-1} - r^{-n}}{n - 1}r^q - p_bv_2.$

The "*General Equation of all Steam-engine Efficiencies,*" therefore, now becomes

$$V = \frac{1}{E'''} = \frac{Av_2 + Bv_2r^{-1}}{2R\left(bp_1v_2\dfrac{nr^{-1} - r^{-n}}{n - 1}r^q - p^bv_2\right)}, \quad . \quad . \quad \text{(A)}$$

which becomes a minimum and makes the *Commercial Efficiency of an Engine*, for the required work, a maximum when, to obtain r_e''', we have made

$$r^q + \frac{q}{M(q - 1)}r^{q-1} - \frac{q - n}{n(q - 1)}r^{q-n+1}$$
$$- \frac{q - n + 1}{Mn(q - 1)}r^{q-n} = \frac{n - 1}{Mnb(q - 1)}\frac{p_b}{p_1}. \quad . \quad \text{(B)}$$

When the ratio of expansion, r_e^{iv}, at "*Maximum Efficiency of a Fixed Plant*" is required, Av_2 is constant, and we may make

$\dfrac{A + \dfrac{C}{v_2}}{B} = N$, and the equation for *Efficiency of Plant* becomes

$$V = \frac{1}{E^{iv}} = \frac{N + r^{-1}}{2B^{-1}R\left(bp_1v_2\dfrac{nr^{-1} - r^{-n}}{n - 1}r^q - p_bv_2\right)}; \quad . \quad \text{(C)}$$

and this gives, similarly, for r^{iv} and a maximum,

$$r^q + \frac{q}{N(q-1)}r^{q-1} - \frac{q-n}{n(q-1)}r^{q-n+1} - \frac{q-n+1}{Nn(q-1)}r^{q-n}$$
$$= \frac{n-1}{Nnb(q-1)}\frac{p_b}{p_1}. \quad \text{(D)}$$

To obtain r_e'' for *Maximum Efficiency of Engine*, we make $N = 0$, and have

$$\frac{q}{q-1}r^{q-1} - \frac{q-n+1}{n(q-1)}r^{q-n} = \frac{n-1}{nb(q-1)}\frac{p_b}{p_1}, \quad . \quad . \quad \text{(E)}$$

and to obtain *Maximum Efficiency of Fluid*, p_b becomes p_3, and

$$\frac{q}{q-1}r^{q-1} - \frac{q-n+1}{n(q-1)}r^{q-n} = \frac{n-1}{nb(q-1)}\frac{p_3}{p_1}, \quad . \quad . \quad \text{(F)}$$

in which r_e' satisfies the equation.

When $b = 1$ and $q = 0$, we have the *ideal* case considered in § 5, and the equation (B) for r_e''' becomes, as before, for the perfect engine,

$$r^{-n} - M\frac{n - nr^{1-n}}{n-1} = \frac{p_b}{p_1} \quad . \quad . \quad . \quad . \quad . \quad \text{(G)}$$

for Maximum Commercial Efficiency; and we again obtain for the ideal case of Maximum Economy of a Given Plant, for r_e^{iv},

$$r^{-n} - N\frac{n - nr^{1-n}}{n-1} = \frac{p_b}{p_1}. \quad . \quad . \quad . \quad . \quad . \quad \text{(H)}$$

For Maximum Efficiency of Engine we now again obtain a value of r_e'', such that

$$r^{-n} = \frac{p_b}{p_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(I)}$$

and finally for ideal Maximum Efficiency of Fluid we find a value of r_e' such that

$$r^{-n} = \frac{p_3}{p_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(J)}$$

precisely as already stated.

By making the assumption considered allowable by Mr. Buel and by Professor C. A. Smith, and apparently justified by the experiments of Emery and the work of the Author, as already remarked (Chapter V), the equations for the ideal engine and the Rankine diagram may sometimes be made to yield substantially accurate and satisfactory results. In such cases the internal wastes are taken as sensibly invariable for all ratios of expansion and can be reckoned as a part of the constant charge in A; and thus the value of FD, Fig. 166, or of M, is increased proportionally. As seen later, this value is usually 2 or 3 per cent in the exact case. M may become, by the addition of internal wastes, 12 or 15 per cent for unjacketed mill-engines, 8 or 10 per cent for jacketed simple engines, as low as 5 to 7 per cent for compound engines, and still less for the higher types. N will be thus increased to a figure 2 or 3 per cent larger than M, for non-condensing engines, in ordinary work, assuming the engines of at least two or three hundred horse-power, and 6 or 8 per cent greater for condensing engines, as seen later, in the tables.

The constants in the formulas should be carefully determined, if possible, by experiment on the class and the size and speed of engine to be designed; but, in the absence of better data, are taken by the Author with moderately large engines, at usual speeds, as follows, for good practice:

	b	q	n
I. Cylinders jacketed, steam superheated at boiler........................	0.90	0.	1.06
II. Cylinders jacketed, steam saturated, but dry at boiler................	0.85	−0.25	1.06
III. Cylinders unjacketed, steam saturated, but dry at boiler................	0.85	−0.3	0.98
IV. Cylinders unjacketed, steam slightly moist........................	0.80	−0.5	0.95

Case I is illustrated by the best work of well-known and successful builders. The value of b is obtained by comparing

the actual results of test with the figures for the perfect engine to determine the waste; that of n is obtained by assuming these engines effectively jacketed, the steam being retained dry and saturated throughout the stroke; and q is taken to be 0, since the rate of transfer of heat to exhaust seems to be nearly constant for such engines, as well as, for the usual ratios of expansion, of minimum amount. The second case is obtained by examining scattered records of somewhat less efficient engines. The values of b and q for III are obtained by studying the performance of good unjacketed engines; while the last, IV, came originally from the results of test of the U. S. S. Michigan, with an allowance of 10 per cent for the unrecorded waste concealed by re-evaporation. In all cases the variations in value, as determined by conditions already fully described (Chapter V) should be considered where the experimental data are taken from engines of a different class or size.

186. Curves of Efficiency for Real Engines.—The correct curve for the diagram, for actual engines, has not yet been expressed by any exact equation. It is very variable in location, in form, and in dimensions, and, as yet, can only be exactly determined by experiment.

In the diagram above given, as is evident, the quantities of steam laid down in arithmetical progression on the base-line cannot now correspond with the ratios of expansion there taken; since in actual engines those values are not in exact, or in constant, inverse proportion. The quantity of steam drawn from the boiler is not measured by the volume of cylinder open to steam up to the point of cut-off; nor is the mean pressure obtained with any given weight of steam drawn from the boiler at each stroke, even approximately, equal to that given by expansion in a non-conducting cylinder. Both these causes operate to depress and flatten the curve of efficiency, and thus, often, to reduce the ratio of economical expansion far below that predicted when the former and impossible conditions were assumed. The vertical scale of pressures and the horizontal scale of ratios of expansion have become altered in

relative magnitude, and the latter becomes for usual cases a variable scale.

To obtain a solution of the actual problem as presented daily to the designing engineer, a new method of procedure must be adopted. The Author has proposed the following :

187. Thurston's Diagrams and Curves of Real Efficiency.—It has become evident that the best ratio of expansion or proper "point of cut-off," and the mean effective pressure to be assumed in designing a proposed engine, for any actual case, is determined, not by the percentage of loss sustained at that point simply, or by the cylinder-condensation there taking place, but by the *method of variation* of such loss all along the curve of efficiency and at other ratios of expansion; since, in the metallic cylinder, the proportion of the water present in the working fluid is constantly varying with change of volume, and the loss of pressure and of work is constantly and proportionally varying, producing a curve of efficiency differing greatly in character, form, and location from that given by a non-conducting cylinder. It is obtained thus:

Assume for the unit of measure so much steam as is drawn from the boiler at one stroke of the piston, without expansion. Draw, Fig. 167, OX, and divide it, as unity of volume or of weight, into a scale of equal fractional parts. Erect at X a perpendicular, XAB, and divide it into any convenient number, say 100, of equal parts. Were there no condensation-wastes, the fluid being worked in a vessel of non-conducting material, instead of an iron steam-cylinder, the mean pressure at full stroke and the work done per cubic foot or per pound of boiler-steam would be measured by XB, and the curve of mean total pressures, or of steam used per "total" horse-power per hour, would be OWB.

Condensation reduces the work at full stroke, and it is actually measured by XA. Were the condensation in constant proportion for all values of the real ratio of expansion, the ordinates of the true curve would be proportional to those of OWB, and the values of $\frac{1}{r}$ would remain proportional to the

expenditure of steam, as in adiabatic expansion. But the amount of condensation usually increases, and often very rapidly, with increasing expansion, and at one half, one quarter, or one eighth cut-off more, and sometimes much more, than

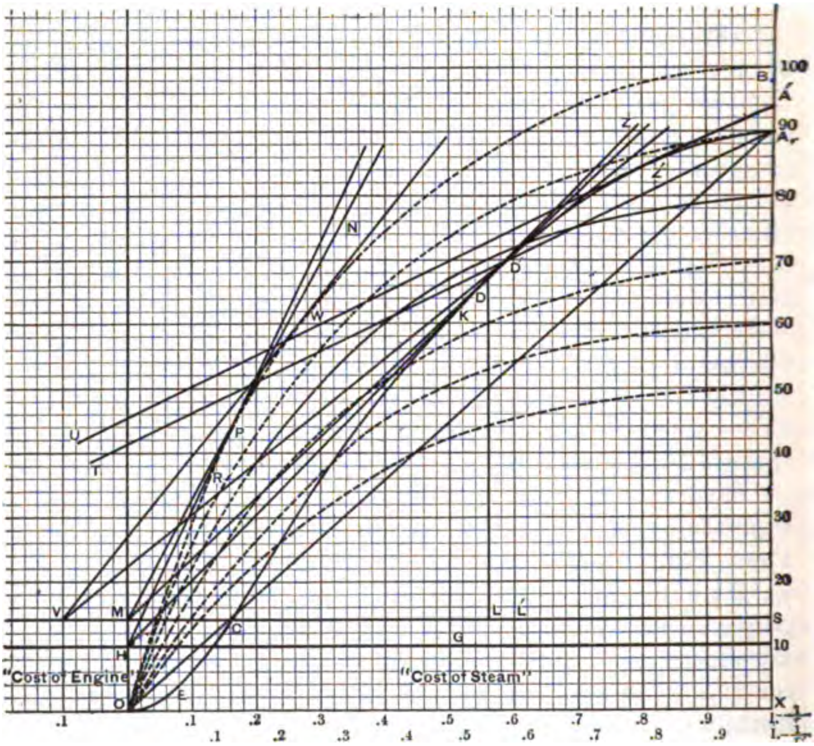


FIG. 167.—THURSTON'S REAL CURVES OF EFFICIENCY.

one half, one quarter, or one eighth as much steam is used as at full stroke. The scale of ratios, $\frac{1}{r}$, is thus not only shifted, but is made a scale of unequal parts, of which the successive values must be located by determining the amount of steam used at each point of cut-off, and placing the value $\frac{1}{r}$ opposite the value of the corresponding amount of steam expended, as has been done in Fig. 166.

It may be remarked here that if, as is sometimes under special circumstances nearly true, the losses by condensation and leakage, or both, are so great as to annul the benefit derived from expansion, the curve flattens down to a straight line, OA. In every engine a point is reached by increasing r, at which the amount of steam used per hour per total horse-power is as great as at full stroke; in every case, therefore, the true curve crosses the line OA, as at C. The line DCE is thus representative of the class of mean-pressure or efficiency curves given by actual engines. Could the variation of expenditure of heat be exactly expressed by an algebraic equation, this equation would be that of the line ACE, and the problem would be capable of exact solution by algebraic methods.

It will be seen that the employment of this curve for the real case by the method previously applied to the ideal case, in the solution of the actual problem, as practically meeting the engineer, results, primarily, in the determination of that quantity of steam per stroke, as a fraction of the conventional unit taken, which will yield the demanded power at minimum cost. The identification of the corresponding, required, ratio of expansion for maximum efficiency is effected after the solution of this problem is completed. The problem solved might have been thus stated:

Required, the quantity of steam, taken as a fraction of that used at full stroke, without either expansion or condensation, which should be worked per stroke to insure minimum total cost of the prescribed power.

This becoming known, the corresponding point of cut-off is at once determinable.

188. Solution of Problems for Actual Engines.—Draw HG at a height above OX, Fig. 167, equal to the back-pressure, p_1; then the tangent line HK identifies a point K, which gives the ratio of expansion and the mean pressure at *maximum efficiency of fluid*—since the ordinate GK measures the work done by the steam HG drawn from the boiler—and the ratio $\frac{GK}{HG}$ becomes a maximum at G. Drawing ML to represent

the pressure demanded to overcome all useless resistance, $p = p_1 + p_f$, a similar construction identifies D as the point corresponding to the ratio of expansion and the mean pressure at *maximum efficiency of engine.* Finally, extending this line to V and making VM proportional to cost of all running expenses, stated in terms of costs of engine and accessories per cubic foot of cylinder, $VM = \frac{A}{B} = M$ for the case of engine working at full stroke, the tangent line VZ meets the curve at a point, D', which gives the ratio of expansion and the mean pressure at *maximum commercial efficiency.* Comparing these values of r with those given by the tangents, HR, MP, VW, drawn to the curve OWB, for dry saturated steam expanded adiabatically, it is seen that the best ratio of expansion, and the mean pressure to be chosen, must be, in each actual example, less than in the hypothetical case, and may even become unity for each kind of efficiency, with very slow piston-speeds, where, were no loss of heat to occur in the manner here considered, considerable expansion would be desirable. These differences all become greater as the back-pressures and current expenditures become less.

Making the value of VM a measure, in the case of an engine in use, of the total current expenses, including the constant as well as variable items of cost, as of attendance, of rent, insurance, etc., which do not depend on size of engine, $VM = \frac{A'}{B} = N$, and a value of r will be obtained which is that real ratio of expansion at which *maximum work is done for a given expenditure*, per hour or per annum, on a plant actually established.

This problem is less frequently presented to the engineer than those already given, and is not the problem of maximum commercial efficiency; since, this ratio and the corresponding power of engine being determined, it will be found, on solving for maximum commercial efficiency, the "designer's problem" as the Author has called it, that another proportion of engine with higher ratio of expansion will supply the power

now demanded at still lower cost. To this new engine the last problem again applies, and the practical conclusion to be drawn from the solution of the interminable succession of problems of this last character which thus follow the first is that the largest amount of power possible should be entrusted to a single engineer, or "engineer's crew," and placed under one roof, etc. In this last case, all items become constant except those dependent upon the quantity of fuel burned.

Finally, the last of these problems may be solved.

To ascertain what ratio of expansion, what mean pressure should be adopted, and what amount of work, as a maximum, can be profitably obtained from an established plant: Compute the net power obtainable from the engine without expansion, and the market value, or otherwise real value to the proprietor, of that power, and estimate the cost of fuel and all items of cost variable therewith. Divide the price of power by this cost. Then lay off, on the base-line appropriate to the given engine, the distance SV, produced, equal to the quotient, taking the distance MS as unity, and from the extremity of this prolonged base-line draw a straight line, TA, to the point A, at the altitude AS equal to the measure of the net power just calculated. Finally draw a line, UA, parallel to this hypothenuse of the triangle so described, and tangent, as at Z', to the curve of efficiency. The point of tangency Z' will identify the *minimum profitable ratio of expansion*, and thus determine the maximum amount of work obtainable from this engine with profit. For, at this point of tangency the ratio of total cost of power to the price obtainable for it, or to its actual value, is that already given as the greatest permitting a fair profit, while the ratio of expansion so determined is that giving that power at that rate of cost.

The value of the *Ratio of Expansion at Maximum Profitable Power* is evidently, in all actual examples, less, and the work done is greater, than in either of the preceding cases, and is dependent upon the market value of that power.

In all cases, the ratio of expansion computed or determined is the *real* ratio; the *apparent* ratio is the former, decreased by

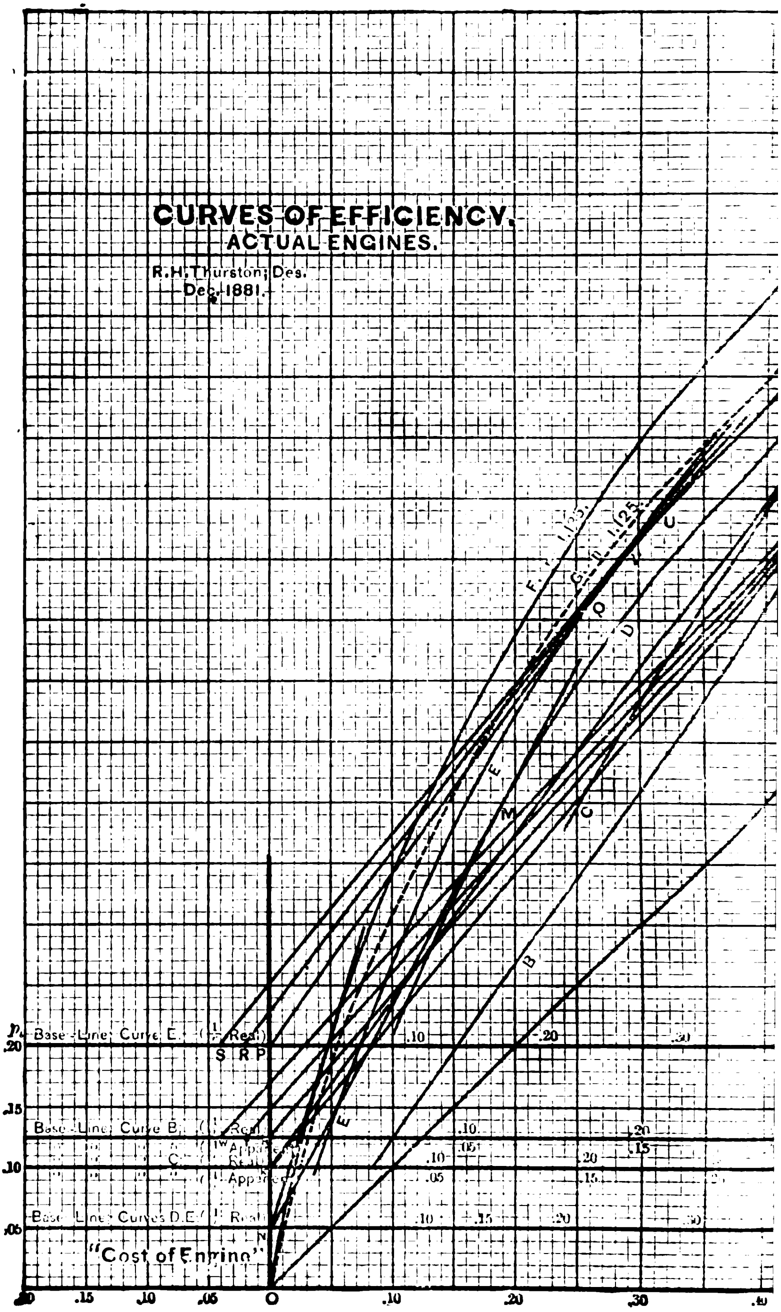
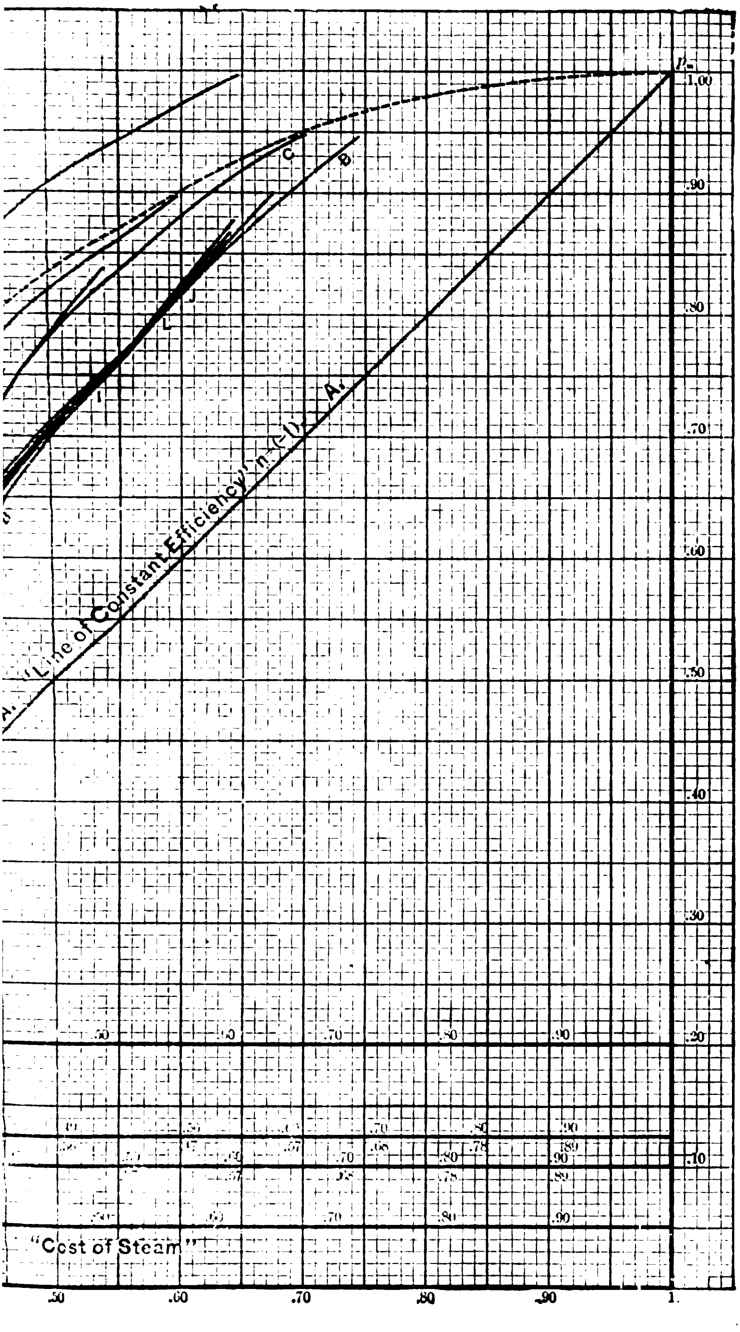


FIG. 166.—REAL CURV



F ENGINE-EFFICIENCIES.

Face page

at maximum commercial economy. The base-line, *VL*, Fig. 167, for maximum efficiency of engine being fixed, the position of the point *V* on that line is readily obtained, and thus the line *VZ* becomes known, and the ratio of expansion at maximum commercial economy is determined. Similarly, by extending the line *VL* until it becomes proportional to the sum of all costs, constant and variable, the ratio of expansion giving maximum work per dollar expended with the given engine may, if desired, be found.

The accompanying plate, Fig. 168, represents a series of real curves of efficiency, several of which are given by working engines. Such curves are here, for the first time, presented. The straight line A_1A, for the case in which $n = -1$, is the line of constant efficiency obtained in an assumed case of no gain and no variation of efficiency with increasing expansion from $r = 0$ to $r = \infty$. The curve marked *G*, and dotted, is the standard curve of efficiency for adiabatic expansion of steam containing initially ten per cent water ($n = 1.125$). The line *F* is the curve of mean pressure or of efficiency for steam initially dry ($n = 1.135$). The other curves are all obtained by reference to experiments on various classes of engines. *B* is the curve of efficiency for the common marine, unjacketed, single-cylinder, condensing engine; *C* is the curve of efficiency for the same engine using moderately superheated steam; *D* is that of a "compound" jacketed, condensing, marine engine; *E* applies almost exactly to both non-condensing engines and compound engines of the best classes, and the curve *F* is practically correct for the last-named class of engines when the steam is kept thoroughly dry by effective superheating, and by reheating in an intermediate receiver.

Curve *B* is thus obtained:

Collating Isherwood's with other experiments made for the United States Navy Department,* we find the following relative measures of steam-consumption at various ratios of expansion, and of work done by it:

* Researches in Engineering; vol. II, Table, p. xxxiv.

Cut-off $\frac{1}{r}$ (real)...............	.1	.3	.5	.7	.9	1.00
" $\frac{1}{r}$ (apparent)...........	.05	.25	.47	.68	.89	1.00
Relative weights of steam......	.16	.41	.60	.76	.92	1.00
" "total work" done....	.21	.56	.82	.97	1.00	1.00

The base-line, B, for this case, in which $\frac{p_b}{p_1} = \frac{1}{8}$, is drawn on the plate, and on this line are a set of values of $\frac{1}{r}$ corresponding to the relative weights of steam as laid down on the bottom-scale, .10 above .16, .30 above .41, etc., etc., and the ordinates erected at these points are made proportional to the mean pressures and the total work done at those ratios of expansion; and, thus carefully laying down these points, the line B_1B is constructed as the curve of efficiency for the engine, of which those of the United States steamers Eutaw, Michigan, and all "American river steamboat engines" are representatives.

In a similar manner, by collating the data obtained by the trial of the Georgiana's engine, using superheated steam, with the experiments of Hirn showing a reduction of exhaust waste by superheating, we obtain the curve of efficiency C_1C and the base-scale accompanying it. A set of experiments on the Bache gives the line D_1D, and the curve E_1E is found, by trial, to meet cases of good work with non-condensing engines, unjacketed, but worked at high piston-speed, and of some of the very best results obtained with compound engines of the most successful types. Curve F seems to meet those cases in which superheating has been so efficient as nearly to prevent all condensation, and the line corresponds closely with the adiabatic for steam, dry initially, and only condensing so much as is due to the performance of work.

In the last figure, the straight line, A, may be taken as measuring the work done in the engine up to the point of cut-off, to which work its ordinates are proportional; while the line of adiabatic mean pressures gives, similarly, the total

work, and their difference the gain by expansion. The several curves exhibit the extent to which this gain is affected by wasteful conditions in the ideal and the various forms of real engine represented.

To obtain an exact solution of these problems, the quantity of steam present in the cylinder at the point of cut-off must be precisely measured and compared with the quantity sent to the engine from the boiler.

190. **Method of Use of the Diagram.**—Comparing curves *F* and *G*, Fig. 168, representing the case of steam expanding in a non-conducting cylinder, i.e., adiabatically, with the other curves, obtained for expansion in real engines, it is seen, at a glance, that the more perfectly exhaust-waste by cylinder-condensation is guarded against, the more closely does the actual engine approach to the perfect engine in its utilization of steam, and the less effective the provision against such loss, the more widely does the curve of efficiency depart both in location and form from the ideal curve, finally approximating to the straight line of constant efficiency A_1A. While the best engines approach comparatively near the curve of maximum possible efficiency, the great majority of condensing engines in use are of the class represented by that giving curve *B*; which latter is, however, by no means a case of remarkably low efficiency. In many cases the curve will be found to fall within the line *B*.

Selecting one of these curves, as *B* or *C*, we may solve either or all of the problems already defined by merely applying a straight-edge to the diagram. For *B* we have $p_1 = 40$; $p_b = 5$; $\frac{p_b}{p_1} = \frac{1}{8} = 0.125$. To determine the best mean pressure and ratio of expansion at maximum efficiency, draw the base-line at the altitude 0.125, and from its junction with the ordinate at the zero point draw the line *HI* tangent to the curve; it touches the curve at *I* and the corresponding mean pressure and ratio of expansion on the base-line beneath is a trifle less than $\frac{1}{r} = 0.4$; $r = 2.5$ nearly—a result confirmed by reference to the original data.

Next ascertain the hourly or annual cost of supplying the engine with steam when worked without expansion, including all items of expense variable with the quantity of steam used, and determine the *variable* part of all running expenses in the engine-room, including interest, insurance, rent, cost of oil, and so much of the wages of the attendants as is properly taken as variable with the size of engine. Suppose, as in a case taken by the Author, that the latter is found to be two per cent of the former, $M = .02$.

From the point T, at the ordinate .02, on the left of the H, draw the tangent to the curve, as TL on the curve B; its point of tangency identifies the best mean pressure and ratio of expansion for commercial efficiency.

Similarly compare the "cost of full steam" with the sum of all other running expenses chargeable to the plant; if the ratio is $N = .04$, draw the tangent line WL from the ordinate .04, and thus find that ratio of expansion which will give most work for the money expended on a plant already installed. The lines PQ, RV, and SU thus determine these three ratios for the curve F, of a well-constructed non-condensing engine, using perfectly dry steam and with a ratio $\frac{p_b}{p_1} = 0.20$. The line NM determines the best mean pressure and ratio of expansion at maximum efficiency for the case D, a compound engine doing good work with $\frac{p_b}{p_1} = .05$.

191. Estimation of Expenses.—The following example illustrates, in detail, the calculation of values of M and of N:

Rated power of given engine and boiler...........	500 H. P.
Working time, per annum........................	3,000 hrs.

(*A*) *Costs of engine* (*variable with size of engine*).

Cost of engine (approximate)............	$10,000	
Annual interest at 6 per cent......................		$600
" cost of repairs and depreciation, 4 p. c..........		400
" " " materials used........................		50
Total annual cost..................................		$1,050

(*B*) *Costs of boiler* (*variable with demanded boiler-power*).

Cost of boiler: actual (approximate)..........$12,000
for "full steam".............. 24,000
Interest on cost, using steam without expansion, at 6 p. c. $1,440
Repairs and depreciation at 15 per cent............... 3,600
Minor expenses per annum, say...................... 200

Total annual maximum cost....................$5,240

(*C*) *Fuel Account* (*variable with size of boiler*).

Coal, per year at the rated power................ 2,000 tons
" " " with no expansion............... 4,000 "
Cost of fuel at "full steam," at $5 per ton.... ...$20,000
" " " transportation and storage at 50c..... 2,000

Total maximum per year.................. $22,000

(*D*) *Attendance* (*wholly or partly constant, or variable*).

(*a*) "Engine-driver's" (engineer's) pay, per year.........$1,000
(*b*) "Firemen's" (stokers') pay, per year ("full steam").... 1,200

$2,200

(*E*) *Incidentals* (*constant as a rule*).

Rent, taxes, insurance, etc., per annum.................$1,000

Studying the statement of costs, the designing engineer decides in each case, and for each problem presented, how the items should be grouped. For the case of a stationary steam-engine, such as is here presented, he would find

$$M = \frac{A}{B + C} = 0.035, \text{ nearly,}$$

if the costs D_a, D_b are not variable within the probable range of variation of expansion; and

$$M = \frac{A}{B + C + D_b} = 0.03, \text{ nearly.}$$

Assuming cost of fire-room labor variable with quantity of steam demanded,

$$N = \frac{A + D + E}{B + C} = 0.15, \text{ nearly,}$$

for the first case, and

$$N = \frac{A + D_a + E}{B + C + D_b} = 0.10, \text{ nearly,}$$

for the second case. In marine engineering, storage becomes an important matter, in items A and D, and in B, as well as very important in C and E, since every cubic foot occupied by machinery, fuel, or attendants displaces a cubic foot of paying load. With very large powers, the items D both become to a certain extent variable, the one, D_a, with magnitude of the whole plant, the other, D_b, with quantity of fuel burned. Correctness in making up the bill of costs will be found to be absolutely essential.*

192. **Statement of Results.**—Laying out these curves on a conveniently large scale and proceeding as just indicated, the Author obtained the results exhibited in Table I, here given. Cases I to VI, inclusive, are obtained from curve E; VII to XII from curve B; XIII to XVIII from E; and XIX to XXIV from the best curve of efficiency, on the plate, F.

The ratio of expansion at Maximum Efficiency of Fluid will be found in column r_e', that at Maximum Efficiency of Engine under r_e'', and the Best Ratio of Expansion for Commercial Efficiency, or for Maximum Efficiency of Capital, is given under r_e'''; M, N, are the ratios of cost. Comparing the first, and especially the second, set with the last, the enormous variation due to cylinder-condensation is readily appreciated. Even the last case is far from the efficiency of the perfect engine.

* For a considerable amount of data in this field, see the concluding chapter of Part II.

The mean pressures and ratios of expansion for superheated steam in the unjacketed (Fig. 168) cylinder are obtainable from curve *C*. Here

p_1 = initial pressures measured from perfect vacuum;
p_2 = back-pressure in cylinder;
p_b = same plus friction;
M = ratio of variable part of cost of engine to variable part of cost of steam, when $r = 1$.

The values here presented for these several cases are not to be taken as exact for other examples, but must always be corrected, in the simple ways already described (Chap. V) for variations of size, speed and temperature variations. They are given as representative illustrations, and the engineer designing new engines should, whenever possible, construct his own more exact diagram and make his own solution of the problem before him.

The determination of the last of the several ratios in the table constitutes the solution of the "Designer's Problem." To finally settle the size of the engine to be designed, on this commercial basis, it is only necessary to ascertain what size of engine, working at the determined ratio of expansion for maximum commercial efficiency, will perform the specified required work.

We have the power required,

$$\text{H. P.} = \frac{p_e A V}{33{,}000};$$

in which the power is prescribed, and the value of p_e, taken as the mean effective pressure corresponding to this power, is known from the stated conditions of the problem and the value of the now determined ratio of expansion; while the velocity, V, of piston is exactly or approximately known, or may be assumed; then the area of piston is

$$A = \frac{33{,}000 \text{ H. P.}}{p_e V}.$$

TABLE I.

RATIOS OF EXPANSION AT MAXIMUM EFFICIENCY OF FLUID, OF ENGINE AND OF CAPITAL.

SINGLE CYLINDERS.

Absolute Initial Pressures.			Case No.	Class I. Non-condensing, High Speed.							Case No.	Class II. Condensing, Moderate Speed.						
p	p_m	Atmospheres.		p_3	p_b	M	$\frac{p_1}{p_b}$	r_e'	r_e''	r_e'''		p_3	p_b	M	$\frac{p_1}{p_b}$	r_e'	r_e''	r_e'''
40	2.8	2⅔	I	18	20	.02	2	2	2	2	VII	3	5	.04	8	2½	2¼	2
60	4.2	4	II	18	20	.02	3	3	3	2¾	VIII	3	5	.04	12	3½	3¼	3
80	5.6	5⅓	III	18	20	.02	4	4	3¾	3¼	IX	3	5	.04	16	4¼	4	3½
100	7.0	6⅔	IV	18	20	.02	5	5	4½	3½	X	3	5	.04	20	4¾	4½	4
120	8.4	8	V	18	20	.02	6	6	5½	4	XI	3	5	.04	24	5½	5	4½
150	10.5	10	VI	18	20	.02	7½	7	6	4½	XII	3	5	.04	30	6	5¼	5

COMPOUND, CONDENSING, JACKETED.

Absolute Initial Pressures.			Case No.	Class III. Saturated Steam.							Case No.	Class IV. Superheated Steam.						
p	p_m	Atmospheres.		p_3	p_b	M	$\frac{p_1}{p_b}$	r_e'	r_e''	r_e'''		p_3	p_b	M	$\frac{p_1}{p_b}$	r_e'	r_e''	r_e'''
40	2.8	2⅔	XIII	3	5½	.04	7	6	5	3	XIX	2½	5	.05	8	8	6	5
60	4.2	4	XIV	3	5½	.04	11	8	7	4½	XX	2½	5	.05	10	11	8	6
80	5.6	5⅓	XV	3	5½	.04	14	9	8	6	XXI	3	5½	.05	13	14	10	7
100	7.0	6⅔	XVI	3	6	.04	17	10	9	7	XXII	3	5½	.05	18	16	12	8
120	8.4	8	XVII	3	6	.04	20	11	10	8	XXIII	3	5½	.05	22	20	15	9
150	10.5	10	XVIII	3	6	.04	25	13	10	9	XXIV	3	6	.05	27	25	17	10

The value of A being thus obtained, in terms of power, mean pressure, and velocity of piston, the diameter of piston and length of stroke are readily settled.

Further investigation will, undoubtedly, sooner or later, establish the curves of efficiency for all standard types of engine and for those special cases for which the engineer can

day only obtain them approximately. Meantime, the plate exhibits a range of variation of curve which extends completely across the field of every-day practice; and an experienced engineer can trust his judgment in the interpolation of the curve of efficiency for any special case arising in his own practice. For example: Cases of best practice in which the engine is worked at higher speed, and with a warmer condenser, and having less friction, will, when corrected for any differences of size, speed, and range of expansion or temperature, give a curve for the class from which *B* was obtained which will fall between *B* and *C*.

The values given of $\frac{p_1}{p_b}$ are interesting in comparison with the values of r_t, as exhibiting the enormous difference between the best ratio of expansion in actual work and the ratio giving maximum efficiency in the ideal case, and also as strikingly presenting to the mind how far we are still, in actual practice, from even an approximation to the conditions exhibited in the perfect, ideal, engine.

TABLE II.

RATIOS OF EXPANSION GIVING MAXIMUM WORK AT MINIMUM COST FOR A GIVEN PLANT OF KNOWN PROPORTIONS.

	Class I.						Class II.					
Cases......	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
N.......	.04	.04	.04	.04	.04	.04	.10	.10	.10	.10	.10	.10
r_t^{iv}......	$1\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{3}{4}$	$3\frac{1}{4}$	$3\frac{1}{2}$	4	$1\frac{3}{4}$	$2\frac{1}{2}$	3	$3\frac{1}{4}$	$3\frac{1}{2}$	4

	Class III.						Class IV.					
Cases.....	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX	XXI	XXII	XXIII	XXIV
N.........	.10	.10	.10	.10	.10	.10	.12	.12	.12	.12	.12	.12
r_t^{iv}.......	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{4}$	$4\frac{1}{2}$	$4\frac{1}{2}$	$4\frac{3}{4}$	4	$4\frac{1}{2}$	$4\frac{3}{4}$	5	5	$5\frac{1}{4}$

Table II gives values, similarly obtained for the cases taken, of that ratio of expansion which gives a maximum quantity of work for the unit of value with a fixed arrangement of plant. These values are seen to be very much

smaller than the ratios for maximum commercial efficiency; and, although they may give more work for such unit than the higher ratios just determined, they do not give maximum efficiency of capital. For:

Assume the engine working at this closely adjusted ratio for the now given power, still more work will be given for the unit of cost if the value of r be *increased* by replacing the given engine by a larger one, in many cases, or in any case by speeding up the engine, or otherwise doing the larger amount of work with a new and higher ratio of expansion. The Author has sometimes accomplished this latter result by both speeding up the engine and carrying higher steam, with an automatic adjustment of expansion. The real limit to this increase of work done by the given engine is determined by quite other considerations than those above noted. It is determined by the money value of the power obtained, and this increase of power finds a limit, as has been seen, only when either the limit of safety in working engine or boiler is reached, or when the money made by the use of additional power is insufficient to pay a fair profit on the additional expense incurred; which latter limit may be obtained at a value of r^{iv} either equal to or less than r_e.

The radical distinction between the problem of maximum efficiency of capital (8) and maximum commercial efficiency of a given plant (9), § 174, is here well brought out by this difference of results. Comparing Nos. 7, 12, 13, and 18 of Table I with the same in Table II, it is seen that, instead of ratios of 2, 5, 3, and 9, we have 1.75, 4, 2.5, and $4\frac{3}{4}$; results which, while absurd as solving the "designer's problem" (8), are perfectly satisfactory as a solution of the "owner's problem" (9).

193. **Relation of Costs and Profits.**—Table III exhibits the effect of variation of actual value of the power in determining the maximum amount profitably obtainable from any engine.

For example: Suppose the cost of a horse-power to be, as is frequently the case, about equal to the cost of fuel (in the furnace) producing that power without expansion; then calling

this value p_m and this cost p_c, the base-line of the diagram will be extended until it measures $\left(\frac{p_m}{p} = 1 = N\right)$ twice the length of OX, and the angle made by the line from its extremity OA, Fig. 168, makes an angle $\theta = 30°$ with OX. On the large-scale drawing, set the triangle against the edge of the T-square, and adjust it to the line here given; find by shifting it along the blade that point on the selected curve of efficiency at which a parallel tangent can be drawn, and then the ratio of expansion, r^v, answering to this case, is found

If an engine, IV of Class I, is selected, it is found to be $r^v = 2\frac{1}{2}$; if No. VII of Class II, $r^v = 2$, etc., etc., as in Table III.

It is particularly interesting and instructive to observe how the importance of waste, as of cylinder-condensation, in its influence on the best ratio of expansion, here diminishes with decreasing expansion, and that, finally, the most economical and the least efficient give nearly identical figures when the point of cut-off approaches half-stroke.

TABLE III.

Effect of Variation of Ratio of Market Value to Cost of Power. Maximum Limiting Values of r^v.

				N'	0.40	0.50	0.60	0.70	0.80	1.00
Class	I	No.	IV		....	....	....	....	3	$2\frac{1}{2}$
"	II	"	VII		....	....	....	....	....	2
"	II	"	X		....	....	....	....	....	2
"	III	"	XV		....	7	5	4	3	$2\frac{1}{2}$
"	III	"	XVII		....	7	5	4	3	$2\frac{1}{2}$
"	IV	"	XXI		9	7	6	4	3	$2\frac{1}{2}$
"	IV	"	XXIV		10	7	6	4	3	$2\frac{1}{2}$
θ					22°	27°	31°	35°	39°	45°

Taking the cost of fuel, *in the furnace*, for the engine working without expansion, at \$50 per annum per horse-power, the above table gives the ratio of expansion below which a loss

will accrue when the cash value of the horse-power is 20, 25, 30, 35, 40, and 50 dollars. At these ratios of expansion, all that is received for power above these sums is profit.

For other costs, the prices obtained must be correspondingly varied to secure a profit.

194. **Profits at any Fixed Expansion.**—Other problems, the converse of the last, may be solved by this construction: "What is the maximum price which can be paid for power without loss at any *given mean pressure or ratio of expansion?*" "What profit is obtainable at a given cost?" "What total cost makes any given ratio of expansion the most economical?"

To solve these problems, draw an ordinate to the line of mean pressures, or the curve of efficiency, at the assumed ratio of expansion; then the abscissa measures the cost, in terms of full steam, of the power measured by the ordinate, above which loss will accrue, when $M = 0$. The difference between the total cost and the higher price measures the profit obtained if the power is sold at the larger figure.

Table IV exhibits the variation of the relative maximum allowable cost of power, with variation of the ratio of expansion; actual cost of expenses variable with fuel, with ratio unity being taken as the unit.

TABLE IV.

Maximum Limit of Relative Allowable Cost. Most Economical Ratio of Expansion assumed as r. Cost of Full Steam = Unity. M or N = 0.1.

			r	1	2	3	4	5	6	8	10
Class	I	No.	IV	1.1	.80	.75	.75	.85	.85	...	...
"	II	"	VII	1.1	.80	.85	1.1	...	...	...	...
"	II	"	X	1.1	.75	.80	.95	...	...	...	...
"	III	"	XV	1.1	.75	.70	.70	.75	.80	.90	1.1
"	III	"	XVII	1.1	.75	.70	.70	.70	.70	.75	.90
"	IV	"	XXI	1.1	.75	.70	.90	.65	.70	.75	.90
"	IV	"	XXIV	1.1	.75	.70	.65	.65	.55	.55	.65

195. Cost of Engine as affecting the Best Ratio of Expansion.—The effect of variation in cost of engine now becomes of interest, and indeed a matter of real importance to the designer. Studying cases arising in practice, he will probably find the value of M or N to fall between .02 and .15, as in those selected above, but it will probably rarely, if ever, exceed 0.20.

The curve being established correctly for any given engine, it becomes the easiest possible matter to determine the effect of variation of this ratio. Table V gives such results as seem most instructive, from the cases here studied.

TABLE V.

Effect of Variation of "Engine-cost Ratio." Best Values of r_e''' or r_e^{iv}.

			M or N	.02	.04	.06	.08	.10	.15	.20
Class	I	Example	IV	3½	3¼	3	2¾	2¾	2½	2¼
"	II	"	VII	..	2	2	1¾	1¾	1⅝	1½
"	II	"	X	..	4	3¾	3½	3¼	2¾	2½
"	III	"	XV	..	6	5	4¼	4	3½	3
"	III	"	XVII	..	8	6	4¾	4½	3¾	3¼
"	IV	"	XXI	..	6½	6	5½	4¾	3¾	3¼
"	IV	"	XXIV	..	9	7	6	5	4	3½

These differences in the value of the mean pressure and ratio of expansion at maximum commercial efficiency are least where the exhaust wastes are greatest, and as their absolute values become smaller. Cases IV, X, XVII, and XXIV have the same initial steam-pressure and are seen to approximate toward the same value of r_e as the value of M or N becomes greater, becoming, for the first two, and for the last two, nearly equal to the maximum value here taken.

It is obvious that the value r_e becomes a good gauge of the economical value of the engine and of its type, and that the greater these values, other things equal, and the nearer r_e'', r_e''', r_e^{iv} approach each other, in any given engine, the better the design.

It is now seen that we have here a method of determining the effect of variations of single variable quantities, while retaining all others constant—a method very greatly needed, but hitherto unknown.

The case just taken is an illustration of its application. The following is another instance of no less importance:

196. Back-pressure as Modifying Economy.—The Effect of Variation in Back-pressure may be studied, by means of this method of investigation, with the same facility.

Table VI exhibits this effect for a wide range of cases.

TABLE V.

Effect of Variation of Initial Pressure and of Back-pressure. Best Values of r_e'.

			$\frac{p_b}{p_1}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{20}$
Class	I	No.	IV	$2\frac{3}{4}$	$3\frac{1}{4}$	$3\frac{1}{2}$	$3\frac{1}{2}$	..	..	..	..
"	II	"	VII	$1\frac{1}{2}$	$1\frac{5}{8}$	$1\frac{5}{8}$	$1\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{1}{2}$	..	..
"	II	"	X	..	..	..	$1\frac{3}{4}$	$2\frac{1}{4}$	$2\frac{1}{2}$	3	4
"	III	"	XVI	..	..	..	$4\frac{1}{2}$	6	7	9	11
"	IV	"	XXII	..	..	..	6	6	8	12	15

These differences in value of r_e are obtained on the assumption that cylinder-condensation and all other conditions remain unchanged while variation occurs in the back-pressure. In all actual cases, the differences would be reduced by the fact that increased condenser-pressure and the reduction of chilling effect which comes with increase of back-pressure so check exhaust waste that the ratio for maximum efficiency becomes somewhat increased and these differences of ratio are thus lessened. The gain from this and other causes becomes sufficient at high pressures to justify the use of the simpler and less expensive non-condensing engine; it will be best appreciated after comparison of Class I with Class II. An independent solution of every actual problem is always desirable.

197. Deductions.—In illustration of the use of this method and of the application of the results, we may observe as in

Table I values of the ratio of expansion for maximum efficiency for any standard type of engine. Thus: Case III is that of an ordinary, standard, non-condensing, drop cut-off engine, steam 65 pounds ($5\frac{1}{3}$ atmospheres) by gauge, and the cut-off occurs, properly, at a little inside $\frac{1}{4}$ stroke, Case V is the same with steam at 105 by gauge (8 atmospheres), and its valve should close a little inside $\frac{1}{6}$ stroke. For maximum *commercial* efficiency those engines should "cut off" at about $\frac{1}{3}$ and $\frac{1}{4}$ respectively. In the second class, Case VII is that of the old naval or modern very low-pressure river-boat engine carrying 25 pounds of steam by gauge ($2\frac{2}{3}$ atmospheres). The valve should drop so as to completely shut off steam at about half-stroke to give minimum expenditure for coal, and a little later to give minimum cost on total account,* a result already reached by the builders of such engines. Case VIII is that of some of our old Hudson River steamboats (steam 45 by gauge), and these two ratios are found to be a little greater and a little less than 3. The irregularity of wheel which a short cut-off produces, however, makes it inadvisable to expand as much as this, even. Case IX is often seen in mill-engines; its valve closes at $\frac{1}{4}$ and $\frac{1}{3}$ for the cases taken. Above this pressure, a comparison of Class I with Class II shows that in the cases taken the non-condensing engine is about as économical as the other—a conclusion justified by Isherwood's comparison of Corliss engines †—but comparing values of $r_{\prime}'''$ it is seen that the condenser may probably be exchanged for the heater with Classes III and IV only at some very high pressure not yet attained with jacketed engines of good design, while the ten per cent gain obtained at the boiler by the higher temperature of feed given by the heater of the non-condensing engine, together with the differences in size of cylinder, brings down the pressure at which total efficiency becomes a minimum to some

* Engines of this class by good builders, having the "Stevens valve-gear," close the valve at 6 feet on a 10 feet stroke, which, allowing for a little throttling, gives exactly this figure. Those fitted with the "Sickles cut-off" drop the valve as near half-stroke as possible; they cannot "follow" further.

† Journal Franklin Institute; Sept. 1881.

lower figure which may be determined, by the method here given, for any given case.

Cases XV and XVI are often illustrated on transatlantic steamers and by good compound pumping-engines. The cut-off takes effect at $\frac{1}{8}$ or $\frac{1}{9}$ for maximum efficiency of engine and fuel, and at $\frac{1}{6}$ or $\frac{1}{7}$ for most economical expenditure of money,* figures already settled upon by the most successful builders. Cases XXII and XXIII represent the most advanced practice in the use of high steam pressure, superheated steam, and re-heating at the intermediate receiver, as is done in the pumping-engines of Cowper, Corliss, and Leavitt. The best ratios of expansion are 12 and 15, if measured by duty attained and fuel saved, simply, and two thirds those values give maximum efficiency of capital. Case XXIV represents most nearly that of Corliss' best pumping-engine, which lies between XXIII and XXIV; its best ratio of expansion lies between 9 and 10, if the curve of efficiency here taken for Class IV suits that case. If nine is the *real* ratio, the *apparent* cut-off will be nearly at one tenth, while for maximum efficiency of engine and maximum "duty" the valve should drop at about one-sixteenth stroke.

It should be kept in mind that the measure of cost, in all problems relating to expense, as here treated, is the total cost per annum, without expansion, of all items of Class 3, i.e., variable with variation of steam-supply.

The problem illustrated by the cases taken up in Table III is of rare occurrence. The following are two such cases:

(1) Where the proprietor of an engine can rent power from an engine already set up, having boiler-power sufficient to supply an ample amount of steam he will obtain the best return from his invested capital by delivering so much power at remunerative prices as will give the values r_e^{iv}, found in Table II. Cases IV, V, and VI are among the most usual, the best point of cut-off averaging about $\frac{1}{8}$ stroke.

Had this quantity of power to be demanded been originally

* *Vide* Clark's Manual for Mechanical Engineers, pp. 888, 890.

known, however, the proprietor would have done better to have ordered, at the first, a larger or a faster running engine with a higher ratio of expansion, and would usually find it economical to alter the engine here assumed to be used—in the manner already described—if possible, so as to deliver the maximum power, working at the shorter cut-off.

(2) The second is that of a naval engine intended to work with maximum efficiency at low power, or on long runs, and only requiring high power for short periods of time. It has sometimes been customary to design such engines to work with high ratios of expansion while cruising, and to develop full power with less expansion when in action, supplying a fan-blast for the latter occasion. For such cases the best ratio at low power would be r_e'', and it might be well to make the expansion variable through as wide a range as from r_e'' to r_e^{iv}, taken with extreme values of M and N. As already stated, in all ordinary work, the ratio of expansion at maximum commercial efficiency is the ratio of expansion to be adopted for any engine.*

The values here given for M and N are based on cost of fuel taken at $5 per ton. The value of the ratios of expansion at maximum efficiency will be less at lower prices and greater at higher costs, the expenses of maintenance of plant being constant, since the values of cost of steam will be directly, and of M inversely, as the price of fuel. With coal at ten dollars per ton, M will be practically one half the figures given above, and the least ratio of expansion correspondingly increased as per Table VI.

Table III may be consulted by the owner of steam-power for cases which, as is usual, fall within the given limits. For exceptional cases he, or his consulting engineer, can, when data are obtainable, always make his own curve of efficiency and obtain a practically exact solution of the case presented.

The curve, B, in the last group of efficiency-curves, may be taken as fairly approximate for simple locomotives.

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* Where coal displaces paying cargo, the best ratio of expansion is greatly augmented.

Mr. H. J. Hotchkiss has collated for the Author a considerable amount of data from reports on the practice of railways in the United States, for the purpose of solving these problems.* Taking the value of engine as \$8000, of which 45 per cent, \$3600, is charged to boiler and tender, yearly mileage 33,000, life locomotive, 25 years, evaporation 7 to 1, the engine costs per mile \$4.50, boiler charges at "full steam" \$4.00, coal per mile 10.5 cents, labor 7.4 cents, $M = 0.32$, and the problem of the designer being solved, the ratio of expansion at maximum commercial efficiency should be $r_e = 0.2$, nearly, and the engine should be given such size and proportions that it may do its ordinary and average work at that point of cut-off. Once constructed, however, it may be employed, with gradually increasing loads, under similar conditions as to costs until its steam is "following" as far as 0.7 stroke and continuously pay better and better, but yet never as well as an engine precisely adopted by the designer for the heavier work.

A very similar case gives:

Cut-off for maximum efficiency of fluid........	0.40
" " " " " engine......	0.48
" " " commercial efficiency.....	0.63
" " " work and "	0.75

the values of the coefficients being $M = 0.27$; $N = 0.89$.

The assumed conditions may be taken as representing a common set for their data, in the United States and Canada.

The following are figures obtained in 1891 in securing the required data for the solution of the "designer's problem" of Chap. VII, Part I.

Three types of engine were proposed for driving the electric machinery of a street railway: (I) simple non-condensing; (II) simple condensing; (III) compound condensing. Their power, market value, etc., were, respectively, as in the table:

* For much of most valuable data, Wellington's Railway Location has been referred to.

COSTS AND POWER OF ENGINES.

Type	I	II	III
I. H. P., rated	105	105	112
D. H. P., "	95	95	96
Cost per I. H. P., in place	$24	$28	$39
" " " transmission	2	2	2
Total cost	$26	$30	$41
Cost of boilers, set, per H. P.	$14.00	$12.00	$9.00
" " chimney, etc.	7.00	6.50	6.00
Total cost	$21.00	$18.50	$15.00
Total cost of engine	$2560	$3040	$3990
" " " boilers	1995	1710	1425
" " " outfit	$4555	$4750	$5415
Coal per I. H. P. per hour	3.5	2.75	2.1

The annual costs, allowing 1.5 per cent tax on a two-thirds valuation, interest 5 per cent, repairs 2 per cent, depreciation of engine 4 per cent, boilers 10 per cent, oil, waste, etc., at 0.0002 per I. H. P. per hour, fuel at $3.00 per ton, amount to about as below:

	I	II	III
Annual costs	$4500	$3770	$3168

and about $500 per annum could be saved by adopting the compound condensing engine, or the interest on $10,000.

Taking curve *EE* on the last figure, Chap. VII, Part I, as satisfactorily approximate for this case, making $p_1 = 100$, $p_b = 3$, $\frac{p_b}{p_1} = 0.033$, $M = 0.07$, the designer finds that he should plan his engine, for its average power, at $r = 7.5$, nearly. Maximum efficiency, as determined by the solution of the "owner's problem," is obtained when $r = 5$, nearly.

We may compare the preceding with the case of a simple "automatic" non-condensing engine of about 75 I. H. P., of such good construction and such high speed as will make its curve substantially the same as the last, the curve *E* on the plate. This engine gives the following data:

FIRST COSTS.

Power, I. H. P.	75
" D. H. P.	67.5
Cost, per I. H. P., engine	$25
" " " shafting	5
" " " total	$30
" " " boilers, set	$12
" " " chimney, etc.	8
" " " total	$20
" total, engine	$2250
" " boilers	1500
" " plant	$3750
Weight, water per I. H. P., per hour, lbs.	25
" coal " " " " "	4

The engine is to work 12 hours a day, 313 days in the year. Water costs nothing.

ANNUAL COSTS.

(1) Invariable:

Building and land	$7000	
Assessment on	4000	
Annual taxes @ 1.5 ¢		$60
Interest @ 5.4 ¢		378
Engine-driver's pay		1000
Fireman's "		700
Total		$2138

(2) Variable with engine:

Interest on cost @ 5.63 %..................	$126.55
Repairs @ 2 %............................	45.00
Depreciation @ 4 %.......................	90.00
Taxes @ 1.5 %, on ⅔ valuation............	24.75
Oil, waste, etc., @ 94 cents per I. H. P.....	70.50
Total..................................	$356.80

(3) Variable with boiler:

Fuel @ $2 per ton, 563.4*t*.................	$1126.80
Interest @ 5.63 %.........................	84.38
Depreciation and repairs @ 15 %...........	225.00
Insurance @ 0.5 % on ⅔ cost...............	20.00
Taxes @ 1.1 % on cost....................	16.50
Total..................................	$1472.68
Total of all annual variable costs (2) and (3)..	$1829.48

Making use of curve *E*, we find, for $p_b = 18$, $p_1 = 95$, $\frac{p_b}{p_1} = 0.19$, and $M = 0.85$.

The results, obtained as before, are:

Ratio of expansion for maximum efficiency of fluid......	4.35
Ditto for efficiency of engine..........................	3.64
" " " " capital..........................	2.94

And the engine should be designed to do its work at cut-off of about 0.3, but will give highest duty when $r = 3.6$, nearly.

198. Variation of Cylinder-condensation.—One other among the numerous problems capable of solution by this method promises to prove both interesting and important:

"*Given* the method of variation of efficiency with varying ratios of expansion or proportions of steam used, to determine the method of cylinder-condensation with varying values of $\frac{1}{r}$."

To solve this problem, construct the curve of efficiency, as

A, *D*, *E*, Fig. 167, and draw the curves of adiabatic mean pressures for various values of x, as in dotted lines in that figure.

The points of intersection of these curves with the curve of efficiency identify the ratios of expansion at which the total condensation amounts to the proportion due to the adiabatic line so cut.

In all problems of maxima or minima solved by the construction here given it will be observed that the item of quantity of expenditure made the independent variable is that dependent upon the quantity of steam or of fuel demanded by the engine.

199. Problems Solved by Inspection of the Diagrams.—An important class of problems of simple character may be solved with ease and rapidity by the use of the curve of efficiency for the class of engine studied in any case, e.g.:

(1) To determine the gain or decrease of power obtainable by change of ratio of expansion or point of cut-off, measure the ordinates of the curve at the present and at the proposed ratio of expansion. Their relative magnitude will be a measure of the relative power of the engine at the two points of cut-off, using the quantity of steam measured by the abscissas.

(2) To determine the quantity of fuel or of steam, per hour per horse-power, to be gained or lost by change of the ratio of expansion, compare the value of ratios of abscissa to ordinate at the existing and proposed points of cut-off; their relation will be that of cost of power in steam or in fuel.

(3) To determine the absolute amount of fuel or of steam, consumed per horse-power per hour, at any assumed rates of expansion, first compute the consumption for the given engine as a thermodynamic problem simply, and multiply by the ratio, $\frac{y}{p_m}$, of the mean pressure in the perfect engine at the given expansion to that shown by the true curve of efficiency for the engine studied. Or, compute the consumption for the engine working without expansion and without waste, and multiply

by the ratio, $\frac{p_1}{p_m y}$, obtaining y and p_m from the diagram N, the given cut-off, and remembering that p_1 measures the mean pressure at full stroke of the given steam used *dry*.

200. Conclusions.—In view of what has preceded, it becomes obvious that the engineer purposing to write a specification for steam machinery on which bids are to be made with guarantee of performance should *first* determine the probable *curve of efficiency* for the type and design of engine called for, and should solve all the several problems relating to its economy. He should prescribe the size of engine, then the mean pressure, or the ratio of expansion at which maximum "duty" is to be obtained, as well as fix the duty expected in regular work; at which ratio the work done will be less than the regular working power of the machine. He must also indicate at what mean pressure, or what degree of expansion, the engine will be required to do its ordinary work at maximum *commercial* efficiency, and should state what limit of economy at that rate of work will be accepted. Finally, it should be prescribed that the engine should be capable, if its work should be increased, of attaining at least its maximum "efficiency of plant" with safety, and with a specified economy which should be reasonably high.

Thus: fixing the mean pressure and the ratio of expansion for the duty-trial, the builder is able to give an intelligently estimated guarantee of performance at highest efficiency; fixing it for maximum commercial efficiency in regular work fixes at the same time the proper size of engine; and the last specification secures ample strength of parts.

The cases which have been here investigated must be taken simply as illustrative and not as affording results to be accepted in any specified case coming up in the practice of the engineer. Every such case should be independently and thoroughly investigated. A considerable amount of data and some further illustrations of the principles which have been here enunciated will be found in the concluding chapter of the second part of this work. (See Appendix, pp. 1002 *et seq.*)

201. Absolute Limits to Expansion.—It has been generally assumed, hitherto, that the best ratio of expansion, whether for maximum efficiency of fluid, of engine, of capital, or of plant, increases with increase of steam-pressure without limit, and that such ratio may be indefinitely increased with decrease of the ratio of back-pressure for any one kind of engine, notwithstanding the fact that the value of the ratio of expansion is modified by variation of the conditions of working, even where the ratio $\frac{p_b}{p_1}$ is the same. But it may be seen that, in every engine operated under the conditions of real work and of usual practice, there exists a limiting value, for any one of these "ratios of maximum efficiency," beyond which it cannot be economically raised, even with a greatly, perhaps infinitely, elevated boiler-pressure. It will be further seen that this "absolute limit" may be readily, and probably often is, passed in every-day practice; that, in the usual forms of steam-engine, an absolute limit exists, within, or not far beyond, the customary working range of expansion, beyond which expansion cannot be carried with economy, however high the steam-pressure adopted; in other words, with infinite pressure, the economical value of the ratio of expansion will be found often not merely finite, but sometimes probably even within the limits of familiar practice. The designing engineer keeps these facts and all the previously described conditions in mind and bases his determinations of the character and the principal dimensions of the engine upon them. These investigations all have for their purpose the solutions of the main problem in finance. Studying the equations, it will be found that, in all except those relating to efficiency of engine or of fluid, it is possible to find finite values of r such that their left-hand members shall reduce to zero; since n nearly always approximates unity; q varies from $q = 0$ to $q = -.3$ in good practice, and b usually ranges between $b = 0.8$ and $b = 0.9$; M or N usually has a value between 0.02 and 0.15.

Thus the form of the function is such that the first member may always be made to disappear for some finite value of r, and

the value of r, at which this condition is obtained, constitutes an "absolute limit," for the case taken, beyond which expansion cannot be carried economically, even with steam increased to infinite tension; beyond this point $\frac{p_b}{p_1}$ becomes negative, indicating the assumption of impossible conditions.

Examining equations relating to the purely thermodynamic problem, we find no such limit; the sign of the first member remains positive for all values of r, and can never become zero for a finite value of that quantity. Thus an important difference here evidently exists between the *ideal* engine, with its non-conducting cylinder, and the *real* engine working steam in a metallic cylinder, as well as between the case of maximum efficiency of engine and that of maximum efficiency of capital. In the case of maximum efficiency of fluid for the ideal perfect engine only, is it true that indefinite increase of steam-pressure permits indefinitely increased expansion. In all other cases an absolute limit exists, fixed for each case, beyond which expansion cannot be economically carried.

For the U. S. steamers Michigan, Georgiana, and Bache, for which three cases the real curves have been obtained, these curves remaining unchanged by increase of pressure, it is impossible economically to increase the ratio of expansion in such engines beyond three, five, and ten, respectively, even with unlimited steam-pressure; i.e., even when $\frac{p_b}{p_1} = 0$.

We conclude:

(1) That in all real engines there exists an "absolute limit to the economical expansion of steam," whether considered with reference to efficiency of fluid, of engine, or of capital; which limit cannot be passed, whatever pressure of steam may be carried up to the point of cut-off.

(2) That this limit is found at higher ratios of expansion as the type of engine is more efficient, but that the limit is indefinitely removed only in the ideal engine, and then only as affecting the ratios of expansion at maximum efficiency of fluid and engine.

(3) That this limit is found at a small value of the ratio of expansion in ordinary and inefficient engines, and may be readily passed in every-day practice.

(4) It is evident that these general propositions are true of all heat-engines having fluid working substances, whether vapors or gases, worked in metallic cylinders.

202. Market Values and Costs will be considered in detail in Part II, Chap. VIII; and it is only necessary to introduce here the following remark: The cost of fuel alone amounts to $1.50 per annum per pound of coal and per dollar paid per ton. Hence, since this is the interest on $30 at 5 per cent, and on $25 at 6 per cent, these sums may be profitably expended, for each horse-power, to save one pound of fuel per horse-power per hour—a saving frequently exceeded several times by substitution of an economical for an inefficient engine, and difference in first cost becomes insignificant.

Steam "plants" for light and power now usually demand from 3 to 8 pounds of fuel per hour per I. H. P.; while minimum figures are about 2.5, 1.75, 1.5, 1.25, for simple, compound, triple, and quadruple expansion engines, respectively, at pressures ranging from 80 to 200 pounds. Mr. Emery shows that, at usual prices, the compound engine frequently excels more efficient engines in final, financial, economy, in consequence of its moderate cost. (See Vol. II, Chap. VIII). He finds that costs of water-supply to cities, including interest and insurance accounts, ought not exceed $10 per annum, per million gallons raised one foot. (1894.)

The fact must be constantly kept in view, however, in computing efficiencies, that where the exhaust steam has a value, and, as in steam-heating, is nearly as valuable as steam direct from the boiler, the efficiency of the system as a whole becomes nearly unity and the solution of the problem becomes that for an engine in which the only wastes are those of engine-friction and so much of radiated heat as may not be of service in warming the engine-room. The thermodynamic problem in this case disappears.

CHAPTER VIII.

APPLIED THEORY OF THE STEAM-ENGINE ILLUSTRATED BY ITS PERFORMANCE.

203. **A Standard of Efficiency,** as already described (§ 115), by which to measure the thermodynamic value of the steam-engine and other heat-motors in their exceedingly various types and forms, is constantly required by the engineer, and the general adoption of a common and correct standard is one of the desirable conventions in all thermodynamic work. A number of standards, in themselves accurate and scientifically available and acceptable, have been proposed, and the question to be settled by common consent is: What one of the various possible standards shall be adopted? *

The Essentials of a Satisfactory Standard are:

(1) Ideal perfection and accuracy.

(2) Invariability under the conditions of its employment.

(3) Convenience.

(4) Special suitability to the class of values which it is to measure.

Where, as is often the case, a number of possible standards are available, all having a common measure and fixed relations among themselves, that one should be chosen which presents the best combination of precise measures for use in the class of problems to which it is to be applied.

Heat-engines, of whatever class, type, or form, simply convert thermal energy into useful dynamic energy, and their

* See paper by Sankey, "The Thermal Efficiency of Steam-Engines," Proc. B. Inst. C. E., March 24, 1896, p. 182, and that by Thurston in The Journal of the Franklin Institute, Dec. 1896–Jan. 1897.

efficiency is therefore measured by the ratio of the amount of energy thus rendered available to the quantity of energy originally supplied for transformation. From this point of view two measures of efficiency are available, and two standards are offered for choice. Both of the latter are invariable for any given case; only one is absolute and invariable without qualification. The latter is the measure of the thermodynamic equivalent; the former is the measure of the maximum ideal conversion-ratio of the most perfect possible thermodynamic cycle. The one is unity of absolute efficiency, the other the quantity measuring the ratio of thermodynamic transformation of the Carnot cycle $(T_1 - T_2)/T_1$, which ratio may be taken as the standard and as the unit of comparison for any other thermodynamic cycle, or for either the ideal or the real engine.

Efficiency Unity, taken as a Standard, infers complete transformation of all heat supplied into mechanical energy, *i.e.*, for example, perfect thermodynamic conversion of the British thermal unit of energy into 778 foot-pounds of dynamic energy, or 427 kilogrammeters per calorie. It is in this measure that the efficiencies of the Carnot, as of every purely thermodynamic, cycle is measured. Thus gauged, the Carnot cycle, within the usual maximum range of temperature of our steam-engines, has an efficiency of about 30 per cent. This is the ultimate and universal standard for all thermodynamic operations.

The Carnot Efficiency, taken as a Standard, gives a measure of the economic relation of any thermodynamic cycle to the maximum ideal efficiency of all heat-engines, whatever the character of the working fluid adopted. It may be taken as unity in such cases, and the relation thus gauged is as absolute and accurate as the preceding, within its defined limitations. It permits the comparison of the measured efficiency of any heat-engine with that of the perfect, ideal, engine within similar temperature-range. It must obviously be carefully distinguished from the standard of perfect thermodynamic transformation-efficiency, unity. The Carnot Effi-

ciency is not an absolute standard in a proper sense; since its limits are not fixed.

The Efficiency of a Chosen Ideal Thermodynamic Cycle is a conventional standard adopted in many cases as the unit of reference. It measures, when thus taken as the unit of efficiency, the relation of the thermodynamic effect of the real to that of the same conventional ideal engine; showing the degree of approximation of the machine as operated to its representative—the ideal engine, working in the stated cycle. This is the kind of standard employed by Rankine and by Clausius.

For example, comparing the efficiency of the best modern steam-engine employing saturated steam, about 0.20, with the Rankine cycle,—this ideal case giving an absolute efficiency, 0·25,—it is found to have, measured by this latter standard, a relative efficiency of 80 per cent; while, compared with the 30 per cent of the Carnot cycle, the figure becomes nearly 70 per cent.

Absolute and Relative Efficiencies must thus be explicitly distinguished, and all other so-called efficiencies than the first of the series should be denoted by the latter term. Efficiency relative to the Carnot, or to the representative Rankine, or other ideal thermodynamic cycle, if properly defined, often proves a valuable datum in comparing the actual with the ideal performance of engines for the purpose of ascertaining how closely the performance of the engine, under test or in regular use, approximates the perfect thermodynamic action of the same machine with all its extra-thermodynamic wastes extinguished.

This standard of efficiency thus is seen to have its own special purpose and use. *The absolute efficiency* of any engine is a measure of the proportion in which thermal energy supplied is converted into work, and permits the comparison of various ideal or real cycles. *The relative efficiency* measures the degree in which the actual performance in the chosen cycle approximates the purely thermodynamic ideal maximum for that cycle; the difference between the real and the ideal

indicating the limiting range of possible further improvement and the degree of imperfection of the machine.

Each of these efficiencies measures a valuable datum, and, in fact, no investigation is entirely complete in which a determination is not made of each. Each has its use, and neither can be thrown out as superfluous.

In still other phrase, the absolute standard, Joule's equivalent of thermodynamic conversion, is the limit of perfect transformation; the Carnot cycle is the gauge of limiting thermodynamic transformation in the most perfect possible heat-engine cycle; the type-cycles of Rankine and Clausius gauge the limits of perfection of the classes of engine which, by their construction, must work in one or another of those particular cycles.

The Carnot Cycle, representing, as it does, the extreme limit of possible economy in the operation of the heat-engine of whatever kind, and having the same value for similar range of temperature, irrespective of the nature of the working fluid, furnishes a standard which was shown by Carnot himself to be that with which all actual cycles and the performances of all real engines should be compared, to ascertain what is the degree of their mechanical and physical perfection or imperfection.* Improvements of the steam-engine, from the earliest days of the modern Watt-engine to the present time, have had for their purpose and as their result the widening of the range of adiabatic expansion, and the restriction of extra-thermodynamic wastes. This means the closer approximation of the cycle and the physical conditions affecting the operation of the machine to those ideal conditions, thus bringing the final efficiency of the engine to coincide more closely, as becomes practicable, with these maxima of the Carnot cycle of similar temperature-range.

The Steam-engine Working in the Carnot Cycle has, obviously, the same efficiency as any other heat-engine operating in the same cycle within the same temperature-

* Carnot's "Reflections on the Motive Power of Heat," Thurston's translation, p. 68. New York: J. Wiley & Sons.

range. The value of this efficiency is always $(T_1 - T_2)/T_1$, and its figures, as given in the tables appended, are the same for all working fluids when the values of T_1 and T_2 are the same. In the computations, the results of which are tabulated, it is assumed that the terminal pressure, p_2, and the back pressure, p_3, are identical, as prescribed by Carnot, and equal, respectively, in the cases paralleling the condensing and the non-condensing engines of the common type of cycle to 4 and to 18 pounds per square inch, absolute. Pressures are taken from 300 pounds downward. The steam is assumed to be initially dry and saturated. Values are computed for the ratio of expansion r, the mean effective pressure p_e, the efficiency E, the B.T.U. expended per horse-power per hour, and the weight of steam and of fuel demanded, per horse-power per hour; each pound of fuel being assumed to supply 10,000 B.T.U. to the steam. The volume traversed by the piston, per minute, per horse-power, is also computed and tabulated. These "ideal efficiencies" and data represent those limits which may be approached by the "real engine," but never actually attained. It is probable, judging from experience already had, that they may be taken as representing about one-half better work than can be expected in even the best practice of the best builders, the same limits of pressure and temperature being assumed.

The Methods adopted in these computations are those original with Rankine, and similar to those applied to the special cases of the non-conducting cylinder and the ideal jacketed engine.*

These methods have been fully described (Chap. V) and the symbols and the formulas employed are also summarized and collected in the Appendix to § 149 (see page 611.) The details of action of each cycle have been also fully described.

Illustrative Examples have thus been worked out, in which the maximum steam-pressure is assumed to be exceptionally

* Rankine's "Steam-engine," chap. IV, p. 104 et seq.

high, 500 pounds per square inch, absolute; the terminal pressures being identical with the back-pressures in the case of the Carnot cycle, and usually 7 pounds per square inch in the condensing and 21 pounds in the non-condensing engine: while the back-pressures are taken at 2 and at 16 pounds, respectively, for the condensing and the non-condensing engines and their Carnot representatives.

The results of computation give an excellent series of data for comparison; bringing out well the relative efficiencies of the several cycles as well as their absolute efficiencies, and serving also to exhibit, in an instructive manner, when compared with the real engine-cycle, the extent and the character of the physical defects of the actual engine.

In these cases, it is assumed that the quality of the fuel and the efficiency of the boiler are such as to permit the evaporation of 10 pounds of feed-water per pound of fuel in the case of the non-condensing engine—which is supposed to employ a heater capable of raising the temperature of the feed-water to 200° F.—of 9 pounds in the case of the condensing engine, taking the feed-water from its hot-well, and 12 pounds in the Carnot engine; the "feed-water" being, in this case, at the temperature of the prime steam, and only demanding its heat of vaporization to transform it into steam of boiler-pressure.

These figures may be profitably compared with those computed by Rankine, at the middle of the nineteenth century, as corresponding to the cycle of the steam-engine of that date (§ 117). The economical gain by the reduction of the back-pressure to the figures now attainable, with such good design and construction as has been readily secured in recent years, is very considerable. Ample port-area and a tight condenser are obviously elements of economy which it is well worth while to insure. For these figures the tables in the Appendix may be consulted.

204. **The Theory of the Carnot Cycle** for vapors has not usually been fully discussed by writers on thermodynamics. The following are the processes adopted by the Author in

deducing the heat supplied, the work done, the efficiency, and the heat, steam, and fuel demanded for this ideal case. The specific volume of liquid water, $v = 0.017$, is neglected.

In FIG. 169 (Fig. 140*a*, § 115), *AB* is the isothermal

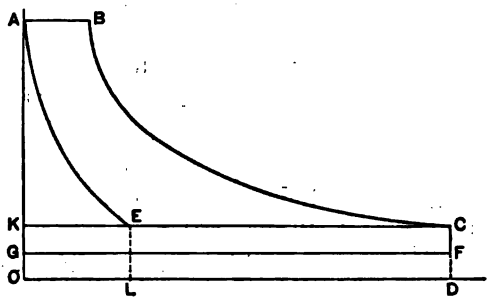

FIG. 169.—TYPE-CYCLES.

induction-line, *BC* is the adiabatic expansion-line characteristic of this cycle, *CE* is the isothermal back-pressure line, and *EA* the compression-line, also adiabatic, along which the fluid returns to its initial, liquid, state. The changes in the fluid working substance are the following: Water at the temperature and pressure assumed as the maximum fills the clearance-space, which is of such volume as to enclose unit weight. Absorbing heat from the furnace, this mass of liquid gradually becomes vaporous by its transformation, molecule by molecule, each particle passing off from the original, liquid, mass, with accession of the heat required for its removal to the comparatively considerable distance from adjacent particles characteristic of the condition of vapor at the constant pressure and temperature of transformation. At *A* the whole mass is liquid; at *B* all is vapor; at intermediate points, the charge is a mixture of steam and water at maximum temperature and pressure. From *B* to *C* adiabatic expansion of the fluid converts it, in part, into liquid, and at *C* the proportion of liquid is not far from the reciprocal of the ratio of expansion. From *C* to *E*, the compression, with abstraction of the heat of vaporization at that temperature and pressure, gradually reduces the fluid, molecule by molecule, to the

liquid state; the whole becoming water at A, however, only after the completion of the final, adiabatic, compression EA, which at the same time elevates its temperature and pressure to the initial figures.

The measures of work, and of heat and efficiency, may be made as follows:

The area of the diagram measures the net work performed by, and the heat supplied is the latent heat of vaporization of, unit-weight of the fluid. The efficiency is the quotient of the former by the latter. The area of the diagram may be taken as the difference between $ABCKA$, and $AKEA$, and, from what has preceded, the values of the volumes, u, at every point along the adiabatic expansion-line, are known in terms of measurable quantities; then (note, App. I, p. 1001):

$$U_1 = ABCKA = \int_{p_2}^{p_1} u\,dp$$

$$= \int_{p_1}^{p_2} 1 \Big/ \frac{dp}{dT}(J \log_e T_1/T_2 + v_1 dp_1/dT_1)dp;$$

$$= \int_{T_1}^{T_2} (J \log_e T_1/T_2 + v_1 dp_1/dT_1)dT;$$

$$= J[T_1 - T_2(I + \log_e T_1/T_2)] + (T_1 - T_2)\frac{v_1 dp_1}{dT_1}. \quad (1)$$

Since the latent heat per unit-weight,

$$H = T\frac{dp}{dT}v; \quad v_1 dp_1/dT_1 = H_1/T_1; \quad . \quad . \quad . \quad (2)$$

and

$$U_1 = J[T_1 - T_2(I + \log_e T_1/T_2)] + \frac{T_1 - T_2}{T_1}H_1. \quad . \quad (3)$$

Further, in the expansion of the liquid from A to E, work would be done precisely equal to that performed in compression from E to A. In this case, the value of the volumes would be such as would be given by the expression employed

for the line BC, *except* that the latent heat of vaporization, $Tvdp/dT$, is zero; whence

$$u = dT/dp \, J \log_e T_1/T_2. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The value of the area $AKEA$ is thus

$$U_2 = AKEA = \int_{p_2}^{p_1} u \, dp = \int_{p_2}^{p_1} \frac{dT}{dp}\left(J \log_e \frac{T_1}{T_2}\right) dp$$
$$= J\left[T_1 - T_2\left(1 + \log_e \frac{T_1}{T_2}\right)\right]. \quad . \quad . \quad (5)$$

The net work performed, $ABCKA$, is thus

$$U = U_1 - U_2 = (3) - (5) = \frac{T_1 - T_2}{T_1} . H_1; \quad . \quad (6)$$

which is precisely the value given in the case of the perfect gas, and the measure of the heat transformed into work, and of the work performed in this cycle; H_1 being the equivalent of the heat supplied the gas during its isothermal expansion, in this case, on AB.

The heat rejected is seen to be $H_2 = \frac{T_1}{T_2} . H_1$, and the efficiency of the cycle is thus found to be measured, as in the case of the gas, by the fraction

$$E = \frac{T_1 - T_2}{T_1} = \frac{U}{H_1}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

On examination of these expressions for work and for heat transformed, it is seen, from (6), that the quantity of work, and of energy transformed per unit-weight of working fluid, is proportional to the magnitude of the latent heat, H, and to the range, $T_1 - T_2$, of temperature worked through; while it is inversely as the absolute temperature of the upper limit of the cycle, and directly as the efficiency.

It is further obvious that the gain by depression of the lower limit of temperature is greater than by equal variation, in the opposite direction, of the upper limit. For example,

let it be assumed possible to either raise the upper limit or to depress the lower by 100° F., thus securing equal gain in range of temperature worked through, as in the following cases:

T_1	T_2	$\frac{T_1 - T_2}{T_1}$	$\frac{T_1 - T_2}{T_2}$
800°	600°	0.250	0.333
900	600	0.333	0.500
800	500	0.375	0.600

it is seen that the efficiencies become, respectively, 0.250, 0.333, and 0.375; and the gain is greatest in the case in which the lower limit is depressed an amount equal to that of elevation of the upper limit in the other case; both being compared with the first case.

Since the work performed per unit-weight is proportional to the latent heat of vaporization, it is obvious that the weight of working substance demanded per H.P. per hour will be proportioned inversely to its latent heat, although the efficiency of heat-transformation is independent of the nature of the fluid. In the various vapor-engines, when worked in the same thermodynamic cycle, equal efficiencies and the same quantities of fuel will be observed per I.H.P. per hour; but each will have a different measure of weight of working fluid circulated, per I.H.P., in the unit of time. Leakage and waste of fluid being prevented, it is evident that the magnitude of this last quantity is a matter of no importance, as bearing upon the cost of production of power.

205. **The Differences between the Representative Carnot and Rankine Cycles**, both having adiabatic expansion, are observed to consist in these defects in the latter:

(1) No adiabatic compression;

(2) Incomplete expansion;

and also, in actual practice, often,

(3) Excessive back-pressure;

all of which are methods of waste of available energy.

The magnitudes of these wastes may be computed as follows:

(1) The work of complete adiabatic compression is precisely equal to that of adiabatic expansion, from T_1, p_1, v_1, of the fluid, when all in the liquid state, to that of a mixture of steam and water, at the final temperature, T_2, and pressure, p_2. Making H_1 zero, in the expression for the work of expansion in the Rankine cycle, complete expansion occurring to T_2, we have, for the measure of this work (of the liquid) for the perfect engine-cycle,

$$U_c = J\left[T_1 - T_2\left(1 + \log_e \frac{T_1}{T_2}\right)\right]. \quad . \quad . \quad . \quad (1)$$

The quantity of heat expended to compensate this gain of work, in the Rankine cycle over that in the Carnot, is $J(T_1 - T_2)$; and this heat might produce by transformation with maximum efficiency, as in the Carnot cycle, the work

$$U = J\left(\frac{T_1 - T_2}{T_1}\right)(T_1 - T_2). \quad . \quad . \quad . \quad . \quad (2)$$

The net loss in the Rankine cycle, due to the fact that it does not restore its working fluid to the initial pressure and temperature by adiabatic compression, by the dynamic heater, is the latter quantity (2) less the former (1):

$$U_1 = U - U_c = J\left[T_2 \log \frac{T_1}{T_2} - \frac{T_2}{T_1}(T_1 - T_2)\right]; \quad . \quad (3)$$

which is the gain to be secured by the conversion of the Rankine cycle with complete adiabatic expansion—the case of the non-conducting cylinder—into a Carnot cycle. It amounts to 8 to 10 per cent. of the work of the Carnot cycle usually, or not far from 2 or 2.5 per cent of the heat sent over from the boiler.

The loss of work, in the Rankine cycle, due to incomplete expansion, is the work of expansion between the terminal pressure, p_2, and the back-pressure, p_3, when exhaust takes place before the back-pressure line is reached. The work of the complete cycle then consists of that portion which

gives that between T_1 and T_2, measured by the expressions already given, $\left(\int_{p_2}^{p_1} u\, dp\right)$, plus the work $(p_2 - p_3)u_2$. The defect is the difference between the work of complete expansion to p_3 and this sum, and is

$$U_e = \frac{T_2 - T_3}{T_2} H + J\left[T_2 - T_3\left(1 + \log_e \frac{T_2}{T_3}\right)\right] - (p_2 - p_3)u_2. \quad (4)$$

The waste by excessive back-pressure comes of the fact that the resistances in the exhaust-ports and the head required to force out the fluid before the piston into the condenser or the atmosphere holds the back-pressure line above the minimum pressure. This loss is

$$U_b = (p_3 - p_c)u_2. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The sum of these three losses is

$$U_t = J\left[T_2 \log_e \frac{T_1}{T_2} - \frac{T_2}{T_1}(T_1 - T_2) + T_2 - T_3\left(1 + \log_e \frac{T_2}{T_3}\right)\right]$$
$$+ \frac{T_1 - T_2}{T_1} H - (p_3 - p_c)u_2. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

These losses by defect of cycle for a condensing engine are, in a fair average case, divided among themselves substantially thus:

Loss by non-compression........	15 per cent.
" " incomplete expansion....	75 " "
" " back-pressure...........	15 " "

As percentages of total work, they ordinarily compare about as follows, dividing total energy into useful and wasted work:

Useful work....................	60 per cent.
Loss by non-compression........	8 " "
" " incomplete expansion....	25 " "
" " back-pressure...........	7 " "

The loss by incomplete expansion, especially, is very variable, however, and these figures can only be taken as illustrative. Each case must be independently computed, in all practical applications. Willans, taking the lower limit in good practice at 170° F. in condensing and 226° F. in non-condensing engines, finds the loss of work to range from 10 to 20 per cent in condensing and from 1 to 3.5 per cent in non-condensing engines, incomplete expansion only being noted, where the initial pressures range from $p_1 = 250$ to $p_1 = 35$ pounds on the square inch.* The loss is necessarily much the greater in the former type of engine.

206. The Process of Heat-storage, more correctly of energy-storage, actual and potential, in the boiler-steam, is, in detail, the following:

During the first step, the water entering the boiler is raised from its temperature as feed-water to that of the steam in the boiler, without change of physical state, and with simple accession of the sensible heat, K_vT, per unit of weight; the expansion of the liquid being neglected. The second step consists in the expansion of the unit weight of liquid into steam, with constant temperature. This is equivalent to saying that no accession of heat occurs; all heat now received,

$$H_e = p_t(v_s - v_r) = T\frac{dp}{dT}\Delta v,$$

being converted, as received, into the mechanical energy performing the work of expansion of the fluid from its volume in the state of water at boiler-temperature into steam at the same pressure and temperature. The resisting force is the sum of the internal and external pressures, p_i and p_e, amounting at an ordinary pressure, as indicated by the gauge, to, for example, one thousand one hundred pounds, $p_t = 1100$, very nearly. These two steps are thus entirely independent; except that there is actually some slight expansion of the liquid in the process of heating it from the lower to the

* Steam-engine Trials; Proc. Inst. C. E., 1893; Author's reprint, p. 12.

higher limit, and some heat received for immediate conversion into mechanical energy and work, in that process. This may be here neglected.

The work performed in the second step results in the storage, in a certain sense, of the equivalent energy in the mass of vapor as potential energy in amount measured by the product of the internal resistance by the change of volume, and in the transfer of the balance of this total "latent" heat to the piston, in the form of mechanical work, and on through the piston to the machinery to be driven.

Thus the steam entering the engine from the boiler contains no more sensible, or actual, heat-energy than was stored in the fluid at the instant at which rise in temperature ceased and its vaporization began. Its "latent heat of vaporization" is usefully applied, so far as useful work is performed by it at all, when, expanding into steam, it displaces an equal volume from the boiler and through the steam-pipe into the engine, which displaced volume performs work on the piston which is measured on the indicator-diagram between the beginning and end of the induction-line. The work to cut-off is thus actually performed by the steam at the moment in process of evolution on the heating-surfaces of the boiler. From the point of cut-off on, the work is performed by transformation of the sensible heat of the steam into mechanical energy. This goes so far as to draw from the fluid more heat than can be supplied by reduction of the total heat with falling pressure, and the consequence is the abstraction of sensible heat to such extent as to compel the partial condensation of the vapor. The amount of this condensation, varying nearly as the ratio of expansion, is computed by a process elsewhere detailed (§ 112, p. 439), and is found, in ordinary cases, to amount to a percentage nearly corresponding with the total ratio of expansion adopted.

207. **The Heat of Combustion** of any fuel is the quantity of heat set free by its union with oxygen, and is a fixed quantity for each, irrespective of the quantity of air supplied, combustion being complete in each case. This heat is dis-

tributed to the mass within which it is evolved, and gives it an increase of temperature above the initial which is measured by the quotient of the heat of combustion by the product of the total weight of products of combustion and their mean specific heat.

The total heat of combustion of the more common fuels and their constituents are here taken as follows:

	Q.	ΔT.
Carbon, *per pound*	14,500	4648
Hydrogen	62,500	3617
Hydrocarbons (gas and oils, petroleum)	20,000	5128
Charcoal and coke (pure and clean)	13,500	4187
Anthracite coals (best)	14,000	4868
Bituminous coals (best)	15,000	5208
Coals, fair average	1,3000	4510
Woods, seasoned	5,500	1910
Gas, *per cubic foot*	650	5128

When, as is usual, the products of combustion are diluted with from fifty to one hundred per cent. more air than is required for complete combustion, it will be found, on computation, that the rise in temperature due to combustion is very nearly inversely proportional to the total air thus introduced.

208. The Energy Required to Supply the Unit of Power or the Unit of Work in the Unit of Time, as per horse-power per hour, may be measured in thermal units, in weight of fuel of a stated quality, in weight of steam demanded at the proposed pressure under prescribed conditions of temperature variation, or, in any case, in dynamic units. Solid fuel, as just shown, stores potential energy to the extent of 15,000 B.T.U. or more per pound, in some cases; but a fair average, for good coals in the market, is more nearly 13,000 B.T.U., or, in round numbers, about 10,000,000 foot-pounds, nearly 2000 mile-pounds, per pound.

Such fuel would therefore be required, if its heat could be completely utilized by a thermodynamic system of which the efficiency is unity, at the rate of

$$W = \frac{1,980,000}{10,000,000} = 0.198 \text{ lb.}$$

per horse-power per hour. An actual efficiency of ten per cent., and a consumption of ten times the above figure, is usual with good steam-engines, and less than six times the computed quantity is rarely reached. One fifth of a pound of solid fuel, per horse-power per hour, may thus be taken as the standard, *for efficiency unity* for the whole system, boiler and engine, in comparisons of performance.

The heat-equivalent of the horse-power, per hour or per minute, in B.T.U., is

$$Q' = \frac{1,980,000}{778} = 2545 \text{ B.T.U.},$$

$$Q'' = \frac{1,980,000}{778 \times 60} = 42.42 \text{ B.T.U.},$$

accordingly as the larger or the smaller unit of time is employed. These quantities of heat would be demanded in a system of efficiency unity and are primary standards.

The Weight of Steam per Horse-power and per unit of time varies with the quantity of heat stored per unit weight, and the conditions both of storage and of energy transformation.

The quantity of heat stored and available in the steam-boiler will be here taken as that which is communicated to the unit-weight of feed-water, and in "making steam," between the temperatures of the entering feed from a condenser, in the case of the condensing engine, or from a heater, with the non-condensing engine; and that of making steam, inclusive of the latent heat of vaporization. In other words, it is that heat which must be added in the conversion of the feed-water into steam when the former is at its highest practicably available temperature, and the latter has the pressure assigned as that of regular operation. This quantity may be expressed in either thermal or dynamic units, and either in foot-pounds per pound of fluid circulating in the unit of time in the system, or in pounds of steam supplying one horse-power of energy. This last is the quantity of working fluid which would be required to be circulated per horse-power per hour,

for example, in the assumed system, could the efficiency be made unity.

The steam-tables give the heat, in thermal units, usually, between the conventional standard temperature—the melting-point of ice—and the temperature of the steam, in heat supplied the fluid in the liquid condition and in the change of physical state, and the total. In the engine, the lower limit, in the two cases assumed, is usually actually not far from 100° F. and 200° F., respectively; and the upper limit ranges up to the temperature of steam of 180 to 200 pounds pressure per square inch, or even, in some cases, higher. It is easy to secure a feed-water temperature approximating the boiling-point by the use of a heater, where the non-condensing engine is used; the hot-well temperature usually fixes the lower limit for the condensing engine. In the following statement it will be assumed that a credit may be deducted from the tabulated figures (H) of 80 thermal units for the latter (H') and of 180 B.T.U. for the former (H''). The stored and available heat thus becomes, for the sample cases here taken, as follows:

HEAT STORED AND AVAILABLE.

p_1	v_1	t_1	H	H'	H''	U'	U''	W'	W''
Lbs. per Sq. In.	Cu. Ft per Lb.	Fahr.	B.T.U.	B.T.U.	B.T.U.	Ft.-lbs.	Ft.-lbs.	Lbs.	Lbs.
100	4.342	328°	1182	1102	1002	857,356	779,556	2.31	2.54
125	3.518	344	1187	1107	1007	861,246	783,441	2.30	2.53
150	2.962	358	1191	1111	1011	864,355	786,558	2.30	2.52
175	2.603	371	1195	1115	1015	867,470	789,670	2.29	2.51
200	2.256	382	1198	1118	1018	869,804	792,004	2.28	2.50
220	2.061	401	1201	1121	1021	872,138	794,338	2.27	2.50

The Stored Heat-energy of Steam in the Boiler, at usual pressures and temperatures, thus may be taken as approximating 1200 B.T.U., as tabulated, and 1000 or 1100 B.T.U. per pound within the temperature-range of the non-condensing or of the condensing engine; the latter having the advantage of wider range of cycle, the former that of less waste due to low temperature of feed-water. In the one case, the stored energy, measured in dynamic units, amounts to appro

mately $U' = 865,000$ foot-pounds; in the other to about $U'' = 785,000$ foot-pounds, averaging about 150 mile-pounds. For the one, $W' = 2.3$ pounds of steam per hour will store the equivalent of one horse-power, and, for the other, $W'' = 2.5$ pounds. In other words, for a thermodynamic system of efficiency unity, from 2.3 to 2.5 pounds of steam will be required, per horse-power and per hour, for such range of temperature as is usual with condensing and for non-condensing engines, respectively. This has been seen to correspond to the complete utilization of the potential energy of one-fifth of a pound of good fuel, and to the expenditure of 42.42 B.T.U. per minute, or 2545 per hour.

These quantities, 2545 B.T.U., 0.2 pound of fuel, 2.3 or 2.5 pounds of steam, per horse-power per hour, for efficiency unity, are the units for comparison in studying the actual efficiencies of cycles or of engines; and their quotients by the actual efficiency, measured or computed for the given case, give the actual consumption of heat, steam, and fuel. The availability of these stored energies is, in each case, determined in its extent by the character of the thermodynamic cycle and by the magnitude of the thermal and dynamic losses experienced.

In the case of the boiler-explosion, the efficiency is thermodynamically limited to about one seventh, 0.14, in the cases assumed above, and the thermodynamic efficiency of the non-condensing engine is usually not far, in the ideal case, from 0.07. Thermal and dynamic wastes in the best condensing engines yet constructed usually limit the actual total efficiency to twenty per cent or less, and the measures of heat, steam, and fuel consumed in practice thus range from 12,500 B.T.U., 11.5 pounds of steam, and something over one pound of fuel, to several times these figures, per horse-power per hour. Reduction of waste heat by the warming of the feed-water reduces the quantity of energy stored per unit of weight while it reduces the weight of fuel required, and thus promotes economy.

The quantity of heat stored in the boiler is a measure of the reserve of energy there kept in stock, and of the regulat-

ing and steadying value of that reservoir, as well as of its destructive power in case of accident. The quantity of energy stored in the unit-weight of the fluid is a measure of the concentration of power attainable by its use and, to a certain extent, a gauge of the limit of weight and volume of the motor operated by it. The expenditure of heat, steam, and fuel for a stated quantity of work is a function, simply, of the efficiency of the system, and a fixed proportion of the total quantity of heat entering the circulating fluid, whatever the amount of heat stored in the unit-weight or volume. It happens that the best steam- and the best gas-engines of the time have substantially equal efficiencies, and demand nearly the same heat-supply, per horse-power, although their working-fluids have vastly different capacities of heat-storage.

In making steam, the heat thus stored, in usual absolute pressures, measured in B.T.U. and in foot-pounds, is seen in the following tables:

ENERGY STORED IN ONE POUND OF STEAM IN B. T. U.

Pressures in Lbs. per Square Inch.	Feed-water Temperatures.					
	110	130	150	170	190	210
90	1101.569	1081.569	1061.569	1041.569	1021.569	1001.569
100	1103.866	1083.866	1063.866	1043.866	1023.866	1003.866
110	1105.986	1085.986	1065.986	1045.986	1025.986	1005.986
120	1107.961	1087.961	1067.961	1047.961	1027.961	1007.961
130	1109.809	1089.809	1069.809	1049.809	1029.809	1009.809
140	1111.555	1091.555	1071.555	1051.555	1031.555	1011.555

ENERGY STORED IN ONE POUND OF STEAM, IN FOOT-LBS.

Pressures in Lbs. per Square Inch.	Feed-water Temperatures.					
	110	130	150	170	190	210
90	857,021	841,461	825,901	810,341	794,781	779.321
100	858,808	843,248	827,688	812,128	796,568	781,008
110	860,457	844,897	829,337	813,777	798,217	782,657
120	861,994	846,434	830,874	815,314	799,754	784,194
130	863,431	847,871	832,311	816,751	801,191	785,631
140	864,790	849,230	833,670	818,110	802,550	786,990

Assuming these energies to be applied in thermodynamic operations, in Carnot cycles of similar total temperature-range, we should obtain the following tabulated efficiencies and quantities of energy utilized in units corresponding to unit-weight of steam:

CARNOT EFFICIENCIES.

Pressures in Lbs. per Square Inch.	Minimum Temperatures.					
	110	130	150	170	190	210
90	.270	.243	.218	.193	.167	.141
100	.276	.250	.225	.200	.174	.149
110	.282	.257	.232	.207	.182	.156
120	.288	.263	.238	.213	.188	.163
130	.294	.269	.244	.219	.194	.169
140	.299	.274	.249	.224	.200	.175

AVAILABLE ENERGY, IN B. T. U., PER POUND OF STEAM.

Pressures in Lbs. per Square Inch.	Minimum Temperatures.					
	110	130	150	170	190	210
90	297.424	262.821	231.422	201.023	170.602	141.221
100	304.667	270.966	239.370	208.773	178.153	149.570
110	311.888	279.098	247.309	216.519	186.729	156.934
120	319.093	286.134	254.175	223.216	193.257	164.298
130	326.284	293.159	261.033	229.908	199.783	170.658
140	332.355	298.386	266.817	235.548	206.311	177.022

AVAILABLE ENERGY, IN FOOT-LBS., PER POUND OF STEAM.

Pressures in Lbs. per Square Inch.	Minimum Temperatures.					
	110	130	150	170	190	210
90	228,096	204,475	180,046	156,396	132,728	109,870
100	236,591	210,812	186,208	162,425	138,603	116,370
110	242,649	217,417	192,406	168,452	145,275	122,021
120	248,254	222,612	197,748	173,662	150,354	127,824
130	253,849	228,078	203,084	178,868	155,431	132,772
140	258,572	232,144	207,584	183,256	160,510	137,729

STEAM PER I. H. P. PER HOUR AT MAXIMUM EFFICIENCIES, AS ABOVE.*

Pressures in Lbs. per Square Inch.	Minimum Temperatures.					
	110	130	150	170	190	210
90	8.68	9.58	10.98	12.64	14.92	18.02
100	8.36	9.39	10.62	12.18	14.30	17.01
110	8.15	9.12	10.28	11.75	13.63	16.22
120	7.97	8.88	10.02	11.41	13.16	15.38
130	7.80	8.68	9.74	11.08	12.73	14.91
140	7.65	8.52	9.54	10.80	12.33	14.38

[See Appendix for more extended tables.]

209. Measures of Efficiency.—What is meant by the measures now coming to be employed in the statement of

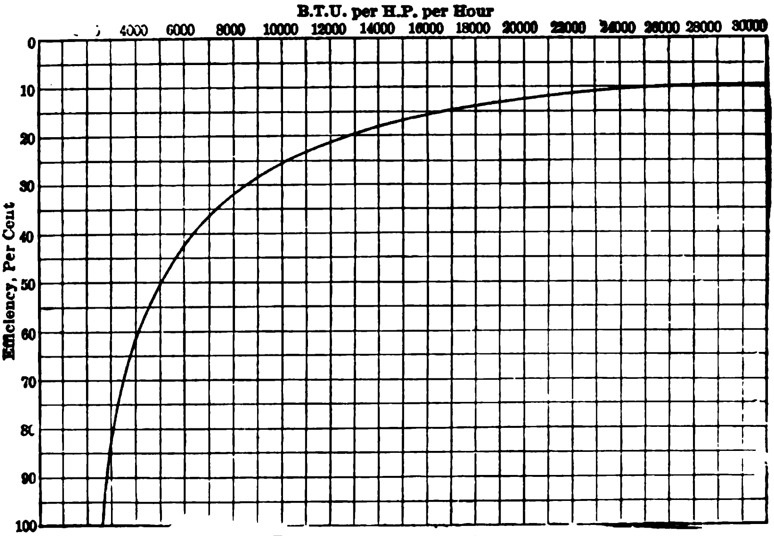

FIG. 170.—THERMAL MEASURE OF EFFICIENCIES.

efficiencies, in the more common units, may be in some degree exhibited by a comparison of those measures. In

* The table exhibits the *steam circulated* in the engine, and equals 1,980-000 foot-pounds, divided by the available energies above.

the accompanying figures (Figs. 170, 171) the variation of quantity of heat and of steam per I.H.P. per hour, with varying efficiency, are exhibited; the primary assumption being that, in a condensing engine, the circulating fluid receives 1100 B.T.U., or 855,800 foot-pounds of energy, per pound vaporized; corresponding, for efficiency unity, to about 2.3 pounds of feed-water or of steam per power-unit. For unity efficiency, the exact figure for $J = 778$, is 2545 B.T.U.,

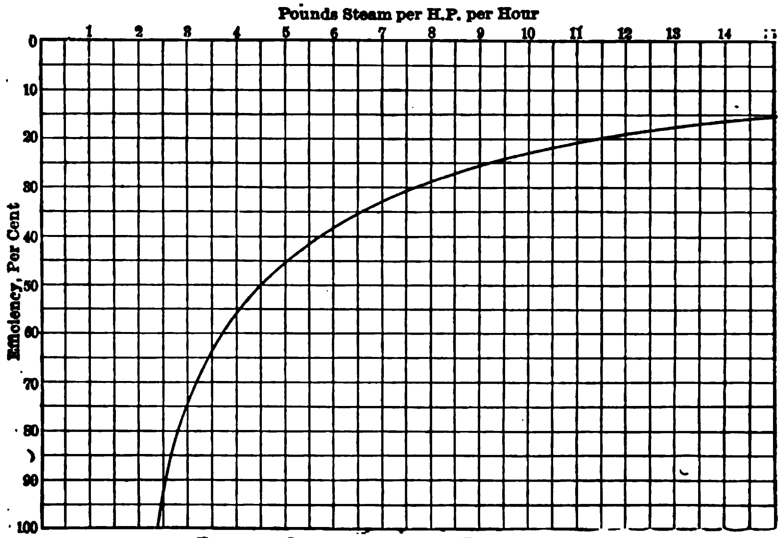

FIG. 171.—STEAM-WEIGHTS AND EFFICIENCIES.

and the values representing a maximum in the best contemporary practice are not far from 20 per cent. efficiency, 12,725 B.T.U. per hour per I.H.P., and 11.6 pounds of feed-water or of steam. One half this efficiency and double these expenditures are considered excellent figures for the average engines of good builders, with steam at now common pressures, between 100 and 125 pounds.

On both scales the limits of the corresponding ideal case may be taken as not far from 25 per cent. efficiency, and rarely as attaining 30 per cent. The latter figure corresponds closely to 8200 B.T.U. and 7.5 pounds of steam per I.H.P.

per hour. Practice has attained to, at best, about 70 per cent of the ideal, thermodynamic, case.

The purpose of employing any stated measure of engine efficiency is always definite, and should be stated in advance, while the unit of comparison should be as precisely defined.

The Absolute Efficiency here given is that which measures the proportion of the total energy supplied, in form of heat, which is transformed into the dynamic form, in the cycle or the series of operations considered.

Any Relative Efficiency, as already seen, measures the ratio of dynamic energy secured for the performance of work, in

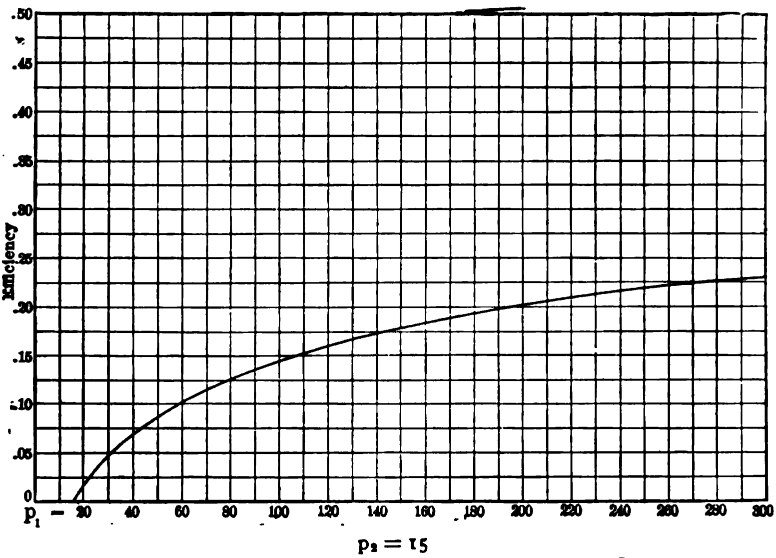

FIG. 172.—CARNOT CYCLE. NON-CONDENSING STEAM-ENGINE CYCLE.

any given cycle or operation, to the quantity which would have been similarly transformed and delivered had the cycle or operation been of equal perfection with one chosen as a standard. This standard is selected for special purposes, and may be more or less perfected or even, in an exactly defined degree, defective, as compared with the pure, the ideal, thermodynamic case most nearly parallel therewith, as may be found desirable or expedient; but its exact nature and its

absolute efficiency should always be known and precisely stated.

In the Carnot cycle the feed-water begins receiving heat from the source of supply at the temperature of vaporization, and only latent heat is demanded; in the other cycles the feed-water is delivered to the boiler at lower points on the scale, and receives heat through a wide range of temperature, in the form of sensible heat, in addition to the supply of latent heat.

The Clausius cycle expands adiabatically to the back-pressure.

The Ideal Limit of Performance, adopting the Carnot as the cycle of maximum efficiency as our standard of compari-

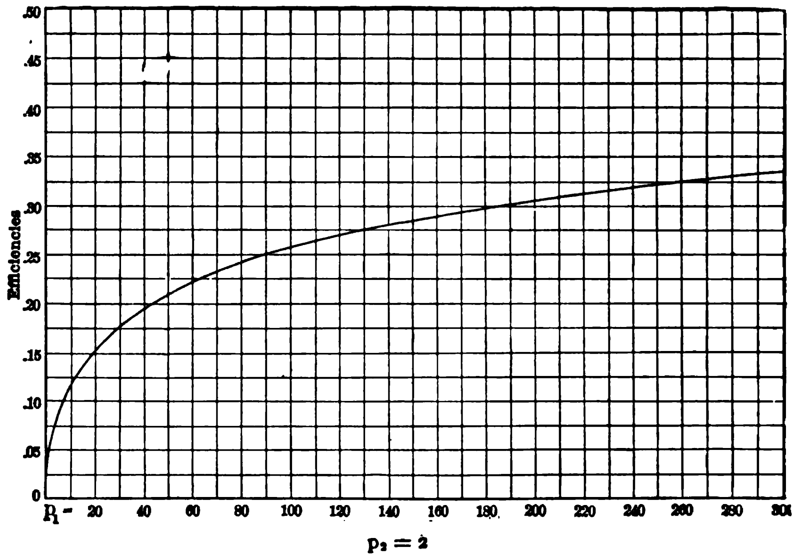

FIG. 173.—CARNOT CYCLE. CONDENSING STEAM-ENGINE CYCLE.

son, is exhibited, for the full range of pressures to-day employed or proposed to be employed by most advanced practitioners, in Figs. 172 and 173. Back-pressure is assumed in the first of these cases—that taken for comparison with the common non-condensing engine-cycle—at 15 pounds per square inch, and in the case of the cycle to be compared

with the best condensing engines, at 2 pounds. In the former case gain is seen to be rapid with increasing pressures up to about the now familiar range of high-pressure machines, thence becoming less and less rapid, and even at 300 pounds pressure reaching only 23 per cent., with final gain at the rate of about 2½ per cent per 100 pounds rise in pressure. A similar method of variation of efficiency with increasing steam-pressures is seen in Fig. 173, in which the range of pressure and temperature is made coincident with that of the best condensing engines. Gains are slow above the now usual maximum of steam-pressure in practice, and at 300 pounds increasing at the rate of about 2½ per cent., per 100 pounds rise, as before. But here the efficiencies have much higher numerical values than in Fig. 172, necessarily, and 25 per cent. at about 100 pounds, 30 per cent. at about 200, and 33 at 300 pounds pressure, are the maxima for even the ideal case and the Carnot cycle. For 500 and 1000 pounds the figures rise to something above 35 per cent, and to about 50 per cent, as seen in more detail later. The diagrams thus exhibit to the eye the data tabulated elsewhere.

210. Internal Heat-wastes, as shown in the text — Chap. V, § 122 et seq., particularly,—are to be added to the thermodynamic expenditures, to determine the total heat-consumption and the actual efficiencies. These wastes may be often most conveniently exhibited and measured by laying down the "saturation-curve" for total steam used, as measured at the engine-trial, over the indicator-diagram representing the mean steam-distribution for the period of the trial. In the case of the multiple-cylinder engine, a mean "combined diagram" must be accurately produced and laid down, as seen below.*

General experience and observation show that the best economical conditions for the steam-engine, as actually operated, are such as cause the steam to leave the cylinder, at the opening of the exhaust, quite dry or very nearly so;

* See papers by the Author: Trans. Brit. Inst. N. A., 1895; Journal Franklin Inst., Oct. 1896. Heat-wastes in Steam-engine Cylinders.

thus, by giving dry surfaces on all parts of the cylinder-wall, preventing loss of heat, without compensation, during the exhaust period, and restricting, to that extent, initial condensation at the opening of the steam-valve for the succeeding stroke of piston. In other words: Could the heat required to prevent thermodynamic condensation, to the extent to which it would occur in adiabatic expansion, be supplied, either by jacketing or by superheating—with perhaps a slight surplus to meet the demand for external radiation and other minor losses—the engine would give its highest efficiency. The tables of thermodynamic condensation will therefore give, at least approximately, the minimum quantities of heat needed to be supplied the working-charge, either in advance or during its expansion period, by superheating or by the jacket, to insure maximum efficiency. This quantity will be seen to vary greatly with the conditions of operation, but, in high expansion engines, to be not far from a percentage measured by the ratio of expansion adopted. This proportion would apparently be independent of size or speed of engine; but since some waste must occur, even with a charge of continuously superheated steam, time and surface would have some effect, and an excess must necessarily be supplied; which excess is proportionally greater as the size of engine and its speed decrease.

In illustration: The best example of the modern steam-engine of high efficiency with saturated boiler-steam and moderate pressures, for our present purposes, may be taken as a pumping-engine, which demands 14,382 B.T.U. per dynamometric horse-power per hour.* Of this 9.22 per cent is lost in friction and 13,056 B.T.U. are required per I.H.P. per hour. This in turn includes extra-thermodynamic wastes amounting to about one fourth, and the ideal representative engine would require 10,000 B.T.U., the same cycle being assumed, but all losses by conduction and radiation being extinguished. Finally, could the form of cycle be altered,

* Maximum Contemporary Economy of the High-pressure Multiple-expansion Steam-engine: Trans. A. S. M. E., 1893.

and the same work done by utilization of an ideal engine working in the last-named cycle, the heat required would fall another 15 per cent, and but 8500 B.T.U. per H.P. per hour would be demanded. The total gain by reduction of wastes and change of cycle would thus be above 40 per cent of the cost of the delivered power of the most efficient of modern engines. Experimental determinations of interna. wastes usually include *leakage* of small but unknown amounts.

211. Computations of Wastes.—The process of measurement and of construction of formulas has been already discussed (Chap. V). One of the first writers to consider and compute the effect of the presence of *water* in the cylinder, in the promotion of initial condensation, and to show that it had precisely the same, and probably a more active, influence on this waste than even the metal of the cylinder-wall, was De Freminville.* His discussion is substantially as follows:

Suppose the cylinder to contain a certain quantity of water, and let us see what occurs during a single revolution of the crank, neglecting, for the present, the influence of the metallic interior surface of the cylinder, which we will suppose perfectly inert.

Let n represent the number of cubic inches of water contained in the cylinder at the end of the stroke;
$T =$ the temperature of that water, which will evidently be that of the entering steam;
$t' =$ the temperature of the condenser;
$H =$ the total heat of steam.

While the cylinder is in communication with the condenser, the mass of water, n, in presence of a vapor of low tension and temperature, commences boiling, and a quantity, x, is evaporated, leaving the water remaining at the temperature, t', of the condenser; this remaining mass being $n - x$.

Then we have the equation

$$nT = (n - x)t' + xH; \quad x = n\left(\frac{T - t'}{H - t'}\right). \quad . \quad . \quad (1)$$

* Cours de Machines à Vapeur; Paris, 1862, p. 121.

There then remains in the cylinder a mass of water, $n - x$, at the temperature t', and at that instant steam enters again; coming in contact with the comparatively cold water, it condenses to a certain amount, x', making the temperature of the resulting mass $n - x + x'$, equal again to T, and we obtain the following equation of the quantities of heat:

$$(n - x + x')T = (n - x)t' + x'H; \quad x' = (n - x)\frac{T - t'}{H - T}; \qquad (2)$$

but

$$n - x = x\frac{H - T^{*}}{T - t'}, \quad \therefore x' = x\frac{H - T}{T - t'} \cdot \frac{T - t'}{H - T} = x; \qquad . \quad (3)$$

consequently, the quantity of steam condensed, on entrance into the cylinder, is equal to the quantity vaporized during the exhaust, and the amount in the cylinder remains unchanged.

If, in the expression $x = x'$, we make $T = 230°$, $t' = 122°$, $H = 1170°$, we obtain $x = n \times \frac{108}{1058} = 0.1n$; the vapor condensed at each stroke of piston and thrown into the condenser, without having produced its full useful effect, is equal to one tenth of the quantity of water in the cylinder.

De Freminville computes the amount of this action in an engine of 55 inches diameter of cylinder and of equal stroke, and working at the low pressures and expansions then customary, and finds that a layer of water 0.2 inch in thickness, covering the bottom of the cylinder—the engine has vertical cylinders—having an area of 2375 square inches, would measure 7 litres (6.5 quarts, 1.63 gallons); one tenth of this quantity, as above, would be equal in weight to the whole quantity of steam supplied per stroke of engine, and the consumption of steam would thus be made double that of the engine with a non-conducting cylinder, and with no moisture

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* Since from (1) $n(T - t') = x(H - t')$,
adding $- x(T - t') = - x(T - t')$,
$(n - x)(T - t') = x(H - t)$.

present. He concludes: "It may be seen from this fact how important is the accumulation of water, however slight in quantity, in the interior of the cylinder." Assuming this amount of water, just taken, to be uniformly spread over cylinder-head, piston, and cylinder proper, it would be but 0.032 inch in thickness over an area of 96.93 square feet; which, as he says, "may be compared to the mist on the window-pane." "Such a deposit may occur wherever the temperature of the cylinder is slightly below that of the entering steam; and this is always the case, in regular working, unless special precautions are taken." He then goes on to show that this layer of mist may transfer heat to and from the metal of the cylinder and thus intensify the effect of cylinder condensation. "This being the situation, when the steam again enters the cylinder it is condensed, yielding its heat to the metal again, until all the cooled mass, and the water of condensation, are restored to the temperature of the entering steam. . . . This effect would not take place if the metallic surfaces of the cylinder could be maintained at a temperature equal to that of the entering steam; in which case there would be no such condensation, and the volume of steam expended would be precisely measured by the volume of the cylinder." He shows that the steam-jacket of Watt corrects this, in part; but "there will always be condensation on the arrival of steam in the cylinder, but this will be much less with than without the steam-jacket." He then proceeds to give a correct account of the action of the jacket and its limitations. Thus De Freminville, thirty years ago and more, saw, as clearly as it is seen to-day, that the gains to be obtained by any stated method of improvement in economy depend quite as much upon the magnitude of the original wastes as upon the intrinsic value of that method of promoting efficiency.

The above method of exhibiting the fact that the quantity of heat entering and rejected from water in the cylinder may evidently also be employed to show the similar fact relative to the metal of the cylinder itself; but algebraic treatment is

entirely unnecessary, as it is obvious that the metal of the cylinder-wall must, in every cycle, when the engine is working steadily, rise and fall through the same range of temperature with each entrance and exit of the working charge, and thus must as necessarily receive and discharge equal amounts of heat, and must condense and re-evaporate constant and nearly equal quantities of steam. The specific heat of iron and steel being one eighth that of water, and its mean density 7.5 to 7.8, the metal stores and re-stores about the same heat per unit of volume or thickness of film as does the water standing upon it. On the other hand, the conductivity of water is so low that it may seriously obstruct the passage of heat into or out of the metal. A film of metal about one tenth of an inch thick has been found, in some cases, in experiments directed by the author, to store or re-store all the heat of initial condensation. In such an instance, the thickness of one thousandth of an inch of water would obstruct the flow of heat into or out of the metal as much as the whole depth of wall affected.

Experiment indicates that the surface-film, in steam-engines operating under customary conditions, does not weigh over about 0.01 pound per square foot, having a thickness of about 0.002 inch as an average throughout the whole area affected. This gives an obstructive power, impeding the passage of heat into and out of the metal, equal to that of a film of iron 0.02 inch in thickness.*

212. **The Equation of Heat-exchanges** in this case is easily deduced, thus (Fig. 174):

Let $w =$ density of the metal of the cylinder-wall, in pounds per cubic foot, or kilogs. per cubic metre:

$a =$ area of surface acting, in square feet or metres;

$c =$ specific heat of the metal;

$d =$ the depth of metal affected sensibly by heat-variations;

* Cotterill; also Callendar and Nicholson, Proc. B. A. A. S., 1897.

t = difference of maximum and minimum temperatures of the elementary film;

x = distance from point of insensible variation, within the mass to the elementary film.

Then t_1 may be taken as the range of temperature of the superficial film, of thickness dx, and distance x_1 from the base-point; and, assuming that, without sensible error, the figure $x_0t_1t_1$ may be taken to be a triangle, we shall have

$$t_1 : t :: x_1 : x; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$t = xt_1/x_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The total quantity of heat absorbed and rejected in each cycle will be measured by the product of mean variation of

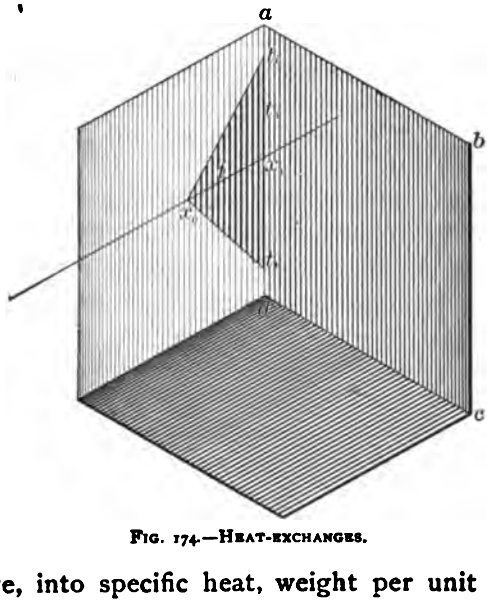

FIG. 174.—HEAT-EXCHANGES.

temperature, into specific heat, weight per unit of volume, and area and depth of metal affected, i.e.,

$$Q = cwa\frac{t_1}{x_1}\int_0^{d=x_1} x\,dx = awct_1d/2; \quad . \quad . \quad (3)$$

which result, in fact, may be read directly from the diagram.

Transforming the equation, we have, also,

$$d = 2Q/awct_1; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In illustration, assume British measures and cast-iron walls;

$C = 0.125$; $w = 450$; $a = 1$; $t_1 = 240°$; $x = .00833 = d$. Then

$$Q = 450 \times 0.125 \times 240 \times 0.00833/2 = 55 \text{ B.T.U.},$$

absorbed per square foot of cylinder-wall, at each cycle, and transferred from the steam to the exhaust side of the engine without performing its share of work, and this is sufficient to seriously impair the efficiency.

In experiments directed by the Author, 30 B. T. U. were absorbed per minute and per degree range of temperature; then, at 60 cycles per minute, and 100° F. range of temperature,

$$Q = 30 \text{ B.T.U. per cycle}; \quad t_1 = 100°;$$

then, the other symbols remaining as before, the thickness of metal affected was not less than, and was probably more than

$$d = 2 \times 30 \div 450 \div 0.125 \div 100 = 0.011, \text{ nearly},$$

or about one hundredth of a foot;* and this proved sufficient to reduce the economy of the machine about twenty-five per cent.

Both theory and experiment show that, under constant conditions of variation of temperature, but with varying speed of engine, the quantity of heat transferred in each exchange, at each revolution, is proportional to the range of temperature of the interior surface of the metal and to the square root of the time of completion of the temperature cycle. Whence the heat per square foot,

$$Q = a\Delta T\sqrt{t} = a\Delta T/\sqrt{N}; \quad . \quad . \quad . \quad . \quad (5)$$

* Manual of the Steam-engine; vol. I, § 130 et seq.

in which, for iron, the value of a is about 2.5, N being revolutions per minute and T Fahrenheit degrees. Thus we have, for various speeds of engine, for the range of temperature of the metal, ΔT taken at 100° F., and in integral numbers, heat absorbed and rejected:

B. T. U. ABSORBED, AT VARIOUS SPEEDS.

N.... ...	50	65	80	100	120	150	200	250	300	400
Q..........	36	30	28	25	23	20	18	15	14	13

The depth to which the heat will penetrate may be taken as approximately

$$x = 0.018Q,$$

and the heat supplied by jacket, when used, as

$$J = 0.003Q;$$

and will x range, in the above cases, from 0.72 inch to 0.25.*

The range of temperature taken above is that of the cylinder-wall at its inner surface, and may or may not coincide with that of the steam in contact with it. With wet steam the two substances are probably often constantly of approximately the same temperature throughout the cycle; but, with dry superheated steam, and especially in fast engines, it is very certain that the range of the temperature-change in the metal is less, and often much less, than that in the fluid, and temperatures coincide while absorbing heat, but may widely differ while rejecting it. This is shown by the fact that, usually, condensation is not increased as rapidly, with depression of the minimum temperature, at low as at high minima, i.e., in condensing as in non-condensing engines.

The Sandy Hook Experiments, and others collated with them, give an average total condensation of 0.02 pound, per square foot, per minute of operation of engine, per Fahrenheit degree range of temperature. This corresponds to 0.000333 pound steam, or 0.3 B.T.U., nearly, per revolu-

* See Cotterill, § 135.

tion, and for one second, which was approximately the actual time of revolution, in the cases taken specially. Assuming, with Fourier, that the total heat-transfer is proportional to the square root of the time of exposure to the lower temperature, the weight of steam condensed will be, per revolution,

$$W = aA\Delta T\sqrt{t}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which the coefficient, a, will have the values given below for the several units of time usually taken.

Coefficient. a	Unit of Time. t
0.000333	Second
0.002582	Minute
0.02	Hour

If N be taken as the number of revolutions in the time taken for computation,

$$W = aA\Delta T\sqrt{tN}; \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and if N be the number of revolutions in the unit of time, t, $Nt = 1$, and $\sqrt{tN} = 1/\sqrt{N}$; the weight condensed in the unit of time chosen thus becomes

$$W = aA\Delta T/\sqrt{N}. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The values of the coefficient, above, are those representative of the operation of an unjacketed engine and of average good practice; they may be reduced to two thirds those values for the best average practice, and for ordinary work with jacketed engines; while for the most efficient simple, jacketed engines they may be divided by two. These differences being due to variable conditions of metal surface, of steam, of action of jacket.

For the same cases, the ratio of condensation to total steam required, as computed for the ideal case, may be taken as approximately

$$\frac{W}{W'} = \frac{a}{d}\sqrt{rt}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

in which a may be taken as 4, for d in inches diameter of cylinder, and time of revolution, t, in seconds. For rough approximations, the weight of steam condensed may be taken as about 15 pounds to the square foot of area of cylinder-wall, and as constant for all ratios of expansion, in the representative case in common mill-practice with simple low-speed non-condensing engines, and as 25 pounds for condensing engines. Two thirds these figures may be taken for ordinary jacketed engines and for unjacketed compounds, and one half for well-jacketed engines.

The quantity of heat wasted may be computed by the use of similar expressions to those for values of W, above. Taking the latent heat of the steam as 900 B.T.U., the quantity of heat wasted will be, per revolution, in B.T.U.

$$Q = aA\Delta T\sqrt{t};$$

for total wastes,

$$Q = cA\Delta T\sqrt{tN};$$

and for N taken as the revolutions in the unit of time,

$$Q = aA\Delta T/\sqrt{N};$$

where the values of the coefficients are

Coefficient. c	Unit of Time. t
0.30	Second
2.32	Minute
18.15	Hour

213. Steam in the Engine, as already shown, when subject to the influences characteristic of the ordinary steam-engine construction, behaves very differently from its assumed method of action in the ideal case. This fact may be perhaps best shown, in a general way, by the accompanying diagram of the action of one pound of steam entering the engine at 100 pounds absolute pressure, and in the dry and saturated condition. Its volume, at entrance, is 4.34 cubic feet and its temperature 327.6° F. If expanded from its initial and

minimum volume and retained, meantime, dry and saturated, its successive volumes, at successive pressures and temperatures, will be given by the steam-tables, and its expansion-line will be *bc*; while the adiabatic will be slightly lower, and as on the diagram, *bc'*; the difference being due to the progressive condensation of the steam expanding and doing work, *i. e.* to thermodynamic condensation.

In the actual engine, however, the steam entering is partly

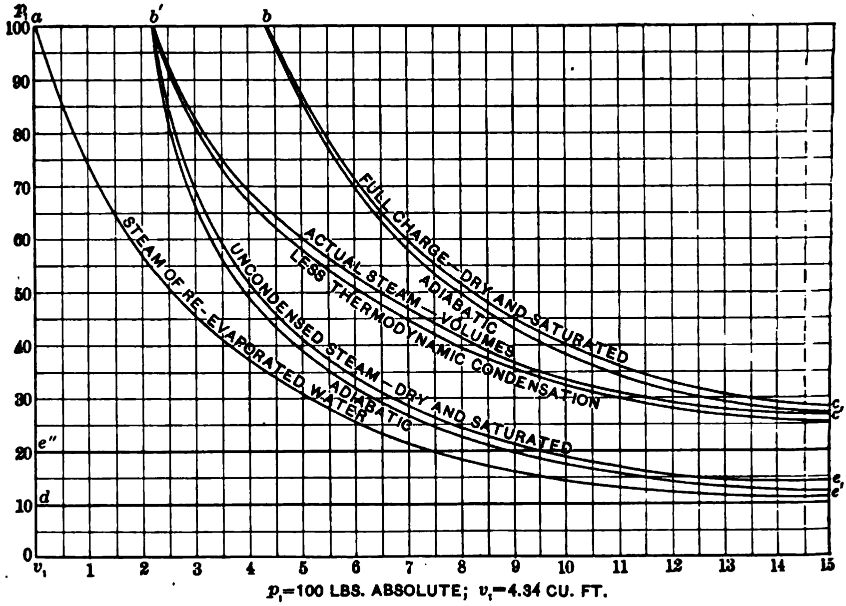

FIG. 175.—ONE POUND OF STEAM IN THE ENGINE.—RANKINE CYCLE.

condensed; and, often, in the simple engine with high ratios of expansion one half may be found, as in the diagram, to be thus reduced to the liquid state. The remaining steam, if expanded as assumed for the full charge, in the preceding case, would give the curve of constant weight, *b'e*, or, if adiabatically expanded, the line *b'e'*; thermodynamic condensation producing loss of pressure continuously. But this does not occur. The water of condensation at entrance commences re-evaporating just after the closing of the cut-off valve, and this produces wet steam continuously: the mix-

ture of this liquid and its vapor thus producing a volume continuously increasing, as along the line *ae'*, of the diagram. It often happens that practically all of the water of "cylinder condensation" is thus re-evaporated before the end of the stroke of the piston is reached. In such case the addition of the steam thus produced to that left uncondensed, at the moment of cut-off, gives the line *b'c*, if thermodynamic condensation is not taken into account. Such condensation does, however, occur in every case, and whatever the circumstances under which heat-exchanges take place, and to the amount of, approximately, a percentage measured by the ratio of expansion, as shown by the line *b'e'*; though in the case here illustrated, perhaps entirely obscured in its effects by the heat-interchanges observed between the fluid and the metal of the cylinder-wall or the water accompanying the steam when not initially dry.

The ideal engine-diagram differs thus from the real, in, often, a very remarkable and striking manner, and the corresponding differences of work performed per unit weight of working fluid are similarly disclosed by comparison of the areas, enclosed between these expansion-lines and the base-line, *ee''* or *e'd*, usual fair terminal pressure-lines respectively, for non-condensing and for condensing engines of good performance. It is this internal form of heat-waste that has constituted the main obstacle to advance in the improvement of the economical action of the steam-engine, and which has impeded approximation to its thermodynamic perfection.*

Superheated Steam, in practice, as has been seen, gives an advantage in three different ways: (1) It conveys more heat to the engine per pound and under better thermodynamic conditions; (2) it carries a supply of sensible heat which must be absorbed by the cylinder-wall before condensation can begin, thus raising the mean temperature and restricting the range of fluctuation, as does the jacket; (3) it insures drier, or if the sensible heat exceeds the thermodynamic condensa-

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* Marine Engineering, April 1897.

tion, superheated, steam during the exhaust or even a part of the period of expansion. All three effects conspire to make the steam a more efficient working substance.

In the case of the real engine, the following distribution of energy will be observed:

Cr.	*Dr.*
Heat, steam, or fuel demanded.	(1) Heat converted into work.
	(*a*) Useful work·
	(*b*) Friction " wasted.
	(2) Heat rejected.
	(*a*) Internal waste;
	(*b*) External "
	(*c*) Thermodynamic waste.

In the case of the jacketed engine the division becomes:

Cr.	*Dr.*
Heat, steam, or fuel demanded.	(1) Heat producing work.
	(*a*) Useful work;
	(*b*) Wasted "
	(2) Heat rejected.
	(*a*) Internal waste;
	(*b*) External "
	(*c*) Jacket "
	(*d*) Thermodynamic waste.

In the latter case the jacket-waste is an added loss; but the internal waste of heat by " initial condensation " is usually more than equally decreased, and the total waste is thus reduced.

The Chain of Efficiencies thus properly begins at the furnace and with the combustion of the fuel, and continues through furnace, boiler, engine, and the complete cycle; the product of the series measuring the total efficiency of the whole. Since every efficiency must necessarily be less than unity, often considerably less, the product may often be found very small, and in ordinary engine-practice ten per cent and less are common figures. The efficiency of furnace,

the product of the efficiency of heat-development by combustion and that of heat-transfer by conduction into the boiler, should be, respectively, not lower than 90 and 80 per cent.; their product, 72 per cent., measuring the proportion of energy latent in the fuel which is applied to the production of steam. The efficiency of the transfer to the engine should be, usually, nearly unity, with short steam-pipes well covered; the efficiency of the better class of engine should be thermodynamically, about 20 per cent., and as a whole, exclusive of friction, about 15 per cent., and including friction 12 to 14 per cent.; the product of the series being thus above 10 per cent. from furnace to driving-belt.

The following represents the distribution of energy and its losses in a system in which high-speed non-condensing engines supplied the power required and in which the available energy is measured at the meter: *

Energy at coal-pile	100			
boiler stop-valve	62	100		
indicated at engine	4.4	7	100	
dynamo	3.6	5.8	83	100
switchboard	3.3	5.3	76	92
meter	2.8	5.5	64	78

In analysis of heat-losses in the test of a Worthington pumping-engine at Hampton, Prof. W. C. Unwin summarizes the results as follows:

	Per cent.
Heat due to coal used	100.00
Given to steam	77.00
Carried off in furnace gases	10.00
Probable loss due to opening fire-doors	1.00
Due to carbon in ashes	1.10
Radiation and unaccounted for	10.8

214. Graphical Illustrations of Real Efficiencies are seen in Fig. 176, where the relation of steam-consumption and

* Milne: Trans. Canadian Elect. Assoc., 1895.

ratio of expansion is shown for 120 pounds absolute pressure. Curve *A* is for the ideal engine of Rankine, with non-conducting cylinder, no condensation and no clearance; *B* for the same engine, with the loss due to 7.6 per cent clearance added; *C* for the same engine, with conducting cylinder and condensation loss; *D* for the brake-power of the engine; i.e.,

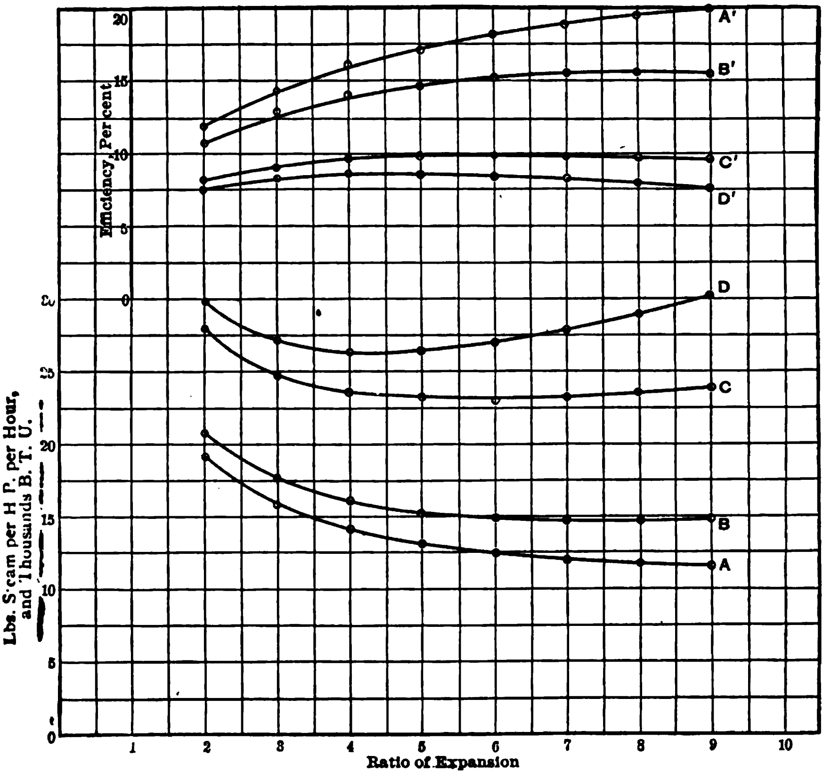

FIG. 176.—REAL ENGINE.—EFFICIENCIES.

with friction-loss. The vertical distance along an ordinate between *A* and *B* measures the increase in steam-consumption due to clearance, between *B* and *C* that due to condensation, between *C* and *D* that due to friction. *A'*, *B'*, *C'*, and *D'* are corresponding efficiency curves, and vertical distances between them measure corresponding losses of effi-

ciency. The radiation loss was too small a variation to be shown on the diagram.

In the graphical representation of the quantities which determine useful and wasteful expenditures and ideal or real

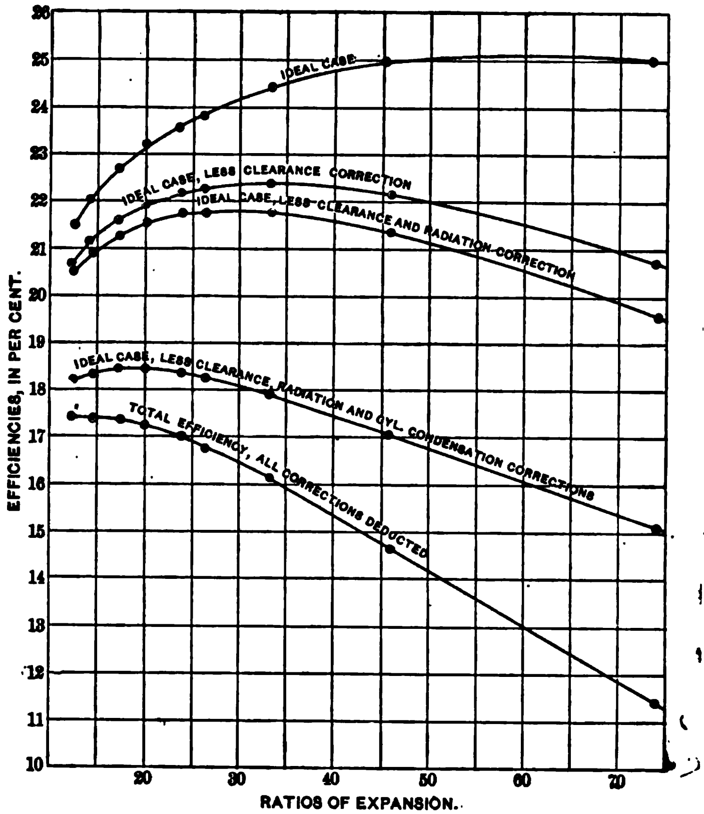

FIG. 177.—REAL ENGINE.—EFFICIENCIES.

efficiencies, the economy of the machine is thus gauged by the amount of steam consumed, and the various items of total expenditure are given in these superposed curves; the magnitude of the ordinate intercepted between each and the next lower curve measuring that particular item indicated in the legend on the curve referred to. The expenditures of a well-known pumping-engine of high efficiency, made up on an

absolute efficiency-scale, are shown in Fig. 177. The pistons were 18 inches, 32 inches, and 48 inches diameter, with a stroke of 36 inches; it was rated at a capacity of 5,000,000 foot-pounds in twenty-four hours, and a duty was guaranteed by its builders of 120,000,000 foot-pounds per 1000 pounds of steam supplied from the boilers. Its actual performance was a duty of 128,108,123 foot-pounds on 100 pounds of dry coal, for a period of nine months. The diagrams showed clearly the manner in which its various wastes varied with varying conditions of operation as computed by securing, from an actual trial, the constants for use in well-established formulas for wastes, and the exact formulas of thermodynamics so far as regarded the ideal case.

215. Practical Results of Thermodynamic Action, as modified by steam-pressure and as determined by experiment

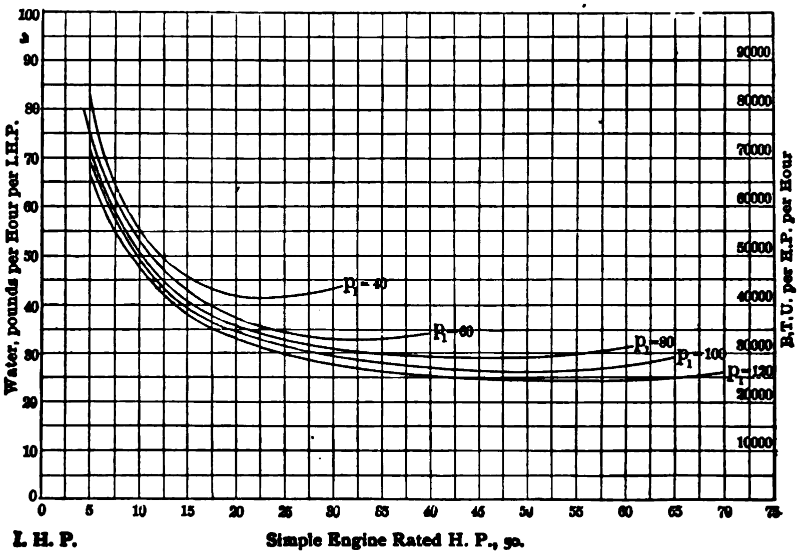

FIG. 178.—STATIONARY CONDENSING-ENGINE EFFICIENCY.

upon engines of ordinarily good construction and performance, are illustrated in Fig. 178, in which the curves of efficiency for an engine rated at 50 horse-power with 100 pounds pressure are given. Collating the results of numerous

tests of this, simple, engine of nine inches diameter of cylinder and of moderate piston-speed, working as a condensing engine, we obtain data which give smooth curves of the character here presented, after careful rectification of curvature and of relation of location.*

The points to be noted are the gradual decrease of the gain attainable by increasing pressures, the location of the power for maximum economy for each pressure, and the extension of its range of practical availability with minimum cost of power as pressures rise. At 40 pounds the best work is done at between 20 and 25 I.H.P., and economy falls off rapidly outside these limits. At 60 pounds the best work is done at between 30 and 40 H.P., and about twice as wide a range is permissible, with the previously assumed allowable variation in economy. At 80 pounds the work is most satisfactory at 45 H.P., and a range of 20 H.P. gives but little variation in the cost of power. At 100 pounds the best power is 50 H.P., and the range of nearly constant economy is still wider; while at 125 pounds, the limit of pressure observed, the engine does its best work at about 55 or 60 H.P., and good work up to 70 H.P.

The same story is told, in a different and perhaps more familiar way, by Fig. 179, in which the ordinates are ratios of expansion, and the abscissæ are costs of power in weights of steam and of feed-water, as before. Pressures ranging from 40 to 120 pounds, the same minima in costs of power are exhibited, and the best work is seen to be effected at ratios of expansion rising in magnitude from 2½ to 4. The range of cut-off giving best work is seen to be more restricted, and variation from that value more costly at the lower than at the higher pressures. Increasing the pressure three times here gives about 50 per cent higher cost of power at the lower than at the higher pressure. These two diagrams illustrate the behavior of the average well-designed and well-constructed simple engine of our day, when of small size and having large

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* Trans. A. S. M. E., vol. XVIII, No. DCCXVIII. Figs. 8, 10.

clearance, and within the range of pressures which have been noted in mill-engine operation during the last half-century; the standard condensing engine of the various dates being taken as the basis of our selection. This may be taken as illustrative, in fact, of the past and present simple mill-engine

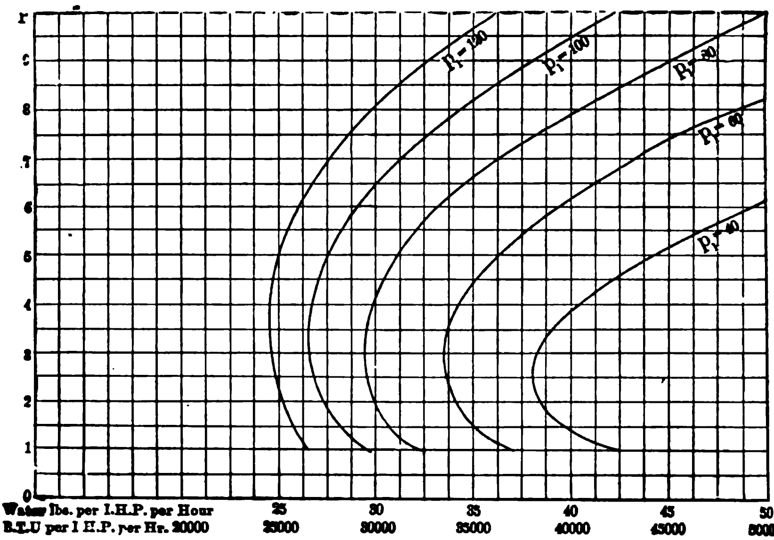


FIG. 179.—ECONOMY WITH VARYING RATIO OF EXPANSION.

practice as pressures have gradually risen in our New England mills from the lower to the upper limit of our range. These data, derived from a small engine, may be taken as representative of either an unusually excellent machine of that size or of a fairly good engine of the average size found in our mills. Large and exceptionally good engines will give better figures; engines taken as they happen to come into the experience of the engineer will fall below these figures, and often very considerably.

Collating a considerable quantity of similar data, from reports on various sizes, types, and constructions of engine, the same form of curve is obtained for all cases, and the mean for good makes of engine under favorable circumstances is usually departed from but little in individual cases. A study

of this field by Professor Carpenter gives the following results: *

The probable actual consumption of feed-water by an engine may be expressed by an equation of the form

$$W' = Wa\sqrt{x};$$

in which W is the demand of the ideal case and x the ratio of the larger to the smaller quantity when the actual and the

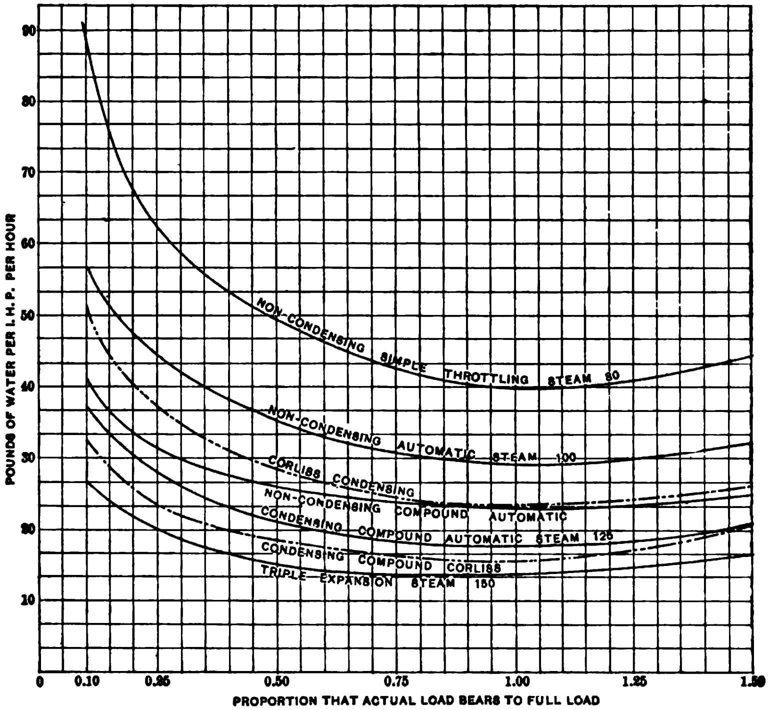


FIG. 180.—WATER CONSUMPTION WITH DIFFERENT LOADS.

best rated load of the engine are given. The value of a ranges from 9 and 10 with compound automatic non-condensing engines to 50 and 100 per cent higher figures, as the

* See Trans. Am. Inst. Elect. Eng'rs, 1893.

machine becomes smaller or for other reasons more wasteful, and from 6 to 10 with condensing engines. W, similarly, ranges from 14 to 20, ordinarily, in the one case, and from 8 to 10 in the other.

Fig. 180 graphically and more fully exhibits these deductions from the results of common good practice. It seems a well-settled fact that, as a general rule, the most economical engine at its rated and assumed best load has the widest range of economical operation, and loses least through internal wastes when such variations occur.

The best work recorded to its date, for a triple-expansion pumping-engine at moderate pressures of steam, give for its equation, deducing this constant a from its best trial-logs,

$$W' = W + a\sqrt{x} = 8 + 3.76\sqrt{x}.$$

The probable water-rate for this engine for different loads would be as follows:

Part of Rated Load.	Water-rate. W'.	Part of Rated Load.	Water-rate. W'.
0.10	17.9	1.00	11.79
0.25	15.6	1.25	12.25
0.75	12.37	1.50	12.65

The "rated load" is assumed to be that giving highest "duty."

The preceding figures for the indicated power become somewhat modified by the friction losses, and the figures given in Chap. V. will serve to show to what extent. The next diagram, Fig. 181, gives a graphical illustration of these modifications comparable with that previously presented, when referring to indicated power.

In this comparison the general character of the curves which represent the variation in water-consumption on this basis are somewhat changed, although the curves of each class, condensing and non-condensing, occupy the same relative positions as before. These curves for the condensing engines deviate more from a right line in this than in the

previous case, and all of them become vertical at a point when the friction of the engine is equal to the indicated horse-power.

The general form of these curves is shown in Fig. 181. It is noted that the simple Corliss engine shows wide variation in economy on the basis of delivered horse-power, and that its curve intersects and rises above that of the compound non-condensing engine for low loads.*

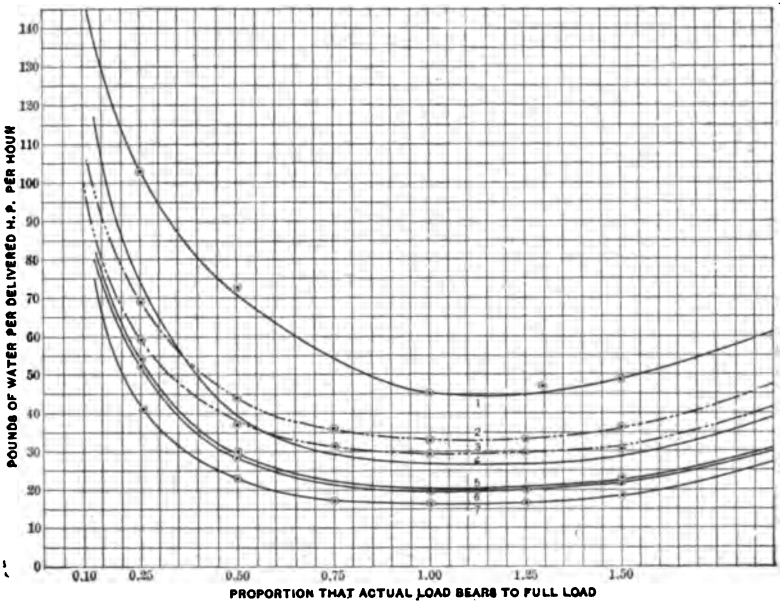


FIG. 181.—WATER CONSUMPTION PER DELIVERED HORSE-POWER.

Curve No. 1.	Non-condensing simple throttling,	steam	80
" " 2.	" " automatic,	"	100
" " 3.	" " compound automatic,	"	125
" " 4.	Condensing, Corliss "	"	80
" " 5.	" compound "	"	125
" " 6.	" " Corliss,	"	100
" " 7.	" Triple-expansion,	"	150

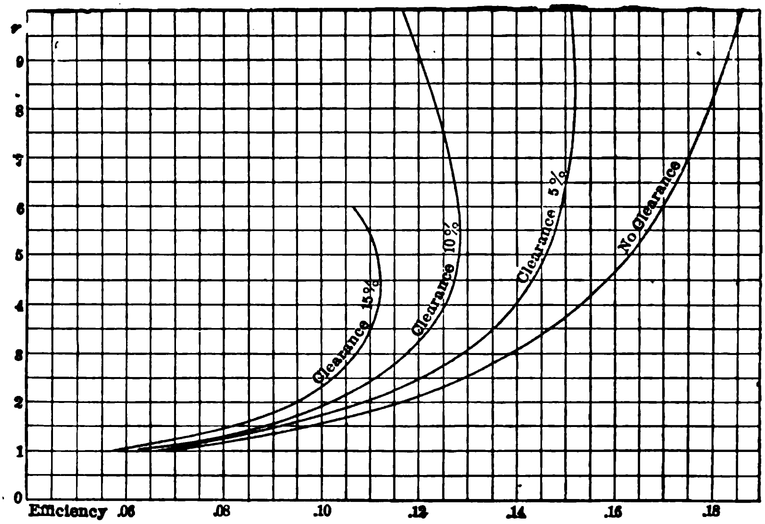
The performance of engines under favorable conditions and with proper adjustment, at their "rated" and presumably best load for economical use of fuel may be roughly

* Ibidem.

taken for good average cases, and best types of engine, as giving fuel-consumption nearly as follows:

Class of Engine.	Lbs. Coal per I.H.P. per Hour.	Steam.
Stationary mill-engine (compound)......	1.5	12
Marine engines (triple).................	1.4	11.2
Locomotive-engines (compound)........	2.5	20.00

The low efficiency of boiler and the comparatively ineffective steam-distribution of the locomotive greatly reduce its efficiency as a whole. Burning 75 pounds of fuel per square



$p_1 = 100$; $p = 4$; Clearance 0, 5, 10, 15, per cent

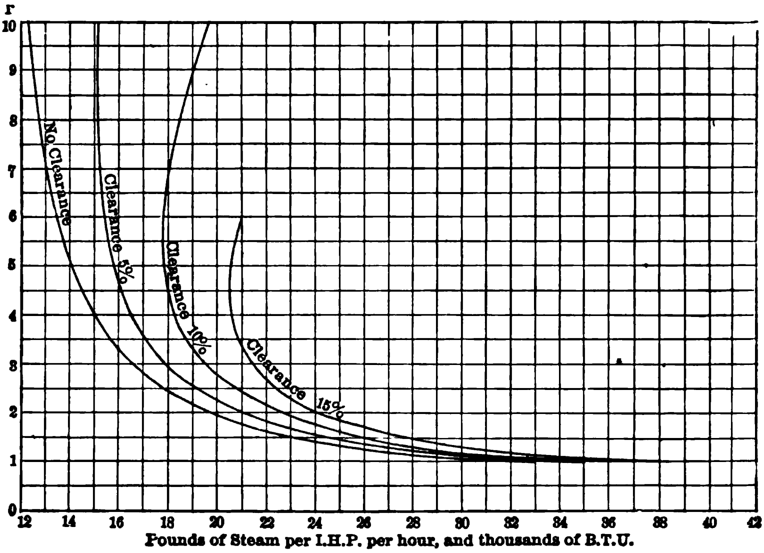
FIG. 182.—EFFICIENCY AND CLEARANCE.

foot of grate and upward, and with steam-pressures such as give high temperature of boiler and with no efficient available system of feed-water heating, the evaporation is usually low, as is the thermodynamic efficiency.

216. **Clearance Losses** are sometimes important, especially in the better class of engines. The accompanying curves, Fig. 182, may be taken as illustrative of the bearing of this defect upon engine performance in the case of the familiar forms of condensing mill-engine. The main facts are

the somewhat rapid increase of this form of waste with increasing clearances, and the obvious deduction that much of the gain noted in recent years in the better classes of mill-engine are unquestionably due to the skill exhibited by their designers in making the "dead spaces" of minimum volume. It is especially interesting to note the influence of decreasing clearance in not only effecting economy and raising the value of the efficiency, but also in raising the value of the ratio of expansion for best performance, and thus permitting more complete utilization of approximately adiabatic expansion and resultant heat-conversion and utilization.

Fig. 183 exhibits the same facts in a more familiar form, and gives the weights of feed-water corresponding to the

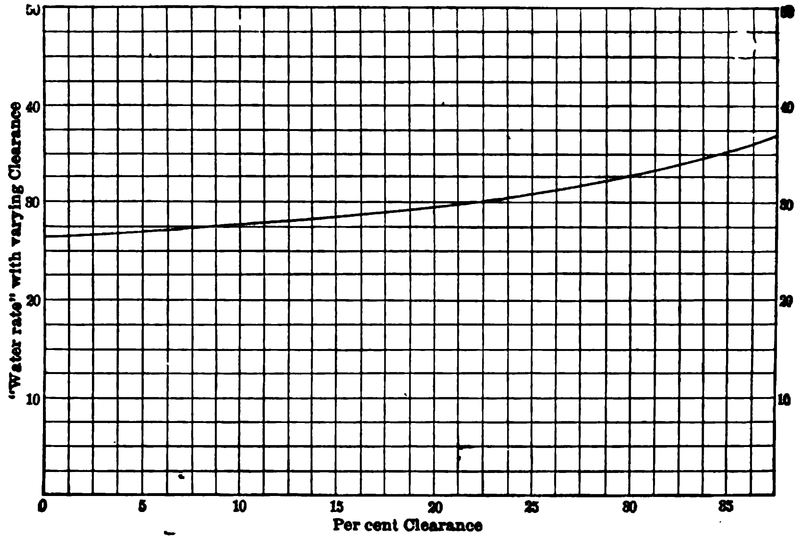


$p = 100$; $p_3 = 4$; Clearance 0, 5, 10, 15 per cent.

FIG. 183.—EFFICIENCY AND CLEARANCE.

efficiencies. Both diagrams are for what may be taken as a representative steam-pressure, 100 pounds per square inch. It is seen that, were the engine to be subject to all other wastes, precisely as at present, it would be practicable, clearance being abolished, to bring up the efficiency to about

20 per cent. The figures of the second set of curves give about 12 pounds of feed-water per power-unit as the limit for the engine with zero clearance, and maximum efficiency would be reached with a higher ratio of expansion than 10. With 5 per cent. clearance, the expansion-ratio is restricted to about 9 for best effect, and the cost of power becomes larger than the minimum by one third. Ten per cent. clearance reduces the available ratio to 5.5, and the steam-consumption becomes 50 per cent. higher than the minimum. Fifteen per



$p_1 = 100$: $p_2 = 18$; Clearance variable

9″ × 36″ at 90 Revolutions.

FIG. 184.—CLEARANCE AND EFFICIENCY, CORLISS ENGINE.

cent. clearance brings down the effective expansion to a cut-off at one fourth, and the steam supplied becomes nearly doubled —21 pounds per I.H.P. per hour. Here also, as in other cases, the conditions restricting economy also restrict the range through which power may be varied, without serious ill-effect, on either side the point of maximum efficiency. Clearance is thus seen to be at once an important element of waste and of restriction of the application of the principles of

maximum economy, and its elimination one of the essential elements of further progress.

Fig. 184 illustrates the same general variation of efficiency with varying clearance for the case of a fairly economical non-condensing engine, and presents the measures of steam-consumption on a diagram in which the curve is, as in the preceding cases, carefully rectified by reference to both the rational and the experimentally determined qualities.

The clearance was here artificially varied from about 35 per cent down to one third that figure; the curve is sufficiently well established to permit tracing it back to zero clearance. The figures for steam-consumption actually varied from 35 pounds down to 28, and, tracing back to zero clearance, we get 26 pounds; but, on a large scale, as 500 H.P. and upwards, these weights should be reduced to one half their present magnitude.

These losses are thus found, in the case here illustrated, to be sensibly proportional to some constant quantity plus the volume of the "clearance" or "dead-spaces," in the ordinary mill-engine of the four-valve type; increasing somewhat more rapidly at extremely high values of those volumes.

The effect of clearance is reduced to a minium, dynamically when the ratios of expansion and compression are equal.

The work performed in an engine without clearance is

$$U_1 = p_1 v_1 (1 + \log_e r_1);$$

where hyperbolic expansion is assumed, and where initial pressure and volume and the ratio of expansion adopted are, respectively, p_1, v_1, and r_1.

When a clearance-space exists of volume v_c, the same volume of steam, v_1, being admitted, the volume of cylinder, instead of $v_2 = r_1 v_1$, will be $r_1 v_1 + v_c$, and the point of cut-off must become

$$c_c = \frac{v_1 - v_c}{r_1 v_1 - v_c},$$

instead of

$$c = \frac{v_1}{v_2} = \frac{1}{r_1},$$

and the expansion is increased, adding the work performed in the added volume v_c at the previous terminal pressure, $p_1/r = p_2$, or

$$U_c' = p_2 v_2 \log_e r_c;$$

where $r_c = \dfrac{v_1 + v_c}{v_1}$.

The total work will then be

$$\begin{aligned} U_c &= U_1 + U_c' - p_1 v_c \\ &= p_1 v_1 (1 + \log_e r_1) + p_2 v_2 \log_e r_c - p_1 v_c \\ &= p_1 v_1 \left(1 + \log_e \frac{v_2}{v_1}\right) + p_2 v_2 \log_e \frac{v_1 + v_c}{v_1} - p_1 v_c. \end{aligned}$$

The net result will be a loss of work measured by

$$U'' = p_1 v_c - p_2 v_2 \log_e \frac{v_1 + v_c}{v_1};$$

or a fraction of the first quantity of resultant work,

$$F_c = \frac{p_1 v_c - p_2 v_2 \log_e r_c}{p_1 v_1 (1 + \log_e r_1)}.$$

This will approximate the value

$$F_c' = \frac{v_c (p_1 - p_2)}{p_1 v_1 (\log_e r_1 + 1)}$$

as the volume of the clearance-space decreases and as the value of r_1 increases.

217. The Study of the Real Case, illustrated by the action of several different classes of engine, may prove useful, as leading to more exact quantitative investigation of the

mathematical expression of quantity and rate of heat-exchange. The method pursued is here that of laying down a "saturation-curve," and placing beside it the expansion-line of the diagram itself. The relative magnitudes of the volumes indicated on the curves measure the proportions of steam existing in the mixture in the engine-cylinder. The variations of this proportion measure, in turn, the variations of composition of the working-charge, and the "quality" of the fluid is thus at every instant determinable as a fraction, completely dry steam being taken as unity.*

In many cases a simple equilateral hyperbola will give a reference-line of sufficient exactness of location for the purposes of the engineer, as the indices of the three standard curves only vary as the numbers 1.000, 1.0646, 1.135, and for moderate ratios of expansion the lines are very nearly coincident. For present purposes the saturation-line, laid down with extreme accuracy, is what is required.†

It is also instructive to draw the adiabatic curve in illustrating the changes which would occur could the same quantity of steam be worked in a non-conducting cylinder, and all wastes by heat-exchange between steam in the engine and the metal of the cylinder thus avoided. This would also show, by its departure from the saturation-curve, with which it would be coincident at its beginning, the amount of thermodynamic condensation—that produced by conversion of internal energy into the work of adiabatic expansion (p. 440).

These methods, and the results of their application to a few specially interesting and typical cases, will be illustrated in the following examples, each of which appertains to a

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* The value of the saturation-curve as a base-line, or reference curve, was first suggested by Professor Cotterill. It has been used extensively in Sibley College work, in connection with such investigations as those here in part described, and it has been made a standard system of measurement of heat and steam variations by Professor Carpenter in the work of the Department of Experimental Engineering.

† This system was referred to by the Author in the discussion of a paper on the pumping-engine, where the question of best methods of combining multiple-diagrams came up (Trans. A. Soc. M. E., vol. xv). For an early example of this application, see pp. 406, 407.

familiar and important class of steam-engines. They are especially instructive, as applied to the multiple-cylinder engine, in which the action of heat-transfer and of heat-transformation has hitherto remained somewhat obscure, in the absence of such studies of the facts of their operation, cylinder by cylinder. It will be noted that the saturation-curve for the combined diagram must usually be discontinuous, as only absolute equality of clearance and compression and of steam-condensation can give perfect continuity.

Fig. 185 presents the diagram from a small, plain slide-valve engine, of 6-inches diameter of piston and 8-inches

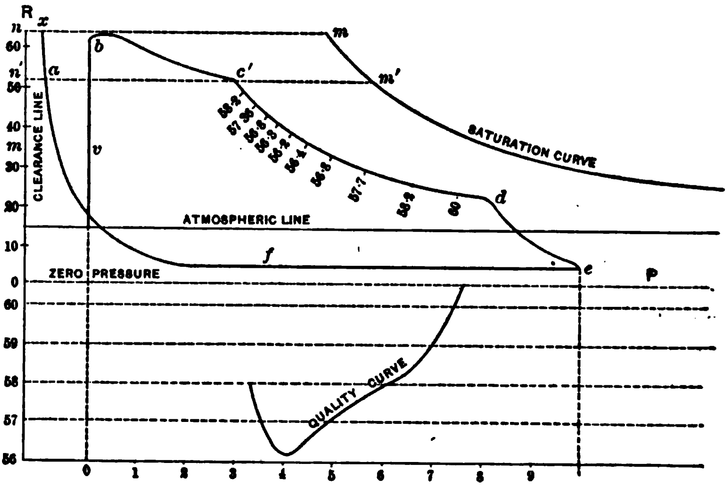


FIG. 185.—SINGLE-VALVE ENGINE-DIAGRAMS.

stroke; cut-off set at three tenths, making 208.4 revolutions per minute, and consuming 39.7 pounds of steam per horse-power per hour. The compression-line of the indicator-diagram is carried up to the steam-line to determine the quantity in the clearance-spaces, and the amount admitted to cut-off; and the saturation-curve for this weight of steam is laid down at the right, in such manner that the co-ordinates of the expansion-line of the latter and those of the saturation-curve shall coincide in magnitude in measuring the varying pressures, and shall have the proportions of volume of mixture to that

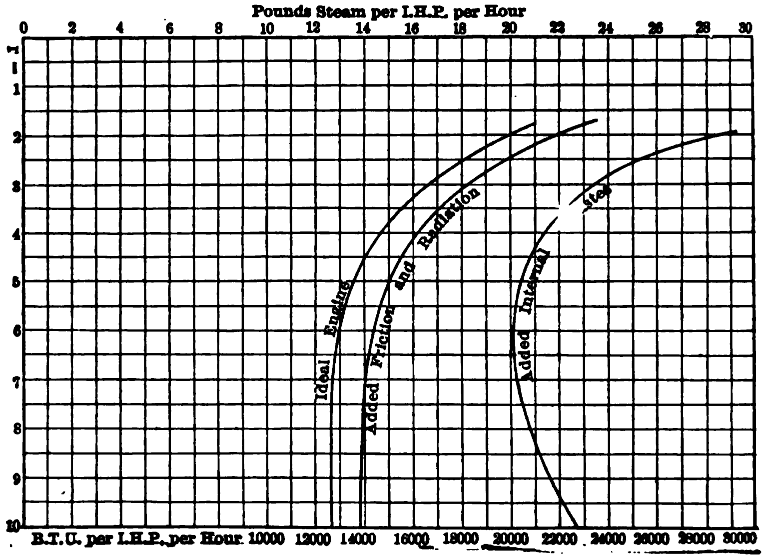
of the fluid when completely vaporous. Below, with corresponding ordinates in line with those of the indicator-diagram, is the "quality-curve," exhibiting the variation of quality of steam, with progressing expansion, from the point of cut-off to the end of the expansion period; outside which range this variation cannot be traced on the diagram.

The interesting facts are the extensive initial condensation, the passage of the fluid through a point of maximum depreciation of quality, i.e., of maximum condensation, after the point of cut-off is passed, and the gradual recovery of quality to the end of the expansion period; and, finally, the fact that, at the opening of the exhaust-valve, the re-evaporation is barely sufficient to restore the quality of the steam at the closing of the cut-off valve. The assumption, commonly made, that the condensation at cut-off represents a complete waste of heat and steam, is here justified. The "quality-curve" enables us to locate the point of inflection and of maximum condensation at almost precisely 0.4 stroke. It is at this point that Isherwood experimentally located the cut-off of maximum effect in engines of this class and in marine engines of the older types. The condensation at cut-off is here 42 per cent, increasing from cut-off, at 0.3 stroke, to the maximum, 44 per cent, at 0.4 stroke, and then decreasing, by re-evaporation, to 40 per cent at the end of the expansion-period. The varying speed of piston and changing temperatures of the gradually uncovered surfaces of the cylinder-wall produce a curious wave in the quality-curve; this may be used to obtain approximate measures of temperatures of metal along the cylinder, and of rate of heat-transfer between metal and steam.*

218. **The Distribution of Energy,** usefully and wastefully, in what is here taken as a representative engine, may be seen in the next set of curves (Fig. 186), in which are

* It will be noticed that the atmospheric line is set at a barometric pressure of 14.42 pounds per square inch. This comes of the fact that the engine (a Sibley College experimental machine) is operated at an elevation of several hundred feet above the sea-level.

shown, after a manner already employed, the variation of quantity of steam and of heat expended in the ideal and the real cases, at ratios of expansion ranging from unity to ten, and with boiler-pressure at 100 pounds, back-pressure 5 pounds, and rated power at 200 I.H.P. The figures are fair average figures for an engine of usual proportions and eco-



$p_1 = 100$: $p_2 = 5$

FIG. 186.—CONDENSING-ENGINE EFFICIENCIES.

nomic value. The ideal thermodynamic case, taken as a Rankine jacketed cycle, would give the lower of the several curves, and a minimum cost of power measured by 12.5 pounds of steam or of feed-water per I.H.P. per hour. Added friction and external thermal losses thus raise the figures above those of the ideal case 1.5 to 2 pounds; internal wastes bring them up to nearly double the ideal figure as a minimum, adding about 6 pounds to the figures for the ratios of expansion and the cut-offs of minimum costs.

The best ratio of expansion is found, finally, at about 6, with little variation between 5 and 7. With a back-pressure of 2 instead of 5 pounds these values of the best adjustment

of that ratio would have been somewhat enlarged, and the economy of the engine considerably increased as well.

A Hirn thermal analysis of the action of this engine, in regular operation, gives the following data:

	Pounds.
Weight of steam used per stroke	0.0126
Weight of steam in clearance	0.00165
	B. T. U.
Total heat supplied	12.857
Loss to metal during admission	5.5114
Returned during expansion	1.3586
Returned during exhaust	3.0365
Returned during compression	0.2973
Waste by radiation	0.8190
	Per Cent.
Heat supplied to the engine	100.00
Stored during admission	42.88
Restored during expansion	10.57
Rejected with exhaust	23.60
Restored during compression	2.31
Lost by radiation from exterior	5.39
Transformed thermodynamically	5.25
Rejected untransformed	94.75
Mechanical efficiency	71.20

The efficiency-curves (Fig. 187) for varying ratios of expansion, as based upon the performance of the engine with 85 pounds pressure, absolute, show the resultant effect where *a*, the lower line, exhibits the varying efficiency of the ideal representative as measured in pounds of steam consumed per hour; *b*, the added tax due to the frictional or dynamic waste, the cost in steam being superposed upon the first curve, and the waste by external radiation added to give *c*, the third line; while the internal wastes similarly added as accessions to the values of the ordinates, giving *d*, the upper curve, we are enabled to read the final costs of operation, in steam consumed in the actual case, for ratios of from unity to twelve.

Thus, this engine, with a back-pressure not exceeding 4 pounds per square inch, could all extra-thermodynamic wastes be extinguished, would have no ratio of expansion for maxi-

mum efficiency within the limits here explored; the friction losses being added, the net power would be obtained at minimum cost in steam used at about $r=8$; external wastes restrict this figure to about 7; and extreme expansion, when internal wastes come into the account, and maximum total efficiency is sought, cannot exceed 4.5 as the highest ratio for

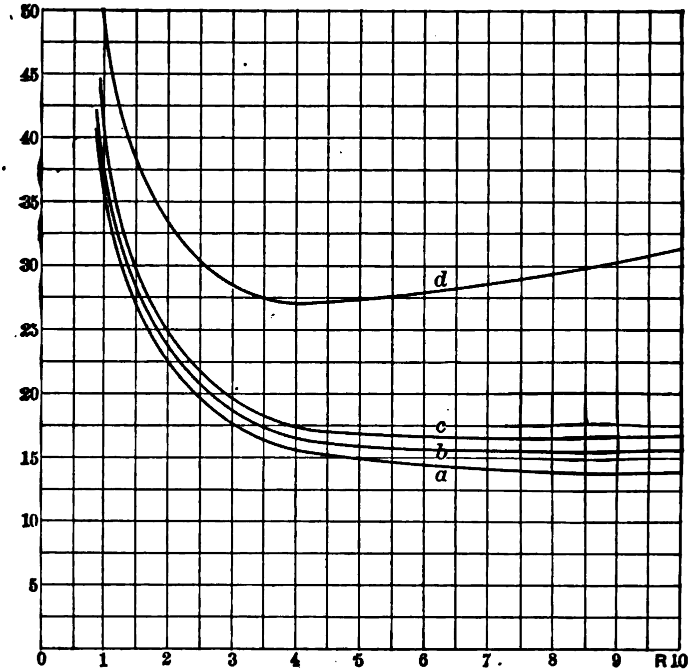
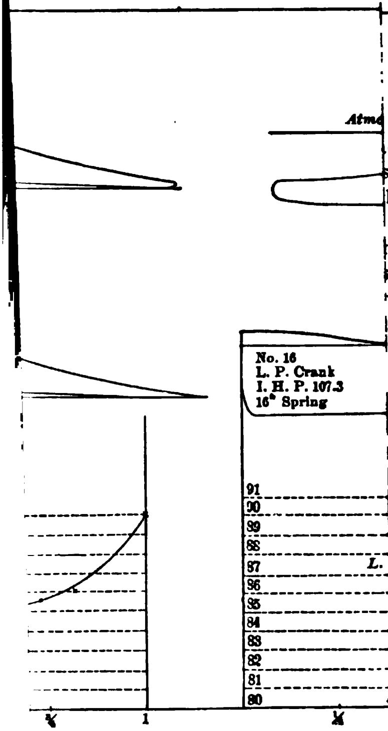


FIG. 187.—EFFICIENCY-CURVES. SMALL ENGINE; THREE-PORTED SLIDE-VALVES.

highest "duty." The ideal thermodynamic case would demand but about 14 pounds of steam per horse-power per hour; while the added wastes of the real case, mainly losses by heat-exchange between steam- and cylinder-wall, restrict the expansion to a ratio not exceeding 4½, and raise the steam-consumption to 27 pounds at highest attainable duty, thus doubling the cost of fuel and steam.

Fig. 188 illustrates the application of these methods to an



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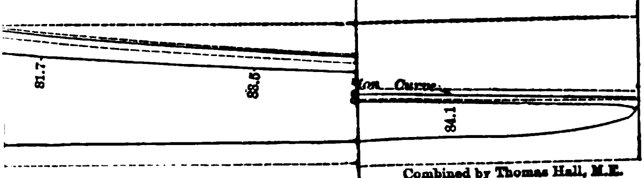

Combined by Thomas Hall, M.E.

...ULTIPLE-CYLINDER DIAGRAMS.

engine of highest type of contemporary construction, three cylinders in series, employing high-pressure steam with a large ratio of expansion. It is the combined diagram of a pumping-engine, beside which are laid down the adiabatic for the total steam used, the saturation-curve for the working-charge, and the hyperbolic line for an equal initial volume of charge. The adiabatic shows the work which would have been performed in a structurally perfect engine of the same cycle, free from extra-thermodynamic wastes by an equal weight of steam from the boiler; the saturation-curve exhibits the effect upon the actual charge of working-steam of the heat transferred from the jacket; and the hyperbolic curve may be taken as a standard of comparison for all. The figures on the margin of the indicator-diagram are those of quality of steam.

The saturation-curve is here discontinuous, for reasons already given; the other curves are continuous. All follow the same general trend; they coincide at about the middle of their length, and there cross, separating slowly to the end, but are never far apart at any point. The hyperbolic line is the most nearly approximate to the path of the expansion-lines of the diagrams given by the indicator. The departure of the saturation and adiabatic curves at their upper limits measures the proportion of jacket-steam, which is included under the latter, but not under the former; since it is desired to exhibit by the saturation-curve the heat-flow between working-charge and jacket-steam, and the variation of quality of the working steam. The zero-line of pressure is here 14.5 pounds below atmospheric pressure. The diagrams are remarkable for the exceptional fulness of the actual, as compared with the ideal, total-work, diagram; for the small amount of initial condensation, for the uniformity of this condensation in the several cylinders, and for the small loss by drop of pressure between cylinders. The engine is remarkable for the smallness of its clearance-volumes, the promptness of cut-off, the effectiveness of its jacketing, and that general excellence of design which enables the ideal to be so nearly approached. The low back-pressure is one of the

most noticeable points to be observed in this case. The most essential quantities are inscribed on the diagram in Fig. 188.*

Computing the efficiency of the ideal representative case for this engine, and the wastes of the real engine, and comparing them with the results of test, the figures given in the succeeding table are obtained. The wastes are computed for the low-pressure cylinder, and the assumption is made that all work is performed in that cylinder. The dynamic waste is taken, at the usual rating of the engine, as 10 per cent of the delivered power; the internal heat-wastes are computed by the formula†

$$c = a\frac{\sqrt{rt}}{d},$$

in which a is taken, as in the Sandy Hook experiments, as 4, and $r = 19.55$, $t = 2.96$;

$$c = 4\sqrt{\frac{2.23 \times 2.96}{74}} = 0.13932.$$

External wastes of heat are taken as 0.5 B.T.U. per square foot of exterior surface, and per degree difference between external and internal temperatures:

COMPARISON OF IDEAL AND REAL ENGINES.

Size of engine	28 + 48 + 74 × 60 inches.
Real ratio of expansion	19.55
Mean effective pressure, ideal case	25.00
Mean effective pressure, real, computed	18.99
Mean effective pressure, real, observed	21.80
I. H. P., ideal	660
I. H. P., real, computed	567
I. H. P., real, observed	573.9
D. H. P., ideal	660
D. H. P., real, computed	501
D. H. P., real, observed	520.9
Dry steam, per I. H. P., per hour, ideal	8.90

* See an account of this engine by the Author in a paper "On the Maximum Economy of the Contemporary High-pressure Steam-engine;" Trans. A. S. M. E., 1893.

† Chapter V. See also Appended Notes, §§ 37, 148.

Dry steam, per I. H. P., per hour, real, computed......	11.73
Dry steam, per I. H. P., per hour, real, observed.......	11.68
Heat, B. T. U., per I. H. P. per minute, ideal..........	167
Heat, B. T. U., per I. H. P. per minute, real, computed.	220
Heat, B. T. U., per I. H. P. per minute, real, observed..	217.6

The curves of efficiencies (Fig. 189) illustrate the action of this engine. The lower curve is that of steam-consumption of the ideal case, the next the dynamic wastes, the third the external thermal loss, and the full line gives the internal

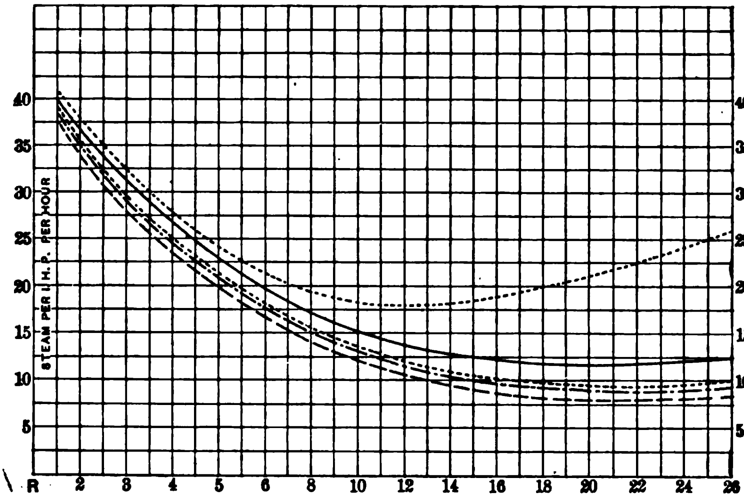

FIG. 189.—EFFICIENCIES OF TRIPLE-EXPANSION ENGINE.

thermal wastes, each quantity being superposed on the preceding in such manner that the ordinates of the highest curve give the computed, and approximately the real, steam-consumption of the engine at the ratios of expansion corresponding to their location. It is seen that the real engine has a maximum efficiency, thus measured, at a ratio of expansion of about 20; the ratio actually adopted by the builders. The fact that its record is one of the best known is evidence that the conclusion is substantially correct. The upper dotted line in the diagram is that representing the internal thermal wastes of the engine, as computed on the assumption that all

work is done in one cylinder. It exhibits the wide difference in final efficiency produced by the cascade action of the multiple-cylinder engine; the effect on the best cut-off is brought out strikingly.

The fac-similes of the diagrams taken from the several cylinders which are combined, and the quality-curves which are obtained from them, are given on Fig. 188. The engine has steam-cylinders 28, 48, and 74 inches in diameter, and 60 inches stroke of piston, carrying 120 pounds steam-pressure at the steam-chest, making 20.3 revolutions per minute when giving a duty of 154,448,000 foot-pounds per 1000 pounds of dry steam supplied. The essential data are inscribed on the diagrams. Below each pair is placed the "quality-curve." These quality-curves exhibit the action of heat-exchange in the elements of the series, and the effect of the jacket in modifying transfer.

In the high-pressure cylinder the condensation ceases more nearly at the point of cut-off than in the preceding case. The cusp is present, reduced by the action of the jacket. In this case the initial condensation is less than 12 per cent. The same wave in the line between the point of the cusp and the terminal of the curve is here seen, as elsewhere observed. The variations of curvature are not extensive, and the heat-exchanges, therefore, correspondingly less serious in reduction of efficiency. The total change of "quality" between cut-off and exhaust is from 88 to 90.5, though the jacket held down initial condensation to 12 per cent. The facts indicate the small amount of heat transferred from jacket to working steam in the high-pressure cylinder, and the correctness of the common assumption that the condensation at cut-off measures internal waste with substantial accuracy. Correcting for the concealed effect of thermodynamic condensation, amounting to about 8 per cent, it appears that the jacket and re-evaporation, together, transfer heat enough to the charge to improve its quality 10 per cent. The cut-off takes place at one-third stroke; the maximum condensation occurs at four-tenths, precisely as in the unjacketed engine.

The intermediate cylinder exhibits the cumulative effect of jacket-action in the reduced extent of the cusp, its earlier occurrence after cut-off, and the larger improvement in quality of steam between cut-off and exhaust, which amounts to above 8 per cent. The same wave is seen as in the preceding cases, indicative of variation of relative velocities of piston and of heat-exchange. Cut-off takes place at 0.34, and the cusp is found at 0.36. Thermodynamic condensation should increase the waste from the 13 or 14 per cent. at cut-off to about 18 per cent.; while the actual change is an improvement of 9 per cent., showing jacket-action and re-evaporation to have transferred to the steam nearly 15 per cent. of the total heat-energy. The curve for the low-pressure cylinder shows still further progress in the same direction. The cusp has disappeared; the steam is continuously drier as the piston moves from cut-off to the end of stroke, and the improvement in quality amounts to about 10 per cent., notwithstanding a further thermodynamic condensation of about 5 per cent. Jacket-action is more effective in the intermediate than in the high-pressure cylinder and receiver, and still more so in the low-pressure receiver and cylinder. In this engine, as a unit, adiabatic condensation is about 17 per cent., and the jacket holds down initial condensation to 12, and the final to 8 per cent., counteracting at the same time the adiabatic condensation, and giving a total heat-transfer sufficient to vaporize a net 20 per cent. at final exhaust, of which 9 per cent., as measured, comes from the jacket itself, and the remainder from the heat previously taken into the cylinder-wall. This latter quantity, 12 per cent. at the start, is thus largely retransferred before exhaust; a fact apparently indicating the complete drying of the surfaces of the cylinder previous to the opening of the exhaust-valve on the low-pressure cylinder.

The final net result may be stated thus: The supply of 9 per cent. of the steam to the jackets, in this case, produces a reduction of initial condensation, from a probable 30 per cent. at the beginning of expansion, to 12, and counteracts thermo-

dynamic condensation to the extent of about 9 per cent. by transfer of heat to the working-steam. Thus checking thermodynamic condensation results in the restriction of internal wastes by about double the amount of heat so expended through the intrinsically wasteful action of the jacket.

The following may be taken as figures illustrating the performance of modern engines, generally:

PERFORMANCE OF ENGINES.

	Best.	Good.	Medium.	Low.
B. T. U. per D. H. P. per hour......	14,500	20,000	30,000	60,000
" " I. H. P. per hour.......	13,000	18,000	27,000	50,000
" " ideal H. P. per hour....	10,000	10,000	10,000	10,000
" " Carnot H. P. per hour...	8,500	8,500	8,500	8,500
" " for efficiency unity.......	2,545	2,545	2,545	2,545

HEAT DISTRIBUTION.

	Best.	Good.	Medium.	Low.
Expended in useful work, per cent....	18	12	9	5
" " friction.................	2—20	1—13	1—10	1—6
" " internal thermal wastes..	20	33	45	55
" " external thermal wastes..	5—25	5—35	5—50	4—59
" " thermodynamic wastes...	55	52	40	35
	100	100	100	100

219. Heat-utilization in Engines in Series is thus, as anticipated in the discussion of the theory, comparatively economical. The fact that the expansion of steam in cylinders in series gives an important advantage, through reduction of heat-wasting area effective in initial condensation, is readily shown by analysis of any good design. We find that in our steam pumping-engine of high efficiency, for example, the steam-cylinders being, respectively, of 28, 48, and 74 inches diameter, and all of 60-inch stroke of piston, the total ratio of expansion is 19.55, and, in each cylinder, from 2.5 to nearly 3. The area of metal surface exposed to cut-off is, in round numbers—

Low-pressure cylinder..........	13,000 sq. in.
Intermediate "	4,000 " "
High-pressure "	2,000 " "

or in the proportion of 6.4 to 2 to 1, or 3.2 to 1 and 2 to 1; and, other things equal, the quantity of steam condensed per stroke or per revolution would be in some such proportion.

The range of temperature in each cylinder and coincident range of pressure is as below:

	p	Δp	T	ΔT
H. P. cylinder...........	135 lbs.	90 lbs.	350° F.	76° F
I. P. cylinder............	45	30	274°	61°
L. P. cylinder...........	15	13	213°	87°
" "	$p_2 = 2$	—	126°	—

The condensations were, in fractions of total steam supplied, h.p. cylinder, 0.12; i.p. cylinder, 0.14,; l.p. cylinder, 0.18; which correspond, in pounds of steam condensed per stroke, very nearly, to

$$W = \frac{A \Delta T \sqrt{pt}}{3,000,000};$$

the symbols as previously used and the constant taken for measures in square feet of cylinder-wall to cut-off, at either end or their mean, degrees Fahr., pounds on the square inch, and seconds occupied in each revolution. In ratio to total supply,

$$C = .15 \sqrt{rt}, \text{ nearly.}$$

Of the total condensation, in the intermediate and low-pressure cylinders, about 7 and 12 per cent. out of the 14 and 18 are due to cumulative thermodynamic or adiabatic liquefaction, leaving, for cylinder-condensation proper, in h.p. cylinder, 0.12; i.p. cylinder, 0.07; l.p. cylinder, 0.06; showing an excess of strictly internal waste in the high-pressure cylinder.

In the expression for waste taken as a percentage of heat,

steam, or fuel computed for its representative thermodynamic, ideal, case,

$$C = \frac{W}{W'} = a\sqrt{rt};$$

for time in seconds per revolution we find, for the high-pressure cylinder of this engine, $a = 0.12$, about one half that for small, unjacketed, engines. For the engine as a whole,

$$C = 0.05\sqrt{rt}, \text{ nearly.}$$

In the last expression, the most successful practice, with jacketed multiple-cylinder engines, seems to approximate

$$C = \frac{W}{W'} = \frac{0.15}{n}\sqrt{rt};$$

where n is the number of cylinders in series, illustrating the fact that such engines should attain nearly an equality of internal wastes, cylinder by cylinder, and thus divide the total for the corresponding simple engine very nearly by the number of cylinders in series. This seems true for the best engines without much regard to the pressure of steam employed, or, if jacketed, the speed of engine.

220. Computation of Proportions of Jacket-water from Trial-data in the engines above discussed, is made thus:*

The jackets on high-pressure and intermediate cylinders and on the two receivers were each supplied with steam at 120 pounds, gauge-pressure, and discharged the water of condensation through a trap at a temperature due to the pressure 350°. Any loss by radiation is assumed to be made up by leakage, which was known to exist. The low-pressure jacket was supplied with steam through a reducing-valve, and kept at a constant pressure of 34 pounds, by gauge. The discharge of the condensing water was from a separate trap, at temperature due to pressure. In the computation, consider that each jacket condenses an equal amount of steam:

* Trans. A. S. M. E., 1893, p. 417; discussion.

whether this is the exact fact or not will not materially affect the result. Computation by product of surface and difference of mean temperatures indicates less than 20 per cent. for the large cylinder.*

For the computation, let y equal the weight of steam condensed in any one jacket, so that $5y$ equals the total weight of jacket-steam. Let x equal the weight of steam discharged from the surface-condenser.

Temperature of discharge from four jackets, 350° F.; from low-pressure jacket, 280° F. Temperature of condensed steam, 108° F. Temperature of feed-water, 130° F. Temperature of lake-water, 34° F.

As the loss of heat by jacket-water equals the gain to the condensed steam, we have

$$4y(350-108)+y(280-108)=(130-108)x+\frac{x}{100}(130-34),$$

$$968y+172y=22x+\frac{96x}{100},$$

$$1140y=22.96x,$$

$$y=.0202x=2.02\%x,$$

$$5y=10.10\%x,$$

$$5y+x=110.10\%x,$$

$$\frac{5y}{5y+x}=\frac{10.10}{110.10}=9.16\%;$$

---

* If we compute the per cent. of jacket-water as proportional to the product of heating-surface exposed and mean difference of temperature, we will obtain the following results:

	Heating-surface. A.	Diff. Mean Temp. B.	Product. A and B.	Per cent Jacket-water to each.
	Sq. Ft.			
High-pressure cylinders....	45	30	1,350	1.9
Intermediate	88.7	95	8,420	11.9
Low	158	84	13,272	18.8
First receiver.............	283	70	19,800	28.1
Second receiver............	205	135	27,680	39.3

which is the per cent. that the jacket-water should bear to the total steam consumed by the engine, calculated by its heating effect on the feed-water.

If the feed-water be taken as 132°, the per cent. of jacket-water required would be 9.95.

The question has often arisen whether it is probable that a jacket can save more steam than it uses itself. The Author, compiling the results of experiments, finds that the average of sixty cases, with simple engines, gives 4 or 5 per cent. of steam used in the jackets, with a saving net about 15 per cent. Compound engines, similarly, average an expenditure of 10 per cent. in the jackets, saving two and a half times that amount at the boiler. Some experiments show a saving of 4 to 5 times the jacket-steam at ratios of expansion ranging from 3 to 6. Cases in the list give ratios of 6, 7, and 8 to 1, and none justify the assumption that it is questionable whether the jacket can save more than it uses. The ratio is less as the engine is more complex, compound giving a smaller ratio than simple, and triple engines lower than compound, the opportunity for saving being less; but all, in the average use of the jacket, give a saving which is a multiple of the expenditure in the jacket.

221. The Distribution of Heat-flow in the course of the two streams of energy entering the working-cylinder and the jacket of the engine from the boiler, in the case of the most economical of modern forms of high-pressure multiple-expansion engine, is illustrated in the Appendix in the results of thermal analysis of this triple-expansion steam pumping-engine when giving a duty of 143,000,000 per 100 pounds of coal, or 154,000,000 per 1000 pounds dry steam, the work performed being 574 I.H.P., 521 D.H.P., the steam used 11.678 pounds per I.H.P. per hour, the heat 217.6 B.T.U. per minute; the steam-pressure being 121.6 pounds above the atmosphere, and the ratio of expansion 19.55.*

* "Maximum Economy of the Contemporary High-pressure Multiple-expansion Steam-engine:" R. H. Thurston, Trans. A. S. M. E., December 1893, vol. XV, No. DLXVI.

The radiation-losses and the quantity of heat rejected from each cylinder, assuming equal distribution of jacket-heat among them, is as follows:

RADIATION LOSSES AND HEAT REJECTED PER 100 REVOLUTIONS.

Total heat admitted to high-pressure cylinder	598,800	B. T. U.
Total heat used in all jackets, assuming one third weight of jacket-steam to be used in each jacket............	40,870	"
Total heat used	639,670	"
Heat rejected from low-pressure cylinder..............	513,420	"
Total work done	119,652	"
Total radiation loss....................................	6,598	"
	639,670	"
Radiation loss from each cylinder assumed to be equal in all cylinders....................................	2,199	

Heat rejected from high-pressure cylinder = heat entering + heat supplied by jacket — radiation loss — work done

$$= 598,800 + 13,360 - 2199 - 36,456$$
$$= 573,505 \text{ B. T. U.}$$

Heat rejected from intermediate-pressure cylinder = heat rejected from high-pressure cylinder + heat supplied by jacket — radiation loss — work done

$$= 572,505 + 13,360 - 2199 - 35,286$$
$$= 549,380 \text{ B. T. U.}$$

Heat rejected from low-pressure cylinder = heat rejected from intermediate-pressure cylinder + heat supplied by jacket — radiation loss — work done

$$= 549,380 + 14,150 - 2200 - 47,910$$
$$= 513,420 \text{ B. T. U.}$$
$$= K + K' \text{ (see analysis in tables).}$$

The tables in the Appendix give the computed distribution of heat in the working-charge of steam, and exhibit the interchange of heat between that charge and the jacket-steam; the quantities being computed as a matter of convenience, as is usual, for 100 revolutions. The real distribution of the radiation-losses among the cylinders, and of the heat received *via* the jacket, is indeterminable; it is assumed that both quantities are equally divided among the three cylinders. The probabilities are that the jacket-supply comes

in much less proportion from the high-pressure jacket than from either the intermediate or the low-pressure; on the latter, the jacket-steam is reduced to a comparatively low pressure by means of a reducing-valve in the passage of supply. It is seen, above, that the heat-supply consists of 598.8 B.T.U. received from the boiler with the working-charge, per revolution, and 408 by the jacket; which is distributed, 66 B.T.U. as radiation loss, 342 B.T.U. as jacket-waste (i.e., lost in the partial suppression of the greater waste by initial condensation), and 1197 B.T.U. converted into work. The sum, 6397 B.T.U., received per revolution, divided by the work per revolution, the equivalent of 1197 B.T.U., is a measure of the cost of the work performed, and its reciprocal measures the efficiency of the system. The jacket-heat constitutes, as seen, 9.22 per cent. of the total from the boiler; and this reduces the initial condensation from what would probably have been above 30 per cent. to 12; returning its own value, plus above 12 per cent., or about 20 per cent. total, through reduction of initial wastes and addition to the work of expansion; the net gain being effected by saving the otherwise anticipated cylinder-condensation. Losses are positive when heat passes from the working-charge to the cylinder-wall, negative when returned to the steam. It is seen, in the first high-pressure table, for example, that the charge yields 62,629 B.T.U. to the metal during admission, regains 46.990 during expansion, and entirely wastes 27,862 units during the period of exhaust, and 1064 during compression. The initial condensation is 98.95 — 86.77 = 12.18 per cent.; the charge losing 0.46 per cent. of its heat to the cylinder-wall in admission, regaining 7.86 and 4.65 per cent. during expansion and exhaust. Of the total, 6 per cent. is utilized in work, 0.37 per. cent. is lost externally, and this one cylinder has 0.68 per cent. of the efficiency of the ideal machine of similar temperature-range.

In the three cylinders, successively, the initial condensation is seen to be, respectively, 12.18, 13.55, and 17.80 per cent., exhibiting in unusual degree the correct apportionment

of this waste among the cylinders.* The more equal these losses, the more perfectly does the engine approximate the ideal action of the multiple-cylinder engine; the heat sup-

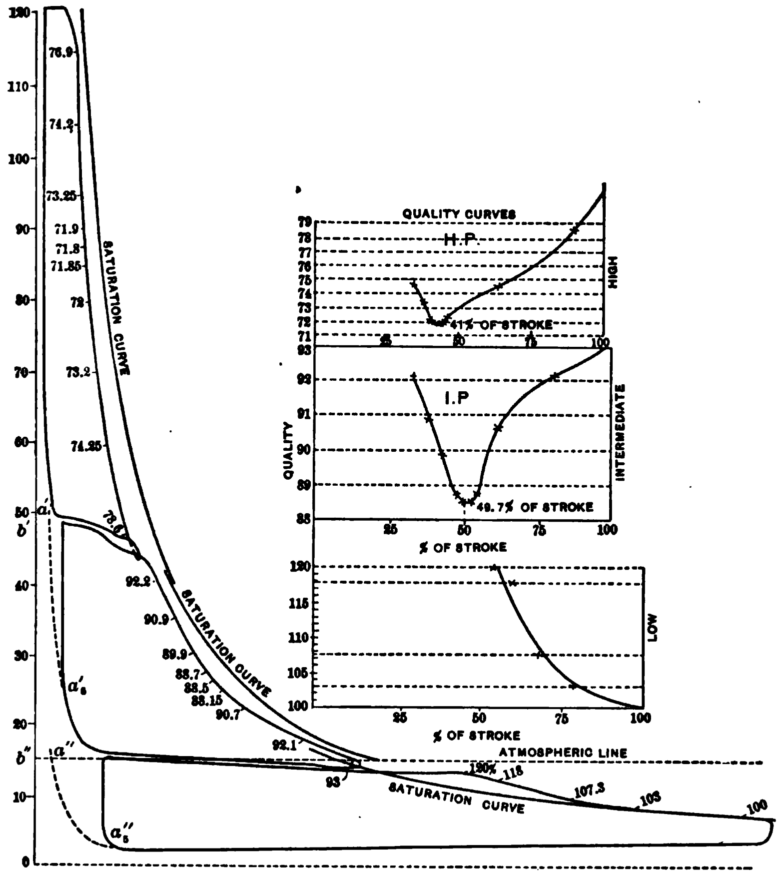

FIG. 190.—QUALITY-CURVES OF TRIPLE-CYLINDER ENGINE.

plied, to be thus wasted from the one cylinder, being rejected from it in precisely the quantity needed to meet the demand for waste heat in this phase of the cycle in the next cylinder. This special feature of operation of the engine, the efficiency

* "Philosophy of the Multiple-cylinder Engine:" R. H. Thurston, Trans. A. S. M. E., vol. XI, No. CCCLXII, 1890.

of its jackets, its effective covering with non-conductors, its tight condenser, and its small clearances (1.4, 1.5, and 0.77 per cent) are the secrets of its great economy.

The next example is, in some respects, still more interesting, novel, and instructive (Fig. 190). The cost of the horse-power was here 13.3 pounds of dry steam per hour. The engine has cylinders 9 inches, 16 inches, and 24 inches in diameter, and a stroke of piston of 36 inches. It is arranged as three independent machines, which can be combined in any manner desired, and operated with or without expansion, and either jacketed or unjacketed.* It here develops 140.2 I.H.P.; 13.72 per cent. of the steam-supply passing through the jackets of cylinders and receivers, all of which had their jackets in action. The saturation-curves on the diagram are laid down for the weight of working-charge, and bring out clearly the action of the jackets and cylinder-walls in heat-exchanges. The points of cut-off were nearly at one-third stroke in the high and intermediate, and three-fifths in the low-pressure, cylinder. The quality-curves are given on the plate, with the combined diagram. The saturation-curves on the latter are seen to be discontinuous, and markedly so, in consequence of the differences in volume of clearance and of compression.

At the instant of closing the cut-off valve on the high-pressure cylinder the steam enclosed contained 25 per cent. water. At four-tenths stroke its quality had depreciated by further cylinder-condensation to 71.8 per cent., and at the end of stroke, by accession of heat from the jackets and restoration from the cylinder-wall, it had risen to a trifle above the quality at the start, to about 80 per cent. Practically, it may be said, as before, the condensation at cut-off measures the waste by heat-exchange between steam and metal. In the intermediate cylinder, the quality at cut-off, raised to 92 per cent. by the drying action of the exhaust period of the high-pressure cylinder and of the receivers and their jackets,

* This is the "experimental" triple-expansion, Reynolds-Corliss engine of Sibley College.

becomes 88 per cent. at half-stroke, as a minimum, and then rapidly improves to the end of the stroke, where it is 93 per cent. As usual, it is found that the waste is here measured by the condensation at cut-off. It will be noted that the saturation-curve for the intermediate cylinder falls inside the line of that for the high-pressure, and the indicator expansion-line falls outside; the two lines being more closely approximated in the case of the intermediate than in that of the high-pressure cylinder.

In the low-pressure cylinder the saturation-curve falls further inside the lines for the other cylinders; the expansion-line of the diagram falls entirely outside the saturation-curve, showing considerable superheating from the transfer of heat by the jackets to the working-steam, in the second receiver and the large cylinder. The quality falls off steadily from the moment of closing the steam-valve. From an initial 120 per cent., it becomes 107 at three-quarters stroke, and attains unity at the end, leaving the engine precisely dry—a condition considered to be that of highest effectiveness of jacketing. The location and the depth of the cusps are indicative of the action of the jackets. The jacket on the high-pressure cylinder is seen to be quite effective in checking heat-exchange; that of the first receiver and that on the intermediate cylinder seem to be of less effect, possibly in consequence of the drier condition of the steam; while the jackets on the second receiver and the low-pressure cylinder are found to be of extraordinary activity.

222. The Ideal and Real Engine Compared.—The following are the results of test of the high-pressure element* of this engine, operated independently as a simple engine, at various ratios of expansion, and similar figures computed by the now familiar methods introduced by the Author. The internal wastes are computed from the expression

$$c = a\sqrt{rt},$$

* The case is that already discussed in the opening paragraphs of this chapter (§ 217).

in which a is taken as 22.5 per cent.* There were noted:

Boiler-pressure, by gauge, pounds per square inch..........	100
Revolutions per minute....................................	86
Quality of steam at engine, per cent.......................	98
Back-pressure, pounds per square inch	5
Friction of engine, per horse-power........................	7.25

ECONOMY OF IDEAL AND OF REAL ENGINE.

(9 IN. × 36 IN.; 86 REVS.)

Expenditures in Pounds of Steam per Horse-power per Hour.

Ratio of Expansion.	Ideal.	Frictional Loss.	Radiation.	Initial Condensation.	Real Engine. Computed.	Real Engine. Total Observed.
	Lbs. Steam.	Lbs. Steam.	Lbs. Steam.			
1	32.91	1.73	.267	7.62	42.63	—
2	20.50	1.75	.268	6.68	29.18	29 1
3	16.70	1.78	.268	6.70	25.45	24.6
4	15.11	1.80	.270	7.01	24.19	23.1
5	14.21	1.83	.271	7.40	23.71	22.6
6	13.52	1.86	.272	7.70	23.35	22.8
8	12.67	1.98	.275	8.27	22.20	23.7
10	12.22	2.12	.280	9.10	24.08	24.0
12	11.90	2.37	.291	9.50	24.06	24.5
15	11.70	2.71	.305	10.30	24.92	25.5
20	11.61	3.06	.331	12.00	27.00	28.0

The engine-trials showed a slightly more rapid gain in the earlier cuts-off, and a greater waste in the latter, than were computed; but the two sets of figures are closely accordant throughout, indicating the practicability of securing a measure of the constants, in the expressions for wastes, by a single trial at any convenient and usual load, and thus a clue to the behavior of the engine at all loads. This fact is of importance where questions of relative efficiency and costs arise in adjusting sizes and steam-distributions of engines to changed or to very variable loads. The two sets of results, computed and observed, are well shown in Fig. 191, arranged like the first presented herewith. It is seen that the maximum

* Chapters IV, V.

efficiency is attained in the computed case at a ratio of expansion of 7, while the engine actually does its best work at about 6. This indicates a greater proportional initial con-

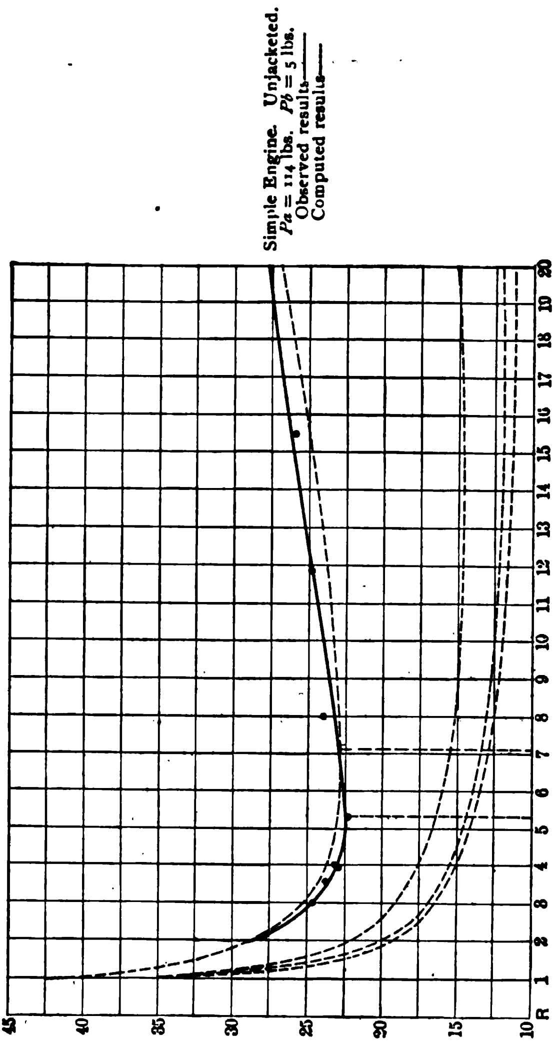

FIG. 191.—EFFICIENCIES OF SMALL SIMPLE ENGINE.

densation at cuts-off near the beginning and the end, and less at the intermediate portions of the stroke, than the assumed formula would give.

As illustrating the modification of efficiencies in this engine produced by compounding, Fig. 192 presents a set of

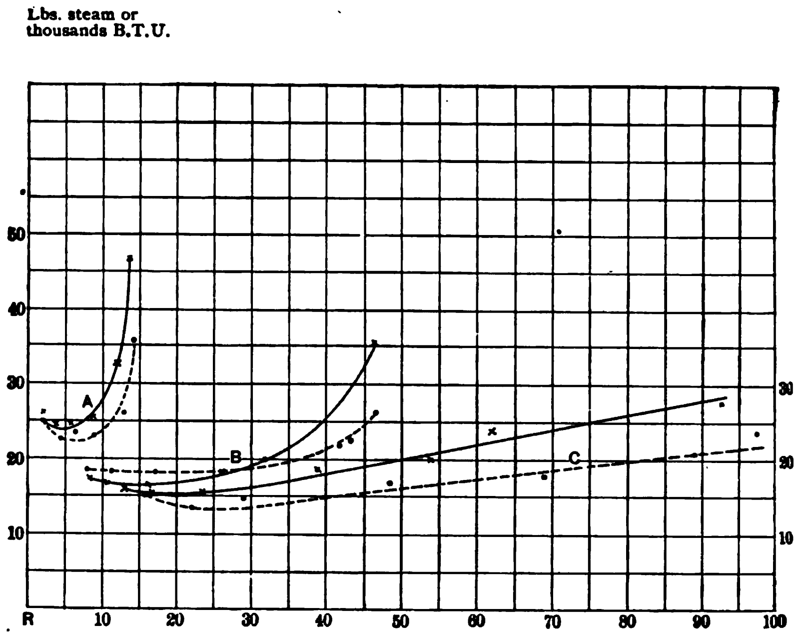

FIG. 192.—EFFICIENCIES OF MULTIPLE-CYLINDER ENGINE.

curves of efficiency. The simple engine does its best work at ratios of about 4 and 6, unjacketed and jacketed, respectively; the minimum cost in steam being 23 pounds steam, about 23,000 B.T.U., per horse-power per hour. The double-cylinder engine brings these figures up to 12 and 1; for the ratio of expansion, and down to 16 and 18 pounds of steam, and exhibits the anomaly of doing its best work with cylinders and receiver unjacketed at the lower, and the reverse at the higher, ratios of expansion. The triple-expansion engine performs its highest duty unjacketed at 20 expansions, jacketed at 22, and at an expenditure of, respectively, a trifle above 15 and 13.3 pounds of steam per hour per horse-power. In the figure the curves A are those obtained from the engine when the high-pressure cylinder is worked alone

[*To face page* 864.]

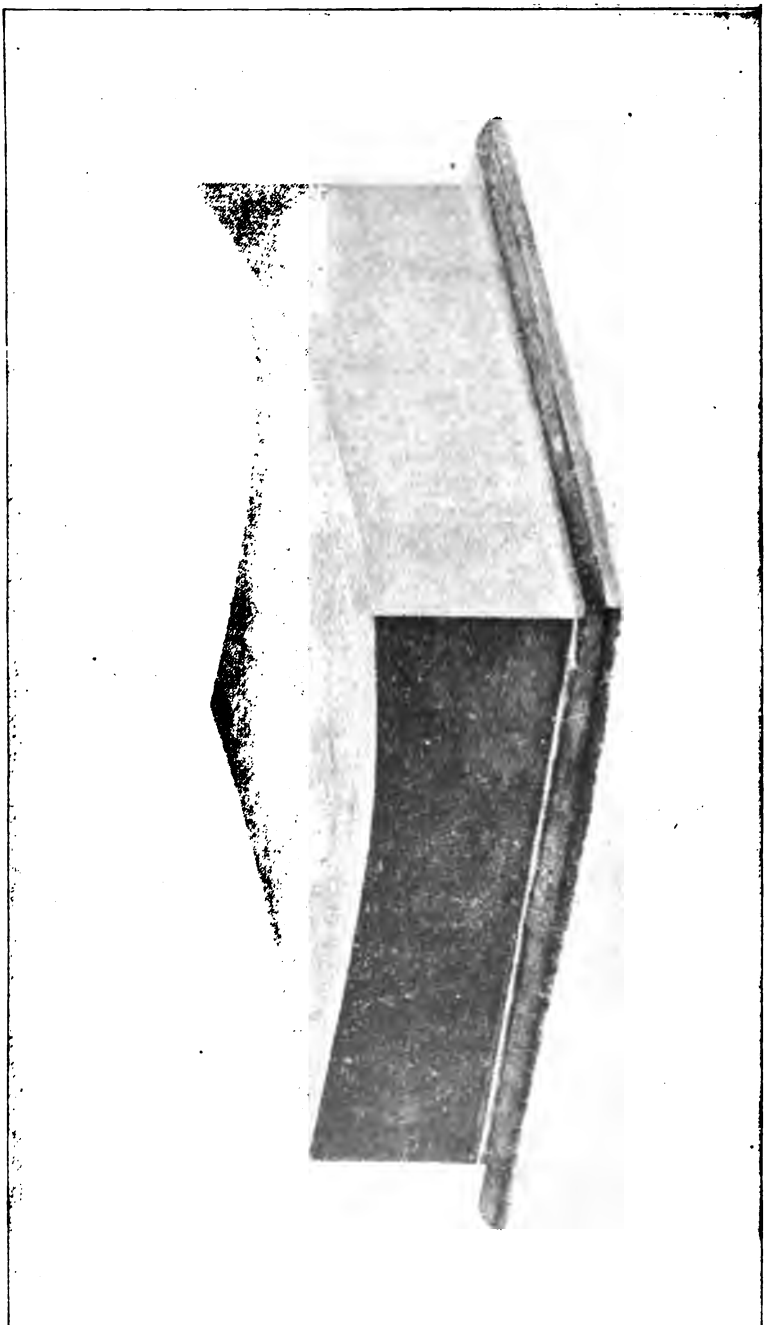

Fig. 192a.—Model of Engine Efficiencies.—Coordinates: Efficiencies, Steam-pressures, Expansion-ratios. (Trans. A. S. M. E., 1898, Thurston.)

as a simple engine, jacketed and unjacketed; *B* is the set of curves obtained from high-pressure and intermediate, working as a compound; and *C* is the set representing the machine as a whole, jacketed and unjacketed, working as a triple-expan-

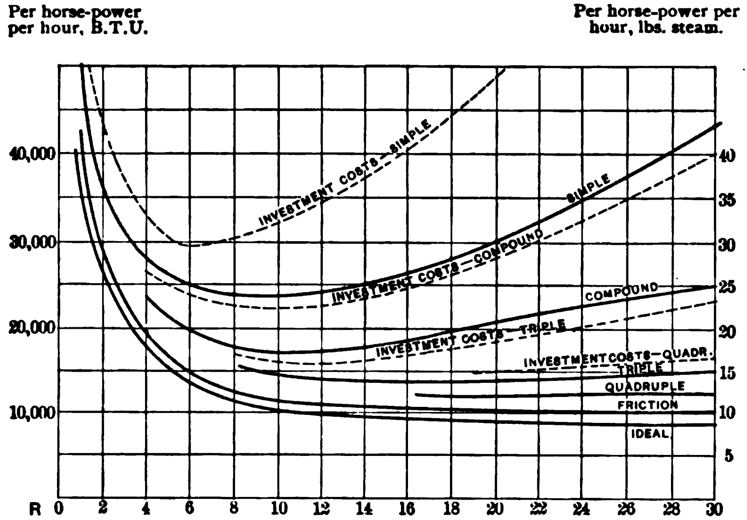

FIG. 193.—COMPARATIVE EFFICIENCY OF ENGINES.

sion engine, but with only 125 pounds pressure. Fig. 192*a* illustrates a group of such curves.

The following are the data obtained when working under best conditions:*

Cylinders	9 + 16 + 24 × 36 inches.
Piston-rods	2.31 inches diameter.
Vacuum	10.8 pounds, 22 inches.
Jacket-water	13.72 per cent.
Total I. H. P	140.2.
Mechanical efficiency	0.88.
B. T. U. per I. H. P. per hour	14,160.
B. T. U. per I. H. P. per minute	236.
Clearances	7.6, 8.93, and 9.35 per cent.
Boiler-pressure	pounds, 125 (abs.); 110 by gauge.
Barometer	29.4, 14.42 inches.

* See Chap. VI, § 200. Trans. A. S. M. E., 1894; "Theory of the Steam-jacket," Discussion, p. 879.

Condensing water, per lb. steam	19 pounds.
D. H. P.	123.4.
Steam per I. H. P. per hour	13.3 pounds.
Steam per D. H. P. per hour	15.1 pounds.
Total ratio of expansion	13.83.

Pressures (absolute) at cut-off, 131, 43, 13.5; at release, 43, 14.5, 2.5.
Jacket-water, per cent, 26.4, 7.05, 28.1 in cylinders; and 9.85, 34.6 in receivers.
Work, per cent, 1, 1.33, and 1.675, in cylinders 1, 2, and 3.
Thermodynamic efficiency of Carnot cycle, 24.7 per cent; actual, 18 per cent; ratio of actual to Carnot, 0.73.
Water-rate of ideal Rankine cycle, 9.6 pounds; ratio to actual, 0.72.

The final effect of the wastes here specially considered, however, can only be discovered, in all their bearings, when the total variation of *costs* of power with variation of output are studied together.

In Fig. 193 are graphically exhibited the elements which finally determine the costs and profits on the use of the engine, in such manner as to give a clue, if not an exact measure, and thus to enable the engineer to ascertain how much improvement in economy and efficiency of engine it is financially practicable to demand, what efficiency it will pay to seek.* In these curves the variations of each element in the problem are shown by a curve, the ordinates of which are in each case measured from the curve immediately below; in other words, the costs of power are shown in steam or fuel or money, either unit of measure being equally available, provided all are reduced to equivalence. The lowest line is the curve of steam expenditure, and is the ideal case, the purely thermodynamic case, as computed by Rankine's method. The second curve, superposed upon the first, is that which exhibits the added cost of friction of engine in the same unit of measure. The curve marked "simple" is one of which the ordinates, measured down to the friction-curve, are proportional to the cost of the wastes in the simple engine by cylinder condensation and leakage, and, by conduction and radiation, the thermal wastes other than thermodynamic.

* See Chap. VII.

The highest curve is the measure in the same way, its ordinates being measured down to the last-named curve, of the investment-costs, which include the interest on first cost, the charges for insurances, rents, and other purely financial charges for capital employed in the purchase, instalment, and preservation of the machine.

It is assumed that the compound engine will approximately divide the cylinder wastes of the simple engine by two, that the triple-expansion engine will have one-third the wastes of the simple, and that, generally, the multiple-cylinder engine will, if properly employed, approximately reduce these internal wastes in the proportion of the number of cylinders employed. This being assumed for an approximation, the curves marked "compound" and "triple" and "quadruple," in full lines, measure, as before, the cylinder wastes of those engines, and the dotted lines, as before, the financial costs of each.

The deductions to be made from this construction are that, while the ideal case, the purely thermodynamic engine, will give increasing efficiencies with increasing expansions indefinitely, the back-pressure not being reached, the introduction of the friction of the engine produces a change, both in the location and the form of the curve, such that, after a time, with increasing expansion, a maximum efficiency is attained, and a minimum cost of power. And, were the curve extended far enough, a point would be found at which the power of the engine would be insufficient, not simply to yield the required power to the belt, but even to turn over the engine-shaft alone. With the introduction of internal and other thermal wastes, the maximum efficiency and minimum costs are found at not only a higher figure, but at an enormously reduced ratio of expansion, which, in the case of the simple engine, as here taken, is not far from 8; in the compound, 12; in the triple, 16; and in the quadruple, 22 or 24. When the investment-costs are brought into the account, we again have the best ratio of expansion reduced in value; and in the simple engine here taken it becomes about

5; in the compound, 8; in the triple, 12; and in the quadruple, 16. These figures vary greatly with the value of fuel and other costs, becoming higher in marine practice than in stationary; for example, other things equal, lower in a coal-mining district than in a country remote from fuel supplies. With coal at $1 per ton in one case, and at $10 per ton in another, we should probably find the simple engine, on the whole, most desirable, from a financial standpoint, in the one instance, and the quadruple-expansion engine the proper selection in the other.

Whatever the location, the conditions of fuel-supply, or the value of money, the methods here indicated may be found useful in securing the best design and construction of engine, and the highest financial returns. *There is always a value of p_1 which insures a maximum return.*

223. Thermal Analyses, both analytically made and resulting from trial, are illustrated in cases reported by the Author,* and are here presented in abstract.

As has been shown, in the text and elsewhere,† the exact expenditure of heat, steam, and fuel under specified representative conditions of this case, including steam-pressure, back-pressure, ratio of expansion, and boiler-efficiencies, can be computed for the thermodynamic, ideal, case; then, knowing the magnitude and conditions of physical operation of the engine, friction included, its wastes of energy, whether thermal or dynamic, can be very closely obtained by computation, as already seen, and these wastes being added to the total thermodynamic expenditure, the gross outlay of energy becomes known and the economical problem can be solved. The following illustrates these methods for the case of an "automatic" simple, condensing, engine, rated at 10 to 15 horse-power; having a cylinder 6 inches in diameter and 8 inches stroke of piston, a speed of 280 revolutions a minute, and proportioned for a steam-pressure of 100 pounds. Compression was assumed complete and leakage insensible.

* Jour. Franklin Inst., Oct. 1892; Oct. 1893.

† Chapters V, VI.

The demand for heat and steam was computed on the assumption of the conditions as to internal wastes being as in the Sandy Hook experiments of 1884.* External wastes were assumed to average 0.5 B.T.U. per square foot of exposed surface, and per degree range of temperature from atmospheric—here taken as 100° Fahr.

The deduction follows that the constants here assumed in the tabulated work are substantially correct for an engine of this class, of good design and construction, and operated under ordinarily favorable conditions. The table of engine-efficiencies given may therefore be taken as a probably safe guide in the design of such engines, assuming that correct proportion of volume of cylinders and the best ratios of expansion are adopted for the cases to be met. Internal wastes were here taken as a fraction of the total steam supplied,

$$w = a/d \,.\, \sqrt{rn},$$

where the coefficient $a = 4$ in the case assumed to be fairly represented of that here considered; d is the diameter of cylinder in inches, r the ratio of expansion, and n the number of revolutions per *second*. Friction wastes were taken as giving an efficiency of engine of 0.85.† J is taken as 778. The following are the data:

DATA.

$p_1 =$	75	95	115	135	155′
$p_2 =$	5	5	5	5	5
$r =$	1.6	2	4	8	16
$c = \frac{1}{r} =$	$\frac{5}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$.

The equations employed were those of Rankine.‡

The results of computation are presented, in part, in Table A.

* Chap. V, § 129, p. 501. † Chap. V, §§ 132–134.
‡ See p. 398, and Chap. V, § 137.

TABLE A.—THERMODYNAMICS OF IDEAL AND REAL ENGINES.

r	Cut-off.	Initial Pressure, Lbs. per Sq. In.	Terminal Pressure, Lbs. per Sq. Inch.	Terminal Pressure, Lbs. per Sq. Ft.	Indicated Power.	Steam per I.H.P. per Hour (Ideal) W	Internal Waste Coef. $\frac{a}{d}\sqrt{\frac{r}{n}}$	Internal Waste, Lbs. per I.H.P. and per Hour.	External Waste, Lbs. per I.H.P. and per Hour	Total Waste, Lbs per I.H.P. and per Hour.	Total Consumption. Lbs. per I.H.P. and per Hour.	Same per D.H.P.
16	1/16	75	3.22	463.7	3.100	15.85	1.234	19.56	3.30	22.86	38.71	45.54
8	1/8	75	7.08	1020	6.42	15.32	.87287	13.37	1.50	14.93	30.25	35.58
4	1/4	75	15.55	2239	11.77	16.72	.61720	10.32	.847	11.167	27.887	32.80
2.7	3/8	75	24.29	3498	14.97	18.48	.50710	9.88	.666	10.546	30.026	35.32
2	1/2	75	34.15	4918	17.85	21.44	.43650	9.71	.555	10.265	32.51	38.24
1.6	5/8	75	44.00	6336	19.90	24.76	.3900	9.66	.502	10.162	34.92	41.08
16	1/16	95	4.08	588	4.83	12.74	1.234	15.74	1.663	17.393	30.133	35.45
8	1/8	95	8.97	1292	9.31	13.21	.87287	11.53	.969	12.499	25.71	30.24
4	1/4	95	19.70	2837	15.97	15.42	.61720	9.09	.566	9.656	25.076	29.50
2.7	3/8	95	30.77	4431	20.58	17.72	.50710	8.99	.428	9.418	27.14	31.93
2	1/2	95	43.26	6229	24.20	20.34	.43650	8.88	.375	9.255	29.595	34.81
1.6	5/8	95	55.72	8024	26.62	23.11	.3900	9.02	.333	9.353	32.46	38.19
16	1/16	115	4.94	711.8	6.18	11.91	1.234	14.70	1.360	16.060	27.97	32.91
8	1/8	115	10.86	1564	11.62	12.68	.87287	11.07	.755	11.825	24.55	28.82
4	1/4	115	23.84	3433	19.68	14.97	.61720	9.24	.415	9.655	26.48	28.97
2.7	3/8	115	37.25	5364	25.28	17.35	.50710	8.80	.331	9.131	28.836	31.15
2	1/2	115	52.36	7540	29.64	19.88	.43650	8.68	.276	8.956	30.00	33.92
1.6	5/8	115	67.46	9714	32.60	22.60	.3900	8.82	.251	9.071	31.67	37.26
16	1/16	135	5.8	835.6	7.534	11.38	1.234	14.05	1.043	15.093	26.473	31.14
8	1/8	135	12.75	1836	13.91	12.32	.87287	10.75	.504	11.314	23.63	27.80
4	1/4	135	27.99	4031	23.37	14.67	.61720	9.05	.344	9.394	24.06	28.30
2.7	3/8	135	43.73	6297	30.00	16.96	.50710	8.60	.263	8.863	25.82	30.37
2	1/2	135	61.47	8852	35.00	19.54	.43650	8.53	.286	8.756	28.29	33.28
1.6	5/8	135	79.19	11403	38.50	22.25	.3900	8.68	.208	8.888	31.13	36.62
16	1/16	155	6.66	959.4	8.89	10.98	1.234	13.55	.836	14.336	25.36	28.84
8	1/8	155	14.63	2106	16.20	12.05	.87287	10.52	.462	10.982	23.03	27.09
4	1/4	155	32.14	4628	27.00	14.41	.61720	8.89	.274	9.164	23.57	27.73
2.7	3/8	155	50.2	7223	34.58	16.72	.50710	8.48	.213	8.693	25.41	29.89
2	1/2	155	70.58	10163	40.50	19.28	.43650	8.41	.182	8.592	27.87	32.74
1.6	5/8	155	90.92	13092	44.48	21.95	.3900	8.57	.164	8.344	30.68	36.20

Simple, "Automatic," engine 6″ × 8″; 280 r. p. m.

These computations for the ideal case show the consumption of steam to vary from a minimum of about 11 pounds per h.p. and per hour, at 155 pounds, to 15½ at the best cut-off for 75 pounds. The introduction of the wastes being made, the minima were found at approximately

$$r = 0.5 \sqrt{p},$$

or from 7 at 155 pounds to 4, nearly, at 75 pounds, initial pressure; while the steam-consumption became about

$$w = 250 \div \sqrt{p},$$

ranging from 23 to 28 pounds per I.H.P. per hour, and 27 to 33 pounds per D.H.P. These figures may be taken as representative of those to be expected with good practice. With larger engines, wastes may be reduced.*

In the next case, the engine is a compound, is larger and more powerful, and affords better opportunity to effect a close approximation. Its dimensions are:

DIMENSIONS OF ENGINE.

Diameter of high-pressure cylinder........	12 inches.
Diameter of low-pressure cylinder........	20 inches.
Length of stroke........................	14 inches.
Diameter of piston-rod..................	1.9375 inches.
Volume of receiver-space................	1.1455 cubic feet.

The computed weights demanded are reduced very considerably by the fact that the wastes are but about one half those found for the smaller engine. These wastes averaged 8 or 9 pounds for the latter and 4½ for the former.

It will be interesting to now compare these computed results with the actual performance of the machine.†

The engine is well balanced, has good provisions for free lubrication, and is an excellent example of its class.

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* See p. 398, and Chap. V, §§ 129–131.

† For details, see art. 35, p. 142.

The following are the results of test:

DATA AND RESULTS.

Duration of trial	5 hours.
Revolutions per minute	201
Barometer in inches of mercury	29.40
Atmospheric pressure	14.50 pounds.
Boiler-pressure, absolute	112.50 pounds.
Pressure in steam-chest, low-pressure cylinder	34.00 pounds.
Vacuum-gauge, inches of mercury	22.99
Temperature of condensed steam	130°.8
Temperature of injection-water	47°.9
Temperature of discharge-water	106°.7
Quality of steam in steam-pipe	95.50 per cent.
Quality of steam in compression (assumed)	100.00 per cent.
Quality of steam in exhaust	93.30 per cent.
Total weight of condensed steam	11,594.50 pounds.
Cubic feet of condensing-water per minute (by meter)	9.304
Pounds per revolution	3.033

The work of the air-pump, separately measured, was 0.7 horse-power, and, with its accessories, at work, 1.18 horse-power, giving, as a total 101.17 D.H.P.

The indicator-cards, measured by planimeter, gave:

Total I.H.P.	112.28
Total D.H.P.	101.17
Efficiency per cent	90.16
Weight of wet steam per hour	2319.10 pounds.
Weight of dry steam per hour	2234.72 pounds.
Weight of steam per I.H.P. per hour	19.903 pounds.
Weight of steam per D.H.P. per hour	22.35 pounds.

The application of the Hirn analysis to this case involves the determination of the condition of the working fluid as it traverses these receivers and the intermediate steam-passages, as well as at entrance into the high-pressure cylinder and at exit into the condenser. A steam-gauge and a calorimeter provide the means of obtaining the desired data at each of these points.

The data obtained, in the present case, from the trial, are as follows:

HIRN'S ANALYSIS—DATA.

High-pressure Cylinder.

	End.	
	Head.	Crank.
Cut-off, per cent	26.40	19.83
Release, per cent	75.17	62.91
Compression, per cent	12.56	12.56
Absolute pressure at cut-off	105.30	104.50
Absolute pressure at release	56.00	49.00
Absolute pressure at compression	49.00	46.00
Absolute pressure at admission	73.00	81.00
Volume in cubic feet, at cut-off	.40045	.32673
Volume in cubic feet, at release	.76313	.70351
Volume in cubic feet, at compression	.27210	.25903
Volume in cubic feet, at admission	.15716	.14718
External work B.T.U., admission	4.9000	3.5958
External work B.T.U., expansion	5.0681	4.8380
External work B.T.U., exhaust	3.4571	2.5497
External work B.T.U., compression	1.2419	1.2749
External work B.T.U., total	5.2692	4.6092
Steam from boilers, pounds	10.3593	8.8704
Steam in clearance, pounds	2.6906	277.91
Steam, total, pounds	13.0499	11.6495
Heat in exhaust	11373.70	9738.80
Heat supplied to engine	12220.95	10316.00
Sensible heat at admission	741.45	785.99
Internal heat at admission	2207.16	2264.20
Sensible heat at cut-off	3940.42	3510.80
Internal heat at cut-off	7747.50	6279.00
Sensible heat at release	3363.00	2901.30
Internal heat at release	8490.55	6959.30
Cylinder loss during admission	2991.64	3216.82
Cylinder loss during expansion	672.44	554.60
Cylinder loss during exhaust	2535.37	2737.76
Cylinder loss during compression	536.51	181.83

Low-pressure Cylinder.

	End.	
	Head.	Crank.
Cut-off, per cent	36.18	24.48
Release, per cent	88.23	87.72

Compression, per cent	33.82	22.80
Absolute pressure at cut-off	25.50	26.50
Absolute pressure at release	12.00	9.70
Absolute pressure at compression	3.00	3.00
Absolute pressure at admission	22.00	19.00
Volume in cubic feet at cut-off	1.2209	.92491
Volume in cubic feet at release	2.3974	2.3752
Volume in cubic feet at compression	1.0359	.76953
Volume in cubic feet at admission	.3142	.3192
Volume in cubic feet of space in pressure-plate	.12819	.12819
External work B.T.U., admission	5.4233	3.5390
External work B.T.U., expansion	4.1360	4.3582
External work B.T.U., exhaust	.4109	.5811
External work B.T.U., compression	1.5339	.9773
Total	7.6146	6 3388
Steam from boiler, pounds	10.3593	8.8704
Steam clearance, pounds	1.7418	1.5387
Steam, total, pounds	12.1011	10.4091
Heat of condensed steam	1023.50	876.40
Condensing water, pounds	108.937	93.279
Heat given to condensing water	9608.30	8227.20
Heat supplied to engine	11373.70	9738.80
Sensible heat at admission	351.51	298.39
Internal heat at admission	1528.00	1362.50
Sensible heat at cut-off	2599.20	2208.30
Internal heat at cut-off	6768.20	5324.50
Sensible heat at release	1980.00	1611.70
Internal heat at release	6783.50	5694.20
Total heat in steam at beginning of compression	935.66	695.07
Heat confined in pressure-plate	521.69	465.36
Cylinder loss during admission	3343.48	3512.99
Cylinder loss during expansion	331.39	674.28
Cylinder loss during exhaust	2763.37	2434.66
Cylinder loss during compression	268.77	402.73

SUMMARY OF RESULTS.

High-pressure Cylinder.

	End.	
	Head. Per Cent.	Crank. Per Cent.
Heat lost by initial condensation	24.48	31.18
Heat restored during expansion	5.50	5.38
Heat rejected during exhaust	20.75	28.11
Heat lost during compression	4.39	1.76

Heat utilized, work (actual efficiency).......	4.31	4.47
Thermodynamic efficiency..................	8.77	8.77
Efficiency compared with ideal..............	49.10	50.90
Quality of steam entering (per calorimeter).	95.50	95.50
Quality of steam at cut-off (computed)......	74.19	67.33
Quality of steam at release (computed).....	78.01	71.07
Quality of steam at admission (assumed)...	100.00	100.00
Quality of steam in exhaust (computed).....	104.00	104.00

LOW-PRESSURE CYLINDER.

Heat lost by initial condensation...	29.40	36.07
Heat restored during expansion	2.91	6.92
Heat rejected during exhaust..............	24.30	25.00
Heat lost during compression..............	2.36	4.13
Heat utilized, work (actual efficiency)	6.69	6.51
Thermodynamic efficiency	15.66	15.66
Efficiency compared with ideal..............	42.70	41.56
Quality of steam entering (per calorimeter).	93.30	93.30
Quality at cut-off (computed)...............	64.22	50.63
Quality at release (computed)..............	64.76	54.00
Quality at admission (assumed)............	100.00	100.00
Quality of steam in exhaust (computed).....	90.12	102.00

Averaging values, the following are obtained;

	Cylinders. H.P. Per Cent.		Cylinders. L.P. Per Cent.
Quality of steam entering (per calorimeter).	95.50		93.30
Quality of steam at cut-off (computed)	70.76		57.42
Quality of steam at release (computed).....	74.54		59.38
Quality of steam at admission (assumed)...	100.00		100.00
Quality of steam in exhaust (computed)....	104.00		96.06
Heat lost by initial condensation...........	27.83		32.73
Heat restored during expansion	5.44		4.91
Heat rejected during exhaust..............	24.43		24.65
Heat lost during compression..............	3.07		3.24
Heat utilized, work (actual efficiency)	4.39		6.60
Total................................		10.99	
Thermodynamic efficiency..................	8.77		15.66
Total................................		24.43	
Efficiency compared with ideal..............	50.00		42.13
Mean................................		46.07	

Table B presents the collated figures for the ideal beside those of the real case.

TABLE B.—STEAM-ENGINE EFFICIENCIES.

TANDEM COMPOUND, 12" × 20" × 14"; 200 R. P. M.

r	Cut-off.	Pressure, Pounds per Sq. In.	Steam per I.H.P. per Hour (Ideal), W.	Total Waste, Pounds per I.H.P. and per Hour.	Total Consumption, Pounds per I.H.P. and per Hour.	Same per D.H.P. Machine Eff., 0.90.
16	1/16	75	15.85	11.5	27.35	30.6
8	1/8	75	15.32	7.5	22.82	25.4
4	1/4	75	16.72	5.5	22 22	20.7
2.7	3/8	75	18.48	5.3	23.78	26.4
2	1/2	75	21.44	5.3	26.74	29.7
1.6	5/8	75	24.76	5.2	29.96	33.3
16	1/16	95	12.74	8.7	21.44	23.8
8	1/8	95	13.21	6.2	19.41	21.6
4	1/4	95	15.42	4.8	20.26	22.5
2.7	3/8	95	17.72	4.7	22.42	22.7
2	1/2	95	20.34	4.6	24.14	27.0
1.6	5/8	95	23.11	4.6	27.71	30.8
16	1/16	115	11.91	8.0	19.91	22.1
8	1/8	115	12.68	5.9	18.58	20.6
4	1/4	115	14.97	4.8	19.77	22.0
2.7	3/8	115	17.35	4.6	21.95	24.4
2	1/2	115	19.88	4.5	24.38	27.1
1.6	5/8	115	22.60	4.0	26.60	29.9
16	1/16	135	11.38	7.5	18.88	21.0
8	1/8	135	12.32	5.6	17.92	19.9
4	1/4	135	14.67	4.7	19.37	21 5
2.7	3/8	135	16.96	4.4	21.36	23.7
2	1/2	135	19.54	4.4	23.94	26.5
1.6	5/8	135	22.25	4.4	26.65	29.6
16	1/16	155	10.98	7.1	18.08	20.1
8	1/8	155	12.05	5.5	17.55	19.5
4	1/4	155	14.41	4.6	19.01	21.0
2.7	3/8	155	16.72	4.4	21.12	21.1
2	1/2	155	19.28	4.3	23.58	25.1
1.6	5/8	155	21.95	4.1	26.05	28.9

In the Appendix will be found still other and no less interesting and helpful data of this class.

224. **High Steam-pressures** are steadily more and more employed in practice. Until the introduction of the compound engine, at about the middle of the century, the advance in pressures and in the available extent of expansion was slow; but from 1850 the progress is seen to have been not only comparatively rapid, but quite as remarkably and con-

tinuously accelerated in its rate of gain. Experience shows that in the steam-engine, as most efficiently employed, simultaneous increase of expansion with increasing pressure

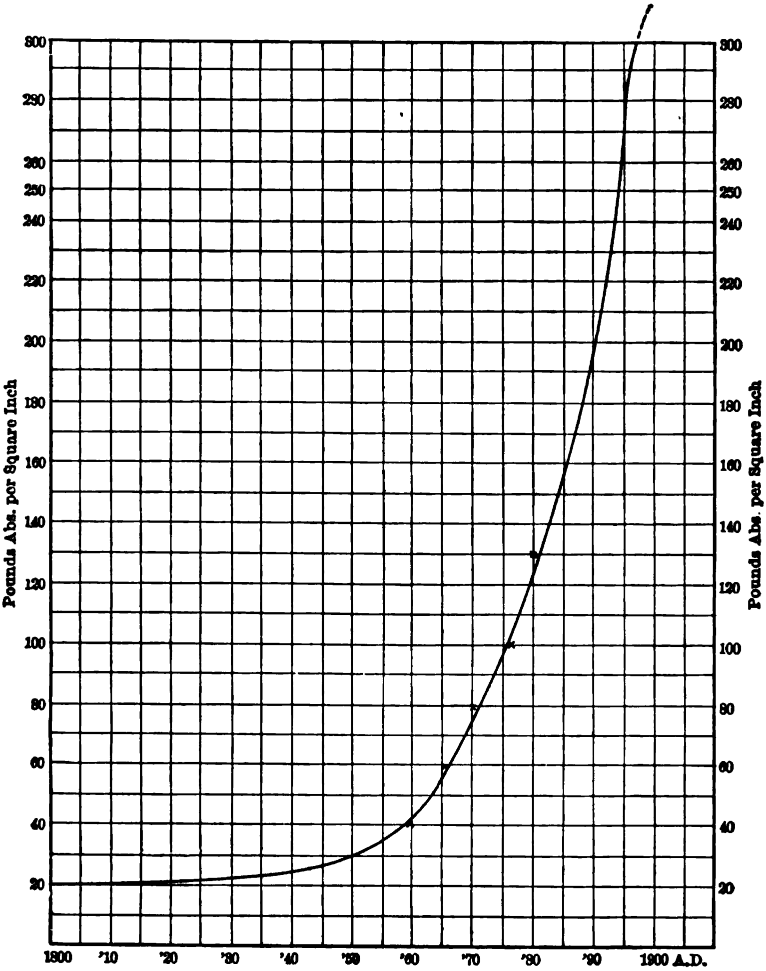

FIG. 194.—RISE IN STEAM-PRESSURES, 1800–1900.

is always observed. The terminal pressure on the expansion line has gradually fallen, in the best engines, from higher figures to about 10 pounds in the square inch, absolute, in

good engines, and to 8, or even to 6, pounds above vacuum in the most economical of modern condensing engines.

As pressures have risen throughout the century, the value of the best ratio of expansion has correspondingly increased, and in still higher ratio, and the best work is now done, in the best of contemporary engines as a rule, at a ratio measured by the quotient $p_1/8$, or a slightly higher value. The Milwaukee pumping-engine, for example, gives $p_1/6.5 = 20$, nearly.

From what has preceded, it is seen that the efficiency, the quantity of work which may be obtained from the unit-weight of steam, may be at least approximately taken as proportional to the logarithm of the ratio of expansion for maximum efficiency, and that consequently the cost of power will be proportional to the quantity

$$W_1 = m / \log p_1,$$

where W or W_1 is weight of steam per I.H.P. per hour, and p pounds per square inch.*

The value of this constant, m, employing common logarithms, was fifteen years ago, about 40, and is now probably not above 30 for good constructions; it has become, in

* The proposition that the maximum efficiency of the fluid, thermodynamically, is proportional to the logarithm of the maximum ratio of expansion may be proven with sufficient definiteness for present purposes thus:

Assuming, for convenience and as sufficiently exact for our purpose, that the expansion is sensibly hyperbolic and the operation purely thermodynamic, the work performed by the fluid at a pressure p_1, and volume v_1, expansion taking place to back-pressure $p_2 = p_3$, is

$$\begin{aligned} U &= p_1 v_1 (1 + \log_e r) - p_2 v_2 \\ &= p_1 v_1 \log_e r; \end{aligned}$$

where r is the ratio of expansion giving a terminal pressure $p_2 = p_3$. The value of the ratio of this work, U, to the quantity, W, of fluid employed, measured sensibly by $p_1 v_1$, is thus

$$\frac{U}{W} \propto p_1 v_1 \log_e r / p_1 v_1 \propto \log_e r;$$

it being, however, noted that the back-pressure, $p_2 = p_3$, in this case.

the case of the best class of engines above referred to, about 25, including all wastes.

Accepting the last measures as limiting figures for the higher pressures of steam which the engineer is coming now rapidly to contemplate and experimentally to investigate, we have approximately the following as probable:

Pressures............	100	200	300	500	1000
Expansion............	15	30	45	75	150
Steam used per I.H.P.	13–15	11–13	10–11	9–10	8–9

225. Quadruple-expansion Engine at pressures of 300 to 500 pounds gives the following:*

The Ideal Case is easily computed for any assumed pressure. Thus, for example, taking the initial pressure as 500 pounds per square inch, the back-pressure in one case as 5 pounds, in another as 2, and the ratio of expansion as 64, in both cases we have the following, adopting Rankine's method of computation:

DATA AND RESULTS.

$p_1 = 500$ lbs ; $p_2 = 6$ lbs.; $p_3 = 5$ lbs.; $r = 64$.

Mean effective pressure..............................	31 lbs.
Efficiency of fluid..................................	0.2575
B. T. U. per I. H. P.................................	10,200
Steam per I. H. P. per hour..........................	9.25 lbs.
Coal per I. H. P. per hour..........	1.03 "

Reducing the back pressure to 2 pounds per square inch, these figures are improved to the extent of 10 per cent, and we have:

Efficiency of fluid..................................	0.28
B. T. U. per I. H. P. per hour.......................	9200
Steam per I. H. P. per hour..........................	8.5 lbs.
Coal per I. H. P. per hour...........................	0.9 "

It is here assumed that 1100 B.T.U. are available per pound of steam with a good boiler and heater system, and that the area of heating-surface is ample to insure an evaporation of 9 pounds of water per pound of good fuel. For unity

* On the Promise and Potency of High-pressure Steam ; R. H. Thurston : Trans. A. S. M. E., vol. XVIII, 1896; No. DCCXVIII.

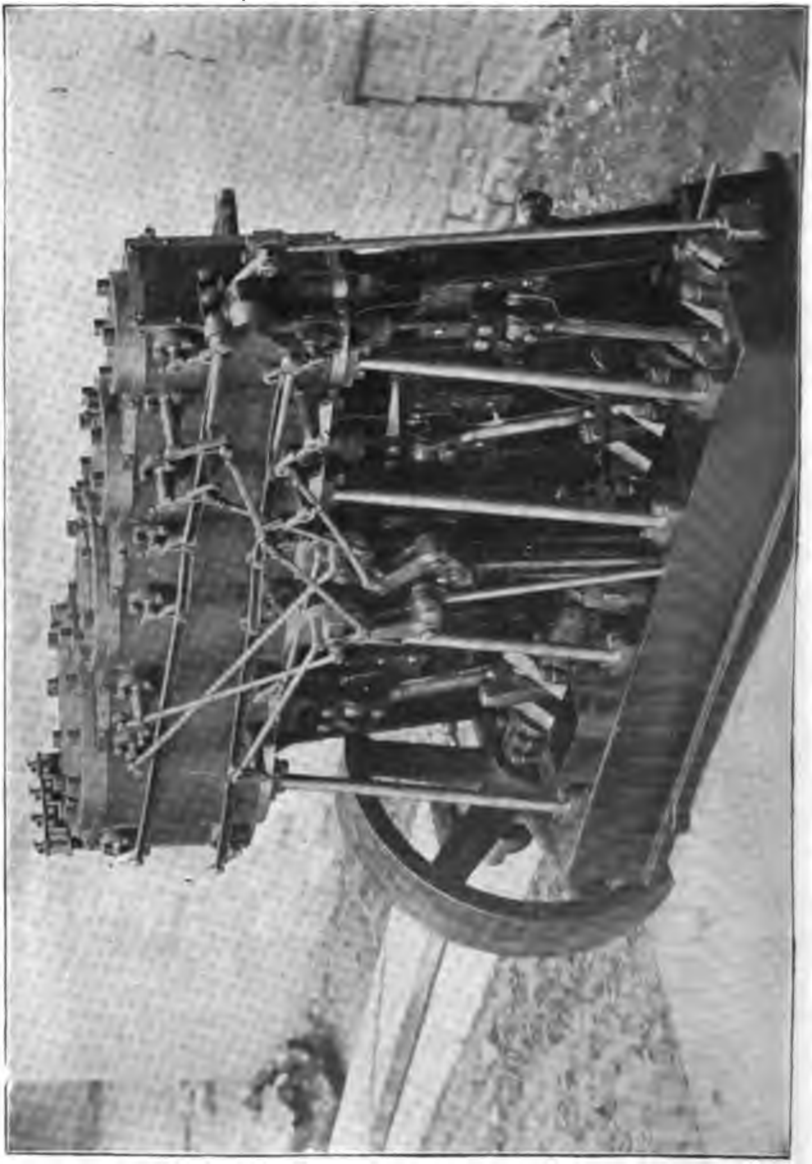

Fig. 195.—Experimental Quadruple-expansion Four-cylinder Engine of Sibley College, C. U.

efficiency this would give about 2.3 pounds of steam and 0.26 pounds of fuel per I.H.P. per hour.

Assuming the total wastes of heat to be measured by the percentage $w = a\sqrt{r}$, and the value of a for an engine of considerable size to be, as found for large mill-engines, about 0.15, we obtain the loss for the simple engine, $w = 0.15 \times 8 = 1.2$, and the total expenditure of feed-water per horse-power per hour here becomes, for the two cases respectively, 20 and 18 pounds. Taking the wastes, however, as inversely as the number of cylinders in series, within the limits here observed, we have the following for the computed and probable costs of power in the two cases:

EFFICIENCIES OF MULTIPLE-CYLINDER ENGINES.

Form of engine.	$p_3 = 5$,	$p_3 = 2$.	
Ideal case...	9.25	8.5	pounds steam per I. H. P. per hour.
Simple jacketed..........	20	18	" " "
Double-expansion.... ...	14	13	" " "
Triple-expansion....... ...	13	11	" " "
Quadruple-expansion	12	10	" " "

Were the same investigation made for steam-pressures approximating 1000 pounds, the results would exhibit but 10 per cent. better figures. Compared with the now usual figure, 100 pounds per square inch of boiler-pressure, the elevation of the pressure 400 per cent would give, as seen already, about 30 per cent gain, and increase to 1000 per cent about 50 per cent gain. It is further obvious that if a well-designed and efficient, yet not excessively expensive, engine could be adapted to the employment of these higher pressures, the gain in efficiency might, in many cases, handsomely repay the costs.

The combined diagram for this case is presented in the inserted plate, Fig. 196, as worked out by careful measurement of the diagrams and comparison with the saturation-curve for the weight of steam inclosed in the cylinder at cut-off.* The saturation-curve shown in the combined diagrams is that

* Trans. A. S. M. E., 1893, vol. XV. p. 313; "The High-pressure Multiple-expansion Engine," R. H. Thurston.

of a weight of dry and saturated steam equal to that worked in the engine in each cycle and without becoming either wet or superheated. The comparison with this of the abscissas of the engine-diagram as obtained with the indicator thus gives a measure of the variation in volume produced by either partial condensation or by superheating, and thus a correct determination is made of the quality of the steam at every instant in its progress through the engine.

In the high-pressure engine, the steam, wet at entrance into the high-pressure cylinder, becomes steadily dryer as expansion progresses, until, at the opening of the exhaust, it approximates dryness, becoming dry and saturated at about three-fourths stroke in the first intermediate cylinder, wet throughout the stroke of the second intermediate, and so continues through the low-pressure cylinder and up to final exhaust into the condenser.

The Calorimetric Analysis of this engine presents many points of interest. The table on page 884, although not exhibiting the highest efficiencies or full pressure, is given as developed in regular work.

The ideal Carnot cycle would demand about 8100 B.T.U. per I.H.P. per hour, and at 1100 B.T.U. supplied per pound by the combustible, the weight of fluid worked would be 7.36, or in the Carnot system about 7, pounds per I.H.P. per hour. The best actual performance indicated a waste of above one-third; and the fuel and steam consumption, as well as the quantity of heat demanded, exceeded the computed figures by 38 per cent—amounting to 9.27 pounds of steam, in the dry and saturated condition, and to 225 B.T.U. per minute per I.H.P. The mechanical efficiency of the engine was excellent for so small a machine—86.88 per cent, at 11 horse-power. The ratio of expansion was probably too large, for best effect at this pressure.

The Actual and Computed Results of operation of this engine, comparing the ideal case with the real engine performance, are shown in the diagrams, Fig. 197. The curve *A* shows the computed efficiencies of the representative ideal

case, as measured in pounds of feed-water per H.P. per hour, and, approximately, in thousands of B.T.U.; the Rankine cycle being assumed, and the feed-water temperature being taken at 40° F. It is seen that the figures range from about $6\frac{1}{2}$ at 500 pounds boiler-pressure to 7 at 300 pounds, to 8 at 150 pounds, and to 9 at 100 pounds.

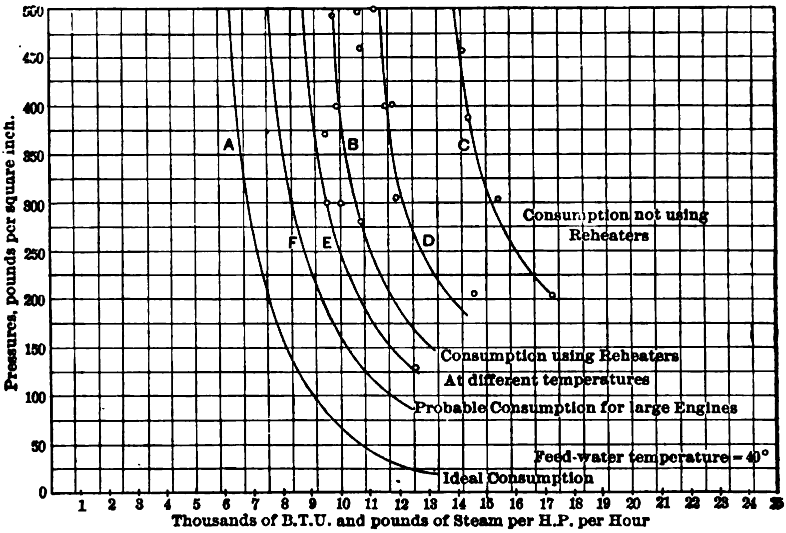

FIG. 197.—EFFICIENCY CURVES, IDEAL AND ACTUAL.

The actual performance, as shown on curve *B*, is irregular, varying somewhat with the action of the boiler and the effectiveness of the reheaters. With most effective boiler-operation and reheating, the very best work is accomplished, in the trials illustrated, between 300 and 400 pounds pressure. From the former figure up to 500, the figures for cost in heat and in fuel range between $9\frac{1}{2}$ and 10, falling to $12\frac{1}{2}$ at 150 pounds—a pressure, however, for which the engine is not well proportioned. As the reheating becomes less effective, the efficiency falls off, until, with reheaters out of use entirely, the curve *C* is obtained; the smooth curvature of the line shows the true law of gain with increasing pressures. The figures measuring economy range from $13\frac{1}{2}$ at 500 down

DATA AND RESULTS. QUADRUPLE-EXPANSION ENGINE.

Made by Messrs. Hall & Treat, Sibley College, Cornell University. **Date, Feb. 14, 1896.**

Kind of engine, quadruple-expansion	
Duration of run ... hours	6¼
Revolutions per minute	286.2
Temperature condensing water, cold ... deg. F.	34.87
" " " warm ... deg. F.	41.27
" condensed steam	58.24
" engine-room	73.80
" external air	31.30
Boiler-pressure gauge ... lbs.	279.2
Barometer, 29.154 inches ... lbs.	14.306
Condenser, 25.43 inches high ... lbs.	124.79
Boiling temp., atmospheric pressure ... deg. F.	210.75
Total steam per hour, C, for engine ... lbs	106.53
Total condensing water per hour	16,632
Wt. condensing water per pound, steam	156.15
Total, I. H. P.	11.095
" D. H. P.	9.606
Mechanical efficiency	86.88
Moisture in steam ... per cent	2.49
Steam per I. H. P. per hour, actual ... lbs.	9.51
" " I. H. P. per hour corrected, cal. ... lbs.	9.27
" " D. H. P. per hour corrected, cal. ... lbs.	10.81
* Water-rate of perfect engine	6.86
Thermodynamic efficiency	81.44
Ratio actual to theoretical water consumption	1.38
Heat supplied per hour B. T. U., by boiler	125,547
" discharged per hour B. T. U.	109,240
" utilized per hour B. T. U.	28,236
B. T. U. per I. H. P. per minute	225.7
Ratio of expansion from combined card	82.9

	H. P.	1st I. P.	2d L. P.	L. P.
Diameter of cylinder ... inches	2.344	3.969	6.977	10.266
Length of stroke ... inches	4.5	4.5	4.5	4.5
Diameter of piston-rod ... inches	.5	.625	.875	1.125
Piston displacement, crank end, cubic inches	18.565	55.403	171.794	371.606
" " head end, cubic inches	19.449	55.710	172.395	372.6
Volume of clearance, head end, per cent	2.14	1.99	2.79	3.94
" " " crank end, per cent	1.88	1.99	2.79	3.94
Make of indicator	Thompson.			
Spring ... lbs. per inch	307	186.8	20 } 30 }	10
Length of brake-arm ... feet	4.8021			
Brake-load ... lbs.	55.33			
Quality of steam at cut-off	See combined diagram.			
" " " " release	"	"		
Cut-off, crank end, per cent, stroke	20.68	30.57	27.56	22.09
" head end, per cent, stroke	26.38	36.82	31.89	31.27
Release, crank end, per cent, stroke	95.84	98.96	95.80	99.92
" head end, per cent, stroke	95.91	98.26	99.40	96.56
Compression, crank, per cent, stroke	3.4	10	13.5	8.3
" head, per cent, stroke	6.77	6.27	12.8	10
Absolute pressure at point of cut-off ... lbs.	264.10	110.325	39.40	10.67
" " " release ... lbs.	115.154	44.43	14.30	4.08
" back-pressure ... lbs.	125.169	44.33	13.38	2.47
I. H. P. head	.868	1.727	1.738	1.441
I. H. P. crank	.804	1.534	1.771	1.212
Total I. H. P.	1.672	3.261	3.509	2.653
Steam per I. H. P. at point cut-off, per diagrams	4.86	8.60	7.71	4.55
" " " " " release, per diagrams	8.99	10.94	11.72	8.07
Distribution of work. Total H. P. units	15.07	29.39	31.63	23.91

* Water-rate of perfect engine taken as $2545 \div (\lambda - q_2)e$, in which λ = total heat of entering steam, q_2 = heat of liquid of exhaust, e = thermodynamic efficiency. Receiver pressure, 1 = 108; 2 = 30.7; 3 = − 1.6. Reheater temperature, 1 = 645; 2 = 472; 3 = 254.6.

to 14 at 400, to 15 at 325, to 16 at 250, and to $17\frac{1}{2}$ at 200 pounds pressure.

In that portion of the scale on C for which the engine is best proportioned, the consumption of steam in pounds, or of heat in thousands of heat-units, is measured with a fair degree of accuracy by the expression

$$w = \frac{a}{\log p_1};$$

the value of a being taken at 30. For the case in which reheating was more or less effective, line B, the value of a becomes more nearly 25, rising at times, with inefficient action of the reheaters, to 30; while, on the line A, the coefficient becomes $a = 18$. Throwing out the reheaters, the efficiency becomes about one-half that of the ideal case at the higher pressures.

226. Summarizing the Case of High-pressure Steam, we may state the following conclusions relating to the promise of high-pressure steam within the limits examined:

(1) With equally excellent design, construction, and management, we may expect the efficiency of the steam-engine, with increasing pressures, to increase nearly as the logarithm of the boiler-pressure.

(2) We may hope to secure, at the highest pressures yet proposed, substantially as close an approximation to the efficiency of the ideal engine as at those pressures which now give our best records—probably 70 per cent of the efficiency of the cycle adopted, considered as an ideal, thermodynamic cycle.

(3) That gain in economy, by increasing pressures simply, must be expected to be slow and to steadily decrease in rate of gain as pressures rise; making the practicable, commercial limit a pressure comparatively low.

(4) That assuming 1000 pounds pressure safely attainable, we cannot expect to reduce the demand for heat and steam below 6000 B.T.U. per I.H.P. per hour, and about 6 pounds

of steam; the probable figures being at least 20 per cent higher.

(5) At 500 pounds pressure, a steam-consumption of 10 pounds and less has been attained under circumstances indicating that, on a large scale, the steam-engine should, under favorable thermal conditions, reduce this figure very considerably.

(6) The directions in which to seek for gain are the reduction of internal wastes and the production of a superheated steam-engine.

227. **The Purposes of Superheating Steam,** as practised in the past and as already studied, may be summarized thus:

(1) Raising the temperature which constitutes the upper limit in the operation of the heat-engine in such manner as to increase the thermodynamic efficiency of the working fluid.

(2) To so surcharge the steam with heat that it may surrender as much as may be required to prevent initial condensation at entrance into the cylinder and still perform the work of expansion without condensation or serious cooling of the surrounding walls of the cylinder.

(3) To make the weight of steam entering the condenser, and its surplus heat-charge, a minimum, with a view to the reduction of the volume of condensing water and the magnitude and cost of the air-pump and condenser system to a minimum.

(4) To reduce the back-pressure and thus to increase the power developed from a given charge of steam and the efficiency of the engine.

(5) To increase the efficiency of the boilers both by the reduction of the quantity of the steam demanded from the original heating-surface and by increasing the area of surface employed to absorb the heat of the furnace and flue gases, and also by evading the waste consequent upon production of wet steam.

The first of these processes has not yet been made successful, and no satisfactory form of steam-engine has been yet constructed in which the increase of thermodynamic efficiency

to be thus gained has been attained by the working of steam as a gas throughout a cycle, or even in which any large increase of working range of temperature above the minimum limit—that of saturation at the temperature of the condenser —has been secured by superheating. The Siemens superheated steam-engine illustrates an attempt in this direction.

The second of these methods of improvement of the steam-engine has been frequently attempted, usually without permanent success, and it remains a promising but unsolved problem. It is this method which is, mainly, the subject of this memoir. Its value consists solely in its power of reduction of the internal wastes of the engine by the substitution of a comparatively gaslike fluid, with reduced heat-transferring power, for the easily condensed and highly-conducting vapor, saturated steam, with consequent diminution by a large proportion of the waste known as initial or cylinder condensation. This it does with undoubted and great success; but the endeavor to utilize this well-recognized method of improvement of steam-engine efficiency is, in practice, found to be fraught with so many and such serious practical difficulties of construction and operation that it has, hitherto, been very little employed, and seldom, if ever, as yet, with complete success.

The third gain by the use of superheated steam is a minor but unquestionable improvement, to be anticipated as incidental to the use of superheat for the other purposes enumerated.

The same remark applies to the fourth item in the list.

The fifth effect of superheating, or rather of the introduction of superheating apparatus, comes simply of the extension of the ratio of the area of boiler-heating surface to the weight of steam to be supplied in the unit of time.

"*Reheating*" between the cylinders of the multiple-expansion engine was practised in England by Cowper a quarter of a century or more ago, and in the United States at about the same time by Leavitt and by Corliss. Receivers, or separators, introduced into the connecting-pipe between a pair of such cylinders, if they are properly designed and proportioned,

permits the complete separation of all the water exhausted from the smaller of the pair, and a fagot of tubes supplied with high-pressure steam enables drying and slight superheating to be secured. In other cases, the receiver is placed in the flue between boiler and chimney, and thus becomes a superheater of the more common class. Both these dispositions of superheating apparatus are now very commonly adopted. Superheating is essential to success.

In the small quadruple-expansion engine already referred to, built in the Sibley College shops for 500 pounds steam-pressure, this latter arrangement is adopted, and the result, although the superheating is but moderate, contributes, presumably, effectively to the remarkable economy there reached—a minimum consumption of less than 10 pounds of steam, or 13,000 B.T.U. per I.H.P. per hour.*

228. **The Economic Value of Heat** applied to the prevention of waste by initial condensation through the preliminary warming of the cylinder-wall up to the temperature, as nearly as may be, of the entering steam, is substantially the same whether it be communicated by jacket, by superheat, or by compression; but there is this difference between jacketing and superheating: the former supplies heat—and wastes heat, necessarily—throughout the period of exhaust and of condensation, while the latter only gives up its heat in that portion of the cycle in which it is needed, and always supplies it precisely when and where required, to the full extent of the supply. The maximum efficiency of action of such heat, as supplied by the jacket, is thus inferior to the minimum effect of heat supplied by superheating. We may thus easily obtain the minimum value of heat supplied the

* "Reheating" was employed by John Bourne as early as 1859, in a marine compound engine, and was illustrated in several of his exhibits at the International Exhibition of 1859. As he states, the "superheater is introduced between the high- and the low-pressure cylinders to replace the heat lost by radiation and by production of power in the high-pressure cylinder," and "promises to be very serviceable in cases where a small consumption of fuel is an object of paramount importance.—*Recent Improvements*, p. 15 (1869).

working-steam in the shape of superheat. The following table gives the best figures that the writer has been able to find, and are presumably illustrative of the most effective employment of heat, by this method, in the prevention of the waste sought to be reduced by superheating.*

ECONOMIC VALUE OF SUPERHEAT (Minimum).

(1) SIMPLE NON-CONDENSING ENGINES.

No. Case.	p_1 Lbs. per Sq. In.	Revolution Per Min.	Ratio of Expansion	Per Cent Heat Expended.	Per Cent of Gain.	Ratio of Economic Value.
2	115	203	...	3.2	23.0	7 to 1
8*b*	110	62	6.2	3.4	22.0	7
8*c*	110	62	5.3	3.1	16.7	5.5
8*d*	110	63	4.4	2.0	16.7	8
7	100	97	3.4	2.7	16.3	6
7*f*	78	93	5.1	3.3	22.4	6.6
8*a*	32	47	7.1	2.5	19.7	8

Average Value, 7.

In the above table the first column gives the numbers in the series reported by the committee of the B. I. M. E., from which this selection has been made of the best cases of economic application of heat to the amelioration of internal wastes. The second, third, and fourth columns give the boiler-pressures, the speed of rotation of the engine, and the ratio of expansion adopted. The fifth column gives the percentage of the total steam-supply measuring the cost of such saving as is given in the sixth column in percentage, also, of the total. The last column is the measure of the economic value of heat supplied in prevention of waste, as measured by the ratio of the quantity furnished for this purpose to the quantity saved by its use.

Since, as above indicated, the best possible work of heat supplied by jacket represents less than the theoretical effect of superheating, the deduction follows with substantial accuracy that, with simple engines, the expenditure of one

* "Theory of the Steam-jacket," R. H. Thurston: Trans. A. S. M. E., vol. XV, XVII, 1894–6, p. 843.

thermal unit in superheating may save from four to eight—the averages being above six and seven—and the percentage of gain thus may become from four to eight times the per cent of heat taken into the superheater and delivered at the cylinder. With compound engines, the proportion becomes about one-half as great as with simple engines; the total ratio of expansion, and the ratio in each cylinder, being ordinarily correspondingly less, and their wastes by initial condensation similarly less. We have little information relative to this point with triple-expansion engines; but it may be taken as the fact with these engines that the unit of heat expended in the manner assumed above may give a return of not less than two units in net saving. The more economical the engine, the less the gain to be secured by these methods of reducing wastes. The final limit in saving is the magnitude of the waste to be reduced. Jacketing may save a large part of this waste; superheating may reduce it almost or quite entirely.

Studying the results of experiments to date determining the magnitude of the internal wastes which superheating is expected to reduce, we shall find that the following may be taken as, roughly, the measure of those wastes, the relative quantities of heat gained by complete extinction and expended in the work, and the extent of the necessary superheat:

(2) SIMPLE CONDENSING ENGINES.

23*a*	110	60	10	2.9	23.0	8
23*b*	110	60	6.4	3.2	20.0	6.3
23*c*	88	59	12	3.0	16.7	5.6
23*d*	89	58	11.3	3.1	17.7	6
23*g*	89	60	5.9	1.5	14.3	9.5
15	80	55	10	3.8	23.7	6
19*a*	67	50	5.7	3.5	21.5	6
19*b*	68	50	12.4	4.8	30.3	6.3
20	58	53	4.7	3.4	15.5	4.7
22	42	20	4.3	4.9	16.6	3.6
16*a*	17	37	3.4	7.25	37.4	4.5
16*b*	16	37	3.4	7.25	31.3	4.3
18*a*	14	41	1.75	2.4	16.3	6.6

Average Value, 6.

(3) COMPOUND CONDENSING ENGINES.

33	80	55	7	6.5	15.4	2.4
24	56	235	4.3	5.2	23.5	4.5
31	47	18	15.8	9.9	34.9	3.5
25	41	36	5.5	8.9	28.6	3.2

Average Value, 3.4.

GAIN BY SUPERHEATING.

Engine.	Steam-pressure, Pounds per Square Inch.	Percentage Steam Condensed, without Superheating.	Relative Heat Lost by Condensation and Expended by Superheating.	Degrees Fahr. Superheat.
Simple	50 to 100	50 to 30	5 to 1	100
Compound	75 to 125	30 to 20	3 to 1	75
Triple	125 to 180	20 to 10	2 to 1	50

229. The Promise of Gain is now thus indicated: Superheating and drying steam simply gives, at best, dry steam

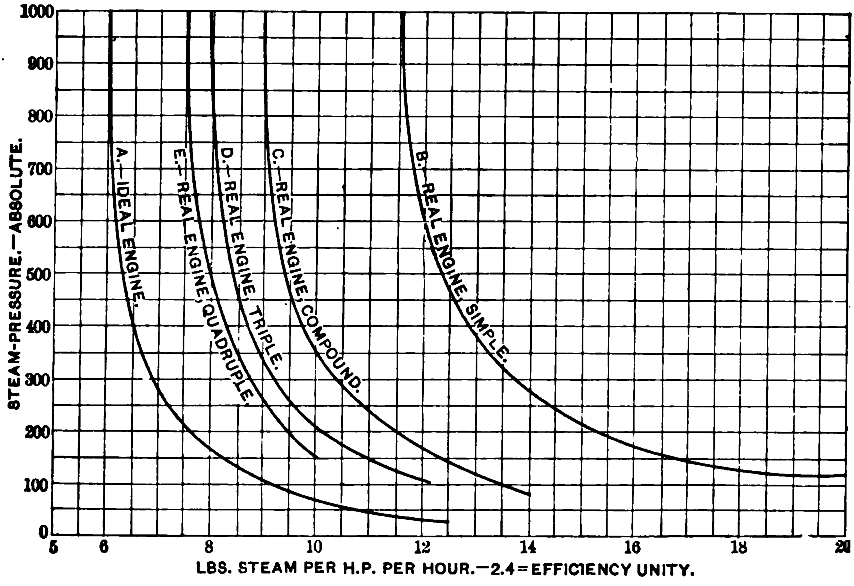

FIG. 198.—ECONOMY OF HIGH STEAM-PRESSURE.

in the engine. Assuming the use of saturated steam, we cannot say how far we shall be able, at as yet unattained pressures in the steam-engine, to approximate the ideal case by reduction of the wastes of the real engine. Assuming, however, that we may anticipate at least as much success in this direction, at higher pressures, in the future, as at the present figures, the accompanying sketch will indicate what may be hoped for, and it may aid in determining whether it will finally pay to struggle for the higher thermodynamic result. In the figure, let the curve at the left represent the performance of the ideal, perfect engine—Fig. 198—perfect in the sense of exhibiting no extra-thermodynamic losses. Let the curve at the right similarly exhibit the costs of power in a real engine, of such proportions that the heat, steam, and fuel supplied will be subject to a loss of one half by extra-thermodynamic action. Experience, so far as it at present guides us, appears to indicate that we may expect to find it practicable to keep down this waste at high pressures to the extent that it is now held down, under usual conditions, in even small engines. Should it prove practicable, also, throughout so wide a range of pressures, to reduce it by compounding or otherwise to one half the amount assumed as a minimum for the simple engine at such pressures, the curve *C* results; and if, by adding a third cylinder, in series, reduction to one third may be hoped for, the curve *D* exhibits the presumable costs of power, in pounds of steam per horse-power per hour, or, if the feed-water is suitably heated by waste heat in the flues or from the exhaust, in, it may be assumed, thousands of B.T.U., or, again, in tenths of a pound of good fuel consumed in good boilers. In the same way, the quadruple-expansion engine, with its added cylinder intercepting the otherwise wasted leakage and cylinder-condensation of its predecessors, in series, may perhaps bring down costs to the figures indicated in curve *E*. A non-conducting cylinder would make them as shown on curve *A*. Any and all of these curves, *A* excepted, would seem to represent practicabilities to the engineer, and curve *E*, at

least, the ultimate probabilities. In any event, the latter will make a good point at which to aim with hope and courage.

It will be interesting to note that, as better performance is insured by improved construction of engine, the advantages of higher thermodynamic efficiency are approximated more and more closely at progressively lower pressures, and that, at the limit *E*, 500 pounds pressure gives very nearly as good results as 1000; and it may prove that, all things considered, it will not be advisable to carry the steam-pressure to as high a figure as would at first thought appear desirable. It does not follow, however, that superheating, or other expedients for the attainment of a wider thermodynamic range of heat-conversion at manageable pressures, may not continue commercially advantageous.

We have already, both in the gas-engine and in the steam-engine, brought down the cost of power, in good coal burned, to about one pound per I.H.P. per hour; it may prove that another third or even half of the remaining wastes of the steam-engine, and even a higher proportion in the gas-engine, may be extinguished. The question, lying beyond this, of the relative standing of these forms of heat-engine, and of the possible substitution of a machine in which the fuel is burned in the engine-cylinder, in which the products of combustion are the working fluids, and in which maximum visible range of thermodynamic action seems possibly attainable, for the familiar steam engine, with its at present superior advantages in mechanical efficiency, in high mean effective pressure, its low cost of maintenance, its long life, its handiness in manipulation, in stopping and starting, and its perfect regulation by economical methods, is one which interests every engineer, but one which none can at present answer positively.*

Professor Unwin, in 1895, gave records to that date, with saturated steam, to which the Author has added those indicated by a dagger,† to date:

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* Marine Engineering, May, 1897; R. H. Thurston.

† Technology Quarterly, 1896; Trans. Am. Soc. M. E., 1896, vol. XVIII.

MINIMUM STEAM CONSUMPTION.

	I. H. P.	Gauge-pressure. Lbs.	Speed of Piston. Ft.	Pounds Steam I. H. P. per Hour.	B. T. U. per I. H. P. per Minute.
Simple Engine:					
Sulzer	284	87	372	18.4	360
Corliss	137	62	...	17.5	350
Compound:					
Dujardin	548	90	570	13.46	255
Sulzer	247	85	493	13.35	250
Wheelock	590	160	612	12.84	235
McIntosh & Seymour†	1076	125	800	12.76	230
Leavitt	643	135	371	12.16	225
Compound; Superheated Steam:					
Schmidt	76	180	380	10.17	202
Triple Expansion:					
Sulzer	615	141	516	11.85	223
Allis	574	120	203	11.68	218
Leavitt†	576	185	240	11.22	210
Quadruple:					
Hall & Treat†	16	300 / 400 / 500	220	10.80 / 9.65 / 9.75	225 / 193 / 195
Nordberg	712	200	255	12.26	186

The outlook, in respect to engine-efficiency and economy in the use of heat, steam, and fuel, may perhaps be fairly well indicated by the data exhibited in the accompanying diagram, Fig. 199, which represents the variation of efficiency and economy in the expenditure of heat, steam, and fuel, in the ideal, purely thermodynamic, case, under the most favorable conditions of actual practice, as seen on the curve *AB*; in the case of an engine in which the total of all wastes is taken at about one fourth the supply, as on the line *CD*; and in cases in which larger proportions of heat fail of useful thermodynamic transformation, as on lines *EF*, *GH*, and *IJ*. Condensation and a good vacuum are assumed, and a feed-water supply at such a temperature as will store about 1000 heat-units in each pound of steam passing through the engine and boiler cycle, where jacket-water is included.

The forms of these curves are substantially exact, their location being representative of good practice, and the area bounded by them being taken as inclusive of the whole field

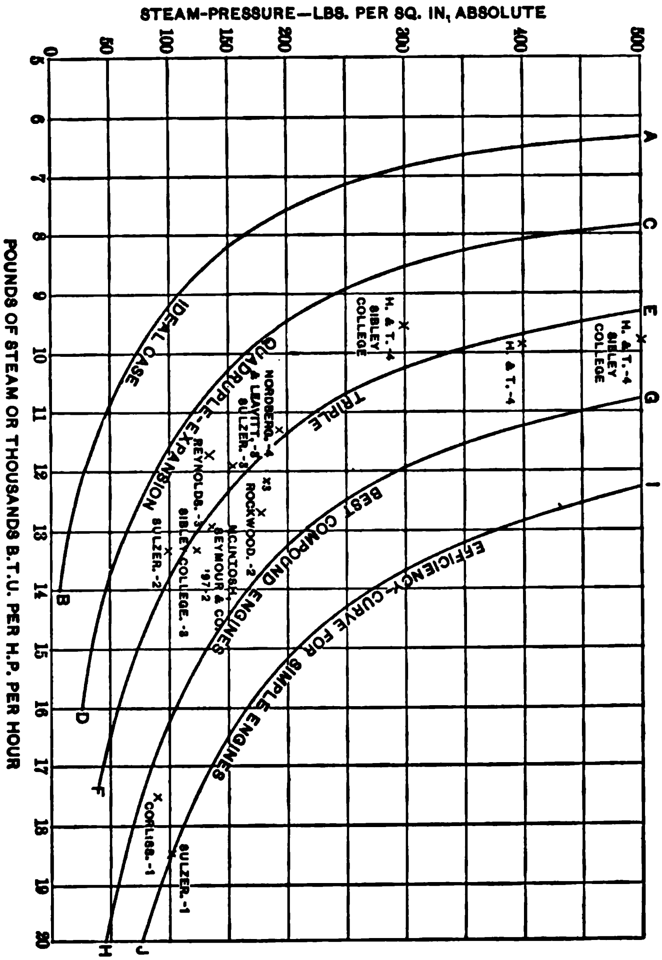

FIG. 199.—EFFICIENCY-CURVES OF ENGINES, 1896-97.

of best recent practice. The limiting curves are those for good modern practice with the simple engine and the case of the perfect thermodynamic machine; while the intermediate

curves may be taken as approximately illustrating past and prospective experience with the multiple-cylinder engines. The data obtained from recent records are illustrated by the points inscribed in this field, the number of cylinders in series, in each representative case, being indicated by the attached figure.

It is seen that the best simple engines recorded are the Sulzer, which gives a point on the curve *IJ*, and the Corliss, a simple engine which excels the former somewhat and gives a point nearly attaining the first line, within the curve *GH*. The best compounds are the Rockwell, which nearly reaches over to a line assigned good triple-expansion engines, and the Sulzer, which excels that standard and approximates that of the compound engines of contemporary practice. The best triple-expansion engine results, judged from this standpoint, are those of the small experimental triple-expansion of Sibley College, designed by Reynolds, working at 113 horse-power, and which comes close up to the standard curve assigned this class; the Leavitt and the Sulzer, which approximate the standard but somewhat excel it; and the Allis, another design by Reynolds, which considerably excels the standard. The Schmidt engine is a small compound machine, brought over into this part of the field by its system of superheating and actually much less effective.

A small quadruple-expansion engine in the set is that of Hall and Treat, built as an experimental engine, and the property of Sibley College. This falls into one or another of our standard curves accordingly as its system of drying and reheating steam is made more or less effective. Were either of the experimental engines here referred to reproduced on the scale of the larger engines with which they are here brought into comparison, the resultant diminution of internal heat-wastes would carry them over near the standard line next the ideal, and they would then demand less steam, heat, and fuel by presumably 25 per cent. at least. The line adjacent to the ideal, here marked quadruple-expansion, is that which

experience seems, in the judgment of the Author, to indicate may be hoped and fairly anticipated to become that of best actual practice when the design, the construction, and the operation of the engine is made in all respects as satisfactory as present knowledge and our improving skill should permit.

The general trend of the curves indicates the gain to be anticipated from increasing pressures when we shall have become as competent to avail ourselves of them as we are to-day in the use of the familiar pressures of our own time. The thermodynamic value of either engine is obviously measured by its economy in the use of heat, and especially by its approximation to the ideal curve. The line *CD* is the estimated position of the quadruple-expansion engine of Sibley College, reproduced on the scale of the large engines with which it comes into competition on the diagram. In the comparison of types and of performances of engines it is now clearly seen that the simple statement of their relative demand for steam or for heat is not a true gauge; the figure must be compared with the curve of efficiencies in the manner here illustrated to determine the real position of the engine, thermodynamically, and the merit of the design.

The success of the steam-engine designer and constructor is not, however, entirely dependent upon his own efforts, or even upon the intrinsic merit of the machine, or on the location of its representative datum upon our diagram: it depends in very large and often controlling degree upon the conditions of the case outside the engine-room. It is only when the load upon the engine is held constant, and at that amount for which the machine is best proportioned, that the maximum efficiency of the engine can be secured. Every variation from its best load, and especially in the direction of reduced demand for power, causes loss of economy of power-supply. The disposition of the plant in such manner as to insure the most uniform possible load upon the engine, and the assignment to each machine of its proper proportion, securing so far as practicable its best load, is a primary requisite of economical operation. In the design of large stations

this problem is less difficult of solution than with smaller loads; but it is usually the principal obstacle to the attainment of highest duty. As the storage-battery becomes more satisfactory in first cost, in endurance, and in operating expenses this problem will become less and less difficult of solution for electric light and power stations, and the perfection of a steam-power system in the distribution of electrical energy will come when this or some other method of giving the engine its best and a constant load can be secured.

230. **The Financial Problem,** as already shown (Chaps. V, VII), remains the final and vital problem in every case, and the questions how to secure the demanded power or stated result of whatever kind, with minimum tax on the business and maximum returns on the amount expended or invested, and how far gain in economy in any given direction is worth paying for, are those to be considered. It has already been seen that, in the case of the steam-engine, the gain by extended expansion finds an early limit, first by increasing heat-wastes; secondly, and still earlier, by costs of construction of the larger engine. The following may be taken as one of the best illustrations of the great and controlling influence of these conditions:—*

In earlier pages are given the data of performance of a

---

* "We should not expect ever to utilize, in practice, all the motive power of combustibles. The attempts made to attain this result would be far more hurtful than useful if they caused other more important considerations to be neglected. The economy of the combustible is only one of the conditions to be fulfilled in the heat-engines. In many cases it is only secondary. It should often give precedence to safety, to strength, to the durability of the engine, to the small space which it must occupy, to small cost of installation, etc.

"To know how to appreciate, in each case, at their true value, the considerations of convenience and of economy which may present themselves; to know how to discern the more important of those which are only accessories; to balance them properly against one another in order to obtain the best results by the simplest methods,—such should be the leading characteristics of the man called to direct, to co-ordinate among themselves, the labors of his comrades, to make them co-operate toward one useful end, of whatsoever sort it may be." [CARNOT, p. 126, THURSTON'S TRANSLATION.]

steam pumping-engine, built for maximum "duty," and at the time of its construction the most successful on record. The question may now be asked, What would have been the limit of economical expansion had a machine of similar general design and equal cost per unit volume of cylinder been built for the same work, but designed to make total costs of water-supply a minimum?

The following are the data and method; the results immediately follow. Costs of engines and boilers and of fuel are only considered; all other expenses being, in this case, assumed to be unaffected by such changes of proportions as are here contemplated.

In the construction of this curve the one available set of data obtained by the one complete engine-trial of which records are available may be used. These give the value of a in the formulas of §§ 130, 137, and 219, as given in the latter article; or assuming, as actually here proposed, a constant absolute waste throughout the whole anticipated possible range of cut-off, permit, as in the diagram, Fig. 198, the reduction of the ordinates of the curve of adiabatic mean pressures by the constant quantity thus ascertained.

In either case, as has already been fully shown, the results of a single trial may be made to give the needed quantities for a more or less accurate construction of the real curve of efficiency for the engine.

CONSTRUCTION OF THE "CURVE OF EFFICIENCY."—PUMPING ENGINE.

DATA.

Total steam per hour	=	6700.7 lbs.
Rev. per min	=	20.41
Strokes per hour	=	2449.2
Steam per stroke	=	2.736 lbs.
p_1	=	135 "
Vol. 1 lb. at 135 lbs. press. (absolute)	=	3.272 cu. ft.
2.736 × 3.272 = per stroke	=	8.952 "
Dia. L. P. cyl	=	74 in.
Area L. P. cyl	=	29.861 sq. in.
Stroke	=	5 ft.

Piston-displacement = 29.861 × 5	=	149.305 cu. ft.
Clearance = 1.4 per cent	=	2.09 "
Vol. L. P. cyl. = 149.305 + 2.09	=	151.375 "
Proportion of cyl. which would be filled at each stroke if there were no condensation = 8.952 ÷ 151.375	=	0.059

Relative Vol. of cylinders:	Average M.E.Ps:
H.P. cyl. = 1	H.P. 48.75 lbs.
I.P. cyl. = 2.98	I.P. 14.45 "
L.P. cyl. = 6.98	L.P. 8.85 "

H.P. M.E.P. reduced to L.P. cyl. $= \frac{48.75}{6.98} = 6.98$ lbs.

I.P. M.E.P. reduced to L.P. cyl. $= \frac{14.45}{2.98} = 4.85$ lbs.

L.P. M.E.P.	=	8.85 lbs.
Total M.E.P. reduced to L.P. cyl	=	20.68 lbs.
Condenser press	=	1.25 "
Friction of engine	=	9.22 per cent
Which reduced to back-press. per sq. in. of piston	=	1.9 lbs.

Costs.

Cost of engine	$66,000.00
" " boiler	10,000.00
" " engine and boiler settings	5,000.00
Take cost of engine-setting	2,000.00
" " " boiler-setting	3,000.00
Cost of fuel	16,410.00

Per Cu. Ft. of L. P. Cylinder.

Cost of engine	$449.00
" " boiler	86.00
" " fuel per year (actual)	108.00
" " " " " for "full steam"	1,830.50
" " oil, waste, and packing	4.00

Engine.—Annual Expenses.

Interest on cost at 5 per cent	$22.45
Depreciation at 4 per cent	17.96
Oil and packing, etc	4.00
Repairs	1.37
	$45.78

Boiler.—Annual Expenses.

Interest on cost at 5 per cent	$4.30
Depreciation at 4 per cent	3.44
Repairs	1.37
	$9.11

Then $$\frac{45.78}{9.11 + 1830.5} = .025 = MO.$$

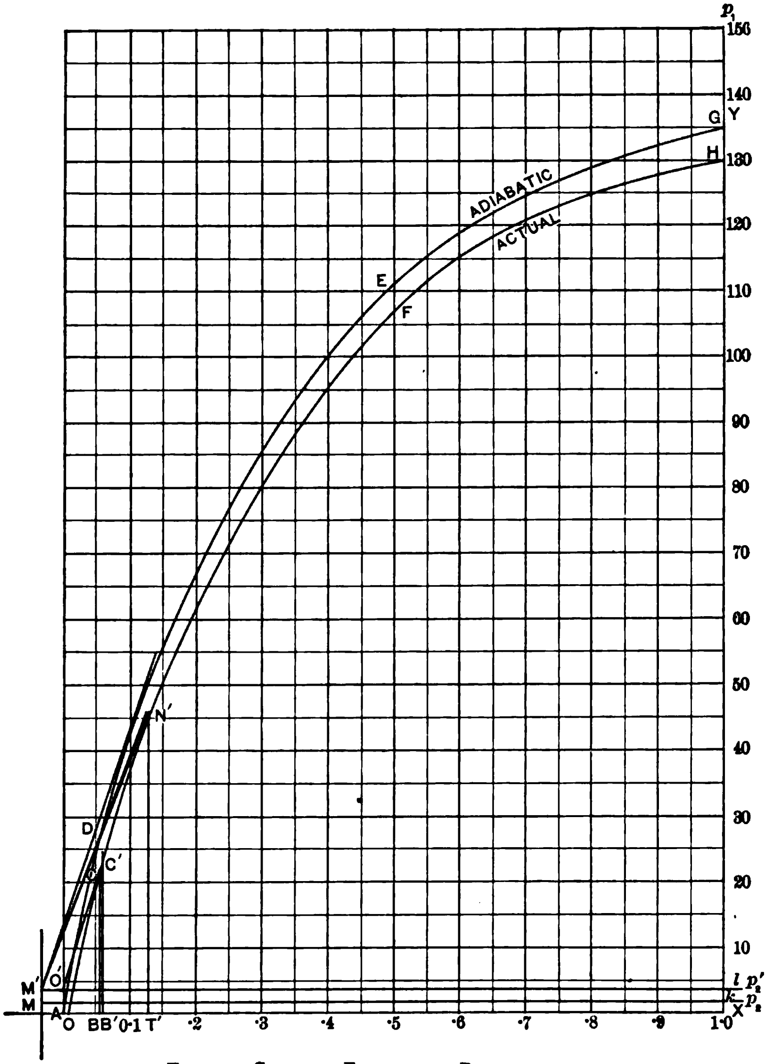

FIG. 200.—CURVE OF EFFICIENCY. PUMPING-ENGINE.

The "ideal curve," *E*, is for the case of truly adiabatic expansion.

To locate point C on the "actual curve," F: Lay off $AB = .059$, and make $BC = 21.93$, which is the M.E.P. $+ p_r$. Make the vertical distances $CD = EF = GH = IJ = KL$, etc., etc. This assumes that it will be practically correct to adopt the assumption of Smith and of Buel, of sensibly constant condensation within the range of expansion considered.

Distance CD is 0.187 of BC; i.e., 18.7 per cent. of the steam is condensed under the conditions of the actual test. As already seen, if there were no condensation, the cylinder would be filled to .059 of its capacity; but 18.7 per cent. of the entering steam is condensed and, in the actual case, the cylinder is filled to only 81.3 per cent of 0.059 = 0.04796 of its capacity, or, in other words, the real cut-off is at 0.04796.

Draw the tangent OC'. This gives $AB' = 0.06$; which corresponds to 0.059 in the above case.

Then the cut-off should be 0.813 of 0.06 = 0.04878.

Point of cut-off, actual test...........	0.04796
" " " as per diagram.......	0.04878

This indicates that the curve must be sensibly correct for the portion employed.

Next for maximum commercial efficiency, lay off $OM = 0.025$ and draw the tangent MN'. This gives $AT' = 0.13$. The condensation at this point is

$$\frac{Q'N'}{N'T'} = \frac{4.1}{4.3} = 0.095,$$

$$1.00 - .095 = 0.905 \quad \text{and} \quad 0.13 \times .905 = 0.11765,$$

the required cut-off.

This corresponds to a ratio of expansion of less than nine.

The abscissas of the diagram measure, primarily, the weights and volumes of steam drawn from the boiler in a stated time, per cubic foot of cylinder. They measure, obviously, with equal exactness, the cost of that steam, per cubic foot of engine supplied, in the ideal case, during the assumed period, as in an hour or a year, or, if for any reason preferable, one stroke. The engine-costs may as obviously

[*To face page* 903.]

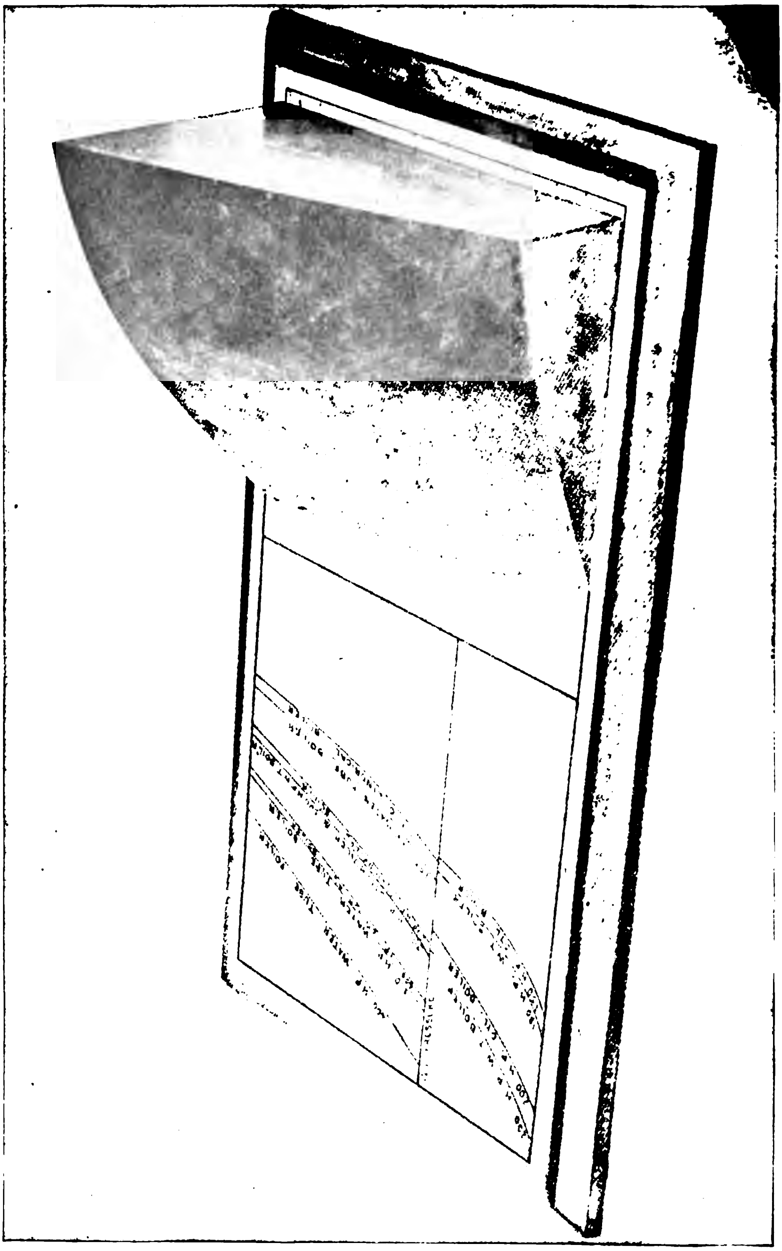

FIG. 200a.—MODEL GROUPING THE EFFICIENCY CURVES. (TRANS. A. S. M. E., 1898, THURSTON.)

be measured in *quantity of steam of equivalent value*, and measured off at the left of the axis of Y. In fact, this is what is always done; and the horizontal measures thus become those of both quantity and value, cubic feet and dollars and cents. It should be particularly observed that the horizontal scale of the diagram is not, except in the ideal case, one *both* of ratios of expansion and quantities of steam, but is primarily and accurately, in all cases, a scale of quantities of steam simply; on which the ratios of expansion, in the real case, must be identified and set off after ascertaining the magnitudes of those ratios coincident with stated quantities of steam taken into the engine-cylinder. This being done, it is easy to lay off, under the scale of steam-volumes and weights, the location of each enumerated ratio of expansion for the actual case.

It is further evident that, since the real engine takes into its cylinder, in all cases, more steam, at any stated ratio of expansion, than does the ideal, the ratios of expansion corresponding to any given quantity of steam entering the engine will be nearer the axis of Y than in the ideal case, and the scale of ratios of expansion will thus be a graded one; the figures nearly coinciding, in most cases, at full stroke, and departing further and further with shorter cut-offs.

Fig. 200*a* shows a model which groups a series of efficiency curves, with common back pressures and varying initial pressures.

Let the *net* work obtainable from unit *value* of heat, steam, or fuel be called X; then we shall have for the ideal case, assuming hyperbolic expansion and zero back-pressure,

$$X = a(1 + \log_e r),$$

where a is a constant dependent upon price, and measuring the cost of work performed without expansion, i.e., with $r = 1$. The maximum return will be secured when X is a maximum, r being the independent variable. Thus:

$$\frac{dX}{dr} = 0 = \frac{a}{r}; \quad r = \text{infinity}.$$

When the cost of wastes due to negative work, performed against a stated back-pressure, are known and are measured by a fraction, $\frac{p_b}{p_1}$ of the costs of work without expansion or back-pressure, as above:

$$X = a(1 + \log_e r) - \frac{arp_b}{p_1},$$

$$\frac{dX}{dr} = 0 = \frac{a}{r} - a\frac{p_b}{p_1}; \quad r_b = \frac{p_1}{p_b},$$

and expansion should continue until the terminal pressure coincides with the back-pressure.

When the costs of wasted energy include those of friction of engine, making the fraction lost

$$\frac{(p_b + p_f)a}{p_1},$$

the same process indicates a ratio of expansion for maximum returns:

$$r_f = p_1 \div (p_b + p_f).$$

When the thermal wastes are added to the back-pressure and friction wastes and costs, we have, in precisely the same manner, representing their cost as a fraction, $\frac{p_k}{p_1}$, of the cost of our unit a for full stroke of engine,

$$r_k = p_1 \div (p_b + p_f + p_k);$$

and the maximum efficiency of the engine is attained, all wastes being taken into account, when this ratio is adopted.

We may go still further in certain cases. Where the costs of operation, outside those of supplying steam, may be taken for the range of doubtful expansion as a constant proportion

of cost of unit volume of steam, or approximately so, and measured by a fraction, $\frac{p_k}{p_1}$, of the expense of the unit a, the ratio of expansion which will give maximum returns on money expended will similarly be

$$r_k = p_1 \div (p_b + p_f + p_h + p_k).$$

In cases studied by the author, values have been found thus:

$$p_b = 0.02p_1 \text{ to } 0.04p_1;\ p_f = 0.02p_1 \text{ to } 0.04p_1,$$
$$p_h = 0.05p_1 \text{ to } 0.10p_1;\ p_k = 0.02p_1 \text{ to } 0.10p_1;$$

whence the values:

$$r_b = 50 \text{ to } 25;\ r_f = 25 \text{ to } 12.5;\ r_h = 11 \text{ to } 5.1;\ r_k = 9 \text{ to } 3.5.$$

which values fairly represent the results of common experience in such practice.*

This investigation leads once more to the conclusion that the best steam-distribution, under very usual conditions of operation, may be essentially different from that giving highest "duty." † In the present case, not only is this found to be true, but it is discovered that an engine of half the volume of low-pressure cylinder would prove better adapted to the place and purpose, presumably decreasing the duty from nearly 150,000,000 to about 100,000,000, increasing the weight of fuel and steam demanded per delivered horse-power and per hour from 1.4 to 2.2 and from 13 to 20 pounds. The further deduction follows that a compound engine, with this same steam-distribution shown on its "combined diagram," would prove a less expensive engine to build for the required power, and this, in turn, would permit some increase of the total ratio of expansion. A compound engine with a total ratio of

* Heat-wastes in Steam-engine Cylinders. R. H. Thurston; Trans. Brit. Inst. N. A., April 4, 1895; Journal Franklin Inst., Oct. 1896.

† These efficiency curves were obtained by Mr. C. L. Hubbard.

about 10 would thus prove a better investment in this case than any triple-expansion engine, all costs considered.*

Measured in electrical energy, the efficiencies attained by the best of modern engines are not far from the following: †

EFFICIENCY ATTAINED IN ELECTRIC-ENERGY DISTRIBUTION.

	Watt-hours per Lb. Coal.	Lbs. Coal per Kw. Hour.
Average of 86 stations	108	7.26
Maximum efficiency	237	4.22
Minimum efficiency	33	30.30
Steam-engine, best equiv. max	557	1.8
	St. per I.H.P. hr.	St. per D.H.P. hr.
Same, best record	11.22	13.4 lbs.

A *Minimum Cost*, including annual charge on account of prime costs and the regular continuous working costs, is always sought by the engineer. Numerous problems are of this character. Of these, that of the proportions of the steam-engine is one of the most interesting, important, and impressive. A minimum is to be in this case sought which must include either the sum of the first costs of the machine and its accessories, and the capitalized costs of its operation, or, if preferred or found more convenient, the sum of the interest on first costs and the running expenses, including maintenance and repair and, in many, probably in most, cases, a sinking fund for its replacement by a similar machine, or to cover the losses due to replacement by more modern machinery. The latter is the more usual method, and the annual charge is commonly summed in the manner indicated.

The two main divisions of the account are thus the expenditure made in the installation of the "plant" and the expenses, as an annual average, incurred in its maintenance, operation, and replacement. Of these two main charges, the wise expenditure of the items collected under the first head will usually result in the reduction of the costs in the second division; the unwise application of the funds available for

* See discussions by the Author; Trans. A. S. M. E., May 1897.

† Electrical Engineer, June 16, 1897.

purchase and establishment will commonly result in extravagant operating costs. That first cost is wisest which makes the sum of all annual charges for interest, maintenance, operation, and replacement the least possible under the circumstances existing at the time of purchase. Thus a certain initial expenditure may sometimes be specified which shall make the costs of power, reduced as above, a minimum, and this is the figure which, in the case of the steam-engine, usually determines the proportions of the machine.

Assuming the cost of the average engine, built by reputable constructors, to be about \$10 a horse-power, as rated, and taking the consumption of fuel as $3\frac{1}{8}$ pounds per H.P. per hour, the working time of a year as 3000 hours, and the price of the fuel as \$5 per "round" ton, the cost of fuel per horse-power per annum would be precisely equal to the first cost of the engine. At this rate, if current prices for money averaged five per cent., net, to the lender, the proprietor of the engine would be justified in paying double price for an engine which should reduce the cost of fuel by five per cent.; and any improvement on his engine which should produce a saving, without adding to annual running expenses, would give him a profit of proportional amount. As a percentage on the investment,

$$p = 100\frac{k}{K}s \text{ per cent.};$$

where K, k, and s, respectively, are the first cost of engine and that of the improvement, per horse-power, and where s is the saving effected, also stated in costs per horse-power, for the assumed time—as for one year.

Thus, assuming the values to be $K = \$10$, $k = \$2$, $s = \$0.50$, the investment in the improvement would pay

$$p = 100 \times 2 \times .5/10 = 10 \text{ per cent.}$$

Thus the saving by steam-jacketing, for example, may pay a very handsome premium, even though that saving be but a small percentage of the total cost of fuel.

The computation of the rate of payment in the aggregation of a sinking fund for the repayment of an invested sum or for the replacement of a "plant" of stated value, or for both repayment of the original investment and renewal of the plant, is made as follows:

The annual payments are assumed to be made in such amount as to permit, when each is placed at interest, immediately on its receipt, at the beginning of the fiscal year, of the aggregation of these successive instalments of principal and their continuous accumulations of interest, realizing at the termination of the prescribed term the full amount to be then employed as proposed, with, properly, some small excess to constitute an insurance against losses by unfortunate investments or occasional failure of interest.

The rate to be established, r, to produce the required sum, p, or the sum to be secured by a stated rate of interest thus compounded, is obtained by an annual payment, s, which is, for n years, given by the expression:

$$s = rp/[(100 + r)(1 + 0.0r)^n - 1] = rp/n;$$
$$r = s/mp.$$

Thus: a thirty-year annual payment, at three per cent interest, should be a sum equal to 0.02041 the sum to be realized in the end. At three and a half per cent interest, computed, the annual payment should be $0.01872p$ or at four per cent it should be $0.01715p$.

Thus the annual charge for accumulation of a stated sum for replacement of the principal in thirty years should be two per cent of that principal, and if for replacement, and also renewal of the plant representing that principal, four per cent.*

The finance of engineering is illustrated in many minute details of daily work and experience, as, for example, in the covering of steam-pipes. Thus: it is found that the heat lost from small steam-pipe amounts to, in some cases investigated

* Trans. A. S. C. E., 1897; Kuichlin on Water-works.

in Sibley College, about 3 B.T.U. per hour, per square foot, per degree difference of temperature between the pipe and the surrounding air. When protected by a good covering, the loss should be reduced at least to one fifth that quantity.

Thus taking 200 feet of 2-inch pipe with steam at 80 pounds pressure, by gauge, and a difference of 250° Fahr. between pipe and air, and the loss, as found by experiment, about 3.15 B.T.U.; then the saving will be the difference between this figure and 0.625, that found for well-covered pipe of the same size and length. This amounts to 2.525 B.T.U. as the unit measure of saving by the covering. The total saving, assuming 3000 hours' use, on the 62.2 feet of surface of the pipe taken in this case, is, in round numbers, 120,000,000 B.T.U. per annum. If the boiler absorb 10,000 B.T.U. per pound of good coal burned in its furnaces, the fuel thus saved will be 12,000 pounds a year, or six tons. This, at $3 per ton, would be $18; while the cost of the covering should not exceed $25; and the investment pays over seventy per cent., exclusive of depreciation. A life of ten years should be fairly expected of any good covering, properly cared for, and this makes the investment, including allowance for depreciation and repair, return probably not less than 60 per cent.

Economy of operating expenses has been gained, by the simple change of pattern of a ship's screw, to the extent, as reported, of saving the expense of the change in a year, and from $50,000 to $75,000 in the life of a vessel of 3000 to 5000 tons, and $100,000 and upward in a ship of 10,000 to 15,000 tons, displacement. This has been effected by the reduction of thickness of blade, mainly, and partly by the increased smoothness, attainable by the adoption of manganese or other bronze in place of cast iron; the power required for propelling the vessel being thus lessened and cost of fuel thus saved, with incidental gains of minor amount.

Wastes of heat, steam, and fuel in engine and boiler-rooms occur, often, to an enormous extent through channels either unsuspected or of unimagined magnitude, and the

proper adjustment of engines and boilers to their work may prove to be of even less consequence than the remedy of such defects. Leakage often amounts to a considerable quantity; condensation in steam-pipes and engines may be as much, and each square foot of unprotected surface of steam-pipes and steam-cylinder may condense a large fraction of a pound of steam each hour, wasting a horse-power through every fifteen to thirty square feet.

Auxiliary engines and distributing transmissions are exceedingly wasteful of energy in its various forms, and not infrequently, as elsewhere shown, constitute a serious tax. Heating is sometimes performed in a very expensive manner by sacrificing power at the engine, raising the back-pressure greatly in the endeavor to utilize exhaust steam. Standing engines and boilers, and the wastes of steam in starting and in preparations for starting, often cause losses that produce observable effects upon net earnings.

The study of the development of the modern steam-engine evidently leads to the following definite deductions:

(1) Economy is gained, other things equal, by increasing steam-pressures and ratios of expansion.

(2) Economy is gained, at any stated pressure, by expedients, such as compounding, steam-jacketing, and superheating, which decrease the extra-thermodynamic, and especially the internal, wastes.

(3) Economy decreases, with departure from the steam-adjustment for maximum thermal—not thermodynamic—efficiency, more rapidly with decreasing than with increasing loads, and an overloaded engine is better than an underloaded machine for this reason, as well as because of decreased cost; the adjustment having been primarily adopted to obtain maximum "duty."

(4) This deduction is modified, when the engine is properly adjusted for maximum financial efficiency, when the selected load is considerably above that load. and usually that rated load, at which maximum "duty" is attained. In this case underloading causes, between the limits of the latter case

and the former, less loss than overloading. Outside these limits the preceding conclusion holds.

(5) The most efficient engines vary less with departure from the properly rated load than uneconomical machines.

(6) In all cases best results are secured when the load is made as nearly as practicable uniform, and adjusted in such manner that the engine shall work at its best adjustment.

The Load-factor is that fraction which measures the ratio of the actual mean load on the source of power, during any specified time, to the full load for which the motor or motors may have been designed. There are several such factors: *The Station Load-factor* is that which measures the ratio of work performed to that which the station is competent to do at its best, and presumably rated, load. *The Dynamo Load-factor* is the ratio, similarly, of the actual mean load on the dynamo, for the stated time, to its rated and best load. *The Engine Load-factor* is the ratio of the work performed by the engine to its rated load. In the latter case, either overload or underload is productive of loss of efficiency at the engine; although it may prove that, at the same time, the deduction would not be true for the station as a whole, and an overloaded and consequently wasteful engine may be found when the station is doing its best work, commercially; in which case the conclusion is, simply, that the engine is not properly adjusted to the work. Its proper adjustment is that which insures most economical power-supply, commercially considered, when the station is, as a whole, doing its best work. *The Plant Load-factor* is the ratio of the actual mean delivery of the plant to its rated and best delivery.

The Factor of Idleness is the difference between the plant-factor and unity, i.e., the ratio of the deficiency of load to full load. It measures the possible, but unutilized, earning power of the station, or of the apparatus to which the factor attaches. In designing power-plants, the engineer seeks to reduce this factor to its lowest figure, and, further, to make the period of idleness of plant, of engine, or of dynamo productive of waste or loss in minimum degree. For example,

the losses and actual wastes differ greatly in the two cases, in one of which one large engine is idle half the time and in the other of which, of two smaller machines, one is idle nearly all the time, or whether one engine is worked half the time at full power or two engines are each worked one half the time at full power, the sizes of engines being suitably adjusted to the one or the other condition, or, again, whether one engine is worked all the time at half power or half the time at full power. The efficiency of the machine varies with every variation in the demand for power, and efficiencies also vary with every variation in the size and rated power of the engine.

The *Factor of Underloading* measures the ratio of the difference between the loads actually carried, and those for which the machine was originally designed, to the latter quantity.

The closer the approximation of all these factors, the last excepted, to unity, and the nearer the last approximates to zero, the better the performance of the plant; it being understood that all efficiencies are reckoned from a commercial standpoint. Station load-factors, in electric plants, may fall as low as 10 or 15 per cent., or they may rise to 50 per cent or more, accordingly as the loads are varied in character and quantity. Increase of load-factor gives almost a proportional increase in economy.

In the Regulation of the Motor and its accessories with a view to insuring steadiness of operation, in many cases—as in the power-supply of electric light and power stations—a serious problem arises from the fact that the demand for power may fluctuate enormously from hour to hour, often from minute to minute or even from second to second; while the maintenance of the motor at its rated and best delivery is a matter of supreme importance, in the endeavor to insure economy of power-supply. In such instances there is often interposed between the source of power and the irregular work some system of storage, as the "storage-battery" or "accumulator," which may receive any amount of energy, at any rate, while delivering it also at any practically desired rate.

Steady accession of energy is thus made compatible with irregular demand and supply on the side of the work. In this arrangement best effect will be secured when the following principle is applied:

In adjusting apparatus to permit steady supply of energy with a rate of supply equal to the mean for the whole period of action while affording any energy-flow to the driven apparatus, however the rate and quantity demanded may vary, the regulating or storage system should be placed as near the point at which the fluctuation of demand occurs as possible.

231. **Multiple-cylinder Engines Compared,** on a financial basis, with each other and with simple engines, once their curves of efficiency are determined, are easily made to exhibit the qualities which will best meet any stated requirements, either in competing for maximum "duty," as hitherto customary or to secure maximum financial results, to obtain highest economy of steam, of fuel, or of money employed.*

The comparison of simple with compound or triple-expansion engines, or other special constructions, the characteristics of each being known, may be easily effected by this construction, in such manner as to permit the selection of that design which will be most likely to prove of maximum efficiency, as seen from the financial point of view, thus:

(1) On the diagram, construct the ideal case, as usual; then determine, from the best available data, the curve of actual efficiency of each of the types of engine to be compared. They will be found to lie under the ideal curve, between the extreme limits constituted by that curve and the curve of efficiency for the most wasteful of engines; the curve for the simple engine lying nearest the latter, that for the triple-expansion nearest the former; the compound lying between the two.

(2) Compute the "engine-costs" for each, and make, for each, the construction determining the best steam-distribution for that case.

* Discussion; R. H. T. in Trans. A. S. M. E., May 1897.

(3) Compare the ratios of works to total costs; then that

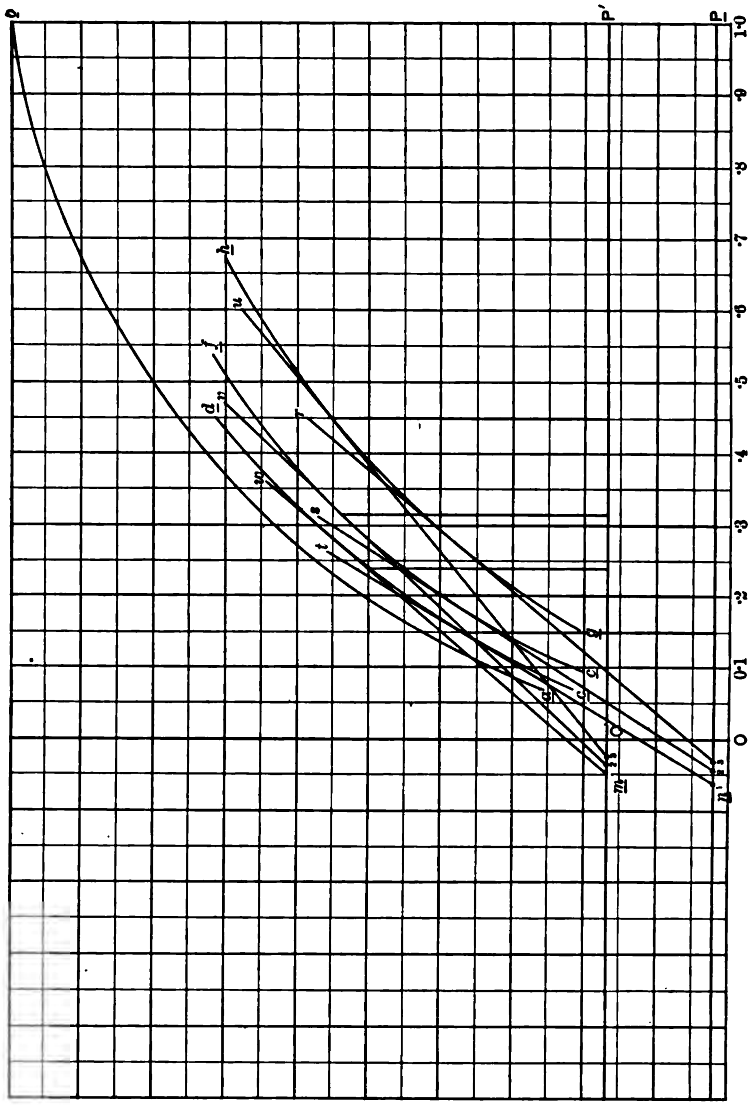

FIG. 201.—RELATIVE EFFICIENCIES (COMMERCIAL).

which gives the highest ratio is the best choice from the point of view of finance. Or, compare the angles made by each

tangent with the base-line; that which is found largest identifies the most suitable type of engine.

Thus, in Fig. 201, let *ab*, *cd*, *ef*, *gh*, respectively, represent the ideal and the actual efficiency-curves of three engines to be compared with a view to ascertaining which, on the whole, will best suit a stated requirement. Let OP, $O'P'$ measure the cost of the unit of steam, and m_1O', m_2O', m_3O', and n_1O, n_2O, n_3O, measure the relative "engine-costs" of the three, non-condensing or condensing as the case may be. Then the tangent-lines as identified by r, s, t, or by u, v, w, will, by their angles with the base, OP, measure the ratios of net work to cost, and that which rises most rapidly will identify the engine of maximum commercial efficiency and financial value.

Mr. Willans found, in his trials of his non-condensing high-speed engines, that, as a matter of maximum fuel-economy, with small powers at least, the compound engine became desirable at as low as 80 pounds absolute pressure, and that the triple-expansion engine became economically available at double that pressure. With slow-working engines, the multiple-expansion types become economically desirable at lower pressures, and every condition exaggerating the internal wastes renders this modification of the machine more advantageous.

On the other hand, in states of the market often illustrated, the total cost of engine and boiler is absolutely less with the compound than with the simple engine, and lower first cost and less cost of operation combine to dictate the adoption of the multiple-cylinder type of engine.

As already seen, three points of view lead to three lines of study of gain in the use of these engines:

(1) To minimize wastes and make total efficiency a maximum.

(2) To equalize work and action (torsion - moments, torques) on the crank-shaft.

(3) To equalize stresses on the running parts as a matter

of economy in first costs and in number of spare parts for the several engines.

The first is related to a correct philosophy of the steam-engine; the others relate to machine-design and the proportioning of details and computations of stresses, and of strengths of parts and of materials.

All these points of view must be assumed and the financial aspect of the case studied from each, and as modifying the proportions of the machine and its uses in all ways.

232. **Possible Improvements** in the thermodynamic action of the steam-engine and its class of ideal heat-engines may occur in either of three principal ways:

(1) Improvements may be effected by change of type-cycle.

(2) Improvements may occur through changes in the form of the cycle, without altering its distinctive characteristics.

(3) Improvement may possibly be produced by change of working-fluid.

In illustration: Should it prove practicable to modify the operation of the steam-engine in such manner as, actually or in effect, to transform the second of the Rankine cycles—as produced in the regular working of the machine, through the use of the steam-jacket upon a metallic cylinder—into the first, that involving the use of a non-conducting cylinder, this change of cycle would usually, even in the thermodynamic case, effect a gain of above 10 per cent. Could still further improvement be made, by converting the cycle, by change of construction of the machine, into that of Carnot, a further gain of 5 to 10 per cent would be produced; these gains increasing with increasing ranges of pressure and temperature.

Changes of the cycle itself may be of advantage when they involve increased temperature- and pressure-range; when they insure more nearly adiabatic expansion; when they produce variations in the direction of approximating better forms of cycle, as when compression, in the common cycle, is made to approximate the action of that part of the Carnot cycle, or when purely "cushion-steam" is reduced in quantity by reduction of clearance and "dead spaces."

Comparing the indicator-diagrams of even the best engines with those of their representative thermodynamic cycles, improvements of this class are seen to be possible in the elevation of the admission-line to more nearly full boiler-pressure, with sharp transfer from parallelism with the initial pressure-line, at the point of cut-off, to a more truly adiabatic expansion-line; in the sharpening of the corners at the end of expansion and at commencement of exhaust; in lowering the back-pressure line and in improvement in the direction of adiabatic compression toward initial pressure. Such changes in the average engine of the day would improve its efficiency by many per cent. Increasing steam-pressures, ratios of expansion, and speeds of engine, with improved forms of valve-gear, are methods of promotion of these improvements; as are all expedients looking to improved non-conductivity of the cylinder-wall, or to exclusion of precipitated water from the engine-cylinder.

Changes in the working fluid may be effected, with steam, for example, by superheating; by introducing an admixture of air, in some cases with the non-condensing engine, in order to reduce conductivity and initial condensation; also by mingling with it mineral oils or thin vapors for the same purpose; and in minor, but useful, degree, by the use of separators, insuring dryness, and, in multiple-cylinder engines, of intermediate reservoirs, to improve the quality of the steam as it passes through the engine.

Changes of working fluid by substitution of ether, carbon-disulphide, petroleum vapor, or other, for steam, have been seen to give in some cases special opportunity for securing increased efficiency by adapting the working substance to the desirable or practicable working range of either temperature or pressure; while these advantages must be weighed against the usually unquestionably serious, sometimes obviously fatal, practical objections to their use, on the score of cost or of danger, or both.

Of these three distinctive methods of improvement of the heat-engine, the first is dependent largely upon invention for

its possible success; the second involves simply improved design or better methods of working of the existing machine; the third, at the moment, mainly calls for experimental research, looking to the complete determination of the practically best methods of working, and the real thermodynamic and commercial value of the several vapors available for use as working fluids. The type of cycle is determined by the construction of the engine; the perfection of the cycle follows upon improvement of the valve-gear and of steam-distribution; the selection of a working fluid, from among the several which are possibly available, involves the thermodynamic and experimental investigation of their economical operation, and the adaptation of the general proportions of the engine, in each case, to the particular fluid employed. Since, in the real engine, the readiness with which the fluid transfers heat to and from the cylinder-wall is known to be an element of extraordinary importance in the determination of its final, net, efficiency, it follows that extra-thermodynamic considerations may possibly ultimately determine the choice of the working fluid, irrespective of the thermodynamic advantage or disadvantage of its use. Oily vapors certainly, probably ammonia, and very possibly other fluids, may prove to be superior to steam in this last respect; and the economy shown in experimental work on these vapor-engines has been such, in some cases, as to indicate a possibility that they may even ultimately prove, under special conditions, desirable forms of heat-engine.

Multiple-fluid Engines, of which the "binary-vapor engine" is an example, are constructed with the object of extending the thermodynamic limits of adiabatic expansion and compression, and the intervening range of temperature, beyond those which are met with in a heat-engine in which a single working fluid is employed. The upper thermal limit is practically limited, in steam, by the impracticability of safely controlling the accompanying high-pressure, and, in fluids like air, by the difficulties of controlling temperature at perfectly controllable pressures. The upper limit of available

pressure does not, in any known fluid, coincide with the higher limit of safe and available temperature. To secure the high values, thermodynamically and dynamically, of the measure of maximum efficiency, $(T_1 - T_2)/T_1$, the range, $T_1 - T_2$, must be carried as high, and as low, as the environment of the machine will permit. This may be accomplished, in some cases, by the adoption of a working fluid which may be employed at the higher limit, and of another which may be used at the lower limit, rejecting the unutilized heat of the one into the other, in such manner as to produce cycles in series, of which the adjacent thermal lines shall be coincident. Du Trembley's steam-and-ether engine is an example of this arrangement, and Ellis' machine of the same class—in which an ordinary non-condensing steam-engine is supplemented by an ammonia-engine operated by the heat of the exhaust-steam —is another. It is conceivable that an indefinite number of such transfers, with a corresponding number of working fluids in series, might be employed, could suitable fluids be found.

In such engines, the construction and arrangement of the working cylinders is the same, in general purpose and arrangement, as in a multiple-cylinder steam-engine of the compound or triple-expansion type; except that in the latter a single working fluid is employed, with cycles in series, transferring the heat, and its vehicle, the fluid itself, from cylinder to cylinder between the higher and lower limits; while, in the multiple-fluid engine, heat only is dropped, step by step, transferring it from one fluid to another, cylinder by cylinder. Thermodynamically, the multiple-cylinder and the multiple-fluid engines are identical. Since the gain of one degree in the range of temperature worked through has more value at the lower than at the higher limit, the introduction, at the lower limit, of working-fluids more volatile than steam, to utilize the rejected heat of the exhaust, now wasted in the atmosphere or the condenser, has especial interest for the engineer. It is further to be noted that, in the multiple-fluid engine, each cycle is necessarily isolated, and its working fluid continuously circulated. Surface condensation thus becomes

an essential feature, and each condenser is a recipient of heat from the working cylinder next above in the series, and a source of heat for the element next in order. Each working fluid constitutes the source of heat-supply for its own cycle and the refrigerating fluid for that next above. Further, with working fluids thus placed in series, the cycle in each cylinder may be made a complete Carnot cycle. This is not true of the multiple-cylinder engine with a single working-fluid, in which series the Carnot cycle is constructed, if it can be produced at all, by the combination of all the cycles of the whole series of cylinders, including boiler and condenser.

A Multiple-vapor Series should evidently consist of fluids of which each will have a boiling-point under a good and manageable pressure, at or below the temperature of the condensation of the fluid next above it in series, and a good mean effective working pressure between that point and the temperature and pressure of its own condensation. The latter temperature should be that of the high-pressure boiling-point of the next lower substance in the series, or above it. The uppermost element of the series should have a manageable pressure at the highest practicable temperature of vaporization. The lowest in the series should have a condensing point, or, boiling-point, somewhat above the temperature of the condensing water, and at that point its vapor should have the least possible pressure. If choice is further allowed, under the circumstances practically controlling the case, the series should have a fairly uniform distribution of the series of pressure-ranges, in order that a division of work as nearly as possible uniform among the cylinders in series may give the desired work-distribution on the crank-pins of the engine.

The qualities demanded, aside from this desirable adjustment of pressures and temperatures of vaporization and condensation, are safety in use, low cost of supply of the volume needed to charge the engine at the start, and the highest possible specific and latent heats. The latter requisite has no important bearing, as has been shown elsewhere, on the

efficiency of the fluid or of the series, but simply determines how much of the fluid is required in the circulation and per unit of time to perform a stated amount of work. The quantity of heat and of fuel depends upon the efficiency of the thermodynamic machine, and is thus independent of the magnitudes of specific and latent heats, except as slightly affected by the form of cycle in other than that of maximum efficiency. The density, also, of the fluid is a matter of no importance in this connection. It varies, in fact, about as the reciprocal of the latent heat of vaporization, and the quantity of any given fluid required in a given cycle to do a stated amount of work varies inversely as its latent heat, i.e., directly as its density at the initial pressure. The higher the density and the lower the latent heat of the vapor, also, the larger must be the working cylinder performing a stated quantity of work, as well as the greater the weight of working fluid employed per unit of time. In the practical use of vapor, any condensibility, its power of exchanging heat with the cylinder-walls, may prove an important element; and the indications seem to be that, in some cases, there exist differences in this respect that may determine a choice between two available vapors that might, on other grounds, take the reverse relation of desirability.

As remarked by Bertrand: There are two opposing influences continually at work in the cylinder of the steam-engine during its working stroke. The expansion of the steam produces cooling; this cooling causes thermodynamic condensation. On the other hand, the expansion permits the stated weight of steam to remain in the state of saturation at a lower temperature than otherwise. It is thus always a question which of these contrary influences will prove the greater. Similarly, in compression, the same opposing influences are observed. The actual result is, in the case of steam, that the performance of work by expansion produces condensation, as with most other of the more familiar vapors available for working fluids in heat-engines; while in others, ether for example, the effect observed at usual pressures is the superheating of

the vapor. The "apparent" specific heat of saturated vapor, in the one case, is negative, in the other positive. The *real* specific heat is, obviously, in all cases positive. This condensation of steam, and superheating of ether vapor, produces a decrease in the quantity of work performed in the one case, and an increase in the other, by giving, at each step in the expansion, a lower or a higher pressure for the given volume.

The Steam Turbine is not subject to such internal wastes as are consequent upon the fluctuations of temperature of the cylinder-wall of the reciprocating engine, all its internal surfaces having constant temperature while in steady working. Nor does the quality of steam or the degree of superheat affect thermodynamic efficiency within the same range of temperature; although the latter has been known by the author greatly to increase the power of the turbine, presumably by permitting delivery of the steam at larger velocity and in greater quantity. The accompanying illustration exhibits the variation of efficiency of the Sibley College "steam-turbine set," engine and dynamo, at varying loads as modified by other conditions.

These are excellent results, and it is not to be expected that the future of this form of motor can promise gain except as steam pressures and temperatures continue to rise, as speeds are reduced by "compounding," and as fluid and solid frictions on journals and on wheel and casing can be minimized. The machine cannot be simplified, and there are no such opportunities for economizing heat now wasted as appear in the reciprocating engine.

Since this engine in steady working is not subject to wastes by "çylinder condensation," it is evident that the large range, still existing in its best examples, between the real and the ideal case, and even between the best steam-turbines and the best reciprocating engines of similar powers and under the same pressures, must be due to non-compliance with the conditions of maximum thermodynamic efficiency and to friction, fluid or solid, leakage, or both, and must be

reduced by more perfect and complete expansion, by reduction of wastes of energy in the stream of fluid, and by lessened

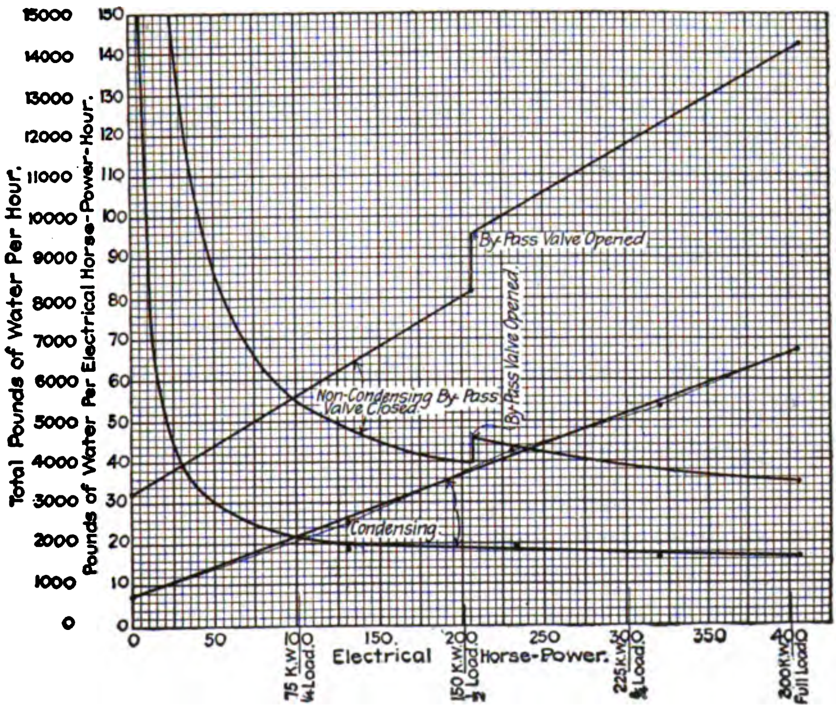

FIG. 202.—EFFICIENCY CURVES OF A STEAM-TURBINE SET.

friction in passages, in bearings, and on the exterior of the whirling disk.

The best results reported to date for the steam-turbine are the following:

STEAM-TURBINE EFFICIENCIES.

Steam Admission Pressure in Front of the Jet-nozzles in Lbs. per Sq. In.	Temperature of Steam at the Admission Valve. Fahr.	Vacuum at Exhaust of Turbine in Inches Mercury.	Number of Jet-nozzles Open.	Number of Revolutions per Minute of the Driving Shafts.	Brake Horse-power.	Steam Consumption per Hour and Brake Horse-power. Pounds.
192.7	453.7°	27.28	7	772	307.8	13.96
196.3	437°	27.64	6	762	259.0	14.46
196.3	440.6°	27.56	5	767	219.9	14.20
196.3	437°	27.64	4	775	175.0	14.29
190.6	426.2°	27.83	3	777	123.3	14.73
196.3	390.2°	28.07	2	775	75.2	17.02
213.4	388.4°	28.54	1	773	31.9	21.30

Thermodynamically, the turbines are equivalent to piston-engines without compression. A mass of steam is pushed

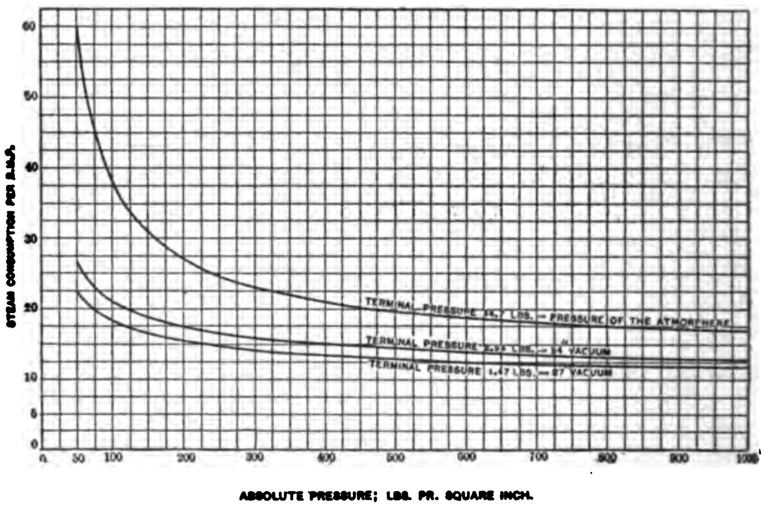

FIG. 203.—STEAM CONSUMPTION, MINIMUM, STEAM-TURBINE.

from the boiler into the engine, work being performed by isothermal expansion. Stored heat-energy is then transformed by free expansion into kinetic energy of similar amount to that expended by adiabatic expansion behind a

piston. This energy is taken up and utilized by the engine of either class.

The preceding diagram exhibits the variation of the efficiency of the steam-turbine with varying steam-pressure, as reported by LaVal, with minimum waste:

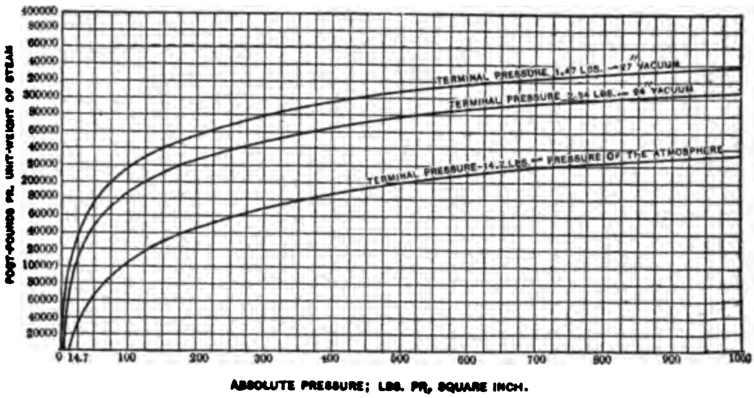

FIG. 204.—ENERGY OF JET OF STEAM.

Were the total energy of expansion converted into kinetic energy of the jet the quantity stored in the process would vary with pressure, as shown in the last diagram. The difference between the actual energy-utilization and this maximum must be attributed to losses by leakage and by solid and fluid friction acting on the exterior of the wheel-disk, and also in retardation of the jet in its distortion, or to waste due to improper design and construction, resulting in incomplete transfer of the energy of the jet into the wheel.

The merits of the turbine system applied to marine propulsion especially appear to be: (1) Greatly increased speed, owing to diminution of weight and smaller steam consumption; (2) increased carrying power of vessel: (3) increased economy in coal consumption; (4) increased facilities for navigating shallow waters; (5) increased stability of vessel; (6) reduced weight of machinery; (7) reduced cost of attendance on machinery; (8) reduced size and weight of screw propellers and shafting; (9) absence of vibration; (10)

lowered centre of gravity of machinery, and reduced risk in time of war.

The Twentieth Century Record began with the production of one horse-power by combustion of 0.97 pound of fuel, on the S. S. *Inchmarlo*, with a five-cylinder quadruple-expansion engine of 1600 I. H. P., using steam superheated to 470° F. and feed-water and air entering furnace heated and cylinders steam-jacketed. The combined diagram is here given (Fig. 205).

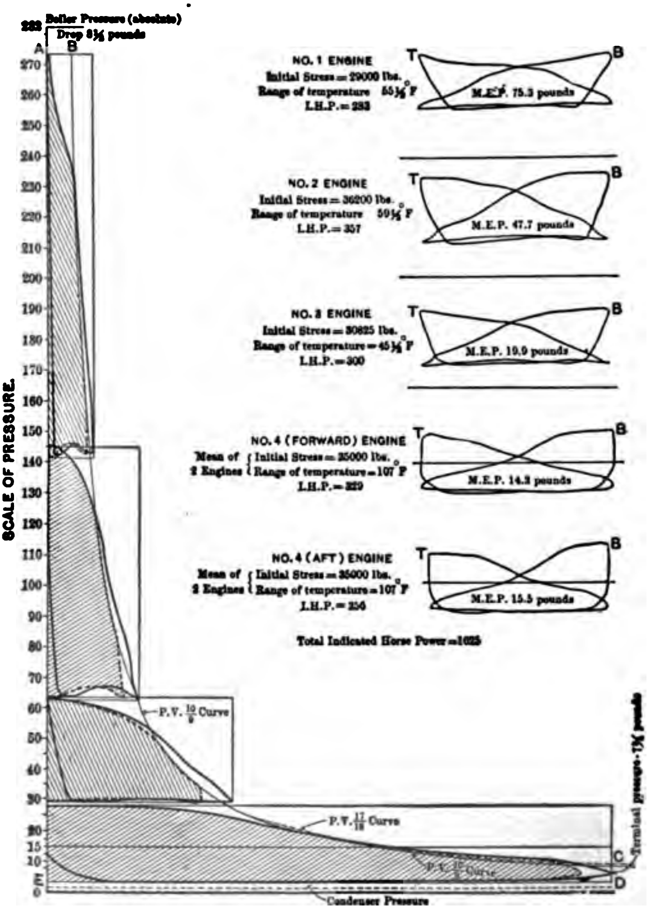

FIG. 205.—INCHMARLO DIAGRAM.

APPENDIX A.

TABLES.

I.

NUMERICAL CONSTANTS.

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
1.0	3.142	0.7854	1.000	1.000	1.0000	1.0000
1.1	3.456	0.9503	1.210	1.331	1.0488	1.0323
1.2	3.770	1.1310	1.440	1.728	1.0955	1.0627
1.3	4.084	1.3273	1.690	2.197	1.1402	1.0914
1.4	4.398	1.5394	1.960	2.744	1.1832	1.1187
1.5	4.712	1.7672	2.250	3.375	1.2247	1.1447
1.6	5.027	2.0106	2.560	4.096	1.2649	1.1696
1.7	5.341	2.2698	2.890	4.913	1.3038	1.1935
1.8	5.655	2.5447	3.240	5.832	1.3416	1.2164
1.9	5.969	2.8353	3.610	6.859	1.3784	1.2386
2.0	6.283	3.1416	4.000	8.000	1.4142	1.2599
2.1	6.597	3.4636	4.410	9.261	1.4491	1.2806
2.2	6.912	3.8013	4.840	10.648	1.4832	1.3006
2.3	7.226	4.1548	5.290	12.167	1.5166	1.3200
2.4	7.540	4.5239	5.760	13.824	1.5492	1.3389
2.5	7.854	4.9087	6.250	15.625	1.5811	1.3572
2.6	8.168	5.3093	6.760	17.576	1.6125	1.3751
2.7	8.482	5.7256	7.290	19.683	1.6432	1.3925
2.8	8.797	6.1575	7.840	21.952	1.6733	1.4095
2.9	9.111	6.6052	8.410	24.389	1.7029	1.4260
3.0	9.425	7.0686	9.00	27.000	1.7321	1.4422
3.1	9.739	7.5477	9.61	29.791	1.7607	1.4581
3.2	10.053	8.0425	10.24	32.768	1.7889	1.4736
3.3	10.367	8.5530	10.89	35.937	1.8166	1.4888
3.4	10.681	9.0792	11.56	39.304	1.8439	1.5037
3.5	10.996	9.6211	12.25	42.875	1.8708	1.5183
3.6	11.310	10.179	12.96	46.656	1.8974	1.5326
3.7	11.624	10.752	13.69	50.653	1.9235	1.5467
3.8	11.938	11.341	14.44	54.872	1.9494	1.5605
3.9	12.252	11.946	15.21	59.319	1.9748	1.5741
4.0	12.566	12.566	16.00	64.000	2.0000	1.5874
4.1	12.881	13.203	16.81	68.921	2.0249	1.6005
4.2	13.195	13.854	17.64	74.088	2.0494	1.6134
4.3	13.509	14.522	18.49	79.507	2.0736	1.6261
4.4	13.823	15.205	19.36	85.184	2.0976	1.6386
4.5	14.137	15.904	20.25	91.125	2.1213	1.6510
4.6	14.451	16.619	21.16	97.336	2.1448	1.6631
4.7	14.765	17.349	22.09	103.823	2.1680	1.6751

CONSTANTS—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
4.8	15.080	18.096	23.04	110.592	2.1909	1.6869
4.9	15.394	18.857	24.01	117.649	2.2136	1.6985
5.0	15.708	19.635	25.00	125.000	2.2361	1.7100
5.1	16.022	20.428	26.01	132.651	2.2583	1.7213
5.2	16.336	21.237	27.04	140.608	2.2804	1.7325
5.3	16.650	22.062	28.09	148.877	2.3022	1.7435
5.4	16.965	22.902	29.16	157.464	2.3238	1.7544
5.5	17.279	23.758	30.25	166.375	2.3452	1.7652
5.6	17.593	24.630	31.36	175.616	2.3664	1.7758
5.7	17.907	25.518	32.49	185.193	2.3875	1.7863
5.8	18.221	26.421	33.64	195.112	2.4083	1.7967
5.9	18.535	27.340	34.81	205.379	2.4290	1.8070
6.0	18.850	28.274	36.00	216.000	2.4495	1.8171
6.1	19.164	29.225	37.21	226.981	2.4698	1.8272
6.2	19.478	30.191	38.44	238.328	2.4900	1.8371
6.3	19.792	31.173	39.69	250.047	2.5100	1.8469
6.4	20.106	32.170	40.96	262.144	2.5298	1.8566
6.5	20.420	33.183	42.25	274.625	2.5495	1.8663
6.6	20.735	34.212	43.56	287.496	2.5691	1.8758
6.7	21.049	35.257	44.89	300.763	2.5884	1.8852
6.8	21.363	36.317	46.24	314.432	2.6077	1.8945
6.9	21.677	37.393	47.61	328.509	2.6268	1.9038
7.0	21.991	38.485	49.00	343.000	2.6458	1.9129
7.1	22.305	39.592	50.41	357.911	2.6646	1.9220
7.2	22.619	40.715	51.84	373.248	2.6833	1.9310
7.3	22.934	41.854	53.29	389.017	2.7019	1.9399
7.4	23.248	43.008	54.76	405.224	2.7203	1.9487
7.5	23.562	44.179	56.25	421.875	2.7386	1.9574
7.6	23.876	45.365	57.76	438.976	2.7568	1.9661
7.7	24.190	46.566	59.29	456.533	2.7749	1.9747
7.8	24.504	47.784	60.84	474.552	2.7929	1.9832
7.9	24.819	49.017	62.41	493.039	2.8107	1.9916
8.0	25.133	50.266	64.00	512.000	2.8284	2.0000
8.1	25.447	51.530	65.61	531.441	2.8461	2.0083
8.2	25.761	52.810	67.24	551.468	2.8636	2.0165
8.3	26.075	54.106	68.89	571.787	2.8810	2.0247
8.4	26.389	55.418	70.56	592.704	2.8983	2.0328
8.5	26.704	56.745	72.25	614.125	2.9155	2.0408
8.6	27.018	58.088	73.96	636.056	2.9326	2.0488
8.7	27.332	59.447	75.69	658.503	2.9496	2.0567
8.8	27.646	60.821	77.44	681.473	2.9665	2.0646
8.9	27.960	62.211	79.21	704.969	2.9833	2.0724

CONSTANTS—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
9.0	28.274	63.617	81.00	729.000	3.0000	2.0801
9.1	28.588	65.039	82.81	753.571	3.0166	2.0878
9.2	28.903	66.476	84.64	778.688	3.0332	2.0954
9.3	29.217	67.929	86.49	804.357	3.0496	2.1029
9.4	29.531	69.398	88.36	830.584	3.0659	2.1105
9.5	29.845	70.882	90.25	857.375	3.0822	2.1179
9.6	30.159	72.382	92.16	884.736	3.0984	2.1253
9.7	30.473	73.898	94.09	912.673	3.1145	2.1327
9.8	30.788	75.430	96.04	941.192	3.1305	2.1400
9.9	31.102	76.977	98.01	970.299	3.1464	2.1472
10.0	31.416	78.540	100.00	1000.000	3.1623	2.1544
10.1	31.730	80.119	102.01	1030.301	3.1780	2.1616
10.2	32.044	81.713	104.04	1061.208	3.1937	2.1687
10.3	32.358	83.323	106.09	1092.727	3.2094	2.1757
10.4	32.673	84.949	108.16	1124.863	3.2249	2.1828
10.5	32.987	86.590	110.25	1157.625	3.2404	2.1897
10.6	33.301	88.247	112.36	1191.016	3.2558	2.1967
10.7	33.615	89.920	114.49	1225.043	3.2711	2.2036
10.8	33.929	91.609	116.64	1259.712	3.2863	2.2104
10.9	34.243	93.313	118.81	1295.029	3.3015	2.2172
11.0	34.558	95.033	121.00	1331.000	3.3166	2.2239
11.1	34.872	96.769	123.21	1367.631	3.3317	2.2307
11.2	35.186	98.520	125.44	1404.928	3.3466	2.2374
11.3	35.500	100.29	127.69	1442.897	3.3615	2.2441
11.4	35.814	102.07	129.96	1481.544	3.3764	2.2506
11.5	36.128	103.87	132.25	1520.875	3.3912	2.2572
11.6	36.442	105.68	134.56	1560.896	3.4059	2.2637
11.7	36.757	107.51	136.89	1601.613	3.4205	2.2702
11.8	37.071	109.36	139.24	1643.032	3.4351	2.2766
11.9	37.385	111.22	141.61	1685.159	3.4496	2.2831
12.0	37.699	113.10	144.00	1728.000	3.4641	2.2894
12.1	38.013	114.99	146.41	1771.561	3.4785	2.2957
12.2	38.327	116.90	148.84	1815.848	3.4928	2.3021
12.3	38.642	118.82	151.29	1860.867	3.5071	2.3084
12.4	38.956	120.76	153.76	1906.624	3.5214	2.3146
12.5	39.270	122.72	156.25	1953.125	3.5355	2.3208
12.6	39.584	124.69	158.76	2000.376	3.5496	2.3270
12.7	39.898	126.68	161.29	2048.383	3.5637	2.3331
12.8	40.212	128.68	163.84	2097.152	3.5777	2.3392
12.9	40.527	130.70	166.41	2146.689	3.5917	2.3453
13.0	40.841	132.73	169.00	2197.000	3.6056	2.3513
13.1	41.155	134.78	171.61	2248.091	3.6194	2.3573
13.2	41.469	136.85	174.24	2299.968	3.6332	2.3633

Constants—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
13.3	41.783	138.93	176.89	2352.637	3.6469	2.3693
13.4	42.097	141.03	179.56	2406.104	3.6606	2.3752
13.5	42.412	143.14	182.25	2460.375	3.6742	2.3811
13.6	42.726	145.27	184.96	2515.456	3.6878	2.3870
13.7	43.040	147.41	187.69	2571.353	3.7013	2.3928
13.8	43.354	149.57	190.44	2628.072	3.7148	2.3986
13.9	43.668	151.75	193.21	2685.619	3.7283	2.4044
14.0	43.982	153.94	196.00	2744.000	3.7417	2.4101
14.1	44.296	156.15	198.81	2803.221	3.7550	2.4159
14.2	44.611	158.37	201.64	2863.288	3.7683	2.4216
14.3	44.925	160.61	204.49	2924.207	3.7815	2.4272
14.4	45.239	162.86	207.36	2985.984	3.7947	2.4329
14.5	45.553	165.13	210.25	3048.625	3.8079	2.4385
14.6	45.867	167.42	213.16	3112.136	3.8210	2.4441
14.7	46.181	169.72	216.09	3176.523	3.8341	2.4497
14.8	46.496	172.03	219.04	3241.792	3.8471	2.4552
14.9	46.810	174.37	222.01	3307.949	3.8600	2.4607
15.0	47.124	176.72	225.00	3375.000	3.8730	2.4662
15.1	47.438	179.08	228.01	3442.951	3.8859	2.4717
15.2	47.752	181.46	231.04	3511.808	3.8987	2.4772
15.3	48.066	183.85	234.09	3581.577	3.9115	2.4825
15.4	48.381	186.27	237.16	3652.264	3.9243	2.4879
15.5	48.695	188.69	240.25	3723.875	3.9370	2.4933
15.6	49.009	191.13	243.36	3796.416	3.9497	2.4986
15.7	49.323	193.59	246.49	3869.893	3.9623	2.5039
15.8	49.637	196.07	249.64	3944.312	3.9749	2.5092
15.9	49.951	198.56	252.81	4019.679	3.9875	2.5146
16.0	50.265	201.06	256.00	4096.000	4.0000	2.5198
16.1	50.580	203.58	259.21	4173.281	4.0125	2.5251
16.2	50.894	206.12	262.44	4251.528	4.0249	2.5303
16.3	51.208	208.67	265.69	4330.747	4.0373	2.5355
16.4	51.522	211.24	268.96	4410.944	4.0497	2.5406
16.5	51.836	213.83	272.25	4492.125	4.0620	2.5458
16.6	52.150	216.42	275.56	4574.296	4.0743	2.5509
16.7	52.465	219.04	278.89	4657.463	4.0866	2.5561
16.8	52.779	221.67	282.24	4741.632	4.0988	2.5612
16.9	53.093	224.32	285.61	4826.809	4.1110	2.5663
17.0	53.407	226.98	289.00	4913.000	4.1231	2.5713
17.1	53.721	229.66	292.41	5000.211	4.1352	2.5763
17.2	54.035	132.35	295.84	5088.448	4.1473	2.5813
17.3	54.350	235.06	299.29	5177.717	4.1593	2.5863
17.4	54.664	237.79	302.76	5268.024	4.1713	2.5913

CONSTANTS—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
17.5	54.978	240.53	306.25	5359.375	4.1833	2.5963
17.6	55.292	243.29	309.76	5451.776	4.1952	2.6012
17.7	55.606	246.06	313.29	5545.233	4.2071	2.6061
17.8	55.920	248.85	316.84	5639.752	4.2190	2.6109
17.9	56.235	251.65	320.41	5735.339	4.2308	2.6158
18.0	56.549	254.47	324.00	5832.000	4.2426	2.6207
18.1	56.863	257.30	327.61	5929.741	4.2544	2.6256
18.2	57.177	260.16	331.24	6028.568	4.2661	2.6304
18.3	57.491	263.02	334.89	6128.487	4.2778	2.6352
18.4	57.805	265.90	338.56	6229.504	4.2895	2.6401
18.5	58.119	268.80	342.25	6331.625	4.3012	2.6448
18.6	58.434	271.72	345.96	6434.856	4.3128	2.6495
18.7	58.748	274.65	349.69	6539.203	4.3243	2.6543
18.8	59.062	277.59	353.44	6644.672	4.3359	2.6590
18.9	59.376	280.55	357.21	6751.269	4.3474	2.6637
19.0	59.690	283.53	361.00	6859.000	4.3589	2.6684
19.1	60.004	286.52	364.81	6967.871	4.3703	2.6731
19.2	60.319	289.53	368.64	7077.888	4.3818	2.6777
19.3	60.633	292.55	372.49	7189.057	4.3932	2.6824
19.4	60.947	295.59	376.36	7301.384	4.4045	2.6869
19.5	61.261	298.65	380.25	7414.875	4.4159	2.6916
19.6	61.575	301.72	384.16	7529.536	4.4272	2.6962
19.7	61.889	304.81	388.09	7645.373	4.4385	2.7008
19.8	62.204	307.91	392.04	7762.392	4.4497	2.7053
19.9	62.518	311.03	396.01	7880.599	4.4609	2.7098
20.0	62.832	314.16	400.00	8000.000	4.4721	2.7144
20.1	63.146	317.31	404.01	8120.601	4.4833	2.7189
20.2	63.460	320.47	408.04	8242.408	4.4944	2.7234
20.3	63.774	323.66	412.09	8365.427	4.5055	2.7279
20.4	64.088	326.85	416.16	8489.664	4.5166	2.7324
20.5	64.403	330.06	420.25	8615.125	4.5277	2.7368
20.6	64.717	333.29	424.36	8741.816	4.5387	2.7413
20.7	65.031	336.54	428.49	8869.743	4.5497	2.7457
20.8	65.345	339.80	432.64	8989.912	4.5607	2.7502
20.9	05.659	343.07	436.81	9129.329	4.5716	2.7545
21.0	65.973	346.36	441.00	9261.000	4.5826	2.7589
21.1	66.288	349.67	445.21	9393.931	4.5935	2.7633
21.2	66.602	352.99	449.44	9528.128	4.6043	2.7676
21.3	66.916	356.33	453.69	9663.597	4.6152	2.7720
21.4	67.230	359.68	457.96	9800.344	4.6260	2.7763
21.5	67.544	363.05	462.25	9938.375	4.6368	2.7806
21.6	67.858	366.44	466.56	10077.696	4.6476	2.7849
21.7	68.173	369.84	470.89	10218.313	4.6583	2.7893

CONSTANTS—*Continued.*

n	πn	$n^2 \frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
21.8	68.487	373.25	475.24	10360.232	4.6690	2.7935
21.9	68.801	376.69	479.61	10503.459	4.6797	2.7978
22.0	69.115	380.13	484.00	10648.000	4.6904	2.8021
22.1	69.429	383.60	488.41	10793.861	4.7011	2.8063
22.2	69.743	387.08	492.84	10941.048	4.7117	2.8105
22.3	70.058	390.57	497.29	11089.567	4.7223	2.8147
22.4	70.372	394.08	501.76	11239.424	4.7329	2.8189
22.5	70.686	397.61	506.25	11390.625	4.7434	2.8231
22.6	71.000	401.15	510.76	11543.176	4.7539	2.8273
22.7	71.314	404.71	515.29	11697.083	4.7644	2.8314
22.8	71.268	408.28	519.84	11852.352	4.7749	2.8356
22.9	71.942	411.87	524.41	12008.989	4.7854	2.8397
23.0	72.257	415.48	529.00	12167.000	4.7958	2.8438
23.1	72.571	419.10	533.61	12326.391	4.8062	2.8479
23.2	72.885	422.73	538.24	12487.168	4.8166	2.8521
23.3	73.199	426.39	542.89	12649.337	4.8270	2.8562
23.4	73.513	430.05	547.56	12812.904	4.8373	2.8603
23.5	73.827	433.74	552.25	12977.875	4.8477	2.8643
23.6	74.142	437.44	556.96	13144.256	4.8580	2.8684
23.7	74.456	441.15	561.69	13312.053	4.8683	2.8724
23.8	74.770	444.88	566.44	13481.272	4.8785	2.8765
23.9	75.084	448.63	571.21	13651.919	4.8888	2.8805
24.0	75.398	452.39	576.00	13824.000	4.8990	2.8845
24.1	75.712	456.17	580.81	13997.521	4.9092	2.8885
24.2	76.027	459.96	585.64	14172.488	4.9193	2.8925
24.3	76.341	463.77	590.49	14348.907	4.9295	2.8965
24.4	76.655	467.60	595.36	14526.784	4.9396	2.9004
24.5	76.969	471.44	600.25	14706.125	4.9497	2.9044
24.6	77.283	475.29	605.16	14886.936	4.9598	2.9083
24.7	77.597	479.16	610.09	15069.223	4.9699	2.9123
24.8	77.911	483.05	615.04	15252.992	4.9799	2.9162
24.9	78.226	486.96	620.01	15438.249	4.9899	2.9201
25.0	78.540	490.87	625.00	15625.000	5.0000	2.9241
25.1	78.854	494.81	630.01	15813.251	5.0099	2.9279
25.2	79.168	498.76	635.04	16003.008	5.0199	2.9318
25.3	79.482	502.73	640.09	16194.277	5.0299	2.9356
25.4	79.796	506.71	645.16	16387.064	5.0398	2.9395
25.5	80.111	510.71	650.25	16581.375	5.0497	2.9434
25.6	80.425	514.72	655.36	16777.216	5.0596	2.9472
25.7	80.739	518.75	660.49	16974.593	5.0695	2.9510
25.8	81.053	522.79	665.64	17173.512	5.0793	2.9549
25.9	81.367	526.85	670.81	17373.979	5.0892	2.9586

CONSTANTS—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
26.0	81.681	530.93	676.00	17576.000	5.0990	2.9624
26.1	81.996	535.02	681.21	17779.581	5.1088	2.9662
26.2	82.310	539.13	686.44	17984.728	5.1185	2.9701
26.3	82.624	543.25	691.69	18191.447	5.1283	2.9738
26.4	82.938	547.39	696.96	18399.744	5.1380	2.9776
26.5	83.252	551.55	702.25	18609.625	5.1478	2.9814
26.6	83.566	555.72	707.56	18821.096	5.1575	2.9851
26.7	83.881	559.90	712.89	19034.163	5.1672	2.9888
26.8	84.195	564.10	718.24	19248.832	5.1768	2.9926
26.9	84.509	568.32	723.61	19465.109	5.1865	2.9963
27.0	84.823	572.56	729.00	19683.000	5.1962	3.0000
27.1	85.137	576.80	734.41	19902.511	5.2057	3.0037
27.2	85.451	581.07	739.84	20123.648	5.2153	3.0074
27.3	85.765	585.35	745.29	20346.417	5.2249	3.0111
27.4	86.080	589.65	750.76	20570.824	5.2345	3.0147
27.5	86.394	593.96	756.25	20796.875	5.2440	3.0184
27.6	86.708	598.29	761.76	21024.576	5.2535	3.0221
27.7	87.022	602.63	767.29	21253.933	5.2630	3.0257
27.8	87.336	606.99	772.84	21484.952	5.2725	3.0293
27.9	87.650	611.36	778.41	21717.639	5.2820	3.0330
28.0	87.965	615.75	784.00	21952.000	5.2915	3.0366
28.1	88.279	620.16	789.61	22188.041	5.3009	3.0402
28.2	88.593	624.58	795.24	2242:.768	5.3103	3.0438
28.3	88.907	629.02	800.89	22 65.187	5.3197	3.0474
28.4	89.221	633.47	806.56	22906.304	5.3291	3.0510
28.5	89.535	637.94	812.25	23149.125	5.3385	3.0546
28.6	89.850	642.42	817.96	23393.656	5.3478	3.0581
28.7	90.164	646.93	823.69	23639.903	5.3572	3.0617
28.8	90.478	651.44	829.44	23887.872	5.3665	3.0652
28.9	90.792	655.97	835.21	24137.569	5.3758	3.0688
29.0	91.106	660.52	841.00	24389.000	5.3852	3.0723
29.1	91.420	665.08	846.81	24642.171	5.3944	3.0758
29.2	91.735	669.66	852.64	24897.088	5.4037	3.0794
29.3	92.049	674.26	858.49	25153.757	5.4129	3.0829
29.4	92.363	678.87	864.36	25412.184	5.4221	3.0864
29.5	92.677	683.49	870.25	25672.375	5.4313	3.0899
29.6	92.991	688.13	876.16	25934.336	5.4405	3.0934
29.7	93.305	692.79	882.09	26198.073	5.4497	3.0968
29.8	93.619	697.47	888.04	26463.592	5.4589	3.1003
29.9	93.934	702.15	894.01	26730.899	5.4680	3.1038
30.0	94.248	706.86	900.00	27000.000	5.4772	3.1072
30.1	94.562	711.58	906.01	27270.901	5.4863	3.1107
30.2	94.876	716.32	912.04	27543.608	5.4954	3.1141

CONSTANTS—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
30.3	95.190	721.07	918.09	27818.127	5.5045	3.1176
30.4	95.505	725.83	924.16	28094.464	5.5136	3.1210
30.5	95.819	730.62	930.25	28372.625	5.5226	3.1244
30.6	96.133	735.42	936.36	28652.616	5.5317	3.1278
30.7	96.447	740.23	942.49	28934.443	5.5407	3.1312
30.8	96.761	745.06	948.64	29218.112	5.5497	3.1346
30.9	97.075	749.91	954.81	29503.629	5.5587	3.1380
31.0	97.389	754.77	961.00	29791.000	5.5678	3.1414
31.1	97.704	759.65	967.21	30080.231	5.5767	3.1448
31.2	98.018	764.54	973.44	30371.328	5.5857	3.1481
31.3	98.332	769.45	979.69	30664.297	5.5946	3.1515
31.4	98.646	774.37	985.96	30959.144	5.6035	3.1548
31.5	98.960	779.31	992.25	31255.875	5.6124	3.1582
31.6	99.274	784.27	998.56	31554.496	5.6213	3.1615
31.7	99.588	789.24	1004.89	31855.013	5.6302	3.1648
31.8	99.903	794.23	1011.24	32157.432	5.6391	3.1681
31.9	100.22	799.23	1017.61	32461.759	5.6480	3.1715
32.0	100.53	804.25	1024.00	32768.000	5.6569	3.1748
32.1	100.85	809.28	1030.41	33076.161	5.6656	3.1781
32.2	101.16	814.33	1036.84	33386.248	5.6745	3.1814
32.3	101.47	819.40	1043.29	33698.267	5.6833	3.1847
32.4	101.79	824.48	1049.76	34012.224	5.6921	3.1880
32.5	102.10	829.58	1056.25	34328.125	5.7008	3.1913
32.6	102.42	834.69	1062.76	34645.976	5.7096	3.1945
32.7	102.73	839.82	1069.29	34965.783	5.7183	3.1978
32.8	103.04	844.96	1075.84	35287.552	5.7271	3.2010
32.9	103.36	850.12	1082.41	35611.289	5.7358	3.2043
33.0	103.67	855.30	1089.00	35937.000	5.7446	3.2075
33.1	103.99	860.49	1095.61	36264.691	5.7532	3.2108
33.2	104.30	865.70	1102.24	36594.368	5.7619	3.2140
33.3	104.62	870.92	1108.89	36926.037	5.7706	3.2172
33.4	104.93	876.16	1115.56	37259.704	5.7792	3.2204
33.5	105.24	881.41	1122.25	37595.375	5.7879	3.2237
33.6	105.56	886.68	1128.96	37933.056	5.7965	3.2269
33.7	105.87	891.97	1135.69	38272.753	5.8051	3.2301
33.8	106.19	897.27	1142.44	38614.472	5.8137	3.2332
33.9	106.50	902.59	1149.21	38958.219	5.8223	3.2364
34.0	106.81	907.92	1156.00	39304.000	5.8310	3.2396
34.1	107.13	913.27	1162.81	39651.821	5.8395	3.2428
34.2	107.44	918.63	1169.64	40001.688	5.8480	3.2460
34.3	107.76	924.01	1176.49	40353.607	5.8566	3.2491
34.4	108.07	929.41	1183.36	40707.584	5.8651	3.2522

CONSTANTS—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
34.5	108.38	934.82	1190.25	41063.625	5.8736	3.2554
34.6	108.70	940.25	1197.16	41421.736	5.8821	3.2586
34.7	109.01	945.69	1204.09	41781.923	5.8906	3.2617
34.8	109.33	951.15	1211.04	42144.192	5.8991	3.2648
34.9	109.64	956.62	1218.01	42508.549	5.9076	3.2679
35.0	109.96	962.11	1225.00	42875.000	5.9161	3.2710
35.1	110.27	967.62	1232.01	43243.551	5.9245	3.2742
35.2	110.58	973.14	1239.04	43614.208	5.9329	3.2773
35.3	110.90	978.68	1246.09	43986.977	5.9413	3.2804
35.4	111.21	984.23	1253.16	44361.864	5.9497	3.2835
35.5	111.53	989.80	1260.25	44738.875	5.9581	3.2866
35.6	111.84	995.38	1267.36	45118.016	5.9665	3.2897
35.7	112.15	1000.98	1274.49	45499.293	5.9749	3.2927
35.8	112.47	1006.60	1281.64	45882.712	5.9833	3.2958
35.9	112.78	1012.23	1288.81	46268.279	5.9916	3.2989
36.0	113.10	1017.88	1296.00	46656.000	6.0000	3.3019
36.1	113.41	1023.54	1303.21	47045.881	6.0083	3.3050
36.2	113.73	1029.22	1310.44	47437.928	6.0166	3.3080
36.3	114.04	1034.91	1317.69	47832.147	6.0249	3.3111
36.4	114.35	1040.62	1324.96	48228.544	6.0332	3.3141
39.5	114.67	1046.35	1332.25	48627.125	6.0415	3.3171
36.6	114.98	1052.09	1339.56	49027.896	6.0497	3.3202
36.7	115.30	1057.84	1346.89	49430.863	6.0580	3.3232
36.8	115.61	1063.62	1354.24	49836.032	6.0663	3.3262
36.9	115.92	1069.41	1361.61	50243.409	6.0745	3.3292
37.0	116.24	1075.21	1369.00	50653.000	6.0827	3.3322
37.1	116.55	1081.03	1376.41	51064.811	6.0909	3.3352
37.2	116.87	1086.87	1383.84	51478.848	6.0991	3.3382
37.3	117.18	1092.72	1391.29	51895.117	6.1073	3.3412
37.4	117.50	1098.58	1398.76	52313.624	6.1155	3.3442
37.5	117.81	1104.47	1406.25	52734.375	6.1237	3.3472
37.6	118.12	1110.36	1413.76	53157.376	6.1318	3.3501
37.7	118.44	1116.28	1421.29	53582.633	6.1400	3.3531
37.8	118.75	1122.21	1428.84	54010.152	6.1481	3.3561
37.9	119.07	1128.15	1436.41	54439.939	6.1563	3.3590
38.0	119.38	1134.11	1444.00	54872.000	6.1644	3.3620
38.1	119.69	1140.09	1451.61	55306.341	6.1725	3.3649
38.2	120.01	1146.08	1459.24	55742.968	6.1806	3.3679
38.3	120.32	1152.09	1466.89	56181.887	6.1887	3.3708
38.4	120.64	1158.12	1474.56	56623.104	6.1967	3.3737
38.5	120.95	1164.16	1482.25	57066.625	6.2048	3.3767
38.6	121.27	1170.21	1489.96	57512.456	6.2129	3.3796
38.7	121.58	1176.28	1497.69	57960.603	6.2209	3.3825

CONSTANTS—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
38.8	121.89	1182.37	1505.44	58411.072	6.2289	3.3854
38.9	122.21	1188.47	1513.21	58863.869	6.2370	3.3883
39.0	122.52	1194.59	1521.00	59319.000	6.2450	3.3912
39.1	122.84	1200.72	1528.81	59776.471	6.2530	3.3941
39.2	123.15	1206.87	1536.64	60236.288	6.2610	3.3970
39.3	123.46	1213.04	1544.49	60698.457	6.2689	3.3999
39.4	123.78	1219.22	1552.36	61162.984	6.2769	3.4028
39.5	124.09	1225.42	1560.25	61629.875	6.2849	3.4056
39.6	124.41	1231.63	1568.16	62099.136	6.2928	3.4085
39.7	124.72	1237.86	1576.09	62570.773	6.3008	3.4114
39.8	125.04	1244.10	1584.04	63044.792	6.3087	3.4142
39.9	125.35	1250.36	1592.01	63521.199	6.3166	3.4171
40.0	125.66	1256.64	1600.00	64000.000	6.3245	3.4200
40.1	125.98	1262.93	1608.01	64481.201	6.3325	3.4228
40.2	126.29	1269.23	1616.04	64964.808	6.3404	3.4256
40.3	126.61	1275.56	1624.09	65450.827	6.3482	3.4285
40.4	126.92	1281.90	1632.16	65939.264	6.3561	3.4313
40.5	127.23	1288.25	1640.25	66430.125	6.3639	3.4341
40.6	127.55	1294.62	1648.36	66923.416	6.3718	3.4370
40.7	127.86	1301.00	1656.49	67419.143	6.3796	3.4398
40.8	128.18	1307.41	1664.64	67911.312	6.3875	3.4426
40.9	128.49	1313.82	1672.81	68417.929	6.3953	3.4454
41.0	128.81	1320.25	1681.00	68921.000	6.4031	3.4482
41.1	129.12	1326.70	1689.21	69426.531	6.4109	3.4510
41.2	129.43	1333.17	1697.44	69934.528	6.4187	3.4538
41.3	129.75	1339.65	1705.69	70444.997	6.4265	3.4566
41.4	130.06	1346.14	1713.96	70957.944	6.4343	3.4594
41.5	130.38	1352.65	1722.25	71473.375	6.4421	3.4622
41.6	130.69	1359.18	1730.56	71991.296	6.4498	3.4650
41.7	131.00	1365.72	1738.89	72511.713	6.4575	3.4677
41.8	131.32	1372.28	1747.24	73034.632	6.4653	3.4705
41.9	131.63	1378.85	1755.61	73560.059	6.4730	3.4733
42.0	131.95	1385.44	1764.00	74088.000	6.4807	3.4760
42.1	132.26	1392.05	1772.41	74618.461	6.4884	3.4788
42.2	132.58	1398.67	1780.84	75151.448	6.4961	3.4815
42.3	132.89	1405.31	1789.29	75686.967	6.5038	3.4843
42.4	133.20	1411.96	1797.76	76225.024	6.5115	3.4870
42.5	133.52	1418.63	1806.25	76765.625	6.5192	3.4898
42.6	133.83	1425.31	1814.76	77308.776	6.5268	3.4925
42.7	134.15	1432.01	1823.29	77854.483	6.5345	3.4952
42.8	134.46	1438.72	1831.84	78402.752	6.5422	3.4980
42.9	134.77	1445.45	1840.41	78953.589	6.5498	3.5007

CONSTANTS—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
43.0	135.09	1452.20	1849.00	79507.000	6.5574	3.5034
43.1	135.40	1458.96	1857.61	80062.991	6.5651	3.5061
43.2	135.72	1465.74	1866.24	80621.568	6.5727	3.5088
43.3	136.03	1472.54	1874.89	81182.737	6.5803	3.5115
43.4	136.35	1479.34	1883.56	81746.504	6.5879	3.5142
43.5	136.66	1486.17	1892.25	82312.875	6.5954	3.5169
43.6	136.97	1493.01	1900.96	82881.856	6.6030	3.5196
43.7	137.29	1499.87	1909.69	83453.453	6.6106	3.5223
43.8	137.60	1506.74	1918.44	84027.672	6.6182	3.5250
43.9	137.92	1513.63	1927.21	84604.519	6.6257	3.5277
44.0	138.23	1520.53	1936.00	85184.000	6.6333	3.5303
44.1	138.54	1527.45	1944.81	85766.121	6.6408	3.5330
44.2	138.86	1534.39	1953.64	86350.888	6.6483	3.5357
44.3	139.17	1541.34	1962.49	86938.307	6.6558	3.5384
44.4	139.49	1548.30	1971.36	87528.384	6.6633	3.5410
44.5	139.80	1555.28	1980.25	88121.125	6.6708	3.5437
44.6	140.12	1562.28	1989.16	88716.536	6.6783	3.5463
44.7	140.43	1569.30	1998.09	89314.623	6.6858	3.5490
44.8	140.74	1576.33	2007.04	89915.392	6.6933	3.5516
44.9	141.06	1583.37	2016.01	90518.849	6.7007	3.5543
45.0	141.37	1590.43	2025.00	91125.000	6.7082	3.5569
45.1	141.69	1597.51	2034.01	91733.851	6.7156	3.5595
45.2	142.00	1604.60	2043.04	92345.408	6.7231	3.5621
45.3	142.31	1611.71	2052.09	92959.677	6.7305	3.5648
45.4	142.63	1618.83	2061.16	93576.664	6.7379	3.5674
45.5	142.94	1625.97	2070.25	94196.375	6.7454	3.5700
45.6	143.26	1633.13	2079.36	94818.816	6.7528	3.5726
45.7	143.57	1640.30	2088.49	95443.993	6.7602	3.5752
45.8	143.88	1647.48	2097.64	96071.912	6.7676	3.5778
45.9	144.20	1654.68	2106.81	96702.579	6.7749	3.5805
46.0	144.51	1661.90	2116.00	97336.000	6.7823	3.5830
46.1	144.83	1669.14	2125.21	97972.181	6.7897	3.5856
46.2	145.14	1676.39	2134.44	98611.128	6.7971	3.5882
46.3	145.46	1683.65	2143.69	99252.847	6.8044	3.5908
46.4	145.77	1690.93	2152.96	99897.344	6.8117	3.5934
46.5	146.08	1698.23	2162.25	100544.625	6.8191	3.5960
46.6	146.40	1705.54	2171.56	101194.696	6.8264	3.5986
46.7	146.71	1712.87	2180.89	101847.563	6.8337	3.6011
46.8	147.03	1720.21	2190.24	102503.232	6.8410	3.6037
46.9	147.34	1727.57	2199.61	103161.709	6.8484	3.6063
47.0	147.65	1734.94	2209.00	103823.000	6.8556	3.6088
47.1	147.97	1742.34	2218.41	104487.111	6.8629	3.6114
47.2	148.28	1749.74	2227.84	105154.048	6.8702	3.6139

CONSTANTS—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
47.3	148.60	1757.16	2237.29	105823.817	6.8775	3.6165
47.4	148.91	1764.60	2246.76	106496.424	6.8847	3.6190
47.5	149.23	1772.05	2256.25	107171.875	6.8920	3.6216
47.6	149.54	1779.52	2265.76	107850.176	6.8993	3.6241
47.7	149.85	1787.01	2275.29	108531.333	6.9065	3.6267
47.8	150.17	1794.51	2284.84	109215.352	6.9137	3.6292
47.9	150.48	1802.03	2294.41	109902.239	6.9209	3.6317
48.0	150.80	1809.56	2304.00	110592.000	6.9282	3.6342
48.1	151.11	1817.11	2313.61	111284.641	6.9354	3.6368
48.2	151.42	1824.67	2323.24	111980.168	6.9426	3.6393
48.3	151.74	1832.25	2332.89	112678.587	6.9498	3.6418
48.4	152.05	1839.84	2342.56	113379.904	6.9570	3.6443
48.5	152.37	1847.45	2352.25	114084.125	6.9642	3.6468
48.6	152.68	1855.08	.2361.96	114791.256	6.9714	3.6493
48.7	153.00	1862.72	2371.69	115501.303	6.9785	3.6518
48.8	153.31	1870.38	2381.44	116214.272	6.9857	3.6543
48.9	153.62	1878.05	2391.21	116930.169	6.9928	3.6568
49.0	153.94	1885.74	2401.00	117649.000	7.0000	3.6593
49.1	154.25	1893.45	2410.81	118370.771	7.0071	3.6618
49.2	154.57	1901.17	2420.64	119095.488	7.0143	3.6643
49.3	154.88	1908.90	2430.49	119823.157	7.0214	3.6668
49.4	155.19	1916.65	2440.36	120553.784	7.0285	3.6692
49.5	155.51	1924.42	2450.25	121287.375	7.0356	3.6717
49.6	155.82	1932.21	2460.16	122023.936	7.0427	3.6742
49.7	156.14	1940.00	2470.09	122763.473	7.0498	3.6767
49.8	156.45	1947.82	2480.04	123505.992	7.0569	3.6791
49.9	156.77	1955.65	2490.01	124251.499	7.0640	3.6816
50.0	157.08	1963.50	2500.00	125000.000	7.0711	3.6840
51.0	160.22	2042.82	2601.00	132651.000	7.1414	3.7084
52.0	163.36	2123.72	2704.00	140608.000	7.2111	3.7325
53.0	166.50	2206.19	2809.00	148877.000	7.2801	3.7563
54.0	169.64	2290.22	2916.00	157464.000	7.3485	3.7798
55.0	172.78	2375.83	3025.00	166375.000	7.4162	3.8030
56.0	175.93	2463.01	3136.00	175616.000	7.4833	3.8259
57.0	179.07	2551.76	3249.00	185193.000	7.5498	3.8485
58.0	182.21	2642.08	3364.00	195112.000	7.6158	3.8709
59.0	185.35	2733.97	3481.00	205379.000	7.6811	3.8930
60.0	188.49	2827.44	3600.00	216000.000	7.7460	3.9149
61.0	191.63	2922.47	3721.00	226981.000	7.8102	3.9365
62.0	194.77	3019.07	3844.00	238328.000	7.8740	3.9579
63.0	197.92	3117.25	3969.00	250047.000	7.9373	3.9791
64.0	201.06	3216.99	4096.00	262144.000	8.0000	4.0000
65.0	204.20	3318.31	4225.00	274625.000	8.0623	4.0207
66.0	207.34	3421.20	4356.00	287496.000	8.1240	4.0412

CONSTANTS—*Continued.*

n	$n\pi$	$n^2\frac{\pi}{4}$	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$
67.0	210.48	3525.66	4489.00	300763.000	8.1854	4.0615
68.0	213.63	3631.69	4624.00	314432.000	8.2462	4.0817
69.0	216.77	3739.29	4761.00	328509.000	8.3066	4.1016
70.0	219.91	3848.46	4900.00	343000.000	8.3666	4.1213
71.0	223.05	3959.20	5041.00	357911.000	8.4261	4.1408
72.0	226.19	4071.51	5184.00	373248.000	8.4853	4.1602
73.0	229.33	4185.39	5329.00	389017.000	8.5440	4.1793
74.0	232.47	4300.85	5476.00	405224.000	8.6023	4.1983
75.0	235.62	4417.87	5625.00	421875.000	8.6603	4.2172
76.0	238.76	4536.47	5776.00	438976.000	8.7178	4.2358
77.0	241.90	4656.63	5929.00	456533.000	8.7750	4.2543
78.0	245.04	4778.37	6084.00	474552.000	8.8318	4.2727
79.0	248.18	4901.68	6241.00	493039.000	8.8882	4.2908
80.0	251.32	5026.56	6400.00	512000.000	8.9443	4.3089
81.0	254.47	5153.01	6561.00	531441.000	9.0000	4.3267
82.0	257.61	5281.03	6724.00	551368.000	9.0554	4.3445
83.0	260.75	5410.62	6889.00	571787.000	9.1104	4.3621
84.0	263.89	5541.78	7056.00	592704.000	9.1652	4.3795
85.0	267.03	5674.50	7225.00	614125.000	9.2195	4.3968
86.0	270.17	5808.81	7396.00	636056.000	9.2736	4.4140
87.0	273.32	5944.69	7569.00	658503.000	9.3274	4.4310
88.0	276.46	6082.13	7744.00	681472.000	9.3808	4.4480
89.0	279.60	6221.13	7921.00	704969.000	9.4340	4.4647
90.0	282.74	6361.74	8100.00	729000.000	9.4868	4.4814
91.0	285.88	6503.89	8281.00	753571.000	9.5394	4.4979
92.0	289.02	6647.62	8464.00	778688.000	9.5917	4.5144
93.0	292.17	6792.92	8649.00	804357.000	9.6437	4.5307
94.0	295.31	6939.78	8836.00	830584.000	9.6954	4.5468
95.0	298.45	7088.23	9025.00	857375.000	9.7468	4.5629
96.0	301.59	7238.24	9216.00	884736.000	9.7980	4.5789
97.0	304.73	7389.83	9409.00	912673.000	9.8489	4.5947
98.0	307.87	7542.98	9604.00	941192.000	9.8995	4.6104
99.0	311.02	7697.68	9801.00	970299.000	9.9499	4.6261
100.0	314.16	7854.00	10000.00	1000000.000	10.0000	4.6416

II.

LOGARITHMS.

Hyperbolic Logarithms.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
1.00	0.0000	2.30	0.8329	3.60	1.2809	4.90	1.5892	6.40	1.8563
1.05	0.0488	2.35	0.8544	3.65	1.2947	4.95	1.5994	6.50	1.8718
1.10	0.0953	2.40	0.8755	3.70	1.3083	5.00	1.6094	6.60	1.8871
1.15	0.1398	2.45	0.8961	3.75	1.3218	5.05	1.6194	6.70	1.9021
1.20	0.1823	2.50	0.9163	3.80	1.3350	5.10	1.6292	6.80	1.9169
1.25	0.2231	2.55	0.9361	3.85	1.3481	5.15	1.6390	6.90	1.9315
1.30	0.2624	2.60	0.9555	3.90	1.3610	5.20	1.6487	7.00	1.9459
1.35	0.3001	2.65	0.9746	3.95	1.3737	5.25	1.6582	7.20	1.9741
1.40	0.3365	2.70	0.9933	4.00	1.3863	5.30	1.6677	7.40	2.0015
1.45	0.3716	2.75	1.0116	4.05	1.3987	5.35	1.6771	7.60	2.0281
1.50	0.4055	2.80	1.0296	4.10	1.4110	5.40	1.6864	7.80	2.0541
1.55	0.4383	2.85	1.0473	4.15	1.4231	5.45	1.6956	8.00	2.0794
1.60	0.4700	2.90	1.0647	4.20	1.4351	5.50	1.7047	8.25	2.1102
1.65	0.5008	2.95	1.0818	4.25	1.4469	5.55	1.7138	8.50	2.1401
1.70	0.5306	3.00	1.0986	4.30	1.4586	5.60	1.7228	8.75	2.1691
1.75	0.5596	3.05	1.1154	4.35	1.4701	5.65	1.7317	9.00	2.1972
1.80	0.5878	3.10	1.1314	4.40	1.4816	5.70	1.7405	9.25	2.2246
1.85	0.6152	3.15	1.1474	4.45	1.4929	5.75	1.7492	9.50	2.2513
1.90	0.6419	3.20	1.1632	4.50	1.5041	5.80	1.7579	9.75	2.2773
1.95	0.6678	3.25	1.1787	4.55	1.5151	5.85	1.7664	10.00	2.3026
2.00	0.6931	3.30	1.1939	4.60	1.5261	5.90	1.7750	11.00	2.3979
2.05	0.7178	3.35	1.2090	4.65	1.5369	5.95	1.7834	12.00	2.4849
2.10	0.7419	3.40	1.2238	4.70	1.5476	6.00	1.7918	13.00	2.5649
2.15	0.7655	3.45	1.2384	4.75	1.5581	6.10	1.8083	14.00	2.6391
2.20	0.7885	3.50	1.2528	4.80	1.5686	6.20	1.8245	15.00	2.7081
2.25	0.8109	3.55	1.2669	4.85	1.5790	6.30	1.8405	16.00	2.7726

Common Logarithms: 10–1200.

N.	0	1	2	3	4	5	6	7	8	9	Diff.
10	00000	00432	00860	01284	01703	02119	02531	02938	03342	03743	396
11	04139	04532	04922	05308	05690	06070	06446	06819	07188	07555	363
12	07918	08279	08636	08991	09342	09691	10037	10380	10721	11059	335
13	11394	11727	12057	12385	12710	13033	13354	13672	13988	14301	312
14	14613	14922	15229	15534	15836	16137	16435	16732	17026	17319	290
15	17609	17898	18184	18469	18752	19033	19312	19590	19866	20140	278
16	20412	20683	20952	21219	21484	21748	22011	22272	22531	22789	256
17	23045	23300	23553	23805	24055	24304	24551	24797	25042	25285	242
18	25527	25768	26007	26245	26482	26717	26951	27184	27416	27646	229
19	27875	28103	28330	28556	28780	29003	29226	29447	29667	29885	212
20	30103	30320	30535	30750	30963	31175	31387	31597	31806	32015	207
21	32222	32428	32634	32838	33041	33244	33445	33646	33846	34044	198
22	34242	34439	34635	34830	35025	35218	35411	35603	35793	35984	189
23	36173	36361	36549	36736	36922	37107	37291	37475	37658	37840	181
24	38021	38202	38382	38561	38739	38917	39094	39270	39445	39620	174
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	167
26	41497	41664	41830	41996	42160	42325	42488	42651	42813	42975	161
27	43136	43297	43457	43616	43775	43933	44091	44248	44404	44560	156
28	44716	44871	45025	45179	45332	45484	45637	45788	45939	46090	150
29	46240	46389	46538	46687	46835	46982	47129	47276	47422	47567	145

COMMON LOGARITHMS—*Continued.*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
30	47712	47857	48001	48144	48287	48430	48572	48714	48855	48996	140
31	49136	49276	49415	49554	49693	49831	49969	50106	50243	50379	136
32	50515	50651	50786	50920	51055	51188	51322	51455	51587	51720	132
33	51851	51983	52114	52244	52375	52504	52634	52763	52892	53020	128
34	53148	53275	53403	53529	53656	53782	53908	54033	54158	54283	124
35	54407	54531	54654	54777	54900	55023	55145	55267	55388	55509	121
36	55630	55751	55871	55991	56110	56229	56348	56467	56585	56703	117
37	56820	56937	57054	57171	57287	57403	57519	57634	57749	57864	114
38	57978	58092	58206	58320	58433	58546	58659	58771	58883	58995	111
39	59106	59218	59329	59439	59550	59660	59770	59879	59988	60097	109
40	60206	60314	60423	60531	60638	60746	60853	60959	61066	61172	106
41	61278	61384	61490	61595	61700	61805	61909	62014	62118	62221	104
42	62325	62428	62531	62634	62737	62839	62941	63043	63144	63246	102
43	63347	63448	63548	63649	63749	63849	63949	64048	64147	64246	99
44	64345	64444	64542	64640	64738	64836	64933	65031	65128	65225	97
45	65321	65418	65514	65610	65706	65801	65896	65992	66087	66181	95
46	66276	66370	66464	66558	66652	66745	66839	66932	67025	67117	93
47	67210	67302	67394	67486	67578	67669	67761	67852	67943	68034	90
48	68124	68215	68305	68395	68485	68574	68664	68753	68842	68931	89
49	69020	69108	69197	69285	69373	69461	69548	69636	69723	69810	87
50	69897	69984	70070	70157	70243	70329	70415	70501	70586	70672	86
51	70757	70842	70927	71012	71096	71181	71265	71349	71433	71517	84
52	71600	71684	71767	71850	71933	72016	72099	72181	72263	72346	83
53	72428	72509	72591	72673	72754	72835	72916	72997	73078	73159	81
54	73239	73320	73400	73480	73560	73640	73719	73799	73878	73957	80
55	74036	74115	74194	74273	74351	74429	74507	74586	74663	74741	78
56	74819	74896	74974	75051	75128	75205	75282	75358	75435	75511	77
57	75587	75664	75740	75815	75891	75967	76042	76118	76193	76268	76
58	76343	76418	76492	76567	76641	76716	76790	76864	76938	77012	74
59	77085	77159	77232	77305	77379	77452	77525	77597	77670	77743	73
60	77815	77887	77960	78032	78104	78176	78247	78319	78390	78462	72
61	78533	78604	78675	78746	78817	78888	78958	79029	79099	79169	71
62	79239	79309	79379	79449	79518	79588	79657	79727	79796	79865	69
63	79934	80003	80072	80140	80209	80277	80346	80414	80482	80550	68
64	80618	80686	80754	80821	80889	80956	81023	81090	81158	81224	67
65	81291	81358	81425	81491	81558	81624	81690	81757	81823	81889	66
66	81954	82020	82086	82151	82217	82282	82347	82413	82478	82543	65
67	82607	82672	82737	82802	82866	82930	82995	83059	83123	83187	64
68	83251	83315	83378	83442	83506	83569	83632	83696	83759	83822	63
69	83885	83948	84011	84073	84136	84198	84261	84323	84386	84448	63
70	84510	84572	84634	84696	84757	84819	84880	84942	85003	85065	62
71	85126	85187	85248	85309	85370	85431	85491	85552	85612	85673	61
72	85733	85794	85854	85914	85974	86034	86094	86153	86213	86273	60
73	86332	86392	86451	86510	86570	86629	86688	86747	86806	86864	59
74	86923	86982	87040	87099	87157	87216	87274	87332	87390	87448	58
75	87506	87564	87622	87679	87737	87795	87852	87910	87967	88024	58
76	88081	88138	88195	88252	88309	88366	88423	88480	88536	88593	57
77	88649	88705	88762	88818	88874	88930	88986	89042	89098	89154	56
78	89209	89265	89321	89376	89432	89487	89542	89597	89653	89708	55
79	89763	89818	89873	89927	89982	90037	90091	90146	90200	90255	55
80	90309	90363	90417	90472	90526	90580	90634	90687	90741	90795	54
81	90849	90902	90956	91009	91062	91116	91169	91222	91275	91328	53
82	91381	91434	91487	91540	91593	91645	91698	91751	91803	91855	52
83	91908	91960	92012	92065	92117	92169	92221	92273	92324	92376	52
84	92428	92480	92531	92583	92634	92686	92737	92788	92840	92891	51
85	92942	92993	93044	93095	93146	93197	93247	93298	93349	93399	51
86	93450	93500	93551	93601	93651	93702	93752	93802	93852	93902	50
87	93952	94002	94052	94101	94151	94201	94250	94300	94349	94399	50

Common Logarithms—*Continued.*

N.	0	1	2	3	4	5	6	7	8	9	Diff.
88	94448	94498	94547	94596	94645	94694	94743	94792	94841	94890	49
89	94939	94988	95036	95085	95134	95182	95231	95279	95328	95376	49
90	95424	95472	95521	95569	95617	95665	95713	95761	95809	95856	48
91	95904	95952	95999	96047	96095	96142	96190	96237	96284	96332	47
92	96379	96426	96473	96520	96567	96614	96661	96708	96755	96802	47
93	96848	96895	96942	96988	97035	97081	97128	97174	97220	97267	46
94	97313	97359	97405	97451	97497	97543	97589	97635	97681	97727	46
95	97772	97818	97864	97909	97955	98000	98046	98091	98137	98182	45
96	98227	98272	98318	98363	98408	98453	98498	98543	98588	98632	45
97	98677	98722	98767	98811	98856	98900	98945	98989	99034	99078	45
98	99123	99167	99211	99255	99300	99344	99388	99432	99476	99520	44
99	99564	99607	99651	99695	99739	99782	99826	99870	99913	99957	44
100	00000	00043	00087	00130	00173	00217	00260	00303	00346	00389	43
101	00432	00475	00518	00561	00604	00647	00689	00732	00775	00817	43
102	00860	00903	00945	00988	01030	01072	01115	01157	01199	01242	42
103	01284	01326	01368	01410	01452	01494	01536	01578	01620	01662	42
104	01703	01745	01787	01828	01870	01912	01953	01995	02036	02078	42
105	02119	02160	02202	02243	02284	02325	02366	02407	02449	02490	41
106	02531	02572	02612	02653	02694	02735	02776	02816	02857	02898	41
107	02938	02979	03019	03060	03100	03141	03181	03222	03262	03302	41
108	03342	03383	03423	03463	03503	03543	03583	03623	03663	03703	40
109	03743	03782	03822	03862	03902	03941	03981	04021	04060	04100	40
110	04139	04179	04218	04258	04297	04336	04376	04415	04454	04493	39
111	04532	04571	04610	04650	04689	04727	04766	04805	04844	04883	39
112	04922	04961	04999	05038	05077	05115	05154	05192	05231	05269	39
113	05308	05346	05385	05423	05461	05500	05538	05576	95614	05652	38
114	05690	05729	05767	05805	05843	05881	05918	05956	05994	06032	38
115	06070	06108	06145	06183	06221	06258	06296	06333	06371	06408	38
116	06446	06483	06521	06558	06595	06633	06670	06707	06744	06781	37
117	06819	06856	06893	06930	06967	07004	07041	07078	07115	07151	37
118	07188	07225	07262	07298	07335	07372	07408	07445	07482	07518	37
119	07555	07591	07628	07664	07700	07773	07773	07809	07846	07882	36

Log.

Base cf Naperian logarithms, ... $e = 2.7182818$ 0.4342945

Log. e = Modulus of common logarithms, ... $M = 0.4342945$ 9.6377843 − 10.

To obtain common from Naperian or "natural" logarithms, multiply by 2.3026.

III.

MEAN PRESSURES FOR VARIOUS METHODS OF EXPANSION.

Values of $\frac{p_m}{p_1}$. Adiabatic Expansion of Steam.

Ratio of Expansion.	Cut-off, $\frac{1}{r}$.	Percentage of Steam and Value of n.							
		100	90	80	76	70	60	50	100
		1.135	1.125	1.115	1.111	1.105	1.095	1.085	1.333
2	$\frac{1}{2}$	.829	.831	.833	.834	.835	.836	.837	.810
$2\frac{1}{4}$	$\frac{4}{9}$	.785	.787	.788	.789	.790	.791	.793	.754
$2\frac{1}{2}$	$\frac{2}{5}$	.744	.746	.747	.748	.749	.750	.751	.714
$2\frac{3}{4}$	$\frac{4}{11}$	.707	.708	.710	.711	.712	.713	.714	.675
3	$\frac{1}{3}$	.675	.676	.677	.678	.679	.681	.683	.639
$3\frac{1}{4}$	$\frac{4}{13}$	.644	.645	.647	.648	.649	.650	.652	.606
$3\frac{1}{3}$	$\frac{3}{10}$	.633	.635	.636	.637	.639	.641	.643	.600
$3\frac{1}{2}$	$\frac{2}{7}$	.616	.618	.619	.620	.622	.624	.626	.576
$3\frac{3}{4}$	$\frac{4}{15}$	.591	.592	.593	.594	.595	.596	.598	.552
4	$\frac{1}{4}$	.567	.568	.570	.572	.573	.574	.576	.523
$4\frac{1}{2}$	$\frac{2}{9}$	.525	.527	.528	.530	.531	.533	.534	.486
5	$\frac{1}{5}$	.488	.491	.493	.494	.496	.498	.500	.447
$5\frac{1}{2}$	$\frac{2}{11}$	.458	.460	.462	.463	.465	.467	.470	.417
6	$\frac{1}{6}$	.432	.434	.435	.437	.439	.441	.443	.390
$6\frac{1}{2}$	$\frac{2}{13}$	.409	.410	.411	.413	.415	.417	.420	.369
7	$\frac{1}{7}$	.387	.390	.392	.394	.400	.403	.405	.345
8	$\frac{1}{8}$	.355	.356	.357	.358	.360	.361	.363	.312
10	$\frac{1}{10}$	.298	.300	.302	.303	.304	.305	.308	.263
20	$\frac{1}{20}$	.170	.173	.175	.177	.178	.180	.182	.144
50	$\frac{1}{50}$	.080	.082	.083	.084	.084	.085	.086	.063
100	$\frac{1}{100}$	.044	.045	.045	.046	.046	.047	048	.034

III.—(*Continued.*)

MEAN PRESSURES FOR VARIOUS METHODS OF EXPANSION.

Values of $\frac{p_m}{p_1}$ for Steam, Air, Gas, and Mixtures.

Ratio of Expansion, r.	Point of cut-off, $\frac{1}{r}$.	Steam Expanding, Dry and Saturated, n, 1.046.	Moist Air in Compressors, n, 1.20.	Steam and Leakage, Actual Engines.		Gas and Vapor in Gas-engine, n, 1.60.	Gases.	
				n, 0.50.	n, 0.75.		Isothermal, n, 1.00.	Adiabatic, n, 1.41.
2	$\frac{1}{2}$	.841	.825	.914	.875	.783	.846	.801
$2\frac{1}{4}$	$\frac{4}{9}$	.793	.787	.888	.844	.733	.804	.753
$2\frac{1}{2}$	$\frac{2}{5}$	.760	.745	.866	.800	.683	.765	.707
$2\frac{3}{4}$	$\frac{4}{11}$	.717	.700	.846	.785	.638	.731	.668
3	$\frac{1}{3}$	.695	.665	.824	.752	.598	.699	.638
$3\frac{1}{4}$	$\frac{4}{13}$	.665	.635	.802	.732	.578	.670	.596
$3\frac{1}{3}$	$\frac{3}{10}$	.652	.625	.796	.716	.568	.661	.588
$3\frac{1}{2}$	$\frac{2}{7}$	.632	.605	.782	.704	.548	.642	.566
$3\frac{3}{4}$	$\frac{4}{15}$	.608	.580	.775	.684	.515	.616	.538
4	$\frac{1}{4}$	.587	.550	.750	.664	.486	.566	.518
$4\frac{1}{2}$	$\frac{2}{9}$	.540	.510	.720	.624	.441	.555	.473
5	$\frac{1}{5}$	.510	.482	.695	.600	.406	.522	.428
$5\frac{1}{2}$	$\frac{2}{11}$	.478	.455	.674	.560	.371	.492	.406
6	$\frac{1}{6}$	.454	.420	.650	.530	.349	.465	.378
$6\frac{1}{2}$	$\frac{2}{13}$	.430	.390	.632	.515	.326	.441	.358
7	$\frac{1}{7}$	.409	.375	.612	.500	.303	.421	.337
8	$\frac{1}{8}$	.372	.340	.697	.468	.276	.385	.302
10	$\frac{1}{10}$	.326	.284	.532	.412	.225	.330	.253
20	$\frac{1}{20}$	.192	.165	.396	.272	.103	.200	.138
50	$\frac{1}{50}$	.091	.074	.245	.193	.050	.098	.060
100	$\frac{1}{100}$	.053	.040	.180	.134	.025	.056	.032

III.—(*Continued.*)

MEAN PRESSURE RATIOS.

r	*A*	*B*	*C*	*r*	*A*	*B*	*C*	*r*	*A*	*B*	*C*	*r*	*A*	*B*	*C*
1.0	1.000	1.000	1.000	5.3	.478	.503	.488	9.6	.312	.340	.324	17.8	.194	.218	.204
1.1	0.996	0.996	0.996	5.4	.472	.497	.482	9.7	.310	.338	.322	18.0	.192	.216	.202
1.2	0.983	0.983	0.983	5.5	.467	.492	.477	9.8	.307	.335	.319	18.2	.190	.215	.200
1.3	.966	.968	.967	5.6	.461	.486	.471	9.9	.305	.333	.317	18.4	.189	.214	.199
1.4	.947	.952	.950	5.7	.456	.481	.466	10.0	.303	.330	.314	18.6	.187	.212	.197
1.5	.928	.934	.931	5.8	.450	.475	.460	10.2	.299	.325	.310	18.8	.185	.210	.195
1.6	.910	.919	.914	5.9	.445	.470	.455	10.4	.295	.321	.306	19.0	.183	.208	.193
1.7	.890	.900	.895	6.0	.440	.465	.450	10.6	.291	.317	.302	19.2	.182	.207	.192
1.8	.870	.880	.875	6.1	.434	.460	.445	10.8	.287	.313	.298	19.4	.180	.205	.190
1.9	.850	.862	.856	6.2	.429	.455	.440	11.0	.283	.309	.294	19.6	.179	.204	.189
2.0	.833	.846	.840	6.3	.424	.450	.435	11.2	.279	.305	.290	19.8	.178	.202	.187
2.1	.817	.830	.824	6.4	.419	.445	.430	11.4	.275	.301	.286	20.0	.177	.200	.186
2.2	.798	.812	.805	6.5	.414	.441	.426	11.6	.272	.298	.283	20.2	.175	.198	.184
2.3	.780	.795	.787	6.6	.409	.436	.421	11.8	.268	.294	.279	20.4	.174	.196	.183
2.4	.763	.780	.771	6.7	.405	.432	.417	12.0	.264	.290	.275	20.6	.173	.194	.182
2.5	.748	.766	.756	6.8	.401	.428	.413	12.2	.261	.287	.272	20.8	.171	.193	.180
2.6	.732	.750	.740	6.9	.396	.424	.408	12.4	.257	.283	.268	21.0	.169	.192	.178
2.7	.718	.736	.726	7.0	.393	.421	.405	12.6	.254	.280	.265	21.2	.168	.191	.177
2.8	.705	.723	.713	7.1	.389	.417	.401	12.8	.251	.277	.262	21.4	.167	.190	.176
2.9	.692	.710	.700	7.2	.385	.413	.397	13.0	.248	.274	.259	21.6	.165	.188	.174
3.0	.680	.699	.688	7.3	.381	.410	.393	13.2	.245	.271	.256	21.8	.164	.187	.173
3.1	.668	.687	.676	7.4	.377	.406	.390	13.4	.242	.268	.253	22.0	.163	.186	.172
3.2	.656	.675	.664	7.5	.373	.402	.386	13.6	.239	.265	.250	22.2	.162	.185	.171
3.3	.645	.664	.653	7.6	.370	.399	.383	13.8	.236	.262	.247	22.4	.161	.184	.170
3.4	.634	.653	.642	7.7	.367	.396	.380	14.0	.234	.260	.245	22.6	.160	.183	.169
3.5	.622	.642	.631	7.8	.363	.392	.376	14.2	.231	.257	.242	22.8	.159	.182	.168
3.6	.612	.632	.621	7.9	.360	.389	.373	14.4	.228	.254	.239	23.0	.158	.180	.167
3.7	.602	.622	.611	8.0	.356	.385	.370	14.6	.225	.251	.236	23.2	.156	.179	.165
3.8	.593	.613	.602	8.1	.353	.382	.367	14.8	.223	.249	.234	23.4	.155	.178	.164
3.9	.584	.604	.593	8.2	.350	.379	.364	15.0	.221	.247	.232	23.6	.154	.177	.163
4.0	.572	.596	.583	8.3	.347	.376	.361	15.2	.219	.245	.230	23.8	.153	.176	.162
4.1	.565	.587	.575	8.4	.344	.373	.358	15.4	.217	.242	.227	24.0	.151	.174	.160
4.2	.556	.578	.566	8.5	.341	.371	.355	15.6	.215	.240	.225	24.2	.150	.173	.159
4.3	.548	.570	.558	8.6	.338	.368	.352	15.8	.213	.238	.223	24.4	.149	.172	.158
4.4	.540	.563	.550	8.7	.335	.364	.349	16.0	.211	.236	.221	24.6	.148	.171	.157
4.5	.532	.555	.542	8.8	.332	.361	.346	16.2	.209	.234	.219	24.8	.147	.170	.156
4.6	.525	.548	.535	8.9	.330	.358	.34	16.4	.207	.232	.217	25.0	.146	.169	.155
4.7	.518	.542	.528	9.0	.327	.355	.340	16.6	.205	.230	.215				
4.8	.511	.535	.521	9.1	.324	.353	.337	16.8	.203	.228	.213				
4.9	.504	.528	.514	9.2	.322	.351	.335	17.0	.201	.226	.211				
5.0	.496	.522	.506	9.3	.320	.348	.332	17.2	.199	.224	.209				
5.1	.490	.515	.500	9.4	.317	.345	.329	17.4	[illegible]	.222	.207				
5.2	.484	.509	.494	9.5	.315	.343	.327	17.6	.105	.220	.205				

Column r, the ratio of expansion $= \frac{v_2}{v_1}$

" A, ratio of mean to initial pressure, $\frac{p_m}{p_1} = \frac{10 - 9r^{-\frac{1}{9}}}{r}$ — For dry steam, expanded without gain or loss of heat, in a non-conducting cylinder.

" B, " " " " " $\frac{p_m}{p_1} = \frac{1 + \text{hyp. log. } r}{r}$ — For damp steam, expanded receiving heat.

" C, " " " " " $\frac{p_m}{p_1} = \frac{17 - 16r^{-\frac{1}{16}}}{r}$ — For dry steam, expanded receiving heat sufficient to prevent liquefaction.

RULE.—To find the mean pressure exerted throughout the stroke, multiply the initial pressure by the number opposite the ratio of expansion, in the column corresponding with the conditions of expansion. (From Northcott.)

IV.

TERMINAL PRESSURE RATIOS $\frac{p_1}{p_2}$.

r	A	B	C	r	A	B	C	r	A	B	C	r	A	B	C
1.0	0.00	0.0	0.00	4.7	5.58	4.7	5.18	8.3	10.5	8.3	9.47	13.8	18.5	13.8	16.2
1.1	1.11	1.1	1.11	4.8	5.70	4.8	5.29	8.4	10.6	8.4	9.59	14.0	18.8	14.0	16.5
1.2	1.22	1.2	1.21	4.9	5.84	4.9	5.41	8.5	10.7	8.5	9.64	14.2	19.1	14.2	16.8
1.3	1.34	1.3	1.32	5.0	5.98	5.0	5.52	8.6	10.9	8.6	9.76	14.4	19.4	14.4	17.0
1.4	1.45	1.4	1.43	5.1	6.11	5.1	5.64	8.7	11.0	8.7	9.88	14.6	19.7	14.6	17.2
1.5	1.57	1.5	1.54	5.2	6.24	5.2	5.76	8.8	11.2	8.8	10.0	14.8	20.0	14.8	17.5
1.6	1.69	1.6	1.65	5.3	6.38	5.3	5.88	8.9	11.3	8.9	10.2	15.0	20.3	15.0	17.8
1.7	1.80	1.7	1.75	5.4	6.51	5.4	6.00	9.0	11.5	9.0	10.3	15.2	20.6	15.2	18.0
1.8	1.92	1.8	1.87	5.5	6.64	5.5	6.12	9.1	11.6	9.1	10.4	15.4	20.9	15.4	18.2
1.9	2.04	1.9	1.98	5.6	6.78	5.6	6.23	9.2	11.8	9.2	10.6	15.6	21.2	15.6	18.5
2.0	2.16	2.0	2.08	5.7	6.91	5.7	6.35	9.3	11.9	9.3	10.7	15.8	21.5	15.8	18.7
2.1	2.28	2.1	2.20	5.8	7.05	5.8	6.47	9.4	12.0	9.4	10.8	16.0	21.8	16.0	19.0
2.2	2.40	2.2	2.31	5.9	7.18	5.9	6.59	9.5	12.2	9.5	10.9	16.2	22.1	16.2	19.3
2.3	2.52	2.3	2.42	6.0	7.32	6.0	6.71	9.6	12.3	9.6	11.0	16.4	22.4	16.4	19.5
2.4	2.64	2.4	2.53	6.1	7.45	6.1	6.83	9.7	12.5	9.7	11.1	16.6	22 7	16.6	19.8
2.5	2.76	2.5	2.64	6.2	7.59	6.2	6.95	9.8	12.6	9.8	11.3	16.8	23.0	14.8	20.0
2.6	2.89	2.6	2.76	6.3	7.73	6.3	7.07	9.9	12.8	9.9	11.4	17.0	23.3	17.0	20.3
2.7	3.01	2.7	2.87	6.4	7.86	6.4	7.18	10.0	12.9	10.0	11.5	17.2	23.6	17.2	20.5
2.8	3.14	2.8	2.99	6.5	8.00	6.5	7.30	10.2	13.2	10.2	11.7	17.4	23.9	17.4	20.8
2.9	3.26	2.9	3.10	6.6	8.14	6.6	7.42	10.4	13.5	10.4	12.0	17.6	24.2	17.6	21.0
3.0	3.39	3.0	3.21	6.7	8.27	6.7	7.54	10.6	13.8	10.6	12.3	17.8	24.5	17.8	21.3
3.1	3.51	3.1	3.32	6.8	8.41	6.8	7.66	10.8	14.1	10.8	12.5	18.0	24.8	18.0	21.6
3.2	3.64	3.2	3.43	6.9	8.55	6.9	7.78	11.0	14.3	11.0	12.8	18.2	25.1	18.2	21.8
3.3	3.77	3.3	3.55	7.0	8.69	7.0	7.90	11.2	14.6	11.2	13.0	18.4	25.4	18.4	22.0
3.4	3.89	3.4	3.67	7.1	8.83	7.1	8.02	11.4	14.9	11.4	13.3	18.6	25.7	18.6	22.3
3.5	4.02	3.5	3.79	7.2	8.96	7.2	8.14	11.6	15.2	11.6	13.5	18.8	26.0	18.8	22.5
3.6	4.15	3.6	3.90	7.3	9.10	7.3	8.27	11.8	15.5	11.8	13.7	19.0	26.3	19.0	22.8
3.7	4.28	3.7	4.01	7.4	9.24	7.4	8.38	12.0	15.8	12.0	14.0	19.2	26.6	19.2	23.1
3.8	4.41	3.8	4.13	7.5	9.38	7.5	8.49	12.2	16.1	12.2	14.2	19.4	26.9	19.4	23.3
3.9	4.54	3.9	4.25	7.6	9.52	7.6	8.62	12.4	16.4	12.4	14.5	19.6	27.2	19.6	23.6
4.0	4.66	4.0	4.36	7.7	9.66	7.7	8.74	12.6	16.7	12.6	14.8	19.8	27.5	19.8	23.9
4.1	4.79	4.1	4.47	7.8	9.80	7.8	8.87	12.8	17.0	12.8	15.0	20.0	27.9	20.0	24.1
4.2	4.91	4.2	4.60	7.9	9.94	7.9	8.99	13.0	17.3	13.0	15.2	21.0	29.5	21.0	25.4
4.3	5.05	4.3	4.71	8.0	10.1	8.0	9.11	13.2	17.6	13.2	15.5	22.0	31.0	22.0	26.7
4.4	5.18	4.4	4.82	8.1	10.2	8.1	9.23	13.4	17.9	13.4	15.7	23.0	32.6	23.0	28.0
4.5	5.32	4.5	4.95	8.2	10.3	8.2	9.35	13.6	18.2	13.6	16.0	24.0	34.1	24.0	29.3
4.6	5.45	4.6	5.06												

Column r, ratio of expansion $= \frac{v_2}{v_1}$

" A, ratio of initial to final pressure, $p_2 = \frac{p_1}{r^{\frac{17}{16}}}$ .. { For dry steam, expanded without gain or loss of heat in a non-conducting cylinder.

" B, " " " " $p_2 = \frac{p_1}{r}$ .. { For damp steam, expanded receiving heat.

" C, " " " " $p_2 = \frac{p_1}{r^{\frac{15}{16}}}$ .. { For dry steam, expanded receiving sufficient heat to prevent liquefaction.

RULE.—To find the final pressure obtaining with any ratio of expansion, divide the initial pressure by the number opposite the ratio of expansion, in the column corresponding with the conditions of expansion.

III.—(*Continued.*)

MEAN PRESSURE RATIOS.

r	*A*	*B*	*C*	*r*	*A*	*B*	*C*	*r*	*A*	*B*	*C*	*r*	*A*	*B*	*C*
1.0	1.000	1.000	1.000	5.3	.478	.503	.488	9.6	.312	.340	.324	17.8	.194	.218	.204
1.1	0.996	0.996	0.996	5.4	.472	.497	.482	9.7	.310	.338	.322	18.0	.192	.216	.202
1.2	0.983	0.983	0.983	5.5	.467	.492	.477	9.8	.307	.335	.319	18.2	.190	.215	.200
1.3	.966	.968	.967	5.6	.461	.486	.471	9.9	.305	.333	.317	18.4	.189	.214	.199
1.4	.947	.952	.950	5.7	.456	.481	.466	10.0	.303	.330	.314	18.6	.187	.212	.197
1.5	.928	.934	.931	5.8	.450	.475	.460	10.2	.299	.325	.310	18.8	.185	.210	.195
1.6	.910	.919	.914	5.9	.445	.470	.455	10.4	.295	.321	.306	19.0	.183	.208	.193
1.7	.890	.900	.895	6.0	.440	.465	.450	10.6	.291	.317	.302	19.2	.182	.207	.192
1.8	.870	.880	.875	6.1	.434	.460	.445	10.8	.287	.313	.298	19.4	.180	.205	.190
1.9	.850	.862	.856	6.2	.429	.455	.440	11.0	.283	.309	.294	19.6	.179	.204	.189
2.0	.833	.846	.840	6.3	.424	.450	.435	11.2	.279	.305	.290	19.8	.178	.202	.187
2.1	.817	.830	.824	6.4	.419	.445	.430	11.4	.275	.301	.286	20.0	.177	.200	.186
2.2	.798	.812	.805	6.5	.414	.441	.426	11.6	.272	.298	.283	20.2	.175	.198	.184
2.3	.780	.795	.787	6.6	.409	.436	.421	11.8	.268	.294	.279	20.4	.174	.196	.183
2.4	.763	.780	.771	6.7	.405	.432	.417	12.0	.264	.290	.275	20.6	.173	.194	.182
2.5	.748	.766	.756	6.8	.401	.428	.413	12.2	.261	.287	.272	20.8	.171	.193	.180
2.6	.732	.750	.740	6.9	.396	.424	.408	12.4	.257	.283	.268	21.0	.169	.192	.178
2.7	.718	.736	.726	7.0	.393	.421	.405	12.6	.254	.280	.265	21.2	.168	.191	.177
2.8	.705	.723	.713	7.1	.389	.417	.401	12.8	.251	.277	.262	21.4	.167	.190	.176
2.9	.692	.710	.700	7.2	.385	.413	.397	13.0	.248	.274	.259	21.6	.165	.188	.174
3.0	.680	.699	.688	7.3	.381	.410	.393	13.2	.245	.271	.256	21.8	.164	.187	.173
3.1	.668	.687	.676	7.4	.377	.406	.390	13.4	.242	.268	.253	22.0	.163	.186	.172
3.2	.656	.675	.664	7.5	.373	.402	.386	13.6	.239	.265	.250	22.2	.162	.185	.171
3.3	.645	.664	.653	7.6	.370	.399	.383	13.8	.236	.262	.247	22.4	.161	.184	.170
3.4	.634	.653	.642	7.7	.367	.396	.380	14.0	.234	.260	.245	22.6	.160	.183	.169
3.5	.622	.642	.631	7.8	.363	.392	.376	14.2	.231	.257	.242	22.8	.159	.182	.168
3.6	.612	.632	.621	7.9	.360	.389	.373	14.4	.228	.254	.239	23.0	.158	.180	.167
3.7	.602	.622	.611	8.0	.356	.385	.370	14.6	.225	.251	.236	23.2	.156	.179	.165
3.8	.593	.613	.602	8.1	.353	.382	.367	14.8	.223	.249	.234	23.4	.155	.178	.164
3.9	.584	.604	.593	8.2	.350	.379	.364	15.0	.221	.247	.232	23.6	.154	.177	.163
4.0	.572	.596	.583	8.3	.347	.376	.361	15.2	.219	.245	.230	23.8	.153	.176	.162
4.1	.565	.587	.575	8.4	.344	.373	.358	15.4	.217	.242	.227	24.0	.151	.174	.160
4.2	.556	.578	.566	8.5	.341	.371	.355	15.6	.215	.240	.225	24.2	.150	.173	.159
4.3	.548	.570	.558	8.6	.338	.368	.352	15.8	.213	.238	.223	24.4	.149	.172	.158
4.4	.540	.563	.550	8.7	.335	.364	.349	16.0	.211	.236	.221	24.6	.148	.171	.157
4.5	.532	.555	.542	8.8	.332	.361	.346	16.2	.209	.234	.219	24.8	.147	.170	.156
4.6	.525	.548	.535	8.9	.330	.358	.34	16.4	.207	.232	.217	25.0	.146	.169	.155
4.7	.518	.542	.528	9.0	.327	.355	.340	16.6	.205	.230	.215				
4.8	.511	.535	.521	9.1	.324	.353	.337	16.8	.203	.228	.213				
4.9	.504	.528	.514	9.2	.322	.351	.335	17.0	.201	.226	.211				
5.0	.496	.522	.506	9.3	.320	.348	.332	17.2	.199	.224	.209				
5.1	.490	.515	.500	9.4	.317	.345	.329	17.4	[illegible]	.222	.207				
5.2	.484	.509	.494	9.5	.315	.343	.327	17.6	.105	.220	.205				

Column r, the ratio of expansion $= \frac{v_2}{v_1}$

" A, ratio of mean to initial pressure, $\frac{p_m}{p_1} = \frac{10 - 9r^{-\frac{1}{9}}}{r}$ { For dry steam, expanded without gain or loss of heat, in a non-conducting cylinder.

" B, " " " " " $\frac{p_m}{p_1} = \frac{1 + \text{hyp. log. } r}{r}$ { For damp steam, expanded receiving heat.

" C, " " " " " $\frac{p_m}{p_1} = \frac{17 - 16r^{-\frac{1}{16}}}{r}$ { For dry steam, expanded receiving heat sufficient to prevent liquefaction.

RULE.—To find the mean pressure exerted throughout the stroke, multiply the initial pressure by the number opposite the ratio of expansion, in the column corresponding with the conditions of expansion. (From Northcott.)

IV.

TERMINAL PRESSURE RATIOS $\frac{p_1}{p_2}$.

r	A	B	C	r	A	B	C	r	A	B	C	r	A	B	C
1.0	0.00	0.0	0.00	4.7	5.58	4.7	5.18	8.3	10.5	8.3	9.47	13.8	18.5	13.8	16.2
1.1	1.11	1.1	1.11	4.8	5.70	4.8	5.29	8.4	10.6	8.4	9.59	14.0	18.8	14.0	16.5
1.2	1.22	1.2	1.21	4.9	5.84	4.9	5.41	8.5	10.7	8.5	9.64	14.2	19.1	14.2	16.8
1.3	1.34	1.3	1.32	5.0	5.98	5.0	5.52	8.6	10.9	8.6	9.76	14.4	19.4	14.4	17.0
1.4	1.45	1.4	1.43	5.1	6.11	5.1	5.64	8.7	11.0	8.7	9.88	14.6	19.7	14.6	17.2
1.5	1.57	1.5	1.54	5.2	6.24	5.2	5.76	8.8	11.2	8.8	10.0	14.8	20.0	14.8	17.5
1.6	1.69	1.6	1.65	5.3	6.38	5.3	5.88	8.9	11.3	8.9	10.2	15.0	20.3	15.0	17.8
1.7	1.80	1.7	1.75	5.4	6.51	5.4	6.00	9.0	11.5	9.0	10.3	15.2	20.6	15.2	18.0
1.8	1.92	1.8	1.87	5.5	6.64	5.5	6.12	9.1	11.6	9.1	10.4	15.4	20.9	15.4	18.2
1.9	2.04	1.9	1.98	5.6	6.78	5.6	6.23	9.2	11.8	9.2	10.6	15.6	21.2	15.6	18.5
2.0	2.16	2.0	2.08	5.7	6.91	5.7	6.35	9.3	11.9	9.3	10.7	15.8	21.5	15.8	18.7
2.1	2.28	2.1	2.20	5.8	7.05	5.8	6.47	9.4	12.0	9.4	10.8	16.0	21.8	16.0	19.0
2.2	2.40	2.2	2.31	5.9	7.18	5.9	6.59	9.5	12.2	9.5	10.9	16.2	22.1	16.2	19.3
2.3	2.52	2.3	2.42	6.0	7.32	6.0	6.71	9.6	12.3	9.6	11.0	16.4	22.4	16.4	19.5
2.4	2.64	2.4	2.53	6.1	7.45	6.1	6.83	9.7	12.5	9.7	11.1	16.6	22.7	16.6	19.8
2.5	2.76	2.5	2.64	6.2	7.59	6.2	6.95	9.8	12.6	9.8	11.3	16.8	23.0	14.8	20.0
2.6	2.89	2.6	2.76	6.3	7.73	6.3	7.07	9.9	12.8	9.9	11.4	17.0	23.3	17.0	20.3
2.7	3.01	2.7	2.87	6.4	7.86	6.4	7.18	10.0	12.9	10.0	11.5	17.2	23.6	17.2	20.5
2.8	3.14	2.8	2.99	6.5	8.00	6.5	7.30	10.2	13.2	10.2	11.7	17.4	23.9	17.4	20.8
2.9	3.26	2.9	3.10	6.6	8.14	6.6	7.42	10.4	13.5	10.4	12.0	17.6	24.2	17.6	21.0
3.0	3.39	3.0	3.21	6.7	8.27	6.7	7.54	10.6	13.8	10.6	12.3	17.8	24.5	17.8	21.3
3.1	3.51	3.1	3.32	6.8	8.41	6.8	7.66	10.8	14.1	10.8	12.5	18.0	24.8	18.0	21.6
3.2	3.64	3.2	3.43	6.9	8.55	6.9	7.78	11.0	14.3	11.0	12.8	18.2	25.1	18.2	21.8
3.3	3.77	3.3	3.55	7.0	8.69	7.0	7.90	11.2	14.6	11.2	13.0	18.4	25.4	18.4	22.0
3.4	3.89	3.4	3.67	7.1	8.83	7.1	8.02	11.4	14.9	11.4	13.3	18.6	25.7	18.6	22.3
3.5	4.02	3.5	3.79	7.2	8.96	7.2	8.14	11.6	15.2	11.6	13.5	18.8	26.0	18.8	22.5
3.6	4.15	3.6	3.90	7.3	9.10	7.3	8.27	11.8	15.5	11.8	13.7	19.0	26.3	19.0	22.8
3.7	4.28	3.7	4.01	7.4	9.24	7.4	8.38	12.0	15.8	12.0	14.0	19.2	26.6	19.2	23.1
3.8	4.41	3.8	4.13	7.5	9.38	7.5	8.49	12.2	16.1	12.2	14.2	19.4	26.9	19.4	23.3
3.9	4.54	3.9	4.25	7.6	9.52	7.6	8.62	12.4	16.4	12.4	14.5	19.6	27.2	19.6	23.6
4.0	4.66	4.0	4.36	7.7	9.66	7.7	8.74	12.6	16.7	12.6	14.8	19.8	27.5	19.8	23.9
4.1	4.79	4.1	4.47	7.8	9.80	7.8	8.87	12.8	17.0	12.8	15.0	20.0	27.9	20.0	24.1
4.2	4.91	4.2	4.60	7.9	9.94	7.9	8.99	13.0	17.3	13.0	15.2	21.0	29.5	21.0	25.4
4.3	5.05	4.3	4.71	8.0	10.1	8.0	9.11	13.2	17.6	13.2	15.5	22.0	31.0	22.0	26.7
4.4	5.18	4.4	4.82	8.1	10.2	8.1	9.23	13.4	17.9	13.4	15.7	23.0	32.6	23.0	28.0
4.5	5.32	4.5	4.95	8.2	10.3	8.2	9.35	13.6	18.2	13.6	16.0	24.0	34.1	24.0	29.3
4.6	5.45	4.6	5.06												

Column r, ratio of expansion $= \frac{v_2}{v_1}$

" A, ratio of initial to final pressure, $p_2 = \frac{p_1}{r^{\frac{10}{9}}}$ .. { For dry steam, expanded without gain or loss of heat in a non-conducting cylinder.

" B, " " " " " $p_2 = \frac{p_1}{r}$ .. { For damp steam, expanded receiving heat.

" C, " " " " " $p_2 = \frac{p_1}{r^{\frac{17}{16}}}$ .. { For dry steam, expanded receiving sufficient heat to prevent liquefaction.

RULE.—To find the final pressure obtaining with any ratio of expansion, divide the initial pressure by the number opposite the ratio of expansion, in the column corresponding with the conditions of expansion.

V.

WORKING OF STEAM.—(NORTHCOTT.)

HEAT-TRANSFER AND TRANSFORMATION.

Initial absolute pressure per sq. in.	Ratio of Expansion.	Mean total pressure.	Mean Effective Pressure.	Mean Back Pressure.	Pressure at Release.	Indicated Work per lb. of Steam.	Steam per Indicated Horse-power per Hour.	Piston Displacement per lb. of Steam.	Piston Displacement per Indicated Horse-power per Hour.	Piston Area per Indicated Horse-power with speed of 330 ft. per minute.	Heat entering Cylinder per lb. of Steam.	Heat imparted during Expansion per lb. of Steam.	Heat expended per lb. of Steam.	Heat converted into Motive Power indicated per lb. of Steam.	Heat carried off with the Exhaust Steam per lb.	Heat expended per Indicated Horse-power per Hour.	Efficiency of Steam.	Coal per Indicated Horse-power per Hour with Boiler of .71 efficiency.
p_1.	r.	p_m.	p_e.	p_3.	p_2.	Ft. lbs.	Lbs.	Cu. ft.	Cu. ft.	Sq. in.	Units.	Units.	Units.	Units.	Units.	Units.	E.	Lbs.
Class 1											From 212° F.		From 212° F.		From 212° F.	From 212° F.		
60	1	60	44	16	60	44,660	44.4	7.0500	313.0	2.27	991	0	991	57.8	933.2	44,001	.0582	4.40
80	1	80	64	16	80	49,698	39.9	5.3926	215.0	1.56	997	0	997	64.4	932.6	39,781	.0644	3.98
100	1	100	84	16	100	52,952	37.4	4.3777	163.7	1.19	1,002	0	1,002	68.6	933.4	37,475	.0684	3.75
120	1	120	104	16	120	55,234	35.8	2.6887	132.0	0.96	1,006	0	1,006	71.5	934.5	36,015	.0712	3.61
150	1	150	134	16	150	57,673	34.4	2.9888	102.8	0.75	1,011	0	1,011	74.7	936.3	34,779	.0737	3.48
200	1	200	184	16	200	60,475	32.7	2.2834	74.6	0.54	1,018	0	1,018	78.3	939.7	33,289	.0770	3.33
250	1	250	234	16	250	62,454	31.7	1.8533	58.7	0.43	1,024	0	1,024	80.9	943.1	32,461	.0790	3.25
300	1	300	284	16	300	64,016	30.9	1.5653	48.3	0.35	1,029	0	1,029	82.9	946.1	31,797	.0806	3.18
Class 2											From 212° F.		From 212° F.		From 212° F.	From 212° F.		
60	2	50.7	34.7	16	30	70,445	29.5	14.1000	396.2	2.89	991	0	991	91.3	899.7	29,227	.0877	2.93
80	2	67.6	51.6	16	40	80,139	25.9	10.7852	266.4	1.94	997	0	997	103.8	893.2	25,823	.0993	2.59
100	2	84.6	68.6	16	50	86,490	24.0	8.7554	200.5	1.46	1,002	0	1,002	112.0	890.0	24,049	.1066	2.41
120	2	101.5	85.5	16	60	90,840	22.9	7.3774	160.8	1.17	1,006	0	1,006	117.6	888.4	23,038	.1113	2.31
150	2	126.9	110.9	16	75	95,442	21.8	5.9776	124.3	0.90	1,011	0	1,011	123.6	887.4	22,040	.1118	2.21
200	2	169.2	153.2	16	100	100,603	20.7	4.5648	89.9	0.65	1,018	0	1,018	130.3	887.7	21,073	.1217	2.11
250	2	211.5	195.5	16	125	104,406	19.9	3.7066	70.4	0.51	1,024	0	1,024	135.2	888.8	20,478	.1252	2.05
300	2	253.8	237.8	16	150	107,202	19.4	3.1306	57.9	0.42	1,029	0	1,029	138.8	890.2	19,963	.1280	1.99

Class 3											From 212° F.		From 212° F.		From 212° F.	From 212° F.		
60	3.4	38.0	22.0	16	16	77,200	25.7	24.32	625.0	4.54	991	77	1,068	100	968	27,448	.0934	2.75
80	4.5	43.1	27.1	16	16	94,956	20.9	24.32	508.3	3.69	997	94	1,091	123	968	22,802	.1120	2.28
100	5.5	47.1	31.1	16	16	108,852	18.2	24.32	441.6	3.21	1,002	107	1,109	141	968	20,184	.1270	2.02
120	6.6	50.6	34.6	16	16	121,204	16.3	24.32	396.4	2.89	1,006	119	1,125	157	968	18,338	.1398	1.84
150	8.1	54.1	38.1	16	16	133,556	14.8	24.32	359.9	2.61	1,011	130	1,141	173	968	16,887	.1518	1.69
250	10.6	59.6	43.6	16	16	152,856	12.9	24.32	313.7	2.28	1,018	148	1,166	198	968	15,042	.1705	1.51
200	13.1	65.8	47.8	16	16	167,524	11.8	24.32	287.0	2.08	1,024	161	1,185	217	968	13,984	.1834	1.40
300	15.5	67.1	51.1	16	16	179,104	11.0	24.32	267.5	1.94	1,029	171	1,200	232	968	13,200	.1943	1.32
Class 4											From 102° F.		From 102° F.		From 102° F.	From 102° F.		
60	3.4	38.0	35.0	3	16	122,826	16.1	24.32	391.5	2.84	1,101	77	1,178	159.0	1,019	18,966	.1352	1.90
80	4.5	43.1	40.1	3	16	140,582	14.0	24.32	330.4	2.40	1,107	94	1,201	182.0	1,019	16,815	.1465	1.69
100	5.5	47.1	44.1	3	16	154,478	12.8	24.32	311.2	2.26	1,112	107	1,219	200.0	1,019	15,604	.1624	1.57
120	6.6	50.6	47.6	3	16	166,830	11.8	24.32	286.9	2.08	1,116	119	1,235	216.0	1,019	14,573	.1759	1.46
150	8.1	54.1	51.1	3	16	179,182	11.0	24.32	267.5	1.94	1,121	130	1,251	232.0	1,019	13,761	.1856	1.38
200	10.7	59.6	56.6	3	16	198,482	9.9	24.32	240.7	1.75	1,128	148	1,276	257.0	1,019	12,633	.2002	1.27
250	13.1	65.8	62.8	3	16	213,150	9.2	24.32	223.7	1.62	1,134	161	1,295	276.0	1,019	11,914	.2152	1.20
300	15.5	67.1	64.1	3	16	224,730	8.8	24.32	214.0	1.55	1,139	171	1,310	291.0	1,019	11,528	.2138	1.16
Classes 5 & 6											From 102° F.		From 102° F.		From 102° F.	From 102° F.		
60	16.7	12.8	9.8	3	3	166,752	11.9	118	1,404	10.2	1,101	170	1,271	216	1,055	15,125	.1695	1.52
80	23.7	13.8	10.8	3	3	184,508	10.7	118	1,262	9.2	1,107	184	1,294	239	1,055	13,846	.1852	1.39
100	26.9	14.7	11.7	3	3	198,404	10.0	118	1,180	8.5	1,112	200	1,312	257	1,055	13,120	.1954	1.32
120	32.0	15.4	12.4	3	3	210,756	9.4	118	1,109	8.0	1,116	212	1,328	273	1,055	12,484	.2052	1.25
150	39.5	16.2	13.2	3	3	223,108	8.9	118	1,050	7.6	1,121	223	1,344	289	1,055	11,962	.2144	1.20
200	51.7	17.3	14.3	3	3	242,408	8.2	118	968	7.0	1,128	241	1,369	314	1,055	11,226	.2284	1.13
250	63.6	18.2	15.2	3	3	257,076	7.7	118	900	6.6	1,134	254	1,388	333	1,055	10,688	.2399	1.07
300	75.3	18.8	15.8	3	3	268,656	7.4	118	873	6.1	1,139	264	1,403	348	1,055	10,383	.2470	1.04

CLASS NO.	KIND OF ENGINE.
1	Non-condensing; $r = 1$.
2	" " $r = 2$.
3	" " expanding to back-pressure.
4	Condensing; moderate expansion.
5	" jacketed, full expansion.
6	" jacketed, compound expansion complete.

VI.

COMPARISON OF THERMOMETERS.

Celsius.	Réaumur.	Fahrenheit.	Celsius.	Réaumur.	Fahrenheit.	Celsius.	Réaumur.	Fahrenheit.
−20	−16	−4	25	20.0	77.0	70	56.0	158.0
−19	−15.2	−2.2	26	20.8	78.8	71	56.8	159.8
−18	−14.4	−0.4	27	21.6	80.6	72	57.6	161.6
−17	−13.6	1.4	28	22.4	82.4	73	58.4	163.4
−16	−12.8	3.2	29	23.2	84.2	74	59.2	165.2
−15	−12.0	5.0	30	24.0	86.0	75	60.0	167.0
−14	−11.2	6.8	31	24.8	87.8	76	60.8	168.8
−13	−10.4	8.6	32	25.6	89.6	77	61.6	170.6
−12	−9.6	10.4	33	26.4	91.4	78	62.4	172.4
−11	−8.8	12.2	34	27.2	93.2	79	63.2	174.2
−10	−8.0	14.0	35	28.0	95.0	80	64.0	176.0
−9	−7.2	15.8	36	28.8	96.8	81	64.8	177.8
−8	−6.4	17.6	37	29.6	98.6	82	65.6	179.6
−7	−5.6	19.4	38	30.4	100.4	83	66.4	181.4
−6	−4.8	21.2	39	31.2	102.2	84	67.2	183.2
−5	−4.0	23.0	40	32.0	104.0	85	68.0	185.0
−4	−3.2	24.8	41	32.8	105.8	86	68.8	186.8
−3	−2.4	26.6	42	33.6	107.6	87	69.6	188.6
−2	−1.6	28.4	43	34.4	109.4	88	70.4	190.4
−1	−0.8	30.2	44	35.2	111.2	89	71.2	192.2
0	0	32.0	45	36.0	113.0	90	72.0	194.0
1	0.8	33.8	46	36.8	114.8	91	72.8	195.8
2	1.6	35.6	47	37.6	116.6	92	73.6	197.6
3	2.4	37.4	48	38.4	118.4	93	74.4	199.4
4	3.2	39.2	49	39.2	120.2	94	75.2	201.2
5	4.0	41.0	50	40.0	122.0	95	76.0	203.0
6	4.8	42.8	51	40.8	123.8	96	76.8	204.8
7	5.6	44.6	52	41.6	125.6	97	77.6	206.6
8	6.4	46.4	53	42.4	127.4	98	78.4	208.4
9	7.2	48.2	54	43.2	129.2	99	79.2	210.2
10	8.0	50.0	55	44.0	131.0	100	80.0	212.0
11	8.8	51.8	56	44.8	132.8	101	80.8	213.8
12	9.6	53.6	57	45.6	134.6	102	81.6	215.6
13	10.4	55.4	58	46.4	136.4	103	82.4	217.4
14	11.2	57.2	59	47.2	138.2	104	83.2	219.2
15	12.0	59.0	60	48.0	140.0	105	84.0	221.0
16	12.8	60.8	61	48.8	141.8	106	84.8	222.8
17	13.6	62.6	62	49.6	143.6	107	85.6	224.6
18	14.4	64.4	63	50.4	145.4	108	86.4	226.4
19	15.2	66.2	64	51.2	147.2	109	87.2	228.2
20	16.0	68.0	65	52.0	149.0	110	88.0	230.0
21	16.8	69.8	66	52.8	150.8	111	88.8	231.8
22	17.6	71.6	67	53.6	152.6	112	89.6	233.6
23	18.4	73.4	68	54.4	154.4	113	90.4	235.4
24	19.2	75.2	69	55.2	156.2	114	91.2	237.2

COMPARISON OF THERMOMETERS—*Continued.*

Celsius.	Réaumur.	Fahrenheit.	Celsius.	Réaumur.	Fahrenheit.	Celsius.	Réaumur.	Fahrenheit.
115	92.0	239.0	127	101.6	260.6	139	111.2	282.2
116	92.8	240.8	128	102.4	262.4	140	112.0	284.0
117	93.6	242.6	129	103.2	264.2	141	112.8	285.8
118	94.4	244.4	130	104.0	266.0	142	113.6	287.6
119	95.2	246.2	131	104.8	267.8	143	114.4	289.4
120	96.0	248.0	132	105.6	269.6	144	115.2	291.2
121	96.8	249.8	133	106.4	271.4	145	116.0	293.0
122	97.6	251.6	134	107.2	273.2	146	116.8	294.8
123	98.4	253.4	135	108.0	275.0	147	117.6	296.6
124	99.2	255.2	136	108.8	276.8	148	118.4	298.4
125	100.0	257.0	137	109.6	278.6	149	119.2	300.2
126	100.8	258.8	138	110.4	280.4	150	120.0	302.0

VII.

DENSITIES AND VOLUMES OF WATER.

KOPP; CORRECTED BY PORTER.

Temperature. F.	Temperature. C.	Volume, Kopp.	Corrected Volume.	Differences.	Differences.
39.2	4	1.00000	1.00000		
41.0	5	1.00001	1.00001		
51.8	10	1.00025	1.00025	24	
59.0	15	1.00082*	1.00083	58	34
68.0	20	1.00169	1.00171	88	30
77.0	25	1.00284	1.00286	115	27
86.0	30	1.00423	1.00425	139	24
95.0	35	1.00583	1.00586	161	22
104.0	40	1.00768	1.00767	181	20
113.0	45	1.00967	1.00967	200	19
122.0	50	1.01190	1.01186	219	19
131.0	55	1.01423	1.01423	237	18
140.0	60	1.01672	1.01678	255	18
149.0	65	1.01943	1.01951	273	18
150.0	70	1.02238	1.02241	290	17
167.0	75	1.02554	1.02548	307	17
176.0	80	1.02871	1.02872	324	17
185.0	85	1.03202	1.03213	341	17
194.0	90	1.03553	1.03570	357	16
203.0	95	1.03921	1.03943	373	16
212.0	100	1.04312	1.04332	389	16

WEIGHTS AND VOLUMES.

Temperature.	Ratio of volume to that of equal weight at maximum density.	Weight of a cubic foot.	Temperature.	Ratio of volume to that of equal weight at maximum density.	Weight of a cubic foot.	Temperature.	Ratio of volume to that of equal weight at maximum density.	Weight of a cubic foot.
Fahr.		Lbs.	Fahr.		Lbs.	Fahr.		Lbs.
32.°	1.000129	62.417	210.°	1.04226	59.894	390.°	1.15538	54.030
39.1	1.000000	62.425	212.	1.04312	59.707	400.	1.16366	53.635
40.	1.000004	62.423	220.	1.04668	59.641	410.	1.17218	53.255
50.	1.000253	62.409	230.	1.05142	59.372	420.	1.18090	52.862
60.	1.000929	62.367	240.	1.05633	59.096	430.	1.18982	52.466
70.	1.001981	62.302	250.	1.06144	58.812	440.	1.19898	52.065
80.	1.00332	62.218	260.	1.06679	58.517	450.	1.20833	51.662
90.	1.00492	62.129	270.	1.07233	58.214	460.	1.21790	51.256
100.	1.00686	62.000	280.	1.07809	57.903	470.	1.22767	50.848
110.	1.00902	61.867	290.	1.08405	57.585	480.	1.23766	50.438
120.	1.01143	61.720	300.	1.09023	57.259	490.	1.24785	50.026
130.	1.01411	61.556	310.	1.09661	56.925	500.	1.25828	49.611
140.	1.01690	61.388	320.	1.10323	56.584	510.	1.26892	49.195
150.	1.01995	61.204	330.	1.11005	56.236	520.	1.27975	48.778
160.	1.02324	61.007	340.	1.11706	55.883	530.	1.29080	48.360
170.	1.02671	60.801	350.	1.12431	55.523	540.	1.30204	47.942
180.	1.03033	60.587	360.	1.13175	55.158	550.	1.31354	47.522
190.	1.03411	60.366	370.	1.13942	54.787			
200.	1.03807	60.136	380.	1.14729	54.411			

VIII.

TEMPERATURES AND PRESSURES, SATURATED STEAM.

IN METRIC MEASURES AND FROM REGNAULT.

Temperature.	Steam-pressure.		Temperature.	Steam-pressure.	
	In Centimetres.	In Atmospheres		In Centimetres.	In Atmospheres
− 32° *C.*	0.0320	0.0004	+ 14° *C.*	1.1908	0.016
31	0.0352	0.0005	15	1.2699	0.017
30	0.0386	0.0005	16	1.3536	0.018
29	0.0424	0.0006	17	1.4421	0.019
28	0.0464	0.0006	18	1.5357	0.020
27	0.0508	0.0007	19	1.6346	0.022
26	0.0555	0.0007	20	1.7391	0.023
25	0.0605	0.0008	21	1.8495	0.024
24	0.0660	0.0009	22	1.9659	0.026
23	0.0719	0.0009	23	2.0888	0.028
22	0.0783	0.0010	24	2.2184	0.029
21	0.0853	0.0011	25	2.3550	0.031
20	0.0927	0.0012	26	2.4988	0.033
19	0.1008	0.0013	27	2.5505	0.034
18	0.1095	0.0014	28	2.8101	0.037
17	0.1189	0.0015	29	2.9782	0.039
16	0.1290	0.0017	30	3.1548	0.042
15	0.1400	0.0018	31	3.3406	0.044
14	0.1518	0.0020	32	3.5359	0.047
13	0.1646	0.0022	33	3.7411	0.049
12	0.1783	0.0024	34	3.9565	0.052
11	0.1933	0.0025	35	4.1827	0.055
10	0.2093	0.0027	36	4.4201	0.058
9	0.2267	0.0030	37	4.6691	0.061
8	0.2455	0.0032	38	4.9302	0.065
7	0.2658	0.0035	39	5.2039	0.068
6	0.2876	0.0038	40	5.4906	0.072
5	0.3113	0.0041	41	5.7910	0.076
4	0.3368	0.0044	42	6.1055	0.080
3	0.3644	0.0048	43	6.4346	0.085
2	0.3941	0.0052	44	6.7790	0.089
1	0.4263	0.0056	45	7.1391	0.094
0	0.4600	0.0061	46	7.5158	0.099
+ 1	0.4940	0.0065	47	7.9093	0.104
2	0.5302	0.0070	48	8.3204	0.109
3	0.5687	0.0073	49	8.7499	0.115
4	0.6097	0.0080	50	9.1982	0.121
5	0.6534	0.0086	51	9.6661	0.127
6	0.6998	0.0092	52	10.1543	0.134
7	0.7492	0.0199	53	10.6636	0.140
8	0.8017	0.0107	54	11.1945	0.147
9	0.8574	0.011	55	11.7478	0.155
10	0.9165	0.012	56	12.3244	0.163
11	0.9792	0.013	57	12.9251	0.170
12	1.0457	0.014	58	13.5505	0.178
13	1.1162	0.015	59	14.2015	0.187

TEMPERATURES AND PRESSURES, SATURATED STEAM—*Continued.*

Temperature.	Steam-pressure.		Temperature.	Steam-pressure.	
	In Centimetres.	In Atmospheres		In Centimetres.	In Atmospheres
+60° *C.*	14.8791	0.196	+110° *C.*	107.537	1.415
61	15.5839	0.205	111	111.209	1.463
62	16.3170	0.215	112	114.983	1.513
63	17.0791	0.225	113	118.861	1.564
64	17.8714	0.235	114	122.847	1.616
65	18.6945	0.246	115	126.941	1.670
66	19.5496	0.257	116	131.147	1.726
67	20.4376	0.267	117	135.466	1.782
68	21.3596	0.281	118	139.902	1.841
69	22.3165	0.294	119	144.455	1.901
70	23.3093	0.306	120	149.128	1.962
71	24.3393	0.320	121	153.925	2.025
72	25.4073	0.334	122	158.847	2.091
73	26.5147	0.349	123	163.896	2.157
74	27.6624	0.364	124	169.076	2.225
75	28.8517	0.380	125	174.388	2.295
76	30.0838	0.396	126	179.835	2.366
77	31.3600	0.414	127	185.420	2.430
78	32.6811	0.430	128	191.147	2.515
79	34.0488	0.448	129	197.015	2.592
80	35.4643	0.466	130	203.028	2.671
81	36.9287	0.486	131	209.194	2.753
82	38.4435	0.506	132	215.503	2.836
83	40.0101	0.526	133	221.969	2.921
84	41.6298	0.548	134	228.592	3.008
85	43.3041	0.570	135	235.373	3.097
86	45.0344	0.593	136	242.316	3.188
87	46.8221	0.616	137	249.423	3.282
88	48.6687	0.640	138	256.700	3.378
89	50.5759	0.665	139	264.144	3.476
90	52.5450	0.691	140	271.763	3.576
91	54.5778	0.719	141	279.557	3.678
92	56.6757	0.746	142	287.530	3.783
93	58.8406	0.774	143	295.686	3.890
94	61.0740	0.804	144	304.026	4.000
95	63.3778	0.834	145	312.555	4.113
96	65.7535	0.865	146	321.274	4.227
97	68.2029	0.897	147	330.187	4.344
98	70.7280	0.931	148	339.298	4.464
99	73.3305	0.965	149	348.609	4.587
100	76.000	1.000	150	358.123	4.712
101	76.7590	1.036	151	367.843	4.840
102	81.6010	1.074	152	377.774	4.971
103	84.5280	1.112	153	387.918	5.104
104	87.5410	1.152	154	398.277	5.240
105	90.6410	1.193	155	408.856	5.380
106	93.8310	1.235	156	419.659	5.522
107	97.1140	1.278	157	430.688	5.667
108	100.4910	1.322	158	441.945	5.815
109	103.965	1.368	159	453.436	5.966

TEMPERATURES AND PRESSURES, SATURATED STEAM—*Continued.*

Temperature.	Steam-pressure.		Temperature.	Steam-pressure.	
	In Centimetres.	In Atmospheres		In Centimetres.	In Atmospheres
+160° C.	465.162	6.120	+196° C.	1074.595	14.139
161	477.128	6.278	197	1097.500	14.441
162	489.336	6.439	198	1120.982	14.749
163	501.791	6.603	199	1144.746	15.062
164	514.497	6.770	200	1168.896	15.380
165	527.454	6.940	201	1193.437	15.703
166	540.669	7.114	202	1218.369	16.031
167	554.143	7.291	203	1243.700	16.364
168	567.882	7.472	204	1269.430	16.703
169	581.890	7.656	205	1295.566	17.047
170	596.166	7.844	206	1322.112	17.396
171	610.719	8.036	207	1349.075	17.751
172	625.548	8.231	208	1376.453	18.111
173	640.660	8.430	209	1404.252	18.477
174	656.055	8.632	210	1432.480	18.848
175	671.743	8.839	211	1461.132	19.226
176	687.722	9.049	212	1490.222	19.608
177	703.997	9.263	213	1519.748	19.997
178	720.572	9.481	214	1549.717	20.391
179	737.452	9.703	215	1580.133	20.791
180	754.639	9.929	216	1610.994	21.197
181	772.137	10.150	217	1642.315	21.690
182	789.952	10.394	218	1674.090	22.027
183	808.084	10.633	219	1706.329	22.452
184	826.540	10.876	220	1739.036	22.882
185	845.323	11.123	221	1772.213	23.319
186	864.435	11.374	222	1805.864	23.761
187	883.882	11.630	223	1839.994	24.210
188	903.668	11.885	224	1874.607	24.666
189	923.795	12.155	225	1909.704	25.128
190	944.270	12.425	226	1945.292	25.596
191	965.093	12.699	227	1981.376	26.071
192	986.271	12.977	228	2017.961	26.552
193	1007.804	13.261	229	2055.048	27.040
194	1029.701	13.549	230	2092.640	27.535
195	1051.963	13.842			

IX.

METRIC STEAM AND WORK TABLE.

Absolute pressures in Atmosphere.	Specific volumes v_o in Cu. meters.	Product $p_o v_o$.	$W = \frac{26127.34}{1000\, p_o v_o}$	$W \cdot p_o$.
0.1	14.504	1.450	18.010	1.801
0.2	7.525	1.505	17.418	3.483
0.3	5.128	1.540	16.960	5.088
0.4	3.908	1.560	16.750	6.700
0.5	3.165	1.580	16.530	8.265
0.6	2.665	1.600	16.339	9.803
0.7	2.304	1.610	16.230	11.361
0.8	2.031	1.620	16.120	12.896
0.9	1.818	1.630	16.020	14.418
1.0	1.646	1.646	15.870	15.870
1.1	1.505	1.655	15.780	17.385
1.2	1.386	1.663	15.710	18.852
1.3	1.285	1.670	15.640	20.332
1.4	1.199	1.680	15.540	21.756
1.5	1.123	1.684	15.510	23.265
1.6	1.057	1.691	15.450	24.720
1.7	0.999	1.699	15.370	26.129
1.8	0.946	1.703	15.340	27.612
1.9	0.899	1.708	15.290	29.051
2.0	0.857	1.714	15.243	30.486
2.1	0.819	1.718	15.208	31.937
2.2	0.784	1.725	15.146	33.321
2.3	0.751	1.727	15.128	34.794
2.4	0.722	1.733	15.076	36.182
2.5	0.695	1.741	15.002	37.505
2.6	0.670	1.742	14.990	38.974
2.7	0.646	1.744	14.970	40.190
2.8	0.625	1.750	14.929	41.801
2.9	0.604	1.752	14.921	43.271
3.0	0.586	1.758	14.861	44.583
3.1	0.568	1.761	14.838	45.998
3.2	0.551	1.763	14.818	47.417
3.3	0.535	1.765	14.790	48.807
3.4	0.521	1.771	14.749	50.146
3.5	0.507	1.774	14.723	51.330
3.6	0.493	1.775	14.720	52.992
3.7	0.481	1.780	14.680	54.316
3.8	0.469	1.782	14.660	55.708
3.9	0.458	1.786	14.630	57.057
4.0	0.447	1.788	14.61	58.440
4.1	0.437	1.792	14.58	59.778
4.2	0.427	1.793	14.56	61.152
4.3	0.418	1.797	14.53	62.479
4.4	0.409	1.799	14.52	63.888

METRIC STEAM AND WORK TABLE—*Continued.*

Absolute pressure p_o in Atmospheres.	Specific volumes v_o in Cu. meters.	Product p_o v_o.	$W = \frac{26127.34}{1000\, p_o\, v_o}$	W. p_o.
4.5	0.400	1.800	14.51	65.295
4.6	0.392	1.803	14.49	66.654
4.7	0.384	1.805	14.45	67.915
4.8	0.377	1.810	14.43	69.264
4.9	0.370	1.813	14.41	70.609
5.0	0.363	1.815	14.39	71.950
5.1	0.356	1.816	14.38	73.338
5.2	0.350	1.820	14.36	74.672
5.3	0.343	1.821	14.35	76.055
5.4	0.337	1.823	14.33	77.382
5.5	0.332	1.825	14.31	78.705
5.6	0.326	1.826	14.30	80.080
5.7	0.321	1.829	14.26	81.282
5.8	0.316	1.833	14.25	82.650
5.9	0.311	1.835	14.24	84.016
6.0	0.306	1.836	14.23	85.380
6.25	0.294	1.838	14.21	88.812
6.5	0.284	1.845	14.16	92.040
6.75	0.273	1.848	14.13	95.377
7.0	0.265	1.855	14.10	98.700
7.25	0.256	1.856	14.07	100.997
7.5	0.248	1.860	14.04	105.300
7.75	0.241	1.867	13.99	108.422
8.0	0.234	1.872	13.96	111.680
8.25	0.227	1.873	13.95	114.077
8.5	0.221	1.878	13.91	118.235
8.75	0.215	1.881	13.89	121.537
9.0	0.209	1.883	13.86	124.740
9.25	0.204	1.887	13.84	128.020
9.5	0.199	1.891	13.81	131.195
9.75	0.194	1.893	13.80	134.550
10.0	0.190	1.900	13.75	137.500

X.

PROPERTIES OF SATURATED STEAM.

NOTE.—The following table gives the data required by the engineer in this connection as based upon the experiments of Regnault. The temperatures, pressures, and heat-measures are all from Regnault's experiments. The other quantities were calculated by Mr. R. H. Buel.* adopting the formulas of Rankine already given to obtain quantities not ascertained by direct experiment. The two parts of the latent heat of vaporization are separately determined, and the internal thus distinguished from the external work of expansion. British measures are adopted. The nomenclature is sufficiently well explained by the table-headings.

Pressure above a vacuum, in pounds per square inch.	Temperature, Fahrenheit degrees.	Quantities of Heat. In British Thermal Units. Required to raise the temperature of the water from 32° to $T°$.	Internal latent heat.	External latent heat.	Latent heat of evaporation at pressure P $= I + E$.	Total heat of evaporation above 32° $= S + L$.	Total heat of evaporation above 32°, in units of evaporation.	Weight of a cubic foot of steam, in pounds.	Volume. Of a pound of steam in cubic feet.	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density.	Pressure above a vacuum, in pounds per square inch.
P	t	S	I	E	L	H	U	W	C	V	P
1	102.018	70.040	981.396	61.619	1043.015	1113.055	1.1522	.003027	330.4	20,623	1
2	126.302	94.368	961.980	64.114	1026.094	1120.462	1.1599	.005818	171.9	10,730	2
3	141.654	109.764	949.725	65.655	1015.380	1125.144	1.1647	.008522	117.3	7,325	3
4	153.122	121.271	940.597	66.773	1007.370	1128.641	1.1683	.011172	89.51	5,588	4
5	162.370	130.563	933.239	67.660	1000.899	1131.462	1.1712	.013781	72.56	4,530	5
6	170.173	138.401	927.038	68.403	995.441	1133.842	1.1737	.016357	61.14	3,816	6
7	176.945	145.213	921.654	69.041	990.695	1135.908	1.1758	.018908	52.89	3,302	7
8	182.952	151.255	916.883	69.602	986.485	1137.740	1.1777	.021436	46.65	2,912	8
9	188.357	156.699	912.584	70.106	982.690	1139.389	1.1794	.023944	41.77	2,607	9
10	193.284	161.660	908.672	70.560	979.232	1140.892	1.1810	.026437	37.83	2,361	10
11	197.814	166.225	905.083	70.967	976.050	1142.275	1.1824	.028911	34.59	2,159	11
12	202.012	170.457	901.766	71.332	973.098	1143.555	1.1837	.031376	31.87	1,990	12
13	205.929	174.402	898.683	71.663	970.346	1144.748	1.1849	.033828	29.56	1,845	13
14	209.604	178.112	895.784	71.973	967.757	1145.869	1.1861	.036265	27.58	1,72	14
14.69	212.000	180.531	893.894	72.175	966.069	1146.600	1.1869	.037928	26.37	1,646	14.69
15	213.067	181.608	893.044	72.274	965.318	1146.926	1.1872	.038688	25.85	1,614	15
16	216.347	184.919	890.458	72.549	963.007	1147.926	1.1882	.041109	24.33	1,519	16
17	219.452	188.056	888.007	72.811	960.818	1148.874	1.1892	.043519	22.98	1,434	17

* Weisbach's Mechanics, vol. ii., part ii., Dubois' translation. N. Y.: J. Wiley & Sons. 1884.

P	t	S	I	E	L	H	U	W	C	V	P
18	222.424	191.058	885.661	73.060	958.721	1149.779	1.1901	.045920	21.78	1,359	18
19	225.255	193.918	883.427	73.298	956.725	1150.643	1.1910	.048312	20.70	1,292	19
20	227.964	196.655	881.289	73.525	954.814	1151.469	1.1919	.050696	19.73	1,231	20
21	230.565	199.285	879.239	73.739	952.978	1152.263	1.1927	.053074	18.84	1,176	21
22	233.069	201.817	877.267	73.942	951.209	1153.026	1.1935	.055446	18.04	1,126	22
23	235.479	204.258	875.368	74.136	949.504	1153.762	1.1943	.057812	17.30	1,080	23
24	237.803	206.610	873.538	74.323	947.861	1154.471	1.1950	.060171	16.62	1,038	24
25	240.053	208.887	871.767	74.503	946.270	1155.157	1.1957	.062524	16.00	998.4	25
26	242.225	211.089	870.052	74.678	944.730	1155.819	1.1964	.064870	15.42	962.3	26
27	244.333	213.223	868.391	74.847	943.238	1156.461	1.1971	.067210	14.88	928.8	27
28	246.376	215.293	866.780	75.011	941.791	1157.084	1.1978	.069545	14.38	897.6	28
29	248.363	217.308	865.215	75.168	940.383	1157.691	1.1984	.071875	13.91	868.5	29
30	250.293	219.261	863.700	75.319	939.019	1158.280	1.1990	.074201	13.48	841.3	30
31	252.171	221.165	862.221	75.466	937.687	1158.852	1.1996	.076522	13.07	815.8	31
32	254.002	223.021	860.781	75.608	936.389	1159.410	1.2002	.078839	12.68	791.8	32
33	255.782	224.827	859.382	75.745	935.127	1159.954	1.2008	.081152	12.32	769.2	33
34	257.523	226.594	858.013	75.878	933.891	1160.485	1.2013	.083461	11.98	748.0	34
35	259.221	228.316	856.680	76.007	932.687	1161.003	1.2018	.085766	11.66	727.9	35
36	260.883	230.001	855.375	76.133	931.508	1161.509	1.2023	.088067	11.36	708.8	36
37	262.505	231.650	854.099	76.255	930.354	1162.004	1.2028	.090364	11.07	690.8	37
38	264.093	233.261	852.852	76.375	929.227	1162.488	1.2033	.092657	10.79	673.7	38
39	265.647	234.840	851.629	76.493	928.122	1162.962	1.2038	.094946	10.53	657.5	39
40	267.168	236.386	850.432	76.608	927.040	1163.426	1.2043	.097231	10.28	642.0	40
41	268.660	237.902	849.261	76.719	925.980	1163.882	1.2048	.099514	10.05	627.3	41
42	270.122	239.389	848.113	76.827	924.940	1164.329	1.2053	.101794	9.826	613.3	42
43	271.557	240.846	846.988	76.932	923.920	1164.766	1.2058	.104071	9.609	599.9	43
44	272.965	242.275	845.884	77.035	922.919	1165.194	1.2062	.106345	9.403	587.0	44
45	274.347	243.680	844.799	77.136	921.935	1165.615	1.2066	.108616	9.207	574.7	45
46	275.704	245.061	843.733	77.235	920.968	1166.029	1.2070	.110884	9.018	563.0	46
47	277.036	246.418	842.687	77.331	920.018	1166.436	1.2074	.113149	8.838	551.7	47
48	278.348	247.752	841.659	77.425	919.084	1166.836	1.2078	.115411	8.665	540.9	48
49	279.637	249.064	840.647	77.517	918.164	1167.228	1.2082	.117670	8.498	530.5	49
50	280.904	250.355	839.653	77.607	917.260	1167.615	1.2086	.119927	8.338	520.5	50
51	282.151	251.624	838.675	77.696	916.371	1167.995	1.2090	.122181	8.185	510.9	51
52	283.381	252.875	837.720	77.784	915.494	1168.369	1.2094	.124433	8.037	501.7	52
53	284.589	254.106	836.762	77.870	914.632	1168.738	1.2098	.126682	7.894	492.8	53
54	285.781	255.321	835.827	77.954	913.781	1169.102	1.2102	.128928	7.756	484.2	54
55	286.955	256.518	834.906	78.036	912.942	1169.460	1.2106	.131172	7.624	475.9	55
56	288.111	257.695	834.001	78.117	912.118	1169.813	1.2110	.133414	7.496	467.9	56
57	289.251	258.857	833.108	78.196	911.304	1170.161	1.2114	.135654	7.372	460.2	57

PROPERTIES OF SATURATED STEAM—(*Continued*).

Pressure above a vacuum, in pounds per square inch.	Temperature, Fahrenheit degrees.	Quantities of Heat. In British Thermal Units. Required to raise the temperature of the water from 32° to $T°$.	Internal latent heat.	External latent heat.	Latent heat of evaporation at pressure P $= I + E$.	Total heat of evaporation above 32° $= S + L$.	Total heat of evaporation above 32°, in units of evaporation.	Weight of a cubic foot of steam, in pounds.	Volume. Of a pound of steam in cubic feet.	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density.	Pressure above a vacuum, in pounds per square inch.
P	t	S	I	E	L	H	U	W	C	V	P
58	290.374	260.002	832.228	78.273	910.501	1170.503	1.2117	.137892	7.252	452.7	58
59	291.483	261.132	831.361	78.348	909.709	1170.841	1.2120	.140128	7.136	445.5	59
60	292.575	262.248	830.507	78.421	908.928	1171.176	1.2123	.142362	7.024	438.5	60
61	293.653	263.348	829.663	78.494	908.157	1171.505	1.2127	.144594	6.916	431.7	61
62	294.717	264.433	828.830	78.566	907.396	1171.829	1.2130	.146824	6.811	425.[illegible]	62
63	295.768	265.506	828.005	78.638	906.643	1172.149	1.2133	.149052	6.709	418.8	63
64	296.805	266.566	827.191	78.709	905.900	1172.466	1.2136	.151277	6.610	412.6	64
65	297.830	267.612	826.388	78.779	905.167	1172.779	1.2140	.153500	6.515	406.6	65
66	298.842	268.644	825.596	78.847	904.443	1173.087	1.2143	.155721	6.422	400.8	66
67	299.843	269.666	824.814	78.913	903.727	1173.393	1.2146	.157940	6.332	395.2	67
68	300.831	270.674	824.042	78.978	903.020	1173.694	1.2149	.160157	6.244	389.8	68
69	301.807	271.669	823.280	79.042	902.322	1173.991	1.2152	.162372	6.159	384.5	69
70	302.774	272.657	822.524	79.105	901.629	1174.286	1.2155	.164584	6.076	379.3	70
71	303.728	273.633	821.778	79.167	900.945	1174.578	1.2158	.166794	5.995	374.3	71
72	304.669	274.597	821.041	79.228	900.269	1174.866	1.2161	.169003	5.917	369.4	72
73	305.603	275.550	820.312	79.288	899.600	1175.150	1.2164	.171210	5.841	364.6	73
74	306.526	276.493	819.589	79.349	898.938	1175.431	1.2167	.173417	5.767	360.0	74
75	307.440	277.427	818.873	79.410	898.283	1175.710	1.2170	.175622	5.694	355.5	75
76	308.344	278.350	818.166	79.469	897.635	1175.985	1.2173	.177825	5.624	351.1	76

P	t	S	I	E	L	H	U	W	C	V	P
77	309.239	279.265	817.468	79.526	896.994	1176.259	1.2176	.180027	5.555	346.8	77
78	310.123	280.170	816.777	79.582	896.359	1176.529	1.2179	.182229	5.488	342.6	78
79	311.000	281.066	816.090	79.639	895.729	1176.795	1.2181	.184429	5.422	338.5	79
80	311.866	281.952	815.413	79.695	895.108	1177.060	1.2184	.186627	5.358	334.5	80
81	312.725	282.830	814.742	79.749	894.491	1177.321	1.2187	.188823	5.296	330.6	81
82	313.576	283.701	814.077	79.802	893.879	1177.580	1.2190	.191017	5.235	326.8	82
83	314.417	284.562	813.419	79.856	893.275	1177.83[illegible]	1.2193	.193210	5.176	323.1	83
84	315.250	285.414	812.768	79.909	892.677	1178.09[illegible]	1.2195	.195401	5.118	319.5	84
85	316.076	286.260	812.122	79.961	892.083	1178.343	1.2198	.197591	5.061	315.9	85
86	316.893	287.096	811.484	80.012	891.496	1178.59[illegible]	1.2200	.190781	5.006	312.5	86
87	317.705	287.927	810.850	80.063	890.913	1178.840	1.2203	.201969	4.951	309.1	87
88	318.510	288.750	810.222	80.113	890.335	1179.085	1.2205	.204155	4.898	305.8	88
89	319.306	289.565	809.601	80.162	889.763	1179.328	1.2208	.206340	4.846	302.5	89
90	320.094	290.373	808.986	80.210	889.196	1179.569	1.2210	.208525	4.796	299.4	90
91	320.877	291.176	808.375	80.258	888.633	1179.809	1.2212	.210709	4.746	296.3	91
92	321.653	291.970	807.770	80.305	888.075	1180.045	1.2215	.212892	4.697	293.2	92
93	322.422	292.758	807.170	80.351	887.521	1180.279	1.2217	.215074	4.650	290.2	93
94	323.183	293.539	806.575	80.397	886.972	1180.511	1.2220	.217253	4.603	287.3	94
95	323.939	294.314	805.985	80.442	886.427	1180.741	1.2222	.219430	4.557	284.5	95
96	324.688	295.083	805.400	80.487	885.887	1180.970	1.2224	.221604	4.513	281.7	96
97	325.431	295.845	804.821	80.531	885.352	1181.197	1.2227	.223778	4.469	279.0	97
98	326.169	296.601	804.245	80.576	884.821	1181.422	1.2229	.225950	4.426	276.3	98
99	326.900	297.350	803.675	80.620	884.295	1181.645	1.2232	.228122	4.384	273.7	99
100	327.625	298.093	803.108	80.665	883.773	1181.866	1.2234	.230293	4.342	271.1	100
101	328.345	298.832	802.544	80.709	883.253	1182.085	1.2236	.232464	4.302	268.5	101
102	329.060	299.566	801.985	80.752	882.737	1182.303	1.2238	.234634	4.262	266.0	102
103	329.769	300.293	801.432	80.794	882.226	1182.519	1.2240	.236803	4.223	263.6	103
104	330.471	301.014	800.884	80.835	881.719	1182.733	1.2242	.238972	4.185	261.2	104
105	331.169	301.731	800.339	80.875	881.214	1182.945	1.2245	.241139	4.147	258.9	105
106	331.862	302.444	799.796	80.916	880.712	1183.156	1.2247	.243304	4.110	256.6	106
107	332.550	303.152	799.258	80.956	880.214	1183.366	1.2249	.245467	4.074	254.3	107
108	333.232	303.854	798.725	80.995	879.720	1183.574	1.2251	.247629	4.038	252.1	108
109	333.911	304.551	798.196	81.034	879.230	1183.781	1.2254	.249789	4.003	249.9	109
110	334.582	305.242	797.672	81.072	878.744	1183.986	1.2256	.251947	3.969	247.8	110
111	335.250	305.927	797.153	81.110	878.263	1184.190	1.2258	.254105	3.935	245.7	111
112	335.914	306.609	796.637	81.147	877.784	1184.393	1.2260	.256263	3.902	243.6	112
113	336.573	307.285	796.125	81.184	877.309	1184.594	1.2262	.258420	3.870	241.6	113
114	337.22[illegible]	307.956	795.617	81.221	876.838	1184.794	1.2264	.260576	3.838	239.6	114
115	337.874	308.621	795.114	81.257	876.371	1184.992	1.2266	.262732	3.806	237.6	115
116	338.518	309.281	794.614	81.293	875.907	1185.188	1.2268	.264887	3.775	235.7	116
117	339.159	309.939	794.114	81.330	875.444	1185.383	1.2270	.267041	3.745	233.8	117

PROPERTIES OF SATURATED STEAM—(Continued).

Pressure above a vacuum, in pounds per square inch.	Temperature, Fahrenheit degrees.	Quantities of Heat. In British Thermal Units. Required to raise the temperature of the water from 32° to T°.	Internal latent heat.	External latent heat.	Latent heat of evaporation at pressure P = I + E.	Total heat of evaporation above 32° = S + L.	Total heat of evaporation above 32°, in units of evaporation.	Weight of a cubic foot of steam, in pounds.	Volume. Of a pound of steam in cubic feet.	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density.	Pressure above a vacuum, in pounds per square inch.
P	t	S	I	E	L	H	U	W	C	V	P
118	339.796	310.598	793.619	81.366	874.985	1185.577	1.2272	.269195	3.715	231.9	118
119	340.430	311.241	793.126	81.403	874.529	1185.770	1.2274	.271348	3.685	230.1	119
120	341.058	311.885	792.637	81.439	874.076	1185.961	1.2276	.273500	3.656	228.3	120
121	341.681	312.524	792.152	81.474	873.626	1186.150	1.2278	.275651	3.628	226.5	121
122	342.300	313.161	791.669	81.509	873.178	1186.339	1.2280	.277801	3.600	224.7	122
123	342.916	313.795	791.189	81.543	872.732	1186.527	1.2282	.279949	3.572	223.0	123
124	343.528	314.425	790.711	81.578	872.289	1186.714	1.2284	.282097	3.545	221.3	124
125	344.136	315.051	790.236	81.612	871.848	1186.899	1.2286	.284243	3.518	219.6	125
126	344.741	315.672	789.765	81.646	871.411	1187.083	1.2288	.286389	3.492	218.0	126
127	345.340	316.289	789.298	81.679	870.977	1187.266	1.2290	.288533	3.466	216.4	127
128	345.936	316.903	788.834	81.711	870.545	1187.448	1.2292	.290677	3.440	214.8	128
129	346.530	317.513	788.374	81.742	870.116	1187.629	1.2293	.292820	3.415	213.2	129
130	347.121	318.121	787.914	81.774	869.688	1187.809	1.2295	.294961	3.390	211.6	130
131	347.706	318.725	787.458	81.805	869.263	1187.988	1.2296	.297102	3.366	210.1	131
132	348.287	319.325	787.004	81.837	868.841	1188.166	1.2298	.299242	3.342	208.6	132
133	348.867	319.922	786.554	81.868	868.422	1188.344	1.2300	.301382	3.318	207.1	133
134	349.443	320.515	786.105	81.900	868.005	1188.520	1.2302	.303521	3.295	205.7	134
135	350.015	321.105	785.659	81.931	867.590	1188.695	1.2304	.305659	3.272	204.2	135
136	350.584	321.692	785.215	81.962	867.177	1188.869	1.2306	.307797	3.249	202.8	136
137	351.149	322.274	784.775	81.992	866.767	1189.041	1.2308	.309934	3.227	201.4	137
138	351.711	322.853	784.339	82.021	866.360	1189.213	1.2309	.312070	3.204	200.0	138

P	t	S	I	E	L	H	U	W	C	V	P
139	352.271	323.429	783.905	82.050	865.955	1189.384	1.2311	.314205	3.182	198.7	139
140	352.827	324.003	783.472	82.080	865.552	1189.555	1.2313	.316338	3.161	197.3	140
141	353.380	324.573	783.042	82.109	865.151	1189.724	1.2315	.318471	3.140	196.0	141
142	353.931	325.141	782.613	82.138	864.751	1189.892	1.2316	.320603	3.119	194.7	142
143	354.478	325.705	782.188	82.166	864.354	1190.059	1.2318	.322735	3.099	193.4	143
144	355.022	326.265	781.766	82.194	863.960	1190.225	1.2320	.324867	3.078	192.2	144
145	355.562	326.823	781.346	82.221	863.567	1190.390	1.2321	.326998	3.058	190.9	145
146	356.100	327.378	780.927	82.249	863.176	1190.554	1.2323	.329128	3.038	189.7	146
147	356.636	327.930	780.510	82.277	862.787	1190.717	1.2324	.331257	3.019	188.5	147
148	357.169	328.479	780.096	82.304	862.400	1190.879	1.2326	.333386	3.000	187.3	148
149	357.697	329.024	779.684	82.332	862.016	1191.040	1.2328	.335515	2.981	186.1	149
150	358.223	329.566	779.275	82.359	861.634	1191.200	1.2330	.337643	2.962	184.9	150
160	363.345	334.850	775.296	82.616	857.912	1192.762	1.2346	.358886	2.786	173.9	160
170	368.226	339.892	771.505	82.854	854.359	1194.251	1.2361	.380071	2.631	164.3	170
180	372.886	344.708	767.891	83.072	850.963	1195.671	1.2376	.401201	2.493	155.6	180
190	377.352	349.329	764.430	83.273	847.703	1197.032	1.2390	.422280	2.368	147.8	190
200	381.636	353.766	761.111	83.462	844.573	1198.339	1.2404	.443310	2.256	140.8	200
210	385.759	358.041	757.916	83.640	841.556	1199.597	1.2417	.464295	2.154	134.5	210
220	389.736	362.168	754.834	83.808	838.642	1200.810	1.2430	.485237	2.061	128.7	220
230	393.575	366.152	751.862	83.966	835.828	1201.980	1.2442	.506139	1.976	123.3	230
240	397.285	370.008	748.988	84.115	833.103	1203.111	1.2454	.527003	1.898	118.5	240
250	400.883	373.750	746.203	84.256	830.459	1204.209	1.2465	.547831	1.825	114.0	250
260	404.370	377.377	743.508	84.388	827.896	1205.273	1.2476	.568626	1.759	109.8	260
270	407.755	380.905	740.891	84.510	825.401	1206.306	1.2487	.589390	1.697	105.9	270
280	411.048	384.337	738.350	84.623	822.973	1207.310	1.2497	.610124	1.639	102.3	280
290	414.250	387.677	735.878	84.731	820.609	1208.286	1.2507	.630829	1.585	99.0	290
300	417.371	390.933	733.470	84.835	818.305	1209.238	1.2517	.651506	1.535	95.8	300
350	431.96	406.26	722.20	85.28	807.48	1213.74	1.256	.754534	1.325	82.7	350
400	444.92	419.76	712.34	85.60	797.94	1217.70	1.260	.857185	1.167	72.8	400
450	456.62	432.18	703.28	85.84	789.12	1221.30	1.264	.959536	1.042	65.1	450
500	467.42	443.52	695.01	86.01	781.02	1224.54	1.267	1.061700	.942	58.8	500
550	477.50	454.14	687.34	86.12	773.46	1227.60	1.270	1.16380	.859	53.6	550
600	486.86	464.22	680.08	86.18	766.26	1230.48	1.273	1.26586	.790	49.3	600
650	495.68	473.58	673.40	86.20	759.60	1233.18	1.276	1.36791	.731	45.6	650
700	504.14	482.40	667.11	86.19	753.30	1235.70	1.279	1.46995	.680	42.4	700
750	512.06	490.86	661.04	86.14	747.18	1238.04	1.282	1.57198	.636	39.6	750
800	519.62	498.88	665.34	86.08	741.42	1240.30	1.285	1.67401	.597	37.1	800
850	526.82	506.66	649.84	86.00	735.84	1242.50	1.287	1.77603	.563	34.9	850
900	533.66	514.03	644.71	85.91	730.62	1244.65	1.289	1.87804	.532	33.0	900
950	540.32	521.30	639.60	85.80	725.40	1246.70	1.291	1.98004	.505	31.4	950
1000	546.80	528.30	634.68	85.68	720.36	1248.66	1.293	2.08203	.480	30.0	1000

The column headed "U" in the table of the properties of saturated steam is useful for reducing the performance of different boilers to a common standard—this standard being that most generally accepted by engineers: the equivalent evaporation at atmospheric pressure and the temperature of boiling water, or, as it is frequently called, the evaporation from and at 212°. In the table it is assumed that the temperature of the feed-water is 32°, and an auxiliary table is added, giving corrections for any temperature of feed from 32° to 212°.

CORRECTION FOR TOTAL HEAT IN UNITS OF EVAPORATION.

Temperature of feed, Fahrenheit degrees.	Correction.	Temperature of feed, Fahrenheit degrees.	Correction.	Temperature of feed, Fahrenheit degrees.	Correction.	Temperature of feed, Fahrenheit degrees.	Correction.	Temperature of feed, Fahrenheit degrees.	Correction.
33	.0010	69	.0383	105	.0756	141	.1129	177	.1504
34	.0021	70	.0393	106	.0766	142	.1140	178	.1514
35	.0031	71	.0404	107	.0777	143	.1150	179	.1525
36	.0041	72	.0414	108	.0787	144	.1160	180	.1535
37	.0052	73	.0424	109	.0797	145	.1171	181	.1545
38	.0062	74	.0435	110	.0808	146	.1181	182	.1556
39	.0073	75	.0445	111	.0818	147	.1192	183	.1566
40	.0083	76	.0456	112	.0829	148	.1202	184	.1577
41	.0093	77	.0466	113	.0839	149	.1213	185	.1587
42	.0104	78	.0476	114	.0849	150	.1223	186	.1598
43	.0114	79	.0487	115	.0860	151	.1233	187	.1608
44	.0124	80	.0497	116	.0870	152	.1244	188	.1618
45	.0135	81	.0507	117	.0880	153	.1254	189	.1629
46	.0145	82	.0518	118	.0891	154	.1264	190	.1639
47	.0155	83	.0528	119	.0901	155	.1275	191	.1650
48	.0166	84	.0538	120	.0911	156	.1285	192	.1660
49	.0176	85	.0549	121	.0922	157	.1296	193	.1670
50	.0186	86	.0559	122	.0932	158	.1306	194	.1681
51	.0197	87	.0569	123	.0943	159	.1316	195	.1691
52	.0207	88	.0580	124	.0953	160	.1327	196	.1702
53	.0217	89	.0590	125	.0963	161	.1337	197	.1712
54	.0228	90	.0601	126	.0974	162	.1348	198	.1723
55	.0238	91	.0611	127	.0984	163	.1358	199	.1733
56	.0248	92	.0621	128	.0994	164	.1368	200	.1743
57	.0259	93	.0632	129	.1005	165	.1379	201	.1754
58	.0269	94	.0642	130	.1015	166	.1389	202	.1764
59	.0279	95	.0652	131	.1025	167	.1400	203	.1775
60	.0290	96	.0663	132	.1036	168	.1410	204	.1785
61	.0300	97	.0673	133	.1046	169	.1420	205	.1796
62	.0311	98	.0683	134	.1057	170	.1431	206	.1806
63	.0321	99	.0694	135	.1067	171	.1441	207	.1817
64	.0331	100	.0704	136	.1077	172	.1452	208	.1827
65	.0342	101	.0714	137	.1088	173	.1462	209	.1837
66	.0352	102	.0725	138	.1098	174	.1473	210	.1848
67	.0362	103	.0735	139	.1109	175	.1483	211	.1858
68	.0372	104	.0746	140	.1119	176	.1493	212	.1869

XI.

TOTAL AVAILABLE ENERGY IN WATER AND STEAM.

Pressure above a vacuum in pounds per square inch.	Same pressure as indicated by steam-gauge, allowing 14.7 pounds for atmospheric pressure.	Absolute pressure in atmospheres.	Number of British thermal units required for the evaporation of one pound of water, known as latent heat of evaporation, *H*.	Temperature in degrees Fahrenheit of the steam and of the water from which it is evaporated.	Temperature in degrees Centigrade of the steam and of the water from which it is evaporated.	Corresponding absolute temperature in degrees Fahrenheit.	Corresponding absolute temperature in degrees Centigrade.	Amount of energy contained in one pound of water which may be liberated by explosion or expansion to 212° Fahr.	Corresponding amount of energy contained in the latent heat of evaporation.	Total amount of energy contained in one pound of steam at corresponding temperatures and pressures.
20	5.3	1.36	954.415	227.9	108.8	689.0	382.8	145.9	16872.9	17018.8
25	10.3	1.70	945.825	240.0	115.5	701.2	389.5	439.7	29156.8	29596.5
30	15.3	2.04	938.925	250.2	121.2	711.4	395.2	813.5	38921.9	39735.4
35	20.3	2.38	932.1523	259.1	126.1	720.3	400.1	1223.4	47054.9	48278.3
40	25.3	2.72	926.4728	267.1	130.1	728.3	404.6	1645.7	54111.7	55757.4
45	30.3	3.06	921.3343	274.2	134.5	735.4	408.5	2112.9	60158.1	62271.0
50	35.3	3.40	916.6316	280.8	138.2	742.0	412.2	2550.4	65613.8	68164.2
55	40.3	3.74	912.2906	286.8	141.5	748.0	415.5	2999.9	70428.7	73428.6
60	45.3	4.08	908.2472	292.5	144.7	753.7	418.7	3449.2	74884.6	78333.8
65	50.3	4.42	904.4621	297.7	147.6	758.9	421.6	3899.8	78850.5	82750.3
70	55.3	4.76	900.8991	302.7	150.4	763.9	424.4	4361.1	82577.7	86938.8
75	60.3	5.10	897.5269	307.3	152.9	768.5	426.9	4815.8	85923.6	90739.4
80	65.3	5.44	894.3304	311.8	155.4	773.0	429.4	5206.5	89138.7	94345.2
85	70.3	5.78	891.2862	316.0	157.7	777.2	431.7	5638.9	92073.3	97712.2
90	75.3	6.12	888.3758	320.0	160.0	781.2	434.0	6058.1	94814.7	100872.8
95	80.3	6.46	885.5887	323.8	162.1	785.0	436.1	6474.2	97447.2	103921.4
100	85.3	6.80	882.9144	327.5	164.1	788.7	438.1	6885.2	99787.6	106672.8
105	90.3	7.14	880.3429	331.1	166.1	792.3	440.1	7290.3	102163.3	109453.6
110	95.3	7.48	877.8653	334.5	168.0	795.7	442.0	7689.0	104334.9	112023.9
115	100.3	7.82	875.4721	337.8	169.8	799.0	443.8	8087.3	106421.7	114509.0
120	105.3	8.16	873.1555	340.9	171.6	802.1	445.6	8483.1	108325.4	116808.5
125	110.3	8.50	870.9115	344.0	173.3	805.2	447.3	8864.9	110219.9	119084.8
130	115.3	8.84	868.7351	347.0	175.0	808.2	449.0	9252.6	112025.6	121278.2
135	120.3	9.18	866.6223	349.9	176.6	811.1	450.6	9627.0	113745.7	123372.7
140	125.3	9.52	864.5661	352.7	178.1	813.9	452.1	9992.6	115382.1	125374.7

TOTAL AVAILABLE ENERGY IN WATER AND STEAM—*Continued.*

Pressure above a vacuum in pounds per square inch.	Same pressure as indicated by steam-gauge, allowing 14.7 pounds for atmospheric pressure.	Absolute pressure in atmospheres.	Number of British thermal units required for the evaporation of one pound of water, known as latent heat of evaporation, H.	Temperature in degrees Fahrenheit of the steam and of the water from which it is evaporated.	Temperature in degrees Centigrade of the steam and of the water from which it is evaporated.	Corresponding absolute temperature in degrees Fahrenheit.	Corresponding absolute temperature in degrees Centigrade.	Amount of energy contained in one pound of water which may be liberated by explosion or expansion to 212° Fahr.	Corresponding amount of energy contained in the latent heat of evaporation.	Total amount of energy contained in one pound of steam at corresponding temperatures and pressures.
145	130.3	9.86	862.5679	355.5	179.7	816.7	453.7	10361.0	117003.5	127364.5
150	135.3	10.20	860.6213	358.1	181.1	819.3	455.1	10536.5	118477.2	129003.7
155	140.3	10.54	858.7276	360.7	182.6	821.9	456.6	11085.9	119939.4	131025.3
160	145.3	10.88	856.8740	363.2	184.0	824.4	458.0	11444.2	121323.6	132767.8
165	150.3	11.22	855.0654	365.7	185.4	826.9	459.4	11823.4	122697.8	134521.2
170	155.3	11.56	853.2942	368.1	186.7	829.3	460.7	12141.3	123995.5	136136.8
175	160.3	11.90	851.5670	370.5	188.0	831.7	462.0	12508.7	125284.7	137793.4
180	165.3	12.24	849.8698	372.8	189.3	834.0	463.3	12821.4	126499.1	139320.5
185	170.3	12.58	848.2086	375.0	190.5	836.2	464.5	13182 0	127642.4	140824.4
190	175.3	12.92	846.5844	377.2	191.7	838.4	465.7	13367.1	128778.8	142145.9
195	180.3	13.26	844.9938	379.4	193.0	840.6	467.0	13844.1	129908.3	143752.4
200	185.3	13.60	843.4326	381.5	194.1	842.7	468.1	14153.3	130967.4	145120.7
210	195.3	14.28	840.3967	385.6	196.4	846.8	470.4	14830.8	133003.2	147834.0
220	205.3	14.66	838.5864	389.8	198.7	851.0	472.7	15463.1	136003.3	151466.4
230	215.3	15.64	833.9691	394.2	201.2	855.4	475.2	16180.3	137134.2	153314.7
240	225.3	16.32	832.6419	397.9	203.3	859.1	477.3	16790.2	139094.3	155884.5
250	235.3	17.00	830.3630	401.0	205.0	862.2	479.0	17314.4	140516.0	157830.4
500	485.3	34.01	780.8592	467.6	242.0	898.8	516.0	30055.5	165892.7	195948.2
1000	985.3	68.02	720.4350	546.8	286.0	1008.0	560.0	48671.5	179212.0	227883.5
2000	1985.3	136.05	643.9049	643.7	339.8	1104.9	613.8	75777.2	194221.3	269998.5
3000	2985.3	204.08	590.8038	708.3	375.7	1169.5	649.7	96116.3	193555.0	289671.3
4000	3985.3	272.10	544.6774	763.7	405.9	1223.9	679.9	114498.6	189201.7	303700.3
5000	4985.3	340.13	505.7339	807.8	431.0	1269.0	705.1	130494.2	183305.9	313800.1
6000	5985.3	408.16	471.8473	845.9	452.1	1307.1	726.1	144413.4	176656.9	321070.3
7000	6985.3	486.19	439.9985	881.2	471.7	1342.4	745.7	157944.2	169333.3	327277.5
8000	7985.3	544.21	409.4533	914.3	490.1	1375.5	764.1	170832.0	161392.6	332224.6
9000	8985.3	612.24	382.6347	945.2	507.3	1406.4	781.5	183998.6	153998.0	337142.6
10000	9985.3	680.27	355.2491	971.4	521.8	1432.6	795.8	193787.0	145376.8	339164.2
100000	99985.3	6802.72	305.3040	2163.4	1184.1	2624.6	1458.1	....	....	.. ..

XII.

FORMULAS RELATING TO PROPERTIES OF STEAM.

QUANTITY.			SYMBOL.	FORMULA.
Pressure.	Above a Vacuum.	Pounds per square inch.	P	$P = \frac{p}{144}$, $\log P = 6.1007 \frac{2731.62}{t} - \frac{396944}{t^2}$
		Pounds per square foot.	p	$p = P \times 144$, $\log p = 8.2591 - \frac{2731.62}{T} - \frac{396944}{T^2}$
		Inches of mercury, at 32° Fahr.	M	$M = P \times 2.03759$
		Feet of distilled water, at temperature of maximum density.	F	$F = P \times 2.306768$
		Atmospheres.	A	$A = P \times 0.0680967$
	Above the atmosphere, in pounds per square inch.		G	$G = P - 14.685$
Temperature.	Fahrenheit's scales.		t	$t = T - 461°.2$
	Absolute scale, Fahrenheit degrees.		T	$T = 1 + \left(\sqrt{\frac{8.2591 - \log p}{396944} + 0.00001184} - 0.003441\right)$
Quantity of heat.	Per pound of steam in British thermal units.	Required to raise the temperature of the water from 32° to t°.	S	$S = t - 32 + 0.000000103(t - 39.1)^3$
		Required to change the water into steam. (Internal latent heat.)	I	$I = L - E$
		Required to overcome the pressure of the surrounding medium. (External latent heat.)	E	$E = p \times \frac{C - v}{772}$
		Latent heat of evaporation, under constant pressure, P.	L	$L = 1091.7 - 0.695(t - 32) - 0.000000103(t - 39.1)^3$
		Total heat of evaporation above 32°.	H	$H = 1091.7 + 0.305(t - 32)$
	Total heat of evaporation per pound of steam, above 32°, in units of evaporation.		U	$U = \frac{H}{966.1}$

FORMULAS RELATING TO PROPERTIES OF STEAM—*Continued.*

QUANTITY.		SYMBOL.	FORMULA.
Foot-pounds of energy, in latent heat of evaporation, per cubic foot of steam.		l	$l = 2.3026 \times p \times \left(\frac{2731.62}{T} + \frac{793888}{T^2}\right)$
Weight.	Of a cubic foot of steam, in pounds.	W	$W = \frac{l}{772 \times L}$
	Of a cubic foot of distilled water, in pounds, at temperature t.	w	$w = \frac{62.425}{v}$
Volume.	Of a pound of steam, in cubic feet.	C	$C = \frac{1}{W}$
	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density.	V	$V = C \times 62.425$
	Ratio of volume of distilled water, at temperature T, to volume of equal weight at temperature of maximum density.	v	For temperatures from 32° to 70°, $v = 1.00012 - 0.000033914(t-32) + 0.0000023822(t-32)^2 - 0.0000000006403(t-32)^3$ For temperatures above 70°, $v = 0.99781 + 0.00006117(t-32) + 0.000001059(t-32)^2$

XIII.
FACTORS OF EVAPORATION.

Temperature of Feed-water in Degrees.		Gauge Pressure in Pounds per Square Inch above the Atmosphere and in Atmospheres.															
		25	30	35	40	45	50	60	70	80	90	100	120	140	160	180	200
F.	C.	1.7	2.0	2.3	2.7	3.0	3.3	4.0	4.7	5.3	6.0	6.7	8.0	9.3	10.7	12	13.3
32	0	1.204	1.206	1.209	1.211	1.212	1.214	1.217	1.219	1.222	1.224	1.227	1.231	1.234	1.237	1.239	1.241
35	1.6	.201	.203	.206	.208	.209	.211	.214	.216	.219	.221	.224	.228	.231	.234	.236	.238
40	4.4	.196	.198	.201	.203	.204	.206	.209	.211	.214	.216	.219	.223	.226	.229	.231	.233
45	7.2	.190	.192	.195	.197	.198	.200	.203	.205	.208	.210	.213	.217	.220	.223	.225	.227
50	10	.185	.187	.190	.192	.193	.195	.198	.200	.203	.205	.208	.212	.215	.218	.220	.222
55	12.7	.180	.182	.185	.187	.188	.190	.193	.195	.198	.200	.203	.207	.210	.213	.215	.217
60	15.5	.175	.177	.180	.182	.183	.185	.188	.190	.193	.195	.198	.202	.205	.208	.210	.212
65	18.3	.170	.172	.175	.177	.178	.180	.183	.185	.188	.190	.193	.197	.200	.203	.205	.207
70	21.1	.165	.167	.170	.172	.173	.175	.178	.180	.183	.185	.188	.192	.195	.198	.200	.202
75	23.5	.160	.162	.165	.167	.168	.170	.173	.175	.178	.180	.183	.187	.190	.193	.195	.197
80	26.6	.154	.156	.159	.161	.162	.164	.167	.169	.172	.174	.177	.181	.184	.187	.189	.191
85	29.4	.149	.151	.154	.156	.157	.159	.162	.164	.167	.169	.172	.176	.179	.182	.184	.186
90	32.2	.144	.146	.149	.151	.152	.154	.157	.159	.162	.164	.167	.171	.174	.177	.179	.181
95	35.0	.139	.141	.144	.146	.147	.149	.152	.154	.157	.159	.162	.166	.169	.172	.174	.176
100	37.7	.134	.136	.139	.141	.142	.144	.147	.149	.152	.154	.157	.161	.164	.167	.169	.171
105	40.5	.128	.130	.133	.135	.136	.133	.141	.143	.146	.148	.151	.155	.158	.161	.163	.165
110	43.3	.123	.125	.128	.130	.131	.133	.136	.138	.141	.143	.146	.150	.153	.156	.158	.160
115	46.1	.118	.120	.123	.125	.126	.120	.131	.133	.136	.138	.141	.145	.143	.151	.153	.155
120	48.8	.113	.115	.118	.120	.121	.123	.126	.128	.131	.133	.136	.140	.143	.146	.148	.150
125	51.6	.108	.110	.113	.115	.116	.118	.121	.123	.126	.128	.131	.135	.138	.141	.143	.145
130	54.4	.102	.104	.107	.109	.110	.112	.115	.117	.120	.122	.125	.129	.132	.135	.137	.139
135	57.2	.097	.099	.102	.104	.105	.107	.110	.112	.115	.117	.120	.124	.127	.130	.132	.134
140	60.0	.092	.094	.097	.099	.100	.102	.105	.107	.110	.112	.115	.119	.122	.125	.127	.129
145	62.7	.087	.089	.092	.094	.095	.097	.100	.102	.105	.107	.110	.114	.117	.120	.122	.124
150	65.5	.082	.084	.087	.089	.090	.092	.095	.097	.100	.102	.105	.109	.112	.115	.117	.119
155	68.3	.076	.078	.081	.083	.084	.086	.089	.091	.094	.096	.099	.103	.106	.109	.111	.113
160	71.1	.071	.073	.076	.078	.079	.081	.084	.086	.089	.091	.094	.098	.101	.104	.106	.108
165	73.8	.066	.068	.071	.073	.074	.076	.079	.081	.084	.086	.089	.093	.096	.099	.101	.103
170	76.6	.061	.063	.066	.068	.069	.071	.074	.076	.079	.081	.084	.088	.091	.094	.096	.098
175	79.4	.056	.058	.061	.063	.064	.066	.069	.071	.074	.076	.079	.083	.086	.089	.091	.093
180	82.2	.050	.052	.055	.057	.058	.060	.063	.065	.068	.070	.073	.077	.080	.083	.085	.087
185	85.0	.045	.047	.050	.052	.053	.055	.058	.060	.063	.065	.068	.072	.075	.078	.080	.082
190	87.7	.040	.042	.045	.047	.048	.050	.053	.055	.058	.060	.063	.067	.070	.073	.075	.077
195	90.5	.035	.037	.040	.042	.043	.045	.048	.050	.053	.055	.058	.062	.065	.067	.070	.072
200	93.3	.030	.032	.035	.037	.038	.040	.043	.045	.048	.050	.053	.057	.060	.063	.065	.067
205	96.1	.025	.027	.030	.032	.033	.035	.038	.040	.043	.045	.048	.052	.055	.058	.060	.062
210	98.8	.020	.022	.025	.027	.028	.030	.033	.035	.038	.040	.043	.047	.050	.053	.055	.057

XIV.

COMPOSITION OF VARIOUS FUELS OF THE UNITED STATES.

	C.	H.	O.	N.	S.	Moisture.	Ash.	Spec. Grav.
Pennsylvania Anthracite	78.6	2.5	1.7	0.8	0.4	1.2	14.8	1.45
Rhode Island "	85.8	10.5		....	3.7	....	....	1.85
Massachusetts "	92.0	6.0		....	2.0	....	....	1.78
North Carolina "	83.1	7.8		....	9.1	....	....	....
Welsh "	84.2	3.7	2.3	0.9	0.9	1.3	6.7	1.40
Maryland Semi-bituminous	80.5	4.5	2.7	1.1	1.2	1.7	8.3	1.33
Pennsylvania "	75.8	20.2		....	....	....	4.0	1.32
" "	59.4	38.8		....	....	....	1.8	1.30
Indiana "	70.0	28.0		....	....	....	2.0	1.24
" "	52.0	39.0		....	....	....	9.0	1.27
Illinois Bituminous	62.6	35.5		....	....	....	1.9	1.30
" (Block) Bituminous	58.2	37.1		....	....	....	4.7	....
Illinois and Indiana (Cannel) Bituminous	59.5	36.6		....	....	....	3.9	1.27
Kentucky (Cannel) Bituminous	48.4	48.8		....	....	....	2.8	1.25
Tennessee Bituminous	71.0	17.0		....	....	....	12.0	1.45
" "	41.5	56.5		....	....	....	2.5	....
Alabama "	54.0	42.6		....	1.0	1.2	1.2	....
Virginia "	55.0	41.0		....	....	....	4.0	....
" "	74.0	18.6		....	....	....	7.4	....
California and Oregon Lignite	50.1	3.9	13.7	0.9	1.5	16.7	13.2	1.32

Coal. State.	Coal. Kind of Coal.	Per Cent. of Ash.	Theoretical Value. In Heat Units.	Theoretical Value. In Pounds of Water Evaporated.
Pennsylvania	Anthracite	3.49	14,199	14.70
"	"	6.13	13,535	14.01
"	"	2.90	14,221	14.72
"	Cannel	15.02	13,143	13.60
"	Connelsville	6.50	13,368	13.84
"	Semi-bituminous	10.77	13,155	13.62
"	Stone's Gas	5.00	14,021	14.51
"	Youghiogheny	5.60	14,265	14.76
"	Brown	9.50	12,324	12.75
Kentucky	Caking	2.75	14,391	14.89
"	Cannel	2.00	15,198	16.76
"	"	14.80	13,360	13.84
"	Lignite	7.00	9,326	9.65
Illinois	Bureau County	5.20	13,025	13.48
"	Mercer County	5.60	13,123	13.58
"	Montauk	5.50	12,659	13.10
Indiana	Block	2.50	13,588	14.38
"	Caking	5.66	14,146	14.64
"	Cannel	6.00	13,097	13.56
Maryland	Cumberland	13.98	12,226	12.65
Arkansas	Lignite	5.00	9,215	9.54
Colorado	"	9.25	13,562	14.04
"	"	4.50	13,866	14.35
Texas	"	4.50	12,962	13.41
Washington	"	3.40	11,551	11.96
Pennsylvania	Petroleum	....	20,746	21.47

ANALYSES OF ASH.

	Specific Grav.	Color of Ash.	Silica.	Alumina.	Oxide Iron.	Lime.	Magnesia.	Loss.	Acids S.&P.
Pennsylvania Anthracite	1.559	Reddish Buff.	45.6	42.75	9.43	1.41	0.33	0.48	....
" Bituminous	1.372	Gray.	76.0	21.00	2.60	....	..	0.40	...
Welsh Anthracite	1.32	....	40.0	44.8	....	12.0	trace	....	2.97
Scotch Bituminous	1.26	...	37.6	52.0	....	3.7	1.1	....	5.02
Lignite	1.27	....	19.3	11.6	5.8	23.7	2.6	....	33.8

TABLE XIV*a*.

JOHNSON'S RESULTS CORRECTED AND COMPARED. (KENT.)

Number of Coal.	Order of Excellence.	NAME OF COAL.	Evaporation per Pound Combustible.*	Fixed C.; Per cent Total C. and Volatile Matter.	Volatile Matter; Per cent Total C. and Volatile Matter.	Evaporation,† Equivalent, Calories.
		Anthracites, Pennsylvania:				
1	15	Beaver Meadow Slope, No. 3	11.15	97.4	2.6	5,988
2	14	" " " " 5	11.29	97.2	2.8	6,063
3	9	Forest Improvement	11.52	95.6	4.4	6,186
4	8	Peach Mountain	11.59	96.8	3.2	6,224
5	23	Lehigh	10.26	94.4	5.6	5,509
6	11	Lackawanna	11.47	95.7	4.3	6,159
7	10	Lykens Valley	11.50	92.4	7.6	6,176
		Semi-Bituminous:				
8	3	New York & Maryland Mining Co., Md	11.95	85.6	14.4	6,417
9	13	Neff's, Cumberland, Md	11.30	85.5	14.5	6,068
10	7	Easby's, Md	11.66	83.6	16.4	6,261
11	1	Atkinson & Templeman, Md	12.39	83.2	16.8	6,653
12	5	Easby & Smith's, Md	11.76	82.7	17.3	6,315
13	4	Dauphin and Susquehanna, Pa	11.91	84.3	15.7	6,396
14	6	Blossburg, Pa	11.68	83.2	16.8	6,272
15	12	Lycoming Creek, Pa	11.43	83.8	16.2	6,138
16	2	Quin's Run, Pa	12.02	80.1	19.9	6,455
17	20	Karthaus, Pa	10.54	79.1	20.9	5,660
18	16	Cambria Co., Pa	10.91	77.2	22.8	5,859
		Bituminous, United States:				
19	17	Barr's Deep Run, Va	10.81	77.5	22.5	5,805
20	21	Crouch & Snead, Va	10.38	71.1	28.9	5,574
21	18	Midlothian (screened), Va	10.63	60.9	39.1	5,708
22	19	Chesterfield Mining Co., Va	10.55	64.3	35.7	5,665
23	28	Tippecanoe, Va	9.15	61.3	38.7	4,914
24	24	Creek Co., Va	9.82	65.0	35.0	5,273
25	27	Clover Hill, Va	9.15	63.8	36.2	4,914
26	26	Pittsburgh, Pa	9.54	59.9	40.1	5,123
27	31	Cannelton, Ind	8.24	63.2	36.8	4,425
		Bituminous, Foreign:				
28	22	Pictou, N. S.	10.35	67.2	32.8	5,558
29	29	Sidney, N. S	9.06	73.9	26.1	4,865
30	30	Liverpool, Eng	8.80	57.8	42.2	4,726
31	25	Newcastle, Eng	9.78	61.4	38.6	5,252
32	32	Scotch, Scotland	8.23	54.9	45.1	4,419
33	33	Dry Pine Wood	5.02	....	....	2,696

XV.

HORSE-POWER PER POUND MEAN PRESSURE.

Diameter of Cylinder. Inches.	Speed of Piston in Feet per Minute.										
	100	240	300	350	400	450	500	550	600	650	750
4	.038	.091	.114	.133	.152	.171	.19	.209	.228	.247	.285
4½	.048	.115	.144	.168	.192	.216	.24	.264	.288	.312	.360
5	.06	.144	.18	.21	.24	.27	.30	.33	.36	.39	.450
5½	.072	.173	.216	.252	.288	.324	.36	.396	.432	.468	.540
6	.086	.205	.256	.299	.342	.385	.428	.471	.513	.555	.641
6½	.102	.245	.307	.391	.409	.464	.512	.563	.614	.698	.800
7	.116	.279	.348	.408	.466	.524	.583	.641	.699	.756	.874
7½	.134	.321	.401	.468	.534	.602	.669	.735	.802	.869	1.002
8	.152	.365	.456	.532	.608	.685	.761	.837	.912	.989	1.121
8½	.172	.413	.516	.602	.688	.774	.85	.946	1.032	1.118	1.290
9	.192	.462	.577	.674	.770	.866	.963	1.059	1.154	1.251	1.444
9½	.215	.515	.644	.751	.859	.966	1.074	1.181	1.288	1.395	1.610
10	.238	.571	.714	.833	.952	1.071	1.190	1.309	1.428	1.547	1.785
10½	.262	.63	.787	.919	1.050	1.181	1.313	1.444	1.575	1.706	1.969
11	.288	.691	.864	1.008	1.152	1.296	1.44	1.584	1.728	1.872	2.160
11½	.314	.754	.943	1.1	1.257	1.414	1.572	1.729	1.886	2.043	2.357
12	.342	.820	1.025	1.195	1.366	1.540	1.708	1.880	2.050	2.222	2.564
13	.402	.964	1.206	1.407	1.608	1.809	2.01	2.211	2.412	2.613	3.015
14	.466	1.119	1.398	1.631	1.864	2.097	2.331	2.564	2.797	3.029	3.495
15	.535	1.285	1.606	1.873	2.131	2.409	2.677	2.945	3.212	3.479	4.004
16	.609	1.461	1.827	2.131	2.436	2.741	3.045	3.349	3.654	3.958	4.567
17	.685	1.643	2.054	2.396	2.739	3.081	3.424	3.766	4.108	4.450	5.135
18	.771	1.849	2.312	2.697	3.083	3.468	3.854	4.239	4.624	5.009	5.780
19	.859	2.061	2.577	3.006	3.436	3.865	4.295	4.724	5.154	5.583	6.442
20	.952	2.292	2.855	3.331	3.807	4.285	4.759	5.234	5.731	6.186	7.138
21	1.049	2.518	3.148	3.672	4.197	4.722	5.247	5.771	6.296	6.820	7.869
22	1.152	2.764	3.455	4.031	4.607	5.183	5.759	6.334	6.911	7.486	8.638
23	1.259	3.021	3.776	4.405	5.035	5.664	6.294	6.923	7.552	8.181	9.44
24	1.370	3.289	4.111	4.797	5.482	6.167	6.853	7.538	8.223	8.908	10.279
25	1.487	3.569	4.461	5.105	5.948	6.692	7.436	8.179	8.923	9.560	11.053
26	1.609	3.861	4.826	5.630	6.435	7.239	8.044	8.848	9.652	10.456	12.065
27	1.733	4.159	5.199	6.066	6.932	7.799	8.666	9.532	10.399	11.265	12.998
28	1.865	4.477	5.596	6.529	7.462	8.395	9.328	10.261	11.193	12.125	13.991
29	2.002	4.805	6.006	7.007	8.008	9.009	10.01	11.011	12.012	13.013	15.015
30	2.142	5.141	6.426	7.497	8.568	9.639	10.71	11.781	12.852	13.923	1[illegible].065
31	2.288	5.486	6.865	8.001	9.144	10.287	11.43	12.573	13.716	14.8[illegible]	17.145
32	2.436	5.846	7.308	8.526	9.744	10.962	12.18	13.398	14.616	15.834	18.270
33	2.590	6.216	7.770	9.065	10.360	11.655	12.959	14.245	15.54	16.835	19.425
34	2.746	6.59	8.238	9.611	10.984	12.357	13.73	15.103	16.476	17.840	20.595
35	2.914	6.993	8.742	10.199	11.656	13.113	14.57	16.027	17.484	18.941	21.855
36	3.084	7.401	9.252	10.794	12.336	13.878	15.42	16.962	18.504	20.046	23.130
37	3.253	7.819	9.774	11.403	13.032	14.861	16.29	17.919	19.548	21.177	24.435
38	3.436	8.246	10.308	12.026	13.744	15.462	17.18	18.898	20.616	22.334	25.770
39	3.620	8.648	10.86	12.67	14.48	16.29	18.1	19.91	21.62	23.53	27.150
40	3.808	9.139	11.424	13.328	15.232	17.136	19.04	20.944	22.848	24.752	28.560
41	4.002	9.604	12.006	14.007	16.008	18.009	20.00	22.011	24.012	26.013	30.015
42	4.198	10.065	12.594	14.693	16.792	18.901	20.99	23.089	25.188	27.287	31.485
43	4.40	10.56	13.20	15.4	17.6	19.8	22.00	24.2	26.4	28.6	33.00
44	4.606	11.046	13.818	16.121	18.424	20.727	23.03	25.333	27.636	29.939	34.545
45	4.818	11.563	14.454	16.863	19.272	21.681	24.09	26.399	28.908	31.317	36.135
46	5.043	12.086	15.128	17.626	20.144	22.662	25.18	27.698	30.216	32.754	37.770
47	5.256	12.614	15.768	18.396	21.024	23.652	26.28	28.908	31.536	34.164	39.420
48	5.482	12.846	16.446	19.187	21.928	24.669	27.41	30.151	31.152	35.633	41.115
49	5.714	12.913	17.142	19.999	22.856	25.713	28.57	31.427	34.284	37.141	42.855
50	5.950	14.28	17.85	20.825	23.8	26.775	29.75	32.725	35.7	38.675	44.625
51	6.180	14.832	18.54	21.665	24.76	27.855	30.95	34.045	37.08	40.205	46.425
52	6.432	15.437	19.296	22.512	25.728	28.944	32.16	35.376	38.592	41.808	48.240
53	6.084	16.041	20.052	23.394	26.736	30.078	33.42	36.762	40.104	43.446	50.130
54	6.940	16.656	20.82	24.29	27.76	31.23	34.7	38.17	41.64	45.11	52.05
55	7.198	17.275	21.594	25.193	28.792	32.391	35.99	39.589	43.188	46.787	53.985
56	7.462	17.909	22.386	26.117	29.848	33.579	37.31	41.041	44.772	48.503	55.965
57	7.732	18.557	23.196	27.062	30.928	34.794	38.66	42.526	46.392	50.258	57.99
58	8.[illegible]	19.214	24.018	28.021	32.024	36.027	40.03	44.033	48.036	52.039	60.045
59	8.284	19.902	24.852	28.964	33.136	37.278	41.42	45.562	48.704	53.846	62.13
60	8.506	20.558	25.698	29.981	34.264	38.547	42.83	47.113	51.396	55.679	64.245

XVI.

REAL RATIOS OF EXPANSION.

Per cent. of Clearance.	Points of Cut-off. .10	.125	.20	.25	.30	.333	.375	.40	.50	.60	.625	.70	.75	.80	.875	.90
.01	9.111	7.481	4.809	3.884	3.258	2.944	2.623	2.463	1.983	1.655	1.590	1.422	1.328	1.246	1.141	1.109
.0125	9	7.363	4.764	3.875	3.24	2.930	2.612	2.454	1.975	1.653	1.588	1.421	1.327	1.246	1.140	1.109
.0150	8.826	7.25	4.720	3.830	3.222	2.916	2.602	2.445	1.970	1.650	1.585	1.419	1.326	1.245	1.140	1.109
.0175	8.659	7.133	4.677	3.803	3.204	2.902	2.592	2.436	1.966	1.647	1.583	1.418	1.325	1.244	1.140	1.108
.02	8.5	7.034	4.635	3.777	3.187	2.889	2.582	2.428	1.961	1.645	1.581	1.416	1.325	1.243	1.138	1.108
.0225	8.346	6.932	4.595	3.752	3.170	2.876	2.574	2.420	1.956	1.642	1.579	1.415	1.324	1.243	1.138	1.108
.0250	8.2	6.833	4.555	3.727	3.153	2.863	2.562	2.411	1.952	1.640	1.576	1.413	1.322	1.242	1.138	1.108
.0275	8.088	6.738	4.516	3.702	3.137	2.850	2.552	2.403	1.947	1.637	1.574	1.412	1.321	1.241	1.138	1.107
.03	7.933	6.645	4.417	3.678	3.121	2.837	2.543	2.395	1.943	1.634	1.572	1.410	1.320	1.240	1.138	1.107
.0325	7.792	6.555	4.440	3.654	3.105	2.824	2.533	2.387	1.938	1.632	1.570	1.409	1.319	1.240	1.138	1.107
.0350	7.666	6.468	4.404	3.631	3.089	2.812	2.524	2.379	1.934	1.629	1.568	1.408	1.318	1.239	1.137	1.106
.0375	7.545	6.390	4.484	3.608	3.074	2.800	2.515	2.371	1.930	1.627	1.566	1.406	1.317	1.238	1.136	1.106
.04	7.428	6.303	4.333	3.58	3.058	2.788	2.506	2.363	1.925	1.625	1.563	1.405	1.316	1.238	1.136	1.106
.0425	7.315	6.229	4.298	3.564	3.043	2.776	2.497	2.355	1.921	1.622	1.561	1.404	1.315	1.237	1.136	1.106
.0450	7.206	6.147	4.256	3.542	3.028	2.764	2.488	2.348	1.917	1.620	1.569	1.402	1.314	1.236	1.135	1.105
.0475	7.102	6.082	4.232	3.521	3.014	2.752	2.479	2.340	1.913	1.617	1.557	1.401	1.313	1.235	1.135	1.105
.05	7	6	4.2	3.5	3	2.741	2.470	2.333	1.907	1.615	1.555	1.400	1.312	1.235	1.135	1.105
.0525	6.901	5.985	4.168	3.478	2.986	2.730	2.461	2.325	1.904	1.613	1.553	1.398	1.311	1.234	1.134	1.104
.0550	6.806	5.861	4.130	3.459	2.971	2.719	2.453	2.318	1.900	1.610	1.551	1.397	1.310	1.233	1.134	1.104
.0575	6.714	5.794	4.106	3.439	2.957	2.708	2.445	2.311	1.896	1.608	1.549	1.396	1.309	1.233	1.134	1.104
.06	6.625	5.729	4.076	3.418	2.944	2.697	2.436	2.304	1.892	1.606	1.547	1.394	1.308	1.232	1.133	1.104
.0625	6.538	5.666	4.047	3.407	2.931	2.686	2.428	2.297	1.888	1.603	1.545	1.393	1.307	1.231	1.133	1.103
.0650	6.454	5.605	4.045	3.380	2.917	2.675	2.420	2.290	1.884	1.601	1.343	1.392	1.306	1.231	1.132	1.103
.0675	6.373	5.545	3.990	3.362	2.904	2.665	2.412	2.283	1.881	1.599	1.541	1.390	1.305	1.230	1.132	1.103
.07	6.294	5.482	3.963	3.342	2.892	5.655	2.404	2.276	1.877	1.597	1.539	1.389	1.304	1.229	1.132	1.103

XVII.

A.

LOG OF TRIAL BY MECHANICAL LABORATORY, DEPARTMENT OF ENGINEERING.

Test made at____________________

____

on____________________

$$x = \frac{U - wh}{H - h}$$

$$\frac{\frac{U}{w} + t - T}{0.48} = \text{Degrees of Superheating.}$$

Time.	Pressures.			Temperatures.					Weights.			Remarks.
										Feed-water.		
	Barometer.	Steam-gauge.	Draught-gauge.	External Air.	Boiler-room.	Fuel.	Feed-water.	Steam.	Fuel.	Per Metre.	Per Tank.	

PRIMING TESTS.

No.	Time.	Steam-pressures.	Calorimeter.					Heat-units per Pound from Boiler.		Heat transferred to Calorimeter. $W \times R = U$	Heat from Steam $H = T - t'$	Heat from Water $h = t - t'$	Steam run into Calorimeter. x	Percentage of Priming. y	Superheating.		Remarks.
			Weights.		Temperature.												
			Condensing Water. W	Wet Steam. w	Initial. t''	Final t'	Range. $R = t' - t''$	Water. t	Steam. T						Degrees.	Heat-units.	

XVII.—(*Continued.*)

B

AVERAGE AND TOTAL RESULTS OF TRIAL, MECHANICAL LABORATORY, DEPARTMENT OF ENGINEERING.

Trial made at____________________ Fuel____________________

on____________________ Composition____________________

Number of Trial.	Date of Trial.	Length of Trial.	Areas.				Ratio of Grate to Heating-surface.	Average Temperatures.				Average Pressures.			Consumption of Fuel.			Ashes.		Total Combustible.	Remarks.
			Grate.	Heating-surface.	Super-heating-surface.	Least Cross-section of Flues.		Boiler-room.	External Air.	Entrance to Chimney.	Feed-water.	Barometer.	Steam-gauge.	Draught-gauge.	Total.	Per square foot of Grate per hour.	Per sq. ft. of Heating-surface per hour.	Total.	Proportion of Total Fuel.		
		Hours.	sq. ft.	sq. ft.	sq. ft.	sq. ft.		Fahr.	Fahr.	Fahr.	Fahr.	ins.	lbs.	ins.	lbs.	lbs.	lbs.	lbs.	per ct.	lbs.	

Apparent Evaporation.									Real Evaporation.											
Per Pound of Fuel.			Per Pound of Combustible.			Per Square Foot of Heating-surface, per Hour.			Per Pound of Fuel.			Per Pound of Combustible.			Per Square Foot of Heating-surface, per Hour.			Square feet of Heating-surface required to Evaporate one Cubic Foot of Water.		
From actual temperature of feed-water and at actual steam-pressure.	Equivalent from and at 212° F.	Equivalent from 212° F. and at actual steam-pressure.	From actual temperature of feed-water and at actual steam-pressure.	Equivalent from and at 212° F.	Equivalent from 212° F. and at actual steam-pressure.	From actual temperature of feed-water and at actual steam-pressure.	Equivalent from and at 212° F.	Equivalent from 212° F. and at actual steam-pressure.	From actual temperature of feed-water and at actual steam-pressure.	Equivalent from and at 212° F.	Equivalent from 212° F. and at actual steam-pressure.	From actual temperature of feed-water and at actual steam-pressure.	Equivalent from and at 212° F.	Equivalent from 212° F. and at actual steam-pressure.	From actual temperature of feed-water and at actual steam-pressure.	Equivalent from and at 212° F.	Equivalent from 212° F. and at actual steam-pressure.	From actual temperature of feed-water and at actual steam-pressure.	Equivalent from and at 212° F.	Equivalent from 212° F. and at actual steam-press u e.
lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	sq. ft.	sq. ft.	sq. ft.

XVII.—(*Continued.*)

B.

Total Water fed to Boiler.			Average Priming	Total Water Primed.	Water Evaporated into Dry Steam.			Remarks.
From actual temperature of feed-water and at actual steam-pressure.	Equivalent from and at 212° F.	Equivalent from 212° F. and at actual steam-pressure.			From actual temperature of feed-water and at actual steam-pressure.	Equivalent from and at 212° F.	Equivalent from 212° F. and at actual steam-pressure.	
lbs.	lbs.	lbs.	per cent.	lbs.	lbs.	lbs.	lbs.	

Average Amount of Superheating.	Evaporation from and at 212° F., Equivalent to Total Heat-units derived from Fuel.			Efficiency.			Values of A and B in $F = A\sqrt{H} + B$.		Horse-power.		Remarks.
	Per Pound of Fuel.	Per Pound of Combustible.	Per sq. ft. of Heating-surface per hour.	Experimental.	Estimated.	$R = \frac{\text{Experimental.}}{\text{Estimated.}}$	A	B	Rated.	Actual.	
Fahr.	lbs.	lbs.	lbs.	per cent.	per cent.						

XVII.

C.

CONDENSED LOG OF ENGINE-TRIAL.

Date,________________1891.

		Revolutions.		Gauge Readings,						Temperatures.										Weights.				Head.		Crank.			Brake.		
				Pounds.			Inches Hg.									Calorim. (Throttling, Temp.).															
Number.	Time.	Continuous Counter.	Speed-indicator.	Boiler.	Steam-pipe.	Steam-chest.	Exhaust.	Condenser.	Barometer.	External Air.	Engine-room.	Condensed Steam.	Feed-water.	Injection-water.	Discharge-water.	Steam-pipe.	Steam-chest.	Cylinder.	Exhaust.	Condensed Steam.	Feed-water.	Injection-water.		M. E. P.	I. H. P.	M. E. P.	I. H. P.	I. H. P. Total.	Load.	D. H. P.	Scale of Spring.

Clearance, Head lbs. Water......	Diam. Piston-rod, inches........	Areas, square inches............	*Test made by*
" Crank " "	Diam. Crank-pin, inches........	Piston...........................	
Brake-arm.	Length Crank-pin	Steam-port	....
Diam. Cylinder, inches..........	Diam. Wrist-pin, inches.........	Exhaust-port	....
Length Stroke..................	Travel-valve, inches............		
	Lap of Valve, inches............	....	....

XVIII.

ELECTRICAL HORSE-POWER TABLE. By H. W. Fisher, M.E.

VOLTS OR AMPERES.

VOLTS OR AMPERES.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
1	.00134	.00268	.00402	.00536	.00670	.00804	.00938	.01072	.01206	.01340	.01474	.01608	.01742	.01876	.02010	.02144	.02278	.02412	.02546	.02680	1
2	.00268	.00536	.00804	.01072	.01340	.01608	.01876	.02144	.02412	.02680	.02948	.03216	.03484	.03752	.04020	.04288	.04556	.04824	.05092	.05360	2
3	.00402	.00804	.01206	.01608	.02010	.02412	.02814	.03216	.03618	.04020	.04422	.04824	.05226	.05628	.06030	.06432	.06834	.07236	.07638	.08040	3
4	.00536	.01072	.01608	.02144	.02680	.03216	.03752	.04288	.04824	.05360	.05896	.06432	.06968	.07504	.08040	.08576	.09112	.09648	.10184	.10720	4
5	.00670	.01340	.02010	.02680	.03350	.04020	.04690	.05360	.06030	.06700	.07370	.08040	.08710	.09380	.10050	.10720	.11390	.12060	.12730	.13400	5
6	.00804	.01608	.02412	.03216	.04020	.04824	.05628	.06432	.07236	.08040	.08844	.09648	.10452	.11256	.12060	.12864	.13668	.14472	.15276	.16080	6
7	.00938	.01876	.02814	.03752	.04690	.05628	.06566	.07504	.08442	.09380	.10318	.11256	.12194	.13132	.14070	.15008	.15946	.16884	.17822	.18760	7
8	.01072	.02144	.03216	.04288	.05360	.06432	.07504	.08576	.09648	.10720	.11792	.12864	.13936	.15008	.16080	.17152	.18224	.19296	.20368	.21440	8
9	.01206	.02412	.03618	.04824	.06030	.07236	.08442	.09648	.10834	.12060	.13266	.14472	.15678	.16884	.18090	.19296	.20502	.21708	.22914	.24120	9
10	.01340	.02680	.04020	.05360	.06700	.08040	.09380	.10720	.12060	.13400	.14740	.16080	.17420	.18760	.20100	.21440	.22780	.24120	.25460	.26800	10
11	.01474	.02948	.04422	.05896	.07370	.08844	.10318	.11792	.13266	.14740	.16214	.17688	.19162	.20636	.22110	.23584	.25058	.26532	.28006	.29480	11
12	.01608	.03216	.04824	.06432	.08040	.09648	.11256	.12864	.14470	.16080	.17688	.19296	.20904	.22512	.24120	.25728	.27336	.28944	.30552	.32160	12
13	.01742	.03484	.05226	.06968	.08710	.10452	.12194	.13936	.15678	.17420	.19162	.20904	.22646	.24388	.26130	.27872	.29614	.31356	.33098	.34840	13
14	.01876	.03752	.05628	.07504	.09380	.11256	.13132	.15008	.16884	.18760	.20636	.22512	.24388	.26264	.28140	.30016	.31892	.33768	.35644	.37520	14
15	.02010	.04020	.06030	.08040	.10050	.12060	.14070	.16080	.18090	.20100	.22110	.24120	.26130	.28140	.30150	.32160	.34170	.36180	.38190	.40200	15
16	.02144	.04288	.06432	.08576	.10720	.12864	.15008	.17132	.19296	.21440	.23584	.25728	.27872	.30016	.32160	.34304	.36448	.38592	.40736	.42880	16
17	.02278	.04556	.06834	.09112	.11390	.13668	.15946	.18224	.20502	.22780	.25058	.27336	.29614	.31892	.34170	.36448	.38726	.41004	.43282	.45560	17
18	.02412	.04824	.07236	.09648	.12060	.14472	.16884	.19296	.21708	.24120	.26532	.28944	.31356	.33768	.36180	.38592	.41004	.43416	.45828	.48240	18
19	.02546	.05092	.07638	.10184	.12730	.15276	.17822	.20368	.22914	.25460	.28006	.30552	.33098	.35644	.38190	.40736	.43282	.45828	.48374	.50920	19
20	.02680	.05360	.08040	.10720	.13400	.16080	.18760	.21440	.24120	.26800	.29480	.32160	.34840	7520	.40200	.42880	.45560	.48240	.50920	.53600	20
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	VOLTS OR AMPERES.

VOLTS OR AMPERES.

HORSE-POWER TABLE.

The product of volts and amperes measures electrical energy or power. Dividing by 746 reduces electrical energy to horse-power.

Examples:—1st. What electrical horse-power corresponds to 19 amperes and 12 volts?
2d. " " " " " " 190 " " 120 "
3d. " " " " " " 1900 " " 120 "

At the juncture of columns 12 and 19 we find .30552, which is the required horse-power for Example No. 1.

In Example No. 2 there are two significant figures after the 19 and 12, hence we move the decimal point two places to the right and the answer is 30.552 H. P.

There are three significant figures in Example No. 3, hence the horse-power is 305.52.

Now suppose we have 2500 volts and 45 amperes. In this case we must take

The horse-power for
- 2000 volts and 40 amperes = 107.2
- 2000 " " 5 " = 13.4
- 500 " " 40 " = 26.8
- 500 " " 5 " = 3.35

Total horse-power = 150.75

XIX.

WATER-COMPUTATION TABLE

T. P.	0	1	2	3	4	5	6	7	8	9
3	117.300	121.015	124.717	128.406	132.083	135.748	139.399	143.075	146.665	150.279
4	153.880	157.514	161.137	164.750	168.353	171.945	175.527	179.098	182.659	186.210
5	189.750	193.336	196.914	200.483	204.044	207.598	211.142	214.679	218.208	221.728
6	225.240	228.799	232.351	235.897	239.437	242.970	246.497	250.017	253.531	257.039
7	260.540	264.056	267.566	271.071	274.570	278.063	281.550	285.031	288.506	291.976
8	295.440	298.922	302.400	305.872	309.338	312.800	316.256	319.708	323.154	326.594
9	330.030	333.488	336.941	340.389	343.833	347.273	350.707	354.137	357.563	360.984
10	364.400	367.842	371.280	374.714	378.144	381.570	384.992	388.410	391.824	395.234
11	398.640	402.064	405.485	408.902	412.315	415.725	419.131	422.534	425.933	429.328
12	432.720	436.120	439.517	442.911	446.301	449.688	453.071	456.451	459.828	463.200
13	466.570	469.950	473.326	476.699	480.068	483.435	486.798	490 159	493.516	496.869
14	500.220	503.596	506.968	510.338	513.706	517.070	520 432	523.790	527.146	530.500
15	533.850	537.213	540.573	543.930	547.285	550.638	553.987	557.334	560.679	564.011
16	567.360	570.713	574.063	577.411	580.757	584.100	587.441	590.780	594.115	597.449
17	600.780	604.109	607.435	610.759	614.081	617.400	620.717	624.031	627.343	630.653
18	633.960	637.265	640.567	643.867	647.165	650.460	653.753	657.043	660.331	663.617
19	666.900	670.200	673.498	676.793	680.086	683.378	686.666	689.953	693.238	696.520
20	699.800	703.098	706.394	709.688	712.980	716.270	719.558	722.844	726.128	729.410
21	732.690	735.968	739.244	742.518	745.790	749.060	752.328	755.594	758.858	762.120
22	765.380	768.660	771.938	775.215	778.490	781.763	785.034	788.303	791.570	794.836
23	798.100	801.362	804.622	807.881	811.138	814.393	817.646	820.897	824.146	827.334
24	830.640	833.908	837.175	840.440	843.703	846.965	850.225	853.484	856.741	859.996
25	863.250	866.502	869.753	873.002	876.249	879.495	882.739	885.982	889.223	892.462
26	895.700	898.936	902.171	905.404	908.635	911.865	915.093	918.320	921.545	924.768
27	927.990	931.210	934.429	937.646	940.831	944.075	947.287	950.498	953.707	956.914
28	960.120	963.352	966.583	969.813	973.041	976.268	979.493	982.717	985.939	989.160
29	992.380	995.598	998.815	1002.031	1005.245	1008.458	1011.669	1014.879	1018.087	1021.294
30	1024.500	1027.704	1030.907	1034.109	1037.309	1040.508	1043.705	1046.901	1050.095	1053.288
31	1056.480	1059.670	1062.859	1066.047	1069.233	1072.418	1075.601	1078.783	1081.963	1085.142

XIX.—(*Continued.*)

WATER-COMPUTATION TABLE—*Continued.*

T. P.	0	1	2	3	4	5	6	7	8	9
32	1088.320	1091.528	1094.736	1097.942	1101.146	1104.350	1107.552	1110.754	1113.954	1117.152
33	1120.350	1123.546	1126.742	1129.936	1133.128	1136.420	1139.510	1142.700	1145.888	1149.074
34	1152.260	1155.444	1158.628	1161.810	1164.990	1168.170	1171.348	1174.526	1177.702	1180.876
35	1184.050	1187.222	1190.394	1193.564	1196.732	1199.900	1203.066	1206.232	1209.396	1212.558
36	1215.720	1218.917	1222.112	1225.307	1228.500	1231.693	1234.884	1238.075	1241.264	1244.453
37	1247.640	1250.827	1254.012	1257.197	1260.380	1263.563	1266.744	1269.925	1273.104	1276.283
38	1279.460	1282.637	1285.812	1288.987	1292.160	1295.333	1298.504	1301.675	1304.844	1308.013
39	1311.180	1314.347	1317.512	1320.677	1323.840	1327.003	1330.164	1333.325	1336.484	1339.643
40	1342.800	1345.957	1349.112	1352.267	1355.420	1358.573	1371.724	1364.875	1368.024	1371.173
41	1374.320	1377.467	1380.612	1383.757	1386.900	1390.043	1393.184	1396.325	1399.464	1402.603
42	1405.740	1408.877	1412.012	1415.147	1418.280	1421.413	1424.544	1427.675	1430.804	1433.933
43	1437.060	1440.230	1443.398	1446.566	1449.734	1452.900	1456.066	1459.230	1462.394	1465.558
44	1468.720	1471.882	1475.042	1478.202	1481.362	1484.520	1487.678	1490.834	1493.990	1497.146
45	1500.300	1503.454	1506.606	1509.758	1512.910	1516.060	1519.210	1522.359	1525.506	1528.654
46	1531.800	1534.946	1538.090	1541.234	1544.378	1547.520	1550.662	1553.802	1556.942	1560.082
47	1563.220	1566.358	1569.494	1572.630	1575.766	1578.900	1582.034	1585.166	1588.298	1591.430
48	1594.560	1597.69	1600.818	1603.946	1607.074	1610.200	1613.326	1616.450	1619.574	1622.698
49	1625.820	1628.942	1632.062	1635.182	1638.302	1641.420	1644.538	1647.654	1650.770	1653.886
50	1657.000	1660.114	1663.226	1666.338	1669.450	1672.560	1675.670	1678.778	1681.886	1684.994
51	1688.100	1691.206	1694.310	1697.414	1700.518	1703.620	1706.722	1709.822	1712.922	1716.022
52	1719.120	1722.218	1725.314	1728.410	1731.506	1734.600	1737.604	1740.786	1743.878	1746.970
53	1750.060	1753.150	1756.238	1759.327	1762.414	1765.500	1768.586	1771.670	1774.754	1777.838
54	1780.920	1784.002	1787.082	1790.162	1793.242	1796.320	1799.398	1802.474	1805.556	1808.626
55	1811.700	1814.829	1817.957	1821.084	1824.211	1827.338	1830.463	1833.588	1836.713	1839.837
56	1842.960	1846.083	1849.205	1852.326	1855.447	1858.568	1861.687	1864.806	1867.925	1871.043
57	1874.160	1877.277	1880.393	1883.508	1886.623	1889.738	1892.851	1895.964	1899.077	1902.189
58	1905.300	1908.411	1911.521	1914.630	1917.739	1920.848	1923.955	1927.062	1930.169	1933.275
59	1936.380	1939.485	1942.589	1945.692	1948.795	1951.898	1954.999	1958.100	1961.201	1964.301
60	1967.400	1970.499	1973.597	1976.694	1979.791	1982.888	1985.983	1989.078	1992.173	1995.267

XX.

HIRN'S ANALYSIS.

DATA AND RESULTS.

Test of Steam-engine made by....................at....................

Kind of engine................Diam. cylinder........Length stroke........

Diam. piston-rod........Vol. cylinder, crank end.......Vol. head end.......

Vol. clearance, cu. ft., head............Clearance in per cent of stroke.......

" " " crank........... " " " ....

Boiler-pressure by gauge............ Barometer..............................

Boiler-pressure absolute............. Boiling temp., atmos. pressure.........

Revolutions per hour................ Steam used during run, lbs............

Quality of steam in steam-pipe....... Quality of steam in steam-chest.......

Quality of steam in compression..... Quality of steam in exhaust..........

Weight of condensed steam per hour...

Pounds of wet steam per stroke...........Head............Crank..........

Temperatures condensed steam...

Temperatures condensing water, cold.................Hot.................

Pounds of condensing water, per hour..............Per stroke..............

SYMBOLS.

To denote different portions of the stroke, the following subscripts are used:

Admission (*a*); expansion (*b*); exhaust (*c*); compression (*d*).

To denote different events of the stroke, the following sub-numbers are used:

Cut-off (1); release (2); compression, beginning of (3); admission, beginning of (0); in exhaust (5).

Quality of steam denoted by X.

Cut-off, crank end per cent of stroke .... Release, crank end............

Cut-off, head end per cent of stroke .... Release, head end.............

Compression, crank end per cent of stroke.... Lbs. steam per I. H. P........

Compression, head end per cent of stroke.... Lbs. steam per brake H. P......

I. H. P.................................. Brake horse-power..............

XX.—(*Continued.*)

DATA AND RESULTS.

Per 100 Strokes.

________________ *Engine.* *Date* ____________________ 189

Quantities.	Symbol.	Formula.	Results	
			Head.	Crank.
Weight steam per 100 strokes, lbs.	M			
Weight of steam in clearance, lbs.	M_0	$\frac{V_0 \text{ (Wt. per cu. ft.)}}{X_0}$.		
Weight of steam, total	$M+M_0$			
Condensing water, lbs.	G			
Heat given to condensing water, B.T.U.	K	$G(S_2 - S_1)$.		
Heat supplied engine, B.T.U.	Q	$M(XL+S)$		
Heat retained by compression, B.T.U.	Q_0	$M_0S_0 + \frac{V_0 I_0}{C_0}$.		
External heat steam at cut-off, B.T.U.	H_1	$(M+M_0)S_1$		
Internal heat steam at cut-off, B.T.U.	H_1'	$(V_0+V_1)\frac{I_1}{C_1}$.		
Cylinder loss during admission, B.T.U.	Q_a	$Q+Q_0-H_1-H_1'-\frac{1}{778}W_a$		
Loss sensible heat during expansion	H_2	$(M+M_0)(S_1-S_2)$.		
Internal heat after expansion	H_2'	$(V_0+V_2)\frac{I_2}{C_2}$.		
Cylinder loss during expansion, B.T.U.	Q_b	$H_2+H_1'-H_2'-\frac{1}{778}W_b$		
Sensible heat at exhaust	H_e	$(M+M_0)S_2$		
External heat at compression	H_3	M_0S_3		
Internal heat at compression	H_3	$(V_0+V_c)\frac{I_3}{C_3}$.		
Heat delivered from condenser	H_e	MS_4		
Heat carried off in exhaust	H_4	$M(XL_4+S_4)$ (per calorimeter)		
Cylinder loss, exhaust, B.T.U.	Q_c	$H_e+H_2'-H_3'-K-H_3-H_e-\frac{W_c}{778}$.		
" " " "	Q_c	$H_e+H_2'-H_3'-H_3-H_4-\frac{W_c}{778}$		
Sensible heat, gain during compression.	H_5	$M_0(S_3-S_0)$		
Internal heat at admission	H''	$V_0\frac{I_0}{C_0}$.		
Cylinder loss during compression, B.T.U.	Q_d	$H_5+H_3'-H''-\frac{W_d}{778}$.		
Heat admitted	Q			
Heat discharged and external work	B	H_e+K+ total $W \div 778$		
Loss	D	$Q-B$		
Loss	D'	$Q_a+Q_b+Q_c+Q_d$		

XXI.

NON-CONDENSING ENGINE, DRY SATURATED STEAM, UNJACKETED CYLINDER.

	Symbol.	Formula.	Point of Cut-off. Full Stroke.	¾	½	⅓	¼	⅙	⅛	$\frac{1}{12}$	Number for Reference.
Effective horse-power to be developed	E	Assumed.	150	150	150	150	150	150	150	150	1
Absolute initial pressure of steam, pounds per square inch.	P	Assumed.	100	100	100	100	100	100	100	100	2
Back-pressure, pounds per square inch	b	Assumed.	17.5	17.5	17.7	17.5	1.7.5	17.5	17.5	17.5	3
Apparent cut-off	$\frac{1}{r}$	Assumed.	1	.75	.5	.33	.25	.17	.125	.083	4
Absolute pressure at point of cut-off, pounds per square inch.	C	Assumed.	90	92.5	95	96.5	97.5	98	98.5	99	5
Clearance at each end, in equivalent length of cylinder, inches. Piston.	(P)	Assumed.	.3125	.3125	.3125	.3125	.3125	.3125	.3125	.3125	6
Port.	(p)	Assumed.	.625	.6875	.75	.8125	.875	1	1.0625	1.375	7
Total.	f	$(P)+(p)$	.9375	1.	1.0625	1.125	1.1875	1.3125	1.375	1.6875	8
Piston-speed, feet per minute	V	Assumed.	450	450	450	450	450	450	450	450	9
Mean absolute pressure up to cut-off, pounds per square inch	m	$\frac{P+C}{2}$	95	96.75	97.5	98.25	98.75	99	99.25	99.5	10
Apparent ratio of expansion.	r	Assumed.	1	1.33	2	3	4	6	8	12	11
Hyperbolic logarithm of apparent ratio of expansion.	hyp log r	From tables.	....	.2877	.6931	1.0986	1.3863	1.7918	2.0794	2.4849	12
Mean effective trial-pressure, pounds per square inch.	T	$\frac{m+C\times \text{hyp log } r}{R}-b$	77.5	75	64.2	50.6	41	28.3	20.5	11.3	13
Effective horse-power for trial-pressure, each square inch of piston area.	k	$\frac{T\times V}{33{,}000}$	1.057	1.023	.739	.690	.562	.386	.280	.154	14
Trial cross-section of cylinder, square inches.	a	$\frac{E}{k}$	141.9	142.7	203	217.4	267	388.6	536	974	15
Trial diameter of cylinder, inches, to nearest quarter-inch.	d	$\sqrt{\frac{a}{.7854}}$	13.5	13.5	16	16.5	18.5	22.25	26.25	35.25	16
Trial stroke of piston, inches	s	$2\times d$	27	27	32	33	37	45	52.5	70.5	17
Fraction of clearance	F	$\frac{f}{s}$	.0347	.037	.0332	.0341	.0321	.0292	.0262	.0239	18

XXI.—(*Continued.*)

NON-CONDENSING ENGINE, DRY SATURATED STEAM, UNJACKETED CYLINDER.

	Symbol.	Formula.	Point of Cut-off.								Number for Reference.
			Full Stroke.	3/4	1/2	1/3	1/4	1/6	1/8	1/12	
Per cent of clearance to nearest quarter per cent.	c	Assumed from F.	3⅛	3⅛	3⅛	3⅛	3¼	3	2⅛	2⅛	19
Real ratio of expansion	R	$\frac{1 + c/100}{1/r + c/100}$	1	1.318	1.935	2.913	3.655	5.228	6.833	9.464	20
Hyperbolic logarithm of real ratio of expansion.	hyp log R	From tables.	......	.2761	.6601	1.0343	1.2961	1.6540	1.9218	2.2475	21
Mean pressure for stroke plus clearance, corrected for back-pressure, pounds per square inch.	M	$\frac{m + C \times \text{hyp log} R}{R} - b$	77.5	75.4	65.3	52.9	44.7	32.4	24.7	16.8	22
Mean pressure corrected for back-pressure and clearance, pounds per square inch.	n	$M - \frac{c}{100} \times (P - M)$	76.7	74.5	64.1	51.3	42.9	30.4	22.8	14.7	23
Ratio of compression. When final cushion-pressure is less than initial pressure.	l	Assumed.	1	1.318	1.935	2.813	3.655	5.228	......	......	24
Ratio of compression. To make final cushion-pressure and initial pressure equal.	l	$\frac{P}{b}$	......	......	......	......	......	......	5.724	5.714	25
Hyperbolic logarithm of ratio of compression.	hyp log l	From tables.	......	.2761	.6601	1.0343	1.2961	1.6540	1.7429	1.7429	26
Final cushion-pressure, pounds per square inch.	L	$b \times l$	17.5	23.1	33.9	49.2	64	91.5	100	100	27
Mean absolute cushion-pressure, pounds per square inch.	k	$L \times \frac{\text{hyp log } l}{l - 1}$	......	20	23.9	28.1	31.2	35.8	37	37	28
Mean pressure, corrected for back pressure, clearance, and cushion, pounds per square inch	t	$n - \frac{c}{100} \times (l - 1) \times (k - b)$	76.7	74.5	63.9	50.6	41.7	28.1	20.5	12.4	29
Probable mean effective pressure, pounds per square inch.	e	$.95 \times t$	72.9	70.8	60.7	48.1	39.6	26.7	19.5	11.8	30
Horse-power for pressure e, each square inch of effective piston-area.	H	$\frac{e \times B}{33,000}$	.995	.967	.828	.656	.540	.364	.266	.161	31

XXI.—(*Continued.*)

NON-CONDENSING ENGINE, DRY SATURATED STEAM, UNJACKETED CYLINDER.

	Symbol.	Formula.	Point of Cut-off.								Number for Reference.
			Full Stroke.	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	
Effective cross-section of cylinder, square inches.	q	$\frac{E}{H}$	150.8	155.1	181.2	228.7	277.8	412.1	563.9	931.7	32
Actual cross-section of cylinder, square inches for effective section q.	A	$\frac{2q}{2-(.018\times\sqrt[4]{P})^2}$	153.2	157.6	184.1	232.4	282.2	418.7	572.9	946.6	33
Diameter of cylinder, inches to nearest quarter-inch.	D	$\sqrt{\frac{A}{.7854}}$	14	14.25	15.25	17.25	19	23	27	34.75	34
Stroke, inches	S	$2\times D$	28	28.5	30.5	34.5	38	46	54	69.5	35
Diameter of piston-rod, inches to nearest sixteenth-inch.	p	$.018\times D\times\sqrt[4]{P}$	2.5	2.5625	2.75	3.125	3.4375	4.125	4.875	6.25	36
Cross-section of cylinder, square inches, revised value.	(A)	$.7854\times D^2$	153.9	159.5	182.7	233.7	283.5	415.5	572.6	948.4	37
Cross-section of piston-rod, square inches	o	$.7854\times p^2$	4.9	5.2	5.9	7.7	9.3	13.4	18.7	30.7	37½
Effective cross-section of cylinder, square inches, revised value.	(q)	$\frac{2\times(A)-o}{2}$	151.5	156.9	179.8	224.9	278.9	408.8	563.3	933.1	38
Probable effective horse-power	(E)	$\frac{e\times(q)\times V}{33{,}000}$	150.6	151.5	148.8	147.5	150.6	148.8	149.8	150.1	39
Clearance in equivalent length of cylinder, inches, revised value.	(c)	$\frac{c}{100}\times S$	.98	1.	1.07	1.21	1.24	1.38	1.35	1.74	40
Volume of clearance-space at each end, cubic feet.	N	$\frac{(c)\times(q)}{1728}$	.0859	.0906	.111	.157	.199	.326	.440	.938	41
Volume of cylinder and clearance at one end up to .95 stroke, cubic feet.	v	$\frac{.95\times(q)\times S}{1728}+N$	2.418	2.549	3.126	4.423	6.026	10.665	17.163	36.591	42
Number of strokes per hour	w	$\frac{V\times 12\times 60}{S}$	11571	11368	10623	9391	8526	7043	6000	4662	43
Absolute pressure at .95 stroke, pounds per square inch.	B	$\frac{1/r+c/100}{.95+c/100}\times C$	91 ass'm'd	83	51.6	36.1	28.1	19.7	15.2	11	44
Weight in pounds of a cubic foot of steam at pressure B.	W	From tables.	.2107	.1932	.1223	.08830	.06978	.04998	.04109	.02891	45

XXI.—(*Continued.*)

NON-CONDENSING ENGINE, DRY SATURATED STEAM, UNJACKETED CYLINDER.

	Symbol.	Formula.	Point of Cut-off. Full Stroke.	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	Number for Reference.
Weight in pounds of a cubic foot of steam at pressure L	w	From tables.	.04472	.05805	.08323	.1181	.1513	.2118	.2303	.2303	46
Pounds of steam used hourly, calculated by piston-displacement	Q	$w \times (v \times W - N \times w)$	5851	5539	3963	3494	3328	3268	3627	3925	47
Mean total pressure during expansion, pounds per square inch	U	$C \times \frac{\text{hyp log } R}{R - 1}$	....	80.3	67.1	55.1	47.6	38.3	32.2	26.3	48
Ratio of mechanical effect during expansion to total mechanical effect	I	$\frac{U \times (r - 1)}{r \times (n + b)}$	....	.218	.411	.534	.591	.666	.699	.749	49
Units of heat required hourly for the work of expansion	i	$\frac{(n + b) \times I \times (q) \times S/12 \times w}{772}$	....	110052	189649	274063	348205	456091	554981	687080	50
Latent heat per pound of steam at pressure B, British thermal units	(l)	From tables.	....	893	916	931	942	955	965	976	51
Pounds of steam condensed hourly for work of expansion	(C)	$\frac{i}{(l)}$	....	124	207	294	370	477	575	704	52
Thickness of piston, inches, to nearest half-inch	T	$\sqrt[4]{D}$	3.5	4	4	4	4.5	5	5	6	53
Internal condensing surface, square feet	(a)	$\frac{2 \times (A)}{144} + 3.1416 \times \frac{D}{12} \times \frac{S + (T) \div 2 \times (P)}{12} + 3.1416 \times \frac{p}{12} \times \frac{S + (P)}{12}$	18.9	19.7	22.4	28.4	34.5	50.3	68.7	113.3	54
Probably condensation hourly, on internal surfaces, pounds	(d)	$15 \times (a)$	284	296	336	426	518	755	1032	1700	55
Probable consumption of steam hourly, pounds — Total..	(w)	$Q + (C) + (d)$	6135	5959	4506	4214	4216	4500	5234	6329	56
Probable consumption of steam hourly, pounds — Per effective horse-power..	(W)	$\frac{(w)}{(E)}$	40.7	39.3	30.3	28.6	28	30.2	34.9	42.2	57

XXI.—(*Continued.*)

CONDENSING ENGINES, DRY SATURATED STEAM, UNJACKETED CYLINDERS.

		Symbol.	Formula.	Point of Cut-off.							
				Full Stroke.	3/4	1/2	1/3	1/4	1/6	1/8	1/12
Mean pressure, pounds per square inch.	Corrected for back-pressure	M	22	90.5	88.4	78.3	65.9	57.7	45.4	37.7	29.8
	Corrected for back-pressure and clearance	N	23	90.2	88.0	77.5	64.7	56.3	43.8	36.1	28.0
Ratio of compression		l	20	1	1.318	1.935	2.813	3.655	5.228	6.833	9.464
Hyperbolic logarithm of ratio of compression		hyp log l	21	—	.2761	.6601	1.0343	1.2961	1.654	1.9218	2.2475
Final cushion-pressure, pounds per square inch		L	27	4.5	5.9	8.7	12.6	16.4	23.5	30.7	42.6
Mean absolute cushion-pressure, pounds per square inch		k	28	—	5.1	6.1	7.2	8.0	9.2	10.1	11.3
Mean pressure, corrected for back-pressure, clearance and cushion, pounds per square inch		t	29	90.2	88.0	77.4	64 5	56.0	43.2	35.3	26.6
Probable mean effective pressure, pounds per square inch		e	30	85.7	83.6	73.5	61.3	53.2	41.0	33.5	25.3
Probable effective horse-power		(E)	39	177	178.9	180.2	188	202.3	228.6	257.3	321.9
Pounds of steam used hourly.	Calculated by piston-displacement	Q	47	5883	5582	4034	3611	3506	3619	4031	4480
	Condensed for the work of expansion	(C)	52	—	124	207	294	370	477	575	704
	Condensed on internal condensing surfaces	(d)	25 (a)	473	493	560	710	863	1258	1718	2833
	Total	(w)	56	6356	6199	4801	4615	4739	5354	6324	8017
	Per effective horse-power	(W)	57	35.9	34.7	26.6	24.5	23.4	23.4	24.6	24.8

XXI.—(*Continued.*)

CONDENSING ENGINES, DRY SATURATED STEAM, UNJACKETED CYLINDERS, 150 EFFECTIVE HORSE-POWER, 100 POUNDS INITIAL PRESSURE, 4½ POUNDS BACK-PRESSURE.

	Symbol.	Formula.	Point of Cut-off. Full Stroke.	¾	½	⅓	¼	⅙	⅛	$\frac{1}{12}$
Trial data. Mean effective pressure	T	13	90.5	88	77.7	64.1	54.5	41.8	.34	24.8
Trial data. Cross-section of cylinder, square inches	a	15	121.55	125	141.57	171.61	201.84	263.16	323.53	443.55
Trial data. Diameter of cylinder, inches, to nearest ¼″	d	16	12½	12½	13½	14¾	16	18¼	20¼	23¾
Trial data. Stroke, inches	s	17	25	25	27	29½	32	36½	40½	47½
Trial data. Fraction of clearance	F	18	.0375	.0400	.0394	.0381	.0311	.0355	.0340	.0355
Per cent of clearance to nearest ¼″	c	19	3¾	4	4	3¾	3¾	3½	3½	3½
Real ratio of expansion and ratio of compression	R. l	20,24	1	1.3165	1.9259	2.7978	3.6087	5.1322	6.4687	8.7465
Hyperbolic logarithm of preceding	hy log R	From tabl's	—	.2753	.6554	1.0289	1.2834	1.6355	1.867	2.1686
Probable mean effective pressure	e	30	85.6	83.4	73.6	61.4	52.7	41.2	34.4	25.8
Diameter of cylinder, inches, to nearest ¼″	D	34	13	13¼	14	15¼	16½	18½	20¼	23½
Stroke, inches	S	35	26	26½	28	30½	33	37	40½	47
Diameter of piston-rod, inches, to nearest $\frac{1}{16}$″	p	36	2$\frac{5}{16}$	2⅝	2½	2¾	3	3$\frac{5}{16}$	3⅝	4¼
Cross-section of cylinder, square inches. Actual	(A)	37	132.73	137.89	153.94	182.65	213.83	268.8	322.06	433.74
Cross-section of cylinder, square inches. Effective	(q)	38	130.63	135.67	151.48	179.68	210.29	264.49	316.9	426.64
Probable effective horse-power	(E)	39	152.5	154.3	152	150.4	151.1	148.6	148.7	150.1
Volume, cubic feet. Cylinder and clearance, up to $\frac{95}{100}$ stroke	v	42	.941	.06	2.43	3.132	3.966	5.578	7.316	11.309
Volume, cubic feet. Clearance	N	41	.0737	.0832	.0982	.119	.151	.198	.26	.402
Absolute pressures. Final cushion	L	27	4.5	5.9	8.7	12.6	16.2	23.1	29.1	39.4
Absolute pressures. At $\frac{95}{100}$ stroke	B	44	91	73.8	51.8	36.2	28.4	20.1	16	11.9
Number of strokes per hour	w	43	12752	12226	11571	10623	9818	.8759	8000	6968

XXI.—(*Continued.*)

CONDENSING ENGINES, DRY SATURATED STEAM, UNJACKETED CYLINDERS, 150 EFFECTIVE HORSE-POWER, 100 POUNDS INITIAL PRESSURE, 4¼ POUNDS BACK-PRESSURE.—(*Continued.*)

	Symbol.	Formula.	Point of Cut-off. Full stroke.	¾	½	⅓	¼	⅙	⅛	1/12
Weight of steam per cubic foot, pounds. At pressure L	w	From tables	.01249	.0161	.02319	.03285	.04159	.05805	.07211	.09586
Weight of steam per cubic foot, pounds. At pressure B	W	From tables	.2107	.173	.124	.08603	.07048	.05093	.04111	.03113
Pounds of steam hourly, calculated by piston-displacement	Q	47	5203	4330	3460	2821	2683	2387	2256	2185
Latent heat per pound of steam at pressure B. British thermal units	(l)	From tables	—	899	916	931	941	955	963	973
Pounds of steam condensed hourly for work of expansion	(C)	52	—	106	194	249	281	313	339	390
Thickness of piston, inches, to nearest ½"	(T)	53	3½	3½	3½	4	4	4½	4½	5
Internal condensing surface, square feet	(a)	54	16.3	16.9	18.8	22.4	25.8	32.8	39.1	52.1
Probable condensation, hourly, on internal surfaces, pounds	(d)	25×(a)	408	424	471	561	645	819	976	1304
Probable consumption of steam, hourly, pounds. Total	(w)	56	5611	4860	4125	3631	3609	3519	3571	3879
Probable consumption of steam, hourly, pounds. Per effective horse-power	(W)	57	36.8	31.5	27.1	24.1	23.9	23.7	24	25.8

XXI.—(*Continued.*)

NON-CONDENSING ENGINES, DRY SATURATED STEAM-JACKETED CYLINDERS.

	Symbol.	Formula.	Point of Cut-off. Full Stroke	¾	½	⅜	¼	⅙	⅛	$\frac{1}{12}$	Number for Reference.
Sixteenth root of real cut-off	$\left(\frac{1}{R}\right)^{\frac{1}{16}}$	$\left(\frac{1}{R}\right)^{\frac{1}{16}}$	1	.98287	.95960	.93747	.92219	.90169	.88682	.87031	58
Mean pressure for stroke, plus clearance, corrected for back-pressure, pounds per square inch.	M	$\frac{m + 16 \times C \times \left[1 - \left(\frac{1}{R}\right)^{\frac{1}{16}}\right]}{R} - b$	77.5	74.7	64.6	51.8	42.7	30.8	23.1	15.3	59
Ratio of compression. When final cushion-pressure is less than initial pressure.	l	Assumed.	1	1.3185	1.9346	2.81	3.6549	......	......	......	60
Ratio of compression. To make final cushion-pressure and initial pressure equal.		$\left(\frac{P}{b}\right)^{\frac{16}{17}}$	......	......	......	......	......	5.1574	5.1574	5.1574	61
Seventeenth power of sixteenth root of ratio of compression.	$l^{\frac{17}{16}}$	$l^{\frac{17}{16}}$	1	1.341	2.016	2.997	3.963	5.808	5.808	5.808	62
Final cushion-pressure, pounds per square inch.	L	$b \times l^{\frac{17}{16}}$	17.5	23.5	35.3	52.5	69.4	100	100	100	63
Sixteenth root of reciprocal of ratio of compression.	$\left(\frac{1}{l}\right)^{\frac{1}{16}}$	$\left(\frac{1}{l}\right)^{\frac{1}{16}}$	1	.98287	.95960	.93747	.92219	.89587	.89587	.89587	64
Absolute cushion-pressure, pounds per square inch.	k	$L \times \frac{16 \times \left[1 - \left(\frac{1}{l}\right)^{\frac{1}{16}}\right]}{l - 1}$	......	20.2	24.4	29.	32.5	37.5	37.5	37.5	65
Ratio of expansion up to .95 stroke.	—	$\left(\frac{\frac{1}{r} + \frac{c}{100}}{.95 + c/100}\right)^{\frac{17}{16}}$	......	.786	.523	.352	.266	.182	.137	.0969	66
Absolute pressure at .95 stroke, pounds per square inch.	B	$C \times \left(\frac{1/2 + c/100}{.95 + c/100}\right)^{\frac{17}{16}}$	91 ass'd.	72.7	49.7	34.	25.9	17.8	13.5	9.6	67
Weight in pounds of a cubic foot of steam at pressure B.	W	From tables.	2107	.1705	.1193	.08346	.06464	.04544	.03505	.02544	68
Weight in pounds of a cubic foot of steam at pressure L.	w	From tables.	.04472	.05899	.08646	.1256	.1633	.2303	.2303	.2303	69

XXI.—(*Continued.*)

NON-CONDENSING ENGINES, DRY SATURATED STEAM-JACKETED CYLINDERS.

	Symbol.	Formula.	POINT OF CUT-OFF. Full Stroke	¾	½	⅜	¼	⅙	⅛	1/12	Number for Reference.
Felted surface, square feet................	(a)	Assumed.	18.9	19.7	22.4	28.4	34.5	50.3	68.7	113.3	70
Unfelted surface, square feet..............	$(\mathcal{S})$	$2 \times \frac{(A)}{144} + 3.1416 \times \frac{d}{12} \times \frac{S+(P)}{12}$	3.7	3.9	4.4	5.7	6.9	10.1	13.8	22.9	71
External temperature of jacket, Fahr.....	(t)	Assumed.	70°	70°	70°	70°	70°	70°	70°	70°	72
Internal temperature of jacket, Fahr......	(I)	From tables.	328°	328°	328°	328°	328°	328°	328°	328°	73
Latent heat of steam in jacket, British thermal units per pound..	(L)	From tables.	884	884	884	884	884	884	884	884	74
Heat lost by condensation hourly, British thermal units.	Q	$1.1 \times \{(a) \times [(I)-(t)] \times .5 + (\mathcal{S}) \times [(I)-(t)] \times 2.5\}$	5307	5563	6300	8074	9791	14304	19359	32325	75
Probable amount of steam condensed in jacket hourly, pounds.	(K)	$\frac{(Q)+1.1 \times i}{(L)}$	6	143	243	350	444	595	713	892	76
Probable consumption of steam hourly, lbs.	(w)	$Q+(K)$	5857	5023	4103	3631	3488	3479	3714	4225	77

	Symbol.	POINT OF CUT-OFF. Full Str'ke	¾	½	⅜	¼	⅙	⅛	1/12	Ref. No.
Mean pressure, corrected for back-pressure and clearance, pounds per square inch.	n	76.7	73.8	63.4	50.1	40.8	28.7	21.2	13.2	23
Mean pressure, corrected for back-pressure, clearance, and cushion, pounds per square inch.	i	76.7	73.8	63.2	49.4	39.5	26.2	19.1	11.1	29
Probable mean effective pressure, pounds per square inch..........................	e	72.9	70.1	60	46.9	37.5	24.9	18.1	10.5	30
Probable effective horse-power	(E)	150.6	150	147.1	143.8	142.6	139.4	139	133.6	39
Pounds of steam used hourly, calculated by piston-displacement....................	Q	5851	4880	3860	3281	3044	2884	3001	3333	47
Probable consumption of steam hourly, per effective horse-power pounds........	(W)	38.9	33.5	27.9	25.2	24.5	25.1	26.7	31.6	57

XXI.—(*Continued.*)

CONDENSING ENGINES—JACKETED CYLINDERS—DRY SATURATED STEAM.

	Symbol.	Formula.	Point of Cut-off. Full stroke	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$
Mean pressure, pounds per square inch. Corrected for back-pressure	M	59	90.5	87.7	77.6	64.8	55.7	43.8	36.1	28.3
Cushion	k	65	.....	5.2	6.3	7.5	8.4	9.7	10.8	12
Corrected for back-pressure, clearance, and cushion	t	29	90.2	87.3	76.7	63.4	54	41.4	33.6	24.8
Probable effective	e	30	85.7	82.9	72.9	60.2	51.3	39.3	31.9	23.6
Probable effective horse-power	(E)	39	177	177.3	178.7	184.6	195.1	219.1	245	300.3
Pounds of steam hourly. By piston-displacement	Q	47	5883	4924	3933	3415	3244	3264	3385	3838
Condensed in jacket	(K)	76	5	141	263	356	426	554	675	950
Total	(w)	77	5888	5065	4196	3771	3670	3818	4060	4788
Per effective horse-power	(W)	57	33.3	28.6	23.5	20.4	18.8	17.4	16.5	15.9

NON-CONDENSING ENGINES—UNJACKETED CYLINDERS—STEAM SUPERHEATED SUFFICIENTLY TO PREVENT CONDENSATION.

	Symbol	Formula.	Point of Cut-off. Full Str'ke	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	Ref. No.
Terminal pressure, pounds per square inch	(r)	$C \times \frac{1}{R}$	......	70.1	49.1	34 4	26.7	18.7	14.4	10.7	78
Volume of 1 lb. of steam, cubic feet. Atmos. pressure (r)	(v)	From tables.	......	6.07	8.49	11.84	15.04	21.02	26.89	35.56	79
" " " " " " " m.	(o)	From tables.	......	4.48	4.45	4.41	4.39	4.38	4.37	4.36	80
Mechanical effect of 1 lb. of steam, during expansion, foot-lbs.	(P)	$[(v)-(o)] \times U \times 144$	......	18386	58361	66959	73342	91552	104328	118161	81
Units of heat condensed for work, per lb. of steam	(p)	$\frac{(P)}{772}$	......	23.8	75.6	86.7	95	119	135	153	82
Total heat, British thermal units per lb., above 32°. Fah., of steam at pressure (r): Saturated steam	(H)	From tables.	......	1174.3	1167.3	1160.5	1156	1150	1146	1142	83
Steam superheated to prevent condensation due to work	(h)	$(H)+(p)$	......	1198.1	1242.9	1247 2	1251	1269	1281	1295	84
Temperature, Fah., of superheated steam, including 50° of superheat to prevent condensation due to radiation	N	$\frac{(h)-1075}{.4805}+32+50+20.403 \times \sqrt[4]{r}$	378°	397°	485°	490°	495°	528°	540°	587°	85
Degrees of superheat, Fahrenheit	(S)	$N-(I)$	50°	69°	157°	162°	167°	200°	212°	259°	86

XXI.—(*Continued.*)

NON-CONDENSING ENGINES, UNJACKETED CYLINDERS, STEAM SUPERHEATED SUFFICIENTLY TO PREVENT CONDENSATION.

	Symbol.	Point of Cut-off. Full Str'ke	3/4	1/2	1/3	1/4	1/6	1/8	1/12	Ref. No.
Probable effective horse-power.	(E)	150.6	151.5	148.8	147 5	150.6	148 8	149.8	150.1	39
Pounds of steam used hourly, calculated by piston-displacement. Total..	Q	5851	5539	3963	3494	3328	3268	3627	3925	47
Pounds of steam used hourly, calculated by piston-displacement. Per effective H. P..	(W)	38 8	36 6	26.6	23.7	22.1	22	24.2	26.1	57

XXII.

SPECIFIC HEAT OF WATER.

(Report of British Association for Advancement of Science., 1899.)

$t°$ C.	Joules.	s_t.	s_0t.	k (*CB.*)	Rowland (reduced).
		Range 0° to 60°. Callendar and Barnes. Formula, $s_t = 0.9982 + 0.0000045(t - 40)^2$.			
0°	4.203	1.0054	......	......	....
5	4.196	1.0037	1.0045	5.023	5.023
10	4.190	1.0022	1.0037	10.037	10.044
15	4.185	1.0010	1.0030	15.045	15.054
20	4.181	1.0000	1.0024	20.048	20.057
25	4.178	0.9992	1.0018	25.045	25.053
30	4.176	0.9987	1 0013	30.039	30.043
35	4.174	0.9983	1.0009	35.032	35.039
40	4.174	0.9982	1.0006	40.024	Peabody
45	4.174	0.9983	1.0003	45.016	45.000
50	4.176	0.9987	1.0001	50.008	50.040
55	4.178	0.9992	1.0000	55.002	55.080
60	4.181	1.0000	1.0000	60.000	60.120
		Range 60° to 220° C. Regnault (corrected). Formula, $s_t = 0.0044 + 0.00004t + 0.0000009t^2$.			
60	4.181	1.0000	1.0000	60.000	60.12
65	4.184	1.0008	1.0000	65.002	65.16
70	4.188	1.0016	1.0001	70.008	70.20
75	4.191	1.0024	1.0002	75.018	75.24
80	4.195	1.0033	1.0004	80.032	80.28
85	4.199	1.0043	1.0006	85.051	85.32
90	4.203	1.0053	1.0008	90.075	90.36
95	4.207	1.0063	1.0011	95.105	95.40
100	4.212	1.0074	1.0014	100.138	100.44
					Shaw
110	4.222	1.0097	1.0020	110 22	110.67
120	4.232	1.0121	1.0028	120.33	120.73
130	4.243	1.0148	1.0036	130.47	130.80
140	4.255	1.0176	1.0045	140.63	140.88
150	4.267	1.0206	1.0055	150.82	151.02
160	4.281	1.0238	1.0066	161.05	161.20
170	4.295	1.0272	1.0077	171.31	171.61
180	4.310	1.0308	1.0089	181.60	182.14
190	4.326	1.0345	1.0102	191.94	......
200	4.342	1.0384	1.0115	202.31	......
210	4.359	1.0425	1.0130	212.72	......
220	4.377	1.0467	1.0145	223.18	......

$$s_t = s_{40}(1 + 0.0000045(t - 40)^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(CB)}$$

which gives for the mean specific heat between 0° and $t°$ the formula:

$$s_0^{\,t} = s_{40}(1.0072 - 0.00018t + 0.0000015 0t^2).$$

XXIII.

PHYSICAL CONDITIONS AND TEMPERATURES (BROWN).

		Fahrenheit.
Bright iron becomes	yellow	435°
	red	500
	indigo	550
	gray	750
Tin melts		445
Mercury boils		660
Lead melts		612
Zinc melts		775
Silver melts		1775
Copper melts		1885
Gold melts		1900
Iron bar, red in	a dark room, just visible	950
	ordinary office	1075
	daylight, open air	1450
Cast Iron melts	white	2075
	gray	2230
Annealing malleable iron		1600 to 1750
" glassware		800 to 1000

BESSEMER PROCESS:	Centigrade.	Fahrenheit.
Running the slag	1580°	2876°
" steel into ladle	1640	2984
" " " mould	1580	2876
Annealing furnace, ingot in	1200	2192
Ingot under hammer	1080	1976
SIEMENS-MARTIN PROCESS:		
Gas from producers	720	1328
" entering regenerator	400	752
" leaving regenerator	1200	2192
Air leaving regenerator	1000	1832
Fumes passing to shaft	300	572
End of fusion of charge, open hearth	1420	2588
Refining the steel	1500	2732
Running into ladle, first	1580	2876
" " " last	1490	2714
Glass furnace, between the pots	1375	2507
In the pots, refining	1310	2390
" " working	1045	1913
Siemens, melted for casting	1310	2390
BLAST-FURNACE, GRAY BESSEMER:		
Front of tuyere	1930	3506
At tapping	1570	2858
SIEMENS FURNACE FOR GAS RETORTS:		
Top of furnace	1190	2174
Bottom of furnace	1045	1913
Retorts at close of distillation: 3 feet from cover	875	1607
Retorts at close of distillation: 4 feet 6 inches from cover	950	1742
Porcelain, baking furnace	1370	2498
Red brick, Hoffman furnace	1100	2012
Electrical, glow lamps	1800	3272

Centigrade degrees + 80% + 32° = Fahrenheit.

APPENDIX B.

NOTES.

§ 97, p. 373.—We may add that, for the first time, so far as the author is aware, the theory of the gas-engine was constructed with the assumption of a variation of specific heats with temperature, following MM. Mallard and Le Chatelier, by Professor Burstall, thus:

$$K_v = a + sT; \qquad K_p = b + sT;$$
$$K_p - K_v = \text{const.} = b - a = R.$$
$$H_v = (w_1 + w_2)\int_{T_1}^{T_2}(a + sT)\delta T;$$
$$= (w_1 + w_2)[a(T_2 - T_1) + s/2\cdot(T_2^2 - T_1^2)];$$
$$H_p = (w_1 + w_2)\int_{T_2}^{T_3}(b + sT)\delta T;$$
$$= (w_1 + w_2)[b(T_3 - T_2) + s/2\cdot(T_3^2 - T_2^2)];$$

where H_v and H_p are the quantities of heat added during the periods of constant volume and pressure, respectively, w_1 and w_2 the weights of air and gas plus the weight of residual products from the previous stroke in the clearance spaces.

The equation of the adiabatic also differs from that for constant values of specific heats, thus:

$$\delta q = K_v\left(\frac{dT}{dp}\right)\delta p + K_p\left(\frac{dT}{dv}\right)\delta v;$$
$$dT/dp = v/R; \quad dT/dv = p/R:$$
$$\delta q = K_v v/R\cdot\delta p + K_p p/R\cdot\delta v = 0.$$
$$(a + sT)vdp + (b + sT)pdv = 0.$$
$$(b - a)\log_e v + a\log_e(pv) + spv/R = \text{const.}$$
$$p^a v^b e^{s\frac{pv}{R}} = \text{const.}$$

The entropy equation becomes, in the latter case,

$$\phi = a\log_e\frac{T}{T_0} + R\log_e\frac{V}{V_0} + S(T - T_0);$$

where V and V_0 are the volumes at temperatures T and T_0, respectively.

§ 117, pp. 454–459.—It should be noted that the computed quantities are, in the first example, taken for one cubic foot,

and in the third for one pound. Either may be reduced to the other by the elimination or introduction of D_1, the weight of the cubic foot of steam at the initial pressure, p_1, and the use of L or of H as the proper measure of the latent heat in foot-pounds for the cubic foot and the pound unit, respectively.

In the third example, also, the problem is solved by reference to Rankine's tables of work per pound from 32° F.; U_1 and U_2 representing these quantities and their difference the work between initial pressure and p_2.

In this case, also, corresponding to that of the jacketed engine, the weights of steam and fuel are computed by Rankine without taking account of the jacket-steam. The total steam demanded may be computed by ascertaining the quantity for unity efficiency and dividing this amount by the actual efficiency. The quantity for unity efficiency is obtained by dividing 2545 *B. T. U.* by the total heat per pound *from* feed-water temperature *at* the boiler temperature; or 1,980,000 ft.-lbs. by the mechanical equivalent of that total heat. For the case here taken, for example,

$$\frac{1,980,000}{851,132} = 2.3 \text{ lbs.}; \quad W^1 = \frac{2.3}{0.1705} = 13.5 \text{ lbs.}$$

The jacket-steam is seen to be $\frac{13.5}{12.2} = 1.107$; or eleven per cent of the working steam.

§ 130, p. 517.—Equation (1) is seen to give the same result irrespective of the ratio of expansion; which is now obviously incorrect. It would seem that a factor, $f(r)$, should be introduced. On the whole, it appears to the Author probable that the best expressions for internal wastes yet available must include functions of time or speed, of ratio of expansion, of size of engine, and of steam-pressure. He has used the form

$$\frac{W}{W'} = \frac{a}{d}\sqrt{\frac{r}{N}};$$

in which, for p_1 in pounds on the square inch, absolute, d in inches, N in revolutions per minute, the value of the coefficient may be taken $a = 30$.

§ 139, p. 586.—Referring to § 130, p. 523, it will be seen that the multiple-cylinder engine not only excels in the fact of its "cascade" action, as French writers term it, but also, usually, in the better proportions of the smaller cylinders, which approximate much more nearly the proportions of minimum internal waste,

$$l = erd,$$

than simple engines or than the large cylinder. The comparatively small expansion in each cylinder, the large ratio of stroke to diameter, and the series action all conspire in favor of the multiple-cylinder engine. It is comparatively easy to approximate a stroke equal to two or two and a half diameters in the latter, but usually not at all practicable to make it four to eight diameters, in the case of the simple engine.

§ 148, p. 604.—The results of such computations as are here illustrated, in the case of the engine referred to in note to § 37, p. 177, and as made in advance of the trial, are as follow.

The data assumed are the following, and are as nearly those of the trial as it was possible to make them, in advance:

THERMODYNAMIC RESULTS.

$p_1 = 121.6 + 14.5 = 136.1$; $p_2 = 2.8$ lbs. per square inch; $T_1 = 811.8°$ F.

r	p_2	T_2	V_2	U	Therm. Efficiency	Steam per H. P. per hour, W'
15	7.68	642.2	48.93	206,600	0.232	9.61
20	5.55	628.0	66.40	214,600	.252	9.23
25	4.41	618.3	82.50	219,000	.258	9.02
30	3.63	610.5	98.90	220,380	.259	8.98
35	3.09	604.1	115.25	222,070	.261	8.92

Taking into account the wastes, the corrected results for the above case, at $r = 20$, become:

DISTRIBUTION OF ENERGIES.

Energy Supplied.		Steam per hour.	
		Per I. H. P.	Per D. H. P.
Useful work........................	.230	2.80	3.10
Thermal wastes....................	.748	9.13	10.12
Dynamic wastes....................	.022	0.27	0.28
Total received....................	1.00	12.20	13.50

As has been seen, the actual figures were 11,678 lbs. per I. H. P., and 12.86 lbs. per D. H. P.; the engine working with slightly less thermal efficiency than computed, but more than compensating this defect by an exceptionally low friction waste.

For this engine the thermal wastes, per stroke, are ap proximately measured by the expression

$$W = \frac{A \Delta T \sqrt{p_1 t}}{3,000,000};$$

in which, for either cylinder, p_1 is the initial, absolute pressure in pounds per square inch; A is in square feet, T is taken on the Fahrenheit scale, and t is time of revolution in seconds. In each cylinder, also, the average fraction of ideal, computed, weight of steam required to meet this loss is

$$C = \frac{W}{W'} = 0.15 \sqrt{rt}, \text{ nearly.}$$

The waste for the engine as a whole, $\bar{r}$ being the total expansion, is

$$C = 0.05 \sqrt{\bar{r}t}, \text{ nearly.}$$

It seems probable that the best engines may now be usually expected to waste not more than

$$C = \frac{W}{W'} = \frac{0.15}{n} \sqrt{rt},$$

where n is the number of cylinders in series, and the unit is the weight of steam computed for the ideal case.

§ 149, p. 611.—*For the Ideal Cases*, the following *résumé* of formulæ and of methods of computation may prove convenient. (See also § 117, text and notes.) The net thermal value of the pound of fuel is assumed at 10,000 *B. T. U.*

I. CARNOT CYCLE:

p_1 = initial pressure in pounds per square foot, absolute. This pressure is to vary by prescribed intervals between the limits of pressures as assigned.

Pressures are to be entered in the proper line of a tabulated form, the lowest initial pressure in its appropriate column.

p_2 = pressure at the end of expansion;

= back-pressure. Expressed in pounds per square foot, absolute;

= 576 pounds per square foot for the equivalent of the condensing cycle;

= 2592 pounds per square foot for equivalent of the non-condensing engine.

v_1 = initial volume;

= volume in cubic feet of one pound of dry and saturated steam at initial pressure.

Take this from steam-tables. (Manual, p. 820.)

v_2 = volume in cubic feet at the end of expansion;

$= v_1\left(\frac{p_1}{p_2}\right)^{.881}$; $.881 = \frac{1}{1.135}$.

T_1 = absolute temperature in degrees *F.*, corresponding to p_1. Add 461° to the temperature given in tables.

T_2 = absolute temperature in degrees *F.*, corresponding to p_2. Take temperature from tables and add 461°.

L_1 = latent heat of evaporation corresponding to p_1. Take this from tables.

r = ratio of expansion;

$= \frac{v_2}{v_1}$.

H_1 = heat expended per pound of steam, expressed in foot-pounds;

$= JL_1$.

$J = 778$.

$$\text{Efficiency} = \frac{T_1 - T_2}{T_1}.$$

U = net work done per pound of steam, expressed in foot-pounds;

$= H_1 \times$ Efficiency.

$$M.E.P.' = \frac{U}{v_2} \text{ in pounds per square foot.}$$

$$M.E.P.' = \frac{M.E.P.'}{144} \text{ in pounds per square inch.}$$

A = $B.T.U.$ per $I.H.P.$ per hour;

$$= \frac{2545}{\text{Efficiency}}.$$

B = pounds of steam per $I.H.P.$ per hour for efficiency unity;

$$= \frac{1{,}980{,}000}{H_1}.$$

C = pounds of steam per $I.H.P.$ per hour for actual efficiency;

$$= \frac{B}{\text{Efficiency}}.$$

W = equivalent water-rate from and at 212° $F.$;

$$= \frac{A}{966.069}.$$

F = pounds of fuel per $I.H.P.$ per hour;

$$= \frac{A}{10{,}000}.$$

D = piston-displacement per $I.H.P.$ per hour;

$= Cv_2$ cu. ft.

D' = piston-displacement per $I.H.P.$ per minute;

$$= \frac{D}{60}.$$

II. RANKINE'S NON-CONDUCTING ENGINE CYCLE:

v_1 = volume in cubic feet at point of cut-off;

= initial volume = volume of one pound of dry and saturated steam at the pressure chosen.

Take this from steam-tables. (Manual, p. 820.)

v_2 = volume at end of expansion;

$= v_1 r$;

r = ratio of expansion.

Take $r = 1, 2, 3, 4, 6, 10, 15$, and 25 for condensing;

$= 1, 2, 3, 4, 5, 6, 8$, and 12 for non-condensing.

In no case expand below back-pressure.

p_1 = initial pressure in pounds per square foot, absolute.

p_2 = pressure at the end of expansion expressed in pounds per square foot, absolute;

$$= p_1 \left(\frac{v_1}{v_2}\right)^{1.135}$$

(See Manual, pp. 398 and 404.)

p_3 = back-pressure in pounds per square foot, absolute;

$= 4 \times 144 = 576$ pounds per square foot for condensing engine;

$= 18 \times 144 = 2592$ pounds per square foot for non-condensing engine.

$J = 778$ foot-pounds per degree *F.*

T_1 = absolute temperature in degrees *F.*, corresponding to p_1.

Take temperature from steam-tables and add 461°.

T_2 = absolute temperature in degrees *F.*, corresponding to p_2;

= Take temperature from steam-tables and add 461°.

T_4 = absolute temperature in degrees *F.* of feed-water;

$= 104 + 461 = 565°$ *F.*, for condensing engine;

$= 210 + 461 = 671°$ *F.*, for non-condensing engine;

λ_1 = total heat of evaporation corresponding to p_1 in *B. T. U.*

Take this from steam-tables.

q_4 = heat in feed-water at T_4 in *B. T. U.*;

$= T_4 - (461 + 32)$.

U = net work done per pound of steam (in foot-pounds);

$$= J\left[T_1 - T_2\left(1 + \log_e \frac{T_1}{T_2}\right)\right] + \frac{T_1 - T_2}{T} H' + v_2(p_2 - p_3)$$

(Manual, p. 445.)

L_1 = latent heat of evaporation, at p_1, in *B. T. U.*

Take from steam-tables..

H' = same in foot-pounds $= J L_1$

H_1 = heat expended per pound of steam in foot-pounds;

$= J(\lambda_1 - q_4)$.

$$\text{Efficiency} = \frac{U}{H_1}.$$

$$M.E.P.' = \frac{U}{rv_1} = \frac{U}{v_2} \text{ in pounds per square foot.}$$

$$M.E.P.'' = \frac{M.E.P.'}{144} \text{ in pounds per square inch.}$$

A = *B. T. U.* per *I. H. P.* per hour;

$$= \frac{2545}{\text{Efficiency}}.$$

B = pounds of steam per *I. H. P.* per hour for efficiency unity;

$$= \frac{1,980,000}{H_1}.$$

C = pounds of steam per *I. H. P.* per hour for actual efficiency;

$$= \frac{B}{\text{Efficiency}}.$$

W = equivalent water-rate from and at 212° *F.*;

$$= \frac{A}{966.069}.$$

F = fuel per *I. H. P.* per hour;

$$= \frac{A}{10,000}.$$

D = piston-displacement per *I. H. P.* per hour;

$= Cv_2$ cu. ft.

D' = piston-displacement per *I. H. P.* per minute;

$$= \frac{D}{60}.$$

III. RANKINE'S JACKETED CYCLE:

v_1 = volume in cubic feet at point of cut-off;
= initial volume = volume of one pound dry and saturated steam at the pressure chosen.
Take this from steam-tables. (Manual, p. 820.)

v_2 = volume at end of expansion;
$= v_1 r$.

r = ratio of expansion.

Take r = 1, 2, 3, 4, 6, 10, 15, and 25 for condensing;
= 1, 2, 3, 4, 5, 6, 8, and 12 for non-condensing.

p_1 = initial pressure in pounds per square foot, absolute.

p_2 = pressure at end of expansion expressed in pounds per square foot, absolute.
Take this from steam-tables.

p_3 = back-pressure in pounds per square foot, absolute;
$= 4 \times 144 = 576$ pounds per square foot for condensing engine;
$= 18 \times 144 = 2592$ pounds per square foot for non-condensing engine.

J = 778 foot-pounds.

a = 1,117,850 foot-pounds.

b = 544.5 foot-pounds.

T_1 = absolute temperature in degrees *F.*, corresponding to p_1.
Take temperature from steam-tables and add 461°.

T_2 = absolute temperature in degrees *F.*, corresponding to p_2.
Take temperature from steam-tables and add 461°.

T_4 = absolute temperature in degrees *F.* of feed-water;
$= 104 + 461 = 565°$ *F.* for condensing engine;
$= 210 + 461 = 671°$ *F.* for non-condensing engine.

H_1 = heat per pound of steam (at boiler) in foot-pounds;
$= J(\lambda_1 - q_4)$.

H_1' = heat expended per pound of working steam, in foot-pounds;
$= H_1$ plus the heat received by the working steam from the jacket;

$$= J(T_1 - T_4) + a\left(1 + \log_e \frac{T_1}{T_2}\right) - bT_1, \text{ nearly.}$$

The distinction between H_1 and H_1' should be carefully observed.

Working steam is the steam passed through the cylinder and not that in jacket.

U = net work done, in foot-pounds, per pound of working steam;

$$= a \log_e \frac{T_1}{T_2} - b(T_1 - T_2) + v_2(p_2 - p_3).$$

(See Manual, p. 452.)

$$\text{Efficiency} = \frac{U}{H_1'}$$

$$M.E.P.' = \frac{U}{rv_1} = \frac{U}{v_2} \text{ in pounds per square foot.}$$

$$M.E.P.'' = \frac{M.E.P.'}{144} \text{ in pounds per square inch.}$$

$$A = B.T.U. \text{ per } I.H.P. \text{ per hour} = \frac{2545}{\text{Efficiency}}.$$

B = pounds of steam per $I.H.P.$, per hour, at boiler (inclusive of jacket-steam) for efficiency unity;

$$= \frac{1{,}980{,}000}{H_1} = \frac{1{,}980{,}000}{J(\lambda - q_4)}.$$

B' = pounds of working steam per $I.H.P.$ per hour for efficiency unity;

$$= \frac{1{,}980{,}000}{H_1'}.$$

C = pounds of steam per $I.H.P.$, per hour, total (jacket and working charge) for actual efficiency;

$$= \frac{B}{\text{Efficiency}}.$$

$C' =$ pounds of working steam per *I.H.P.* per hour for actual efficiency;

$= \dfrac{B'}{\text{Efficiency}}.$

$W =$ equivalent water-rate from and at 212° F.;

$= \dfrac{A}{966.069}.$

$F =$ pounds fuel per *I.H.P.* per hour;

$= \dfrac{A}{10,000}.$

$D =$ piston-displacement per *I.H.P.* per hour;

$= C'v_2$ cu. ft.

$D' =$ piston-displacement per *I.H.P.* per minute;

$= \dfrac{D}{60}.$

NOTE.

To integrate the expression (pp. 434, 450, 769)

$$dT.\log \frac{T}{T} = (\log_e T_1 - \log_e T)dT:$$

$$d.xy = x\,dy - y\,dx; \qquad xy = \int x\,dy - \int y\,dx;$$

$$\int x dy = xy - y\,dx.$$

$$x = \log_e T; \qquad y = T;$$

$$\int \log_e T dT = T\log_e T - \int T d\log_e T$$

$$= T\log_e T + T = T(\log_e T - I).$$

$$\int \log_e T_1 . dT = T\log_e T_1.$$

$$\int \log_e \frac{T_1}{T} . dT = T\log_e T_1 - T\log T + T.$$

$$\int_{T_2}^{T_1} \log_e \frac{T_1}{T_2} . dT = T_1\log_e T_1 - T_2\log_e T_1 - T_1\log_e T_1 + T_2\log T_2 + T_1 - T_2$$

$$= T_1 - T_2\left(I + \log_e \frac{T_1}{T_2}\right).$$

Hence

$$\int_{p_2}^{p_1} u\,dp = \int\left(\int_{T_2}^{T_1}\log_e \frac{T_1}{T_2}6 + \frac{H_1}{T_1}\right)dT = J\left[T_1 - T_2\left(I + \log\frac{T_1}{T_2}\right)\right] + \frac{T_1 - T_2}{T_2}H,$$

as already given in the text as above, eq. 6, p. 434.

IDEAL STEAM-ENGINE EFFICIENCIES.

CARNOT CYCLE.

IDEAL CASE; $p_2 = p_3 = 2$; CORRESPONDING TO $T_2 = 587°$.

CASE I.

p_1	150	200	250	300	350	400	450	500
T_1	819.2	841.6	861.9	878.36	892.9	905.92	917.6	928.4
r	44.8	57.8	70.4	82.63	94.6	106.5	118	129.6
V_2	133	130.4	128.4	126.83	125.4	124.2	123.3	122
H_1	671,000	656,000	646,097	636,604	628,219	620,797	613,000	607,618
U	190,000	198,500	205,000	210,079	218,900	218,500	221,000	222,000
E	28.3	30.3	31.9	33.1	34.2	35.1	36	36.7
M.E.P.′	1,427	1,522	1,604.9	1,695	1,712.1	1,758.2	1,800	1,820
M.E.P.″	10.01	10.56	11.13	11.55	11.85	12.2	12.5	12.7
A	9,000	8,430	7,986	7,730	7,450	7,236	7,050	6,934
B	2.95	3.02	3.08	3.11	3.15	3.19	3.23	3.26
C	10.45	9.98	9.69	9.4	9.2	9.0	8.94	8.89
W	9.32	8.73	8.27	7.93	7.73	7.49	7.32	7.19
F	.90	.832	.799	.773	.745	.724	.705	.698
D	1,390	1,300	1,236	1,190	1,157	1,115	1,100	1,090
D'	22.8	21.7	20.6	19.9	19.28	18.58	18.3	18.1

CARNOT CYCLE.

IDEAL CASE; $p_2' = p_3' = 16$; NON-CONDENSING; $T_2' = 677.35°$.

CASE II.

p_1	150	200	250	300	350	400	450	500
T_1	819.22	842.63	861.88	878.376	892.96	905.92	917.62	927.2
r	7.18	9.25	11.3	13.23	15.1	17.05	18.8	21.02
V_2	21.27	20.87	20.56	20.3	20.07	19.89	19.627	19.51
H_1	679,324	657,080	646,097	636,641	628,150	620,797	613,935	607,618
U	115,966	128,880	138,264	145,727	151,800	157,062	160,728	164,061
E	17.3	19.69	21.4	22.9	24.2	25.3	26.5	27.1
M.E.P.′	5,450	6,175	6,730	7,178	7,570	7,865	8,159	8,360
M.E.P.″	38	42.8	46.6	49.09	54.2	54.6	56.6	57.0
A	14,710	12,990	11,886.9	11,123	10,530	10,059	9721.16	94.25
B	2.95	3.02	3.06	3.2	3.16	3.19	3.24	3.26
C	17.05	15.33	14.3	13.6	13.08	12.6	12.22	12.07
W	15.3	13.43	12.3	11.5	10.9	10.4	10.06	9.47
F	1.48	1.298	1.19	1.112	1.053	1.01	.972	.941
D	353	321	294	278.2	262.5	262.4	240	237.5
D'	5.89	5.35	4.91	4.63	4.37	4.34	4.03	3.92

RANKINE'S NON-CONDENSING CYCLE.

IDEAL CASE; $p_2 = p_3 = 2$; CONDENSING; $T_2 = 587°$.

CASE III.

p	150	200	250	300	350	400	450	500
T	819.22	842.63	861.88	878.369	892.96	905.9	917.62	927.2
r	14.87	19.34	25	27.4	31.4	35.4	39.17	43.02
V	44.04	43.25	42.6	42.06	41.55	41.25	40.82	40.5
H	858,289	863,813	868,411	872,323	875,827	878,890	881,707	885,000
U	197,624	212,305	223,805	233,120	232,231	242,042	256,526	262,050
E	23	24.5	25.87	26.7	26.5	27.5	29.09	29.6
M.E.P.′	4,521	4,908	5188.7	5,260	5,583	5,875	6,170	6,375
M.E.P.″	31.16	34.2	35.5	36.6	38.77	40.75	42.64	45.5
A	11,065	10,386	9,910	9,875	9,603	9,254	8,748	8,600
B	2.3	2.29	2.28	2.27	2.26	2.25	2.25	2.24
C	10	9.35	8.88	8.52	8.54	8.36	7.7	7.57
W	11.4	10.7	10.3	10.2	9.9	9.2	9.0	8.9
F	1.107	1.03	.991	.987	.96	.925	.875	.843
D	440.4	397	387.1	376.2	355	328	316	306.3
D'	7.3	6.7	6.4	6.2	5.9	5.6	5.3	5.1

RANKINE'S NON-CONDUCTING CYCLE.

IDEAL CASE; $p_2' = p_3' = 16$; NON-CONDENSING; $T_2' = 677.35$.

CASE IV.

p_1	150	200	250	300	350	400	450	500
T_1	819.2	842.63	861.88	878.37	892.96	905.9	917.62	928.4
r	5.64	7.28	8.91	10.44	11.9	13.4	14.88	16.3
V_2	16.7	16.4	16.2	16.03	15.79	15.69	15.45	15.35
H_1	796,049	803.500	805,000	809,898	813,585	816,000	821,000	822,500
U	126,555	142,720	151383.76	164,457	169,959	177,380	186,152	192,820
E	15.9	17.3	18.92	19.9	20.8	21.9	22.7	23.4
M.E.P.′	7,754	8,460	9,219	10,200	10,763	10.940	11,310	12.540
M.E.P.″	52.2	58.8	64.02	70.5	74.7	78.7	83.3	87.2
A	16,006	14,750	13,370	12,494	12,188	11,620	11,201	10,870
B	2.4	2.47	2.46	2.44	2.44	2.45	2.41	2.41
C	15.7	14.2	13.6	12.5	12.2	11.6	10.6	10.2
W	16.58	15.28	14.02	13.6	12.6	12.03	11.5	11.2
F	1.6	1.46	1.35	1.27	1.21	1.16	1.12	1.10
D	246.07	233.9	214.75	195.0	191.5	181	165.06	159.6
D'	4.58	3.89	3.57	3.25	3.19	3.12	3	2.66

RANKINE'S JACKETED CYCLE.

IDEAL CASE; $p_2 = p_3 = 2$; CONDENSING; $T_2 = 587°$.

CASE V.

p_1	150	200	250	300	350	400	450	500
T_1	819.2	842.6	861.88	878.37	892.96	905.92	957.62	928.4
r	17.9	23.5	29	34.4	39.9	45.3	50.76	56.1
V_2	52.89	52.89	52.89	52.89	52.89	52.89	52.89	52.89
H_1	858,289	863,843	868,411	872,323	875,794	878,890	881,707	884,197
U	215,400	236,900	251,555	264,900	....	284,560	291,855	295,600
E	21.3	23.3	24.5	25.5	25.8	26.7	27.3	28.2
M.E.P.″.	28.7	31.1	32.9	34.7	3	37.2	38.3	39.2
H_1'	995,526	1,013,915	1,040,650	1,040,806	1,055,534	1,060,938	....	1,072,853
A	12,177	10,920	10,595.3	10,000	11,162	9,520	9,318.9	9,155
B	2.30	2.29	2.28	2.27	2.26	2.25	2.24	2.23
B'	1,976	1.95	1.93	1.91	1.87	1.86	1.85	1.84
C	10.6	9.77	9.30	8.9	8.7	8.42	8.24	8.06
C'	9.22	8.39	7.87	7.45	8.24	6.99	6.75	6.71
W	12.6	11.38	10.77	10.35	....	9.85	9.76	9.60
F	1.21	1.092	1.05	1	....	.952	.944	.930
D	431	425	416.5	394	....	370	358	351
D'	7.97	7.39	6.94	6.57	....	6	5.96	5.86

RANKINE'S JACKETED CYCLE.

IDEAL CASE; $p_2' = p_3' = 16$; NON-CONDENSING; $T_2' = 677.35$.

CASE VI.

p_1	150	200	250	300	350	400	450	500
T_1	819.2	841.63	861.88.	878.37	892.96	905.92	917.62	928.42
r	6.36	8.32	10.32	12.58	14.21	16.15	18.02	20
V_2	18.84	18.84	18.84	18.84	18.84	18.84	18.84	18.84
H_1	795,894	801,600	806,171	810,084	813,585	816,666	819,234	821,988
U	129,240	143,698	166,529	178,878	181,054	198,365	206,734	211,150
E	14.8	16.9	18.2	19.25	20	20.9	21.5	22.1
M.E.P.′.	6,880	8,140	8,850	9,500	10,060	10,540	10,973	11,200
M.E.P.″.	47.3	56.5	61.4	65	70.1	73.18	76.20	78.84
H_1'	881,868	903,990	912,666	930,750	940,878	950,445	958,241	964,840
A	17,370	15,111	14,411	13,200	12,710	12,096	11,798	11,500
B	2.49	2.47	2.46	2.45	2.43	2.42	2.41	2.40
B'	2.24	2.19	2.16	2.13	2.10	2.08	2.06	2.05
C	17	14.93	13.56	12.7	12.07	11.52	11.2	10.9
C'	15.35	13	12.54	11.05	10.9	9.98	9.57	9.32
W	17.8	15.64	14.48	13.6	13.07	12.8	12.2	11.9
F	1.77	1.46	1.398	1.32	1.27	1.20	1.18	1.16
D	267	243.1	230	208	197.8	188	180.2	177
D'	4.77	4.08	3.88	3.47	3.29	2.99	2.92	2.9

APPENDIX C.

APPLICATION OF HIRN'S ANALYSIS.

[FORM No. 1.]

SIBLEY COLLEGE, CORNELL UNIVERSITY.

DATA AND RESULTS.

Test of steam-engine made by E. P. Allis Co., at Milwaukee, Wis.

Which engine—high-pressure cylinder.
Diameter of cylinder, 28 inches.
Length of stroke, 60 inches.
Diameter of piston-rods (2), 4 inches.
Volume of cylinder, crank end, 20.53.
Volume, head end, 21.40.
Volume of clearance, cu. ft., head, .298.
Clearance in per cent of stroke, 1.4.
Volume of clearance, cu. ft., crank, .287.
Clearance in per cent of stroke, 1.4.
Boiler-pressure by gauge, 121.4.
Barometer, 14.5.
Boiler-pressure, absolute, 135.9.
Revolutions per hour, 1218.8.
Quality of steam in steam-pipe, 98.95.
Weight of condensed steam per hour, 6771.8.
Steam used during run, pounds, 162,524.5.
Lbs. of wet steam per revolution, 5.56
Per stroke, head, 2.72.
Per stroke, crank, 2.84.

SYMBOLS.

To denote different portions of the stroke, the following subscripts are used:

Admission (*a*); expansion (*b*); exhaust (*c*); compression (*d*).

To denote different events of the stroke, the following sub-numbers are used: Cut-off (1); release (2); compression, beginning of (3); admission, beginning of (0); in exhaust (5).

Quality of steam denoted by x.

Cut-off, crank end, per cent of stroke, 34.1.
Cut-off, head end, per cent of stroke, 33.4.
Compression, crank end, per cent of stroke, 1.73.
Compression, head end, per cent of stroke, 2.89.
Release, crank end, 100.
Release, head end, 100.
Pounds of dry steam (3 cylinders) per total I. H. P., 11.678.
I. H. P., head, 87.54.
I. H. P., crank, 87.85.
Total I. H. P., 175.39.

HIGH-PRESSURE.

[FORM No. 2.]

ABSOLUTE PRESSURES FROM INDICATOR-DIAGRAMS AND CORRESPONDING PROPERTIES OF SATURATED STEAM.

		Cut-off.	Release.	Beginning.		Symbols. Rankine.	Symbols. Clausius.
				Compression.	Of Admission.		
Subscripts used		1	2	3	0		
Absolute pressure	Head	134.2	45.0	48.2	135.3	P	p
	Crank	132.4	48.7	49.7	132.7		
Heat of liquid	Head	320.63	243.68	248.11	321.27	S	q
	Crank	319.56	248.66	249.97	319.74		
Internal latent heat	Head	786.01	844.80	841.46	785.52	I	ρ
	Crank	786.84	840.98	840.12	786.69		
Latent heat evaporation	Head	867.92	921.93	917.90	867.46	L	r
	Crank	868.67	918.44	916.50	868.55		
Total heat	Head	1,188.55	1,165.61	1,166.00	1,188.73	H	λ
	Crank	1,188.23	1,167.10	1,167.47	1,188.29		
Volume 1 lb. cubic feet	Head	3.290	9.207	8.684	3.265	C	v
	Crank	3.332	8.550	8.350	3.225		
Volumes symbols		$V_c + V_1$	$V_c + V_2$	$V_c + V_3$	$V_0 + V_c$	V_c	..
Volumes head, cubic feet		7.579	21.68	.668	.299	.299	..
Volumes crank, cubic feet		7.434	21.68	.916	.299	.299	..

MEAN PRESSURES AND HEAT EQUIVALENTS OF EXTERNAL WORK.

	Subscripts.	Head End.		Crank End.	
		Mean Pressures.	External Work. B. T. U.	Mean Pressures.	External Work. B. T. U.
Symbols		MEP	AW*	MEP	AW*
Admission	a	118.9	157.1	125.4	162.5
Expansion	b	58.6	155.7	62.8	157.2
Exhaust	c	32.68	129.0	34.81	129.7
Compression	d	24.15	2.765	43.7	6.48
Total, net		......	181.035	.....	183.52

* $A = \frac{1}{778}$. V_c = volume in clearance-spaces.

HIGH-PRESSURE.

HIRN'S ANALYSIS—DATA AND RESULTS PER 100 STROKES.

JACKET-STEAM INCLUDED.

Quantities.	Symbols.	Formulæ.	Head.	Crank.
Steam from boiler, pounds...	M	....	272	284
Steam in clearance, pounds..	Mc	$100(Vc + CcXc)$....................	9.16	8.998
Steam at admission, pounds..	M_0	$100\frac{Vc+V_0}{C_0X_0}$....................	9.16	8.998
Steam, total, pounds.......	$M+M_0$	....	281.16	292.998
Heat of condensed steam ...	K'	MSg..............................	....	.... ..
Condensing-water, pounds...	G	$K + K_1 =$........................	287,081	303,761
Heat given to cond's'g-water	K	$G(Sk - Si)$.......................	....	....
Heat supplied to engine......	Q	$M(XL + S)$.......................	320,050	340,000
Sensible heat at admission....	H_0	M_0S_0..............................	2,941	2,878
Internal heat at admission....	H_0'	$100\frac{Vc+V_0}{C_0}I_0$....................	7,195	7,075
Sensible heat at cut-off.......	H_1	$(M + M_0)S_1$.......................	90,200	93,500
Internal heat at cut-off.......	H_1'	$100\frac{Vc+V_1}{C_1}I_0$....................	181,400	175,500
Sensible heat at release......	H_2	$(M + M_0)S_2$.......................	68,500	72,900
Internal heat at release..... ..	H_2'	$100\frac{Vc+V_2}{C_2}I_2$....................	198,000	213,300
Sensible heat, beginning of compression.............	H_3	M_0S_3..............................	2,277	2,250
Internal heat, beginning of compression.............	H_3'	$100\frac{Vc+V_3}{C_3}I_3$....................	6,475	9,225
Cylinder loss during admission....................	Qa	$Q + H_0 + H_0' - H_1 - H_1' - \frac{Wa}{778}$	42,876	64,703
Cylinder loss during expansion......................	Qb	$H_1 + H_1' - H_2 - H_2' - \frac{Wb}{778}$.....	−10,470	−32,920
Cylinder loss during exhaust	Qc	$H_2 + H_2' - H_3 - H_3' - K - K' - \frac{W}{778}$	−16,433	−16,066
Cylinder loss during compression.................	Qd	$H_3 + H_3' - H_0 - H_0' - Hx - Hx' - \frac{Wd}{778}$	−1,107.5	2,170
Heat discharged, and work..	B	$K + K' + Hx + Hx' + \frac{W}{778}$.......	305,184.5	322,113
Jacket and radiation.........	D	$Q - B$..............................	14,865.5	17,887
Jacket and radiation.........	D'	$Qa + Qb + Qc + Qd$..............	14,865.5	17,887
Quality of steam entering....	X	per calorimeter...........per cent	98.95	98.95
Quality of steam at cut-off...	X_1	$100\frac{Vc+V_1}{(M+M_0)C_1}$........... "	81.85	76.20
Quality of steam at release...	X_2	$100\frac{Vc+V_2}{(M+M_0)C_2}$........... "	83.90	86.20
Quality of steam in exhaust..	X_5	$\left(\frac{K+K'}{M-Mx} - S_5\right) \div L_5$.... "	88.0	89.3
Heat lost, admission.........	a	$Qa \div Q$..................... "	13.41	19.06
Heat restored, expansion....	b	$Qb \div Q$..................... "	−3.266	−9.68
Heat rejected, exhaust.......	c	$Qc \div Q$..................... "	−5.13	−4.728
Heat lost, compression......	d	$Qd \div Q$..................... "	− .3461	.638
Heat utilized, work..........	w	$\frac{W}{778} \div Q$..................... "	5.66	5.40
Heat lost, radiation..........	r	$D \div Q$..................... "	4.645	5.26
Ratio radiation to work......		$r \div w$..................................	.819	.977
Ratio cyl. condensat. to work		$a \div w$..................................	2.339	3.537
Thermodynamic efficiency...	E	$(t - t_5) \div (461 + t)$........per cent	9.175	8.745
Actual efficiency	E_1	$\frac{W}{778} \div Q$..................... "	5.66	5.40
Efficiency compared with ideal	E'	$E_1 \div E$..................... "	61.8	61.8

Special symbols Vc = volume-clearance, t = measured temperature. Subscript 5 applies to exhaust, i to injection, k to discharge, g to air-pump discharge.
Correct for Mx when necessary.

APPLICATION OF HIRN'S ANALYSIS.

INTERMEDIATE.

[FORM No. 1.]

DATA AND RESULTS.

Which engine — intermediate-pressure cylinder.
Diameter of cylinder, 48 inches.
Length of stroke, 60 inches.
Diameter of piston-rods (2), 4 inches.
Volume of cylinder, crank end, 62.01.
Volume head end, 62.830.
Volume of clearance, cu. ft., head, .942.
Clearance in per cent of stroke, 1.5.
Volume of clearance, cu. ft., crank, .932.
Clearance in per cent of stroke, 1.5.
Receiver-pressure by gauge, 32.43.
Revolutions per hour, 1218.8.
Steam used during run, pounds, 162,524.5.
Weight of condensed steam per hour, 6771.8.
Lbs. of wet steam per revolution, 5.56.
Per stroke, head, 2.915.
Per stroke, crank, 2.545.

SYMBOLS.

To denote different portions of the stroke, the following subscripts are used:

Admission (a); expansion (b); exhaust (c); compression (d).

To denote different events of the stroke, the following sub-numbers are used: Cut-off (1); release (2); compression, beginning of (3); admission, beginning of (0); in exhaust (5).

Quality of steam denoted by x.

Cut-off, crank end, per cent of stroke, 32.85.
Cut-off, head end, per cent of stroke, 35.10.
Compression, crank end, per cent of stroke, 2.92.
Compression, head end, per cent of stroke, 1.67.
Release, crank end, 100.
Release, head end, 100.
I. H. P., head, 88.84.
I. H. P., crank, 80.78.
Total I. H. P., 169.62.

INTERMEDIATE.

[FORM No. 2.]

ABSOLUTE PRESSURES FROM INDICATOR-DIAGRAMS AND CORRESPONDING PROPERTIES OF SATURATED STEAM.

		Cut-off.	Release.	Beginning.		Symbols. Rankine.	Symbols. Clausius.
				Compression.	Of Admission.		
Subscripts used		1	2	3	0		
Absolute pressure	Head	41.5	15.3	16.9	50.1	P	p
	Crank	41.5	14.34	17.1	48.1		
Heat of liquid	Head	238.64	182.59	187.74	250.48	S	q
	Crank	238.64	179.16	188.36	247.88		
Internal latent heat	Head	848.64	892.26	888.24	839.55	I	ρ
	Crank	848.64	894.68	887.76	841.55		
Latent heat evaporation	Head	925.46	964.62	961.03	917.17	L	r
	Crank	925.46	967.04	960.59	918.99		
Total heat	Head	1,164.10	1,147.21	1,184.77	1,167.65	H	λ
	Crank	1,164.10	1,146.20	1,148.95	1,166.87		
Volume 1 lb. cubic feet	Head	9.938	25.09	23.11	8.323	C	v
	Crank	9.938	27.05	22.84	8.549		
Volumes symbols		$V_c + V_1$	$V_c + V_2$	$V_c + V_3$	$V_0 + V_c$	V_c	..
Volumes head, cubic feet		23.01	63.77	2.777	.9425	.9425	..
Volumes crank, cubic feet		21.55	63.77	1.990	.9425	.9425	..

MEAN PRESSURES AND HEAT-EQUIVALENTS OF EXTERNAL WORK.

	Subscripts.	Head End.		Crank End.	
		Mean Pressures.	External Work. B. T. U.	Mean Pressures.	External Work. B. T. U.
Symbols		MEP	AW*	MEP	AW*
Admission	a	30.38	123.7	29.25	110.1
Expansion	b	9.92	69.4	8.35	64.15
Exhaust	c	.736	8.41	.265	2.95
Compression	d	3.827	.745	7.19	2.405
Total, net		... ..	183.955	.....	168.895

* $A = \frac{1}{778}$. V_c = volume in clearance-spaces.

INTERMEDIATE

HIRN'S ANALYSIS—DATA AND RESULTS PER 100 STROKES.

JACKET STEAM INCLUDED.

Quantities.	Symbols.	Formulæ.	Head.	Crank.
Steam from boiler, pounds..	M	....	291.5	254.5
Steam in clearance, pounds.	Mc	$100\,(Vc + CcXc)$....................	11.31	11.02
Steam at admission, pounds..	M_0	$100\dfrac{Vc+V_0}{C_0X_0}$....................	11.31	11.02
Steam, total, pounds.........	$M+M_0$	....	302.81	265.52
Heat of condensed steam...	K'	MSg	....	....
Condensing-water, pounds ..	G	$K + K' =$..........................	306,838.5	262,623.5
Heat given to conden'g-water	K	$G\,(Sk-Si)$	....	....
Heat supplied to engine.....	Q	$M\,(XL + S)$..........................	340,100	297,400
Sensible heat at admission...	h_0	M_0S_0	2,810	2,734
Internal heat at admission...	H_0	$100\dfrac{Vc+V_0}{C_0}I_0$....................	9,510	9,270
Sensible heat at cut-off......	H_1	$(M+M_0)\,S_1$........................	72,300	63,400
Internal heat at cut-off.......	H_1'	$100\dfrac{Vc+V_1}{C_1}I_1$....................	197,300	184,100
Sensible heat at release......	H_2	$(M+M_0)\,S_2$........................	55,300	47,600
Internal heat at release......	H_2'	$100\dfrac{Vc+V_2}{C_2}I_2$....................	223,300	210,800
Sensible heat, beginning of compression..............	H_3	$M_0\,S_3$	2,122	2,079
Internal heat, beginning of compression	H_3'	$100\dfrac{Vc+V_3}{C_3}I_3$....................	10,670	7,750
Cylinder loss during admission	Qa	$Q+H_0+H_0'-H_1-H_1'-\dfrac{Wa}{778}$.......	70,450	50,894
Cylinder loss during expansion	Qb	$H_1+H_1'-H_2-H_2'-\dfrac{Wb}{778}$	−15,940	−17,315
Cylinder loss during exhaust.	Qc	$H_2+H_2'-H_3+H_3'-K-K'-\dfrac{Wc}{778}$	−40,189.5	−13,757.5
Cylinder loss during compression..................	Qd	$H_3+H_3'-H_0-H_0'-Hx-Hx'-\dfrac{Wd}{778}$	546.5	−1,934.5
Heat discharged, and work..	B	$K+K'+Hx+Hx'+\dfrac{W}{778}$..............	325,233	279,513
Jacket and radiation.........	D	$Q-B$	14,867.0	17,887
Jacket and radiation..........	D'	$Qa+Qb+QcQd$........................	14,867.0	17,887
Quality of steam at cut-off...	X_1	$100\dfrac{Vc+V_1}{(M+M_0)\,C_1}$............per cent	76.6	81.7
Quality of steam at release...	X_2	$100\dfrac{Vc+V_2}{(M+M_0)\,C_2}$............ "	83.9	88.9
Quality of steam in exhaust.	X_5	$\left(\dfrac{K+K'}{M-Mx}-S_5\right)\div L_5$....... "	90.25	88.25
Heat lost, admission.........	a	$Qa \div Q$........................ "	20.70	17.12
Heat restored, expansion....	b	$Qb \div Q$ .... "	−4.695	−5.83
Heat rejected, exhaust.......	c	$Qc \div Q$ .... "	−11.81	−4.628
Heat lost. compression.......	d	$Qd \div Q$........................ "	.1608	.6505
Heat utilized, work..........	w	$\dfrac{W}{778}\div Q$ .... "	5.41	5.68
Heat lost. radiation..........	r	$D \div Q$........................ "	4.37	6.015
Ratio radiation to work......		$r \div w$	.807	1.059
Ratio cylinder cond. to work		$a \div w$..................................	3.823	3.016
Thermo-dynamic efficiency..	E	$(t-t_2) \div (461 + t)$.........per cent	9.03	8.95
Actual efficiency.............	E_1	$\dfrac{W}{778}\div Q$.................... "	5.41	5.68
Efficiency comp'd with ideal.	E'	$E_1 \div E$ .... "	60.0	63.53

Special symbols Vc = volume clearance, t = measured temperature. Subscript 5 applies to exhaust, i to injection, k to discharge, g to air-pump discharge.

Correct for Mx when necessary.

APPLICATION OF HIRN'S ANALYSIS.

LOW-PRESSURE.

[FORM No. 1.]

DATA AND RESULTS.

Which engine—low-pressure cylinder.
Diameter of cylinder, 74 inches.
Length of stroke, 60 inches.
Diameter of piston-rods (2), 4 inches.
Volume of cylinder, crank end, 148.60.
Volume head end, 149.45.
Volume of clearance, cu. ft., head, 1.150.
Clearance in per cent of stroke, .77.
Volume of clearance, cu. ft., crank 1.144.
Clearance in per cent of stroke, .77.
Receiver-pressure by gauge, 1.3
Barometer, 14.5.
Revolutions per hour, 1218.8.
Steam used during run, pounds, 162,524.5.
Weight of condensed steam per hour, 6771.8.
Per stroke, head, 282.9.
Per stroke, crank, 273.5.
Temperatures of condensed steam, 102.
Temperatures of condensing water, cold, 40.
Hot, 98.
Pounds of condensing water, per hour, 99,600.
Per revolution, 81.8.

SYMBOLS.

To denote different portions of the stroke the following subscripts are used:

Admission (*a*); expansion (*b*); exhaust (*c*); compression (*d*).

To denote different events of the stroke, the following sub-numbers are used: Cut-off (1); release (2); compression, beginning of (3); admission, beginning of (0); in exhaust (5).

Quality of steam denoted by *x*.

Cut off, crank end, per cent of stroke, 39.1.
Cut-off, head end, per cent of stroke, 38.3.
Compression, crank end, per cent of stroke, .64.
Compression, head end, per cent of stroke, 3.13.
I. H. P., head, 116.23.
I. H. P., crank, 112.63.
Total I. H. P., 228.86.
Release, crank end, 98.8.
Release, head end, 98.9

LOW-PRESSURE.

[FORM No. 2.]

ABSOLUTE PRESSURE FROM INDICATOR-DIAGRAMS AND CORRESPONDING PROPERTIES OF SATURATED STEAM.

		Cut-off.	Release.	Beginning. Compression.	Beginning. Of Admission.	Symbols. Rankine.	Symbols. Clausius.
Subscripts used		1	2	3	0		
Absolute pressure	Head	13.3	5.3	1.8	16.2		
	Crank	13.9	5.3	1.5	16.5	P	p
Heat of liquid	Head	175.51	132.93	89.48	185.54		
	Crank	187.77	132.93	82.22	186.48	S	q
Internal latent heat	Head	897.81	931.37	963.80	889.96		
	Crank	896.09	951.37	971.68	889.23	I	ρ
Latent heat evaporation	Head	969.60	999.26	1029.47	962.56		
	Crank	868.01	999.26	1034.55	961.91	L	r
Total heat	Head	1145.11	1132.19	1118.95	1148.11		
	Crank	1145.78	1132.19	1116.77	1148.39	H	λ
Volume 1 lb. cubic feet	Head	28.96	69.24	202.5	240.06		
	Crank	27.78	69.24	251.1	23.66	C	v
Volumes symbols		$V_c + V_1$	$V_c + V_2$	$V_c + V_3$	$V_0 + V_c$	V_c	....
Volumes head, cubic feet		58 349	148.7	5.824	1.149	1.149	....
Volumes crank, cubic feet		59.949	148.7	2.105	1.149	1.149	....

MEAN PRESSURES AND HEAT-EQUIVALENTS OF EXTERNAL WORK.

	Subscripts.	Head End. Mean Pressures.	Head End. External Work. B. T. U.	Crank End. Mean Pressures.	Crank End. External Work. B. T. U.
Symbols		MEP	AW*	MEP	AW*
Admission	a	.5613	5.95	.713	7.64
Expansion	b	− 6.33	−108.54	− 6.38	−106.6
Exhaust	c	−12.66	−338.2	−12.19	−332.6
Compression	d	− 9.77	−8.515	− 7.548	−1.33
Total	....	....	244.125	....	234.97

* $A = \frac{1}{778}$. V_c = volume in clearance-spaces.

LOW-PRESSURE.

HIRN'S ANALYSIS—DATA AND RESULTS PER 100 STROKES.

JACKET-STEAM INCLUDED.

Quantities.	Symbols.	Formulæ.	Head.	Crank.
Steam from boiler, pounds...	M	....	282.9	273.5
Steam in clearance, pounds..	Mc	$100\,(Vc+CcXc)$....................	4.76	4.85
Steam at admission, pounds.	M_0	$100\frac{V+V_0}{C_0X_0}$..........................	4.76	4.85
Steam, total, pounds..........	$M+M_0$	....	287.66	278.35
Heat of condensed steam.....	K'	MSg	19,040	19,880
Condensing water, pounds ..	G	....	4,002	4,182
Heat given to cond. water...	K	$G(Sk-Si)$......................	232,100	242,400
Heat supplied to engine. ...	Q	$M(XL+S)$......................	324,600	314,000
Sensible heat at admission.	H_0	M_0S_0	835	911
Internal heat at admission.	H_0'	$100\frac{Vc+V_0}{C_0}I_0$	4,280	4,310
Sensible heat at cut-off......	H_1	$(M+M_0)S_1$......................	50,450	52.350
Internal heat at cut-off	H_1'	$100\frac{Vc+V_1}{C_1}I_1$	160,500	193,100
Sensible heat at release......	H_2	$(M+M_0)S_2$	38,180	37,000
Internal heat at release......	H_2'	$100\frac{Vc+V_2}{C_2}I_2$	199.900	199,900
Sensible heat, beginning of compression..	H_3	M_0S_3..............................	425.8	398.4
Internal heat, beginning of compression.............	H_3'	$100\frac{Vc+V_3}{C_3}I_3$......................	2,778	816
Cylinder loss during admission....................	Qa	$Q+H_0+H_0'-H_1-H_1'-\frac{Wa}{778}$	118,160	73,007
Cylinder loss during expan'n.	Qb	$H_1+H_1'-H_2-H_2'-\frac{Wb}{778}$.........	−16,276	19,210
Cylinder loss during exhaust.	Qc	$H_2+H_2'-H_3-H_3'-K-K'-\frac{Wc}{778}$..	−50,083.8	−59,854.4
Cylinder loss during compression	Qd	$H_3+H_3'-H_0-H_0'-Hx-Hx'-\frac{Wd}{778}$	−2,752.7	−4,139.6
Heat discharged, and work..	B	$K+K'+Hx+Hx'+\frac{W}{778}$..............	275,552.5	285,777
Jacket and radiation.........	D	$Q-B$	49,047.5	28,223
Jacket and radiation.........	D'	$Qa+Qb+Qc+Qd$..................	49,047.5	28,223
Quality of steam at cut-off...	X_1	$100\frac{Vc+V_1}{(M+M_0)C_1}$ ....per cent.	70.2	77.5
Quality of steam at release...	X_2	$100\frac{Vc+V_2}{(M+M_0)C_2}$ .... "	74.6	77.1
Quality of steam in exhaust..	X_5	$\left(\frac{K-K'}{M-Mx}-S_5\right)+L_5$ .... "	77.8	83.3
Heat lost, admission.........	a	$Qa+Q$. .... "	36.42	23.25
Heat restored, expansion	b	$Qb+Q$.................. "	−5.008	6.120
Heat rejected, exhaust.......	c	$Qc+Q$.................. "	−15.40	−19.10
Heat lost, compression.......	d	$Qd+Q$ .... "	.848	1.308
Heat utilized, work...........	w	$\frac{W}{778}+Q$.................. "	7.525	7.485
Heat lost, radiation.	r	$D+Q$ .... "	15.08	8.99
Ratio, radiation to work.....		$r+w$	1.995	1.201
Ratio, cylinder cond. to work		$a+w$	4.84	2.977
Thermo-dynamic efficiency..	E	$(t-t_3)+(461+t)$ ....per cent.	11.28	10.70
Actual efficiency.............	E_1	$\frac{W}{778}+Q$ .... "	7.525	7.485
Efficiency comp'd with ideal.	E'	E_1+E.................. "	67.8	70.01

Special symbols Vc = volume-clearance, t = measured temperature. Subscript 5 applies to exhaust, i to injection, k to discharge, g to air-pump discharge.

Correct for Mx when necessary.

APPENDIX D.

COMPLETE THERMAL ANALYSIS.

JACKET-HEAT, RADIATION LOSSES, AND HEAT REJECTED FROM EACH CYLINDER PER 100 REVOLUTIONS.

Total heat admitted to high-pressure cylinder..	598,800	B. T. U.
Total heat used in all jackets, assuming ⅓ weight of jacket-steam to be used in each jacket.....	40,870	"
Total heat used	639,670	"
Heat rejected from low-pressure cylinder......	513,420	"
Total work done	119,652	"
Total radiation loss..........................	6,598	"
	639,670	"
Radiation loss from each cylinder assumed to be equal in all cylinders........................	2,199	"

Heat rejected from high-pressure cylinder = heat entering + heat supplied by jacket − radiation loss − work done

$$= 598,800 + 13,360 - 2199 - 36,456$$
$$= 573,505 \text{ B. T. U.}$$

Heat rejected from intermediate-pressure cylinder = heat rejected from high-pressure cylinder + heat supplied by jacket − radiation loss − work done

$$= 573,505 + 13,360 - 2199 - 35,286$$
$$= 549,380 \text{ B. T. U.}$$

Heat rejected from low-pressure cylinder = heat rejected from intermediate-pressure cylinder + heat supplied by jacket − radiation loss − work done

$$= 549,380 + 14,150 - 2200 - 47,910$$
$$= 513,420 \text{ B. T. U.}$$
$= K + K'$ (see low-pressure analysis).

HIRN'S ANALYSIS—DATA AND RESULTS PER 100 REVOLUTIONS. HIGH-PRESSURE CYLINDER (JACKET-STEAM EXCLUDED) WITH FIRST RECEIVER.

Quantities.	Symbols.	Formulæ.	
1. Steam from boiler entering working cylinder, pounds...	M	....	504.5
2. Steam in clearance, pounds.....	Mc	$100\,(Vc + CcXc)$....................	18.158
3. Steam at admission, pounds ...	M_0	$100\,(Vc + Vc + v_0$....................	....
4. Steam used by calorimeter, pds.	....	....	....
5. Steam, total, pounds	$M + M_0$	....	522.658
6. Heat of condensed steam........	K'	Mqg....................................	....
7. Condensed water, pounds.	G	....	....
8. Heat given to condensing water	K	$G(qk - ql)$ (Heat rejected.)........	573.505
9. Heat supplied to engine	Q	$M(xr + q)$..............................	596.800
10. Sensible heat at admission......	H_0	M_0q_0....................................	5,819
11. Internal heat at admission......	H_0'	$100\,\frac{Vc + V_0}{v_0}\rho_0$	14,270
12. Sensible heat at cut-off	H_1	$(M + M_0)q_1$...........................	167,400
13. Internal heat at cut-off..........	H_1'	$100\,\frac{Vc + V_1}{v_1}\rho_1$........................	356,900
14. Sensible heat at release.........	H_2	$(M + M_0)q_2$...........................	128,700
15. Internal heat at release	H_2'	$100\,\frac{Vc + V_2}{v_2}\rho_2$	411,300
16. Sensible heat, beginning of compression...................	H_3	M_0q_3	4,527
17. Internal heat, beginning of compression...................	H_3'	$100\,\frac{Vc + V_3}{v_3}\rho_3$	15,700
18. Cylinder loss during admission..	Qa	$Q+H_0+H_0'-H_1-H_1'-AWa.$ ...	+62,629
19. Cylinder loss during expansion..	Qb	$H_1+H_1'-H_2-H_2'-AWb$..........	−46.990
20. Cylinder loss during exhaust....	Qc	$H_2+H_2'-H_3-H_3'-K-K'-AWc$	−27,862
21. Cylinder loss during compres'n.	Qd	$H_3+H_3'-H_0-H_0'-AWd$..........	−1,064
22. Heat discharged, and work......	B	$K+K'+AW$	+609,961
*23. Jacket and loss..................	D	$Q-B$	−11,161
*24. Jacket and loss............	D'	$Qa+Qb+Qc+Qd$........................	−11,161
25. Quality of steam entering.......	x	per calorimeter .... per cent	98.95
26. Quality of steam at cut-off......	x_2	$100\,\frac{VcV_1}{(M+M_0)v_1}$............ "	86.77
27. Quality of steam at release......	x_2	$100\,\frac{Vc+V_2}{(M+M_0)v_2}$............ "	93.3
28. Quality of steam at compression	....	$100\,\frac{Vc+V_3}{M_0v_3}$................ "	....
29. Quality of steam at admission .	x_0	per calorimeter............ "	....
30. Quality of steam in exhaust.....	x_5	$\left(\frac{K+K'}{M-Mx}-q_5\right)\div r_5$....... "	96.8
31. Heat lost, admission	a	$Qa\div Q$........................ "	10.46
32. Heat restored, expansion.........	b	$Qb\div Q$........................ "	−7.86
33. Heat rejected, exhaust...........	c	$Qc\div Q$........................ "	−4.65
34. Heat lost, compression..........	d	$Qd\div Q$........................ "	.18
35. Heat utilized, work.............	w	$\frac{W}{778}\div Q$ .... "	6.087
36. Heat lost, radiation.............	R	radiation $\div Q$.............. "	.367
37. Ratio, radiation to work	....	$R\div w$..................................	.0603
38. Ratio, cylinder condensation to work	....	$a\div w$	1.72
39. Thermodynamic efficiency......	E	$(t-t_3)\div(460+t)$.... ....per cent	8.96
40. Actual efficiency	E_1	$AW\div Q$........... .... "	6.087
41. Efficiency compared with ideal..	E'	$E_1\div E$....................... "	68.0

Special symbols, Vc = volume clearance, t = measured temperature. Subscript 5 applies to exhaust, i to injection, k to discharge, g to air-pump discharge. $A = \frac{1}{778}$.

Correct for steam used by calorimeter, when necessary.

* This quantity is the difference between the heat lost by radiation and that received from the jacket. The negative sign shows that more heat is received than lost.

HIRN'S ANALYSIS—DATA AND RESULTS PER 100 REVOLUTIONS—INTERMEDIATE CYLINDER (JACKET-STEAM EXCLUDED) WITH SECOND RECEIVER.

Quantities.	Symbols.	Formulæ.	
1. Steam entering cylinder, pounds	M	....	504.5
2. Steam in clearance, pounds.....	Mc	$100\,(Vc+V_0)\div v_0$..........	22.33
3. Steam used by calorimeter, pds.	....	....	....
4. Steam, total, pounds............	$M+M_0$	....	526.83
5. Heat of condensed steam........	K'	Mqg..........	....
6. Heat rejected...................	G	....	549,380
7. Heat given to condensing water	K	$G(qk-qi)$..........	....
8. Heat supplied to engine	Q	$M(xr+q)$..........	573,505
9. Sensible heat at addmission.....	H_0	M_0q_0..........	5,534
10. Internal heat at admission......	H_0'	$100\dfrac{Vc+V_0}{v_0}\rho_0$..........	18,780
11. Sensible heat at cut-off..........	H_1	$(M+M_0)q_1$..........	125,600
12. Internal heat at cut-off..........	H_1'	$100\dfrac{Vc+V_1}{v_1}\rho_1$..........	381,400
13. Sensible heat at release	H_2	$(M+M_0)q_2$..........	95,300
14. Internal heat at release	H_2'	$100\dfrac{Vc+V_2}{v_2}\rho_2$..........	434,100
15. Sensible heat, beginning of compression	H_3	M_0q_3..........	4,201
16. Internal heat, beginning of compression...............	H_3'	$100\dfrac{Vc+V_3}{v_3}\rho_3$..........	18,420
17. Cylinder loss during admission.	Qa	$Q+H_0+H_0'-H_1-H_1'-A\,Wa$.....	67,439
18. Cylinder loss during expansion..	Q	$H_1+H_1'-H_2-H_2'-A\,Wb$.........	−35,755
19. Cylinder loss during exhaust,...	Qc	$H_2+H_2'-H_3-H_3'-K-K'-A\,Wc$	−41,465
20. Cylinder loss during compression	Qd	$H_3+H_3'-H_0-H_0'-A\,Wd$.........	−1,380
21. Heat discharged, and work.....	B	$K+K'+A\,W$..........	584,666
*22. Jacket and loss.................	D	$Q-B$..........	−11,161
*23. Jacket and loss.................	D'	$Qa+Qb+Qc+Qd$..........	−11,161
24. Quality of steam entering.......	x	per calorimeter ....per cent	....
25. Quality of steam at cut-off......	x_1	$100\dfrac{VcV_1}{(M+M_0)v_1}$.......... "	85.4
26. Quality of steam at realease.....	x_2	$100\dfrac{Vc+V_2}{(M+M_0)v_2}$.......... "	93.2
27. Quality of steam at compression	....	$100\dfrac{Vc+V_3}{M_0v_3}$.......... "	....
28. Quality of steam at admission ..	x_0	per calorimeter.......... "	....
29. Quality of steam in exhaust.....	x_5	$\left(\dfrac{K+K'}{M-Mx}-q_5\right)\div r_5$.......... "	94.0
30. Heat lost, admission.	a	$Qa\div Q$.......... "	11.75
31. Heat restored, expansion	b	$Qb\div Q$.......... "	−6.23
32. Heat rejected, exhaust..........	c	$Qc\div Q$.......... "	−7.23
33. Heat lost, compression	d	$Qd\div Q$.......... "	−0.24
34. Heat utilized, work............	w	$\dfrac{W}{778}\div Q$.......... "	6.155
35. Heat lost, radiation	R	Radiation$\div Q$.......... "	.384
36. Ratio, radiation to work	....	$R\div w$..........	.0623
37. Ratio, cylinder condensation to work.....................	....	$a\div w$..........	1.795
38. Thermodynamic efficiency......	E	$(t-t_3)\div(460+t)$..........per cent	8.99
39. Actual efficiency...............	E_1	$A\,W\div Q$.......... "	6.155
40. Efficiency compared with ideal..	E'	$E_1\div E$.......... "	68.55

Special symbols, $Vc =$ volume clearance, t measured temperature. Subscript 5 applies to exhaust, i to injection, k to discharge, g to air-pump discharge. $A = \dfrac{1}{778}$.

Correct for steam used by calorimeter, when necessary.

* Same explanation as for high-pressure cylinder. Radiation assumed equal for all three cylinders, and heat received from jackets assumed equal in high and intermediate pressure cylinders.

HIRN'S ANALYSIS—DATA AND RESULTS PER 100 REVOLUTIONS. LOW-PRESSURE CYLINDER (JACKET-STEAM EXCLUDED).

Quantities.	Symbols.	Formulæ.	
1. Steam entering working cylinder, pounds	M	....	504.5
2. Steam in clearance, pounds	Mc	$100\,(Vc+V_0)+v_0$	9.61
3. Steam used by calorimeter, pds.	....	....	....
4. Steam, total, pounds	$M+M_0$	....	514.11
5. Heat of condensed steam	K'	Mqg	38,920
6. Condensed water, pounds	G	....	8,184
7. Heat given to condensing water.	K	$G(qk-qi)$	474,500
8. Heat supplied to engine.	Q	$M(xr+q)$	549,380
9. Sensible heat at admission	H_0	M_0q_0	1,746
10. Internal heat at admission	H_0'	$100\,\frac{Vc+V_0}{v_0}\rho_0$	8,590
11. Sensible heat at cut-off	H_1	$(M+M_0)q_1$	93,450
12. Internal heat at cut-off	H_1'	$100\,\frac{Vc+V_1}{v_1}\rho_1$	353,600
13. Sensible heat at release	H_2	$(M+M_0)q_2$	68,300
14. Internal heat at release	H_2'	$100\,\frac{Vc+V_2}{v_2}\rho_2$	399,800
15. Sensible heat, beginning of compression	H_3	M_0q_3	824.2
16. Internal heat, beginning of compression	H_3'	$100\,\frac{Vc+V_3}{v_3}\rho_3$	3,594
17. Cylinder loss during admission	Qa	$Q+H_0+H_0'-H_1-H_1'-AWa$	11,307
18. Cylinder loss during expansion	Qb	$H_1+H_1'-H_2-H_2'-AWb$	464
19. Cylinder loss during exhaust	Qc	$H_2+H_2'-H_3-H_3'-K-K'-AWc$	−116,818.2
20. Cylinder loss during compress'n.	Qd	$H_3+H_3'-H_0-H_0'-AWd$	−6,902.8
21. Heat discharged, and work	B	$K+K'+AW$	561,330
*22. Jacket and loss	D	$Q-B$	−11,950
*23. Jacket and loss	D'	$Qa+Qb+Qc+Qd$	−11,950
24. Quality of steam entering	x	per calorimeter .... per cent	....
25. Quality of steam at cut-off	x_1	$100\,\frac{VcV_1}{(M+M_0)v_1}$ "	81.15
26. Quality of steam at release	x_2	$100\,\frac{Vc+V}{(M+M_0)v_2}$ "	83.33
27. Quality of steam at compress'n.	....	$100\,\frac{Vc+V_3}{M_0v_3}$ "	....
28. Quality of steam at admission	x_0	per calorimeter .... "	....
29. Quality of steam in exhaust	x_5	$\left(\frac{K+K'}{M-Mx}-q_5\right)+r_5$ "	89.95
30. Heat lost. admission	a	$Qa+Q$ "	2.056
31. Heat restored, expansion	b	$Qb+Q$ "	.084
32. Heat rejected, exhaust	c	$Qc+Q$ "	−21.22
33. Heat lost, compression	d	$Qd+Q$ "	−1.256
34. Heat utilized, work	w	$\frac{W}{778}+Q$ "	8.7
35. Heat lost, radiation	R	Radiation$+Q$ "	.40
36. Ratio, radiation to work	....	$R+w$	.046
37. Ratio, cylinder condensation to work	....	$a+w$	.236
38. Thermo-dynamic efficiency	E	$(t-t_3)+(460+t)$ .... per cent	10.98
39. Actual efficiency	E_1	$AW+Q$ "	8.70
40. Efficiency compared with ideal.	E^1	E_1+E "	79.18

Special symbols, Vc = volume clearance, = t measured temperature. Subscript 5 applies to exhaust, i to injection, k to discharge, g to air-pump discharge. $A = \frac{1}{778}$.
Correct for steam used by calorimeter, when necessary.

* Same explanation as for high-pressure cylinder.

§ 37, p. 177.—An engine similar to that illustrated in Fig. 84 has been found to have the highest economy recorded to date, 1893. The following figures were obtained by Professor Carpenter in April, 1893, and reported by the Author to the American Society of Mechanical Engineers in December, 1893:

Foot-pounds of work for 100 lbs. dry coal	143,306,470
" " " " " " wet "	135,770,000
" " " " " " combustible	145,438,000
" " " " 1,000 lbs. feed-water	152,448,000
" " " " 1,000 " dry steam	154,048,000
" " " " 1,000,000 B. T. U.	137,656,000
" " " " 1 cwt. coal (112 lbs.)	152,630,000
Kilogrammeters of work per kilo of coal	429,110

While the thermodynamic results are:

Thermodynamic efficiency		0.194
Heat per I. H. P. per hour, and per minute, B. T. U.	13,056;	217.6
Steam per I. H. P. per hour, lbs		11.678
Fuel per I. H. P. per hour, lbs		1.237
Heat per D. H. P. per hour, and per minute, B. T. U.	14,382;	239.7
Steam per D. H. P. per hour, lbs		12.864
Fuel per D. H. P. per hour, lbs		1.364

The above-given efficiency is 0.668, or two thirds, that of a Carnot cycle working through the same range of temperatures and pressures. It is equal to $0.194 \div 0.252 = 0.77$ of the thermodynamic efficiency for the Rankine cycle of the ideal case.

APPENDIX E.

TEST OF QUADRUPLE-EXPANSION PUMPING-ENGINE, WILDWOOD STATION, PITTSBURG, PA., MARCH 1, 1899 (SIBLEY COLLEGE).

PLATE II AND FIG. 61*a*, p. 210, Vol. II.

DIMENSIONS.

Number of steam cylinder	I	III	II	IV
Length of stroke.....inches	42	42	42	42
Diameter of steam cylinder.... "	19.5	49.5	29.0	57.5
Diameter of piston rod, top.... "	3.625	......	3.625	....
Diameter of piston rod, bottom "	4.5	3.625	4.5	3.625
Net area pistons, top side . .sq. in.	288.327	1,924.43	650.2	2,596.73
Net area pistons, bottom side.. "	282.744	1,914.11	644.6	2,586.40
H.P. per 1 lb. M.E.P., 36.5 revs., top, H.P.	1.1162	7.499	2.517	10.052
H.P. per 1 lb. M.E.P., 36.5 revs., bottom H.P.	1.095	7.410	2.496	10.013
Clearance on steam cylinders........ per cent.	1.25	0.55	1.30	0.36

Pump cylinders	A	B
Length of stroke.....inches	42	42
Diameter of plunger..... "	14.75	14.75
Diameter of rod..... "	4.50	4.50

Cylinder displacement per revolution, cubic feet	15.8396
" " " " gallons	118.383
Weight of water pumped per revolution, pounds	988.65
Elevation, top of tunnel above sea level, feet	704
" bottom " " "	698
" bedplate of engine, bottom level, feet	708.21
" centre of gauge on force main, "	717.3
" bedplate of engine, top "	710.63
" centre of gauge on suction main, "	717.98
" weir notch, by specifications, "	1,310.00
" " per James Harlow, "	1,308.92
" " by breadth	3.14

PRESSURES.

Steam Pressures.

Boiler pressure, gauge, pounds per square inch	199.87
Barometric pressure, inches	28.00
" " pounds	14.19
Absolute pressure in boiler, pounds	214.00
Vacuum gauge, inches	26.60
Jacket No. 1, gauge, pounds	199.81

Mr. J. B. Stanwood has compared the gain of the theoretically perfect Nordberg cycle (expansion in four stages) over a perfect Rankine cycle, operating under same range of temperature, with the gain obtained actually in practice as reported.*

By means of the temperature-entropy diagram (Fig. 206), which shows graphically the transfer of heat to and from 1 pound of H_2O as it makes one complete cycle, we obtain the following results: Assuming the highest temperature of H_2O, corresponding to the boiler pressure of 214 pounds absolute, to be 387 degrees Fahr., equal to 848 degrees absolute, and lowest temperature to be 104 degrees Fahr., equal to 565 degrees absolute, corresponding to pressure in condenser of 26.6 inches below a barometric pressure of 28.9 inches of mercury or $1\frac{1}{8}$ pounds absolute, it follows that the efficiency for the Rankine cycle is .3038; and for the Nordberg cycle is .3284, where the receiver temperatures absolute are 788 degrees, 726 degrees, and 668 degrees, corresponding respectively to receiver pressures (as given) of 99.6 pounds, 38.9 pounds, and 13.4 pounds absolute.

The Carnot efficiency for same range of temperature is .3337.

The theoretical gain for the Nordberg over the Rankine cycle in this case is $\frac{.3284 - .3038}{.3038} = 8\frac{1}{10}$ per cent nearly.

In the test the actual gain was about 10 per cent, or nearly 25 per cent more than the above would warrant.

By means of progressive feed-water heaters receiving heat from the receiver of a multi-cylinder expansion engine, a reduction of the free expansion loss at the end of expansion in the low-pressure cylinder can be effected, securing a gain in addition to that obtained by the employment of the better cycle.

The terminal pressure measured on combined diagram as published, without heaters, is nearly 10 pounds absolute. The same measured with heaters is nearly 8 pounds absolute.

The free expansion loss, in the first case, to a condenser pressure of $1\frac{1}{8}$ pounds absolute is 56 thermal units. In the second case the loss is 45 thermal units per pound of H_2O, or a difference of 11 British thermal units. When heaters are employed these heat-units are not drawn from effective work,

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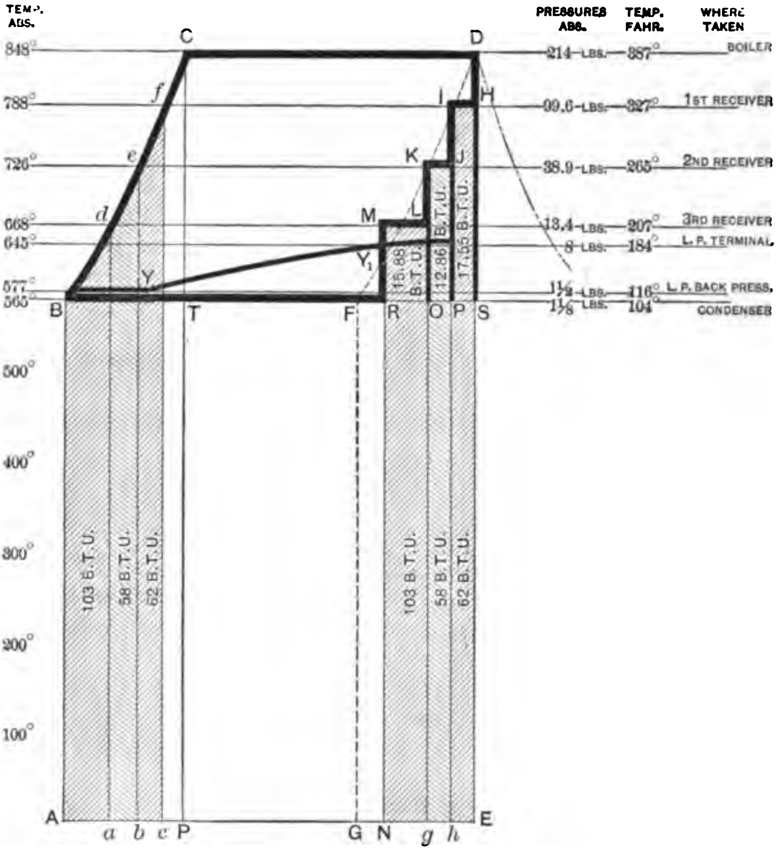
* Trans. A. S. M. E., 1899, vol. XXI., No. 832.

TEMPERATURE-ENTROPY DIAGRAM,

SHOWING TRANSFER OF HEAT FROM RECEIVERS TO FEED-WATER.



Efficiency.

Rankine cycle:
B.T.U. in *BCDS* = 333; in *ABCDE* = 1096... .3038
Carnot cycle:
B.T.U. in *TCDS* = *BCDF* = 283; in *PCDE* = *ABCDFG* = 848... .3337
Nordberg cycle:
In *BCDHIJKLMR* = 287; in *cf CDE* = *ABCDHIJKLMN* = 813... .3284

FIG. 206.—TEMPERATURE-ENTROPY DIAGRAM.

but from heat otherwise rejected, and are utilized in raising the temperature of the feed-water. The heat-units withdrawn from work in low-pressure, second, and first intermediate

cylinders are respectively 15.9, 12.8, and 17.6 thermal units, or a total of 46.3 thermal units. But on account of incomplete expansion, 11 thermal units of this can be secured from otherwise rejected steam, and the deduction from effective work can be reduced about 23 per cent. The space between the base line *BS* and the line BYY_1 shows loss of heat by free expansion in low-pressure cylinder.

The full figure *ABCDE* represents the total thermal units delivered to 1 pound of H_2O in raising it from a feed-water temperature of 104 degrees Fahr. and converting it into steam at 210 pounds absolute pressure, or 1096 thermal units. Area *BCDS* represents the greatest amount of work, in thermal units, that can be obtained by a perfect Rankine cycle under these conditions.

The value in this case is 333 thermal units.

The ratio of areas *BCDS* to *ABCDE* is the efficiency of the cycle = .3038.

In the cycle this heat must be drawn from the receivers to heat the feed-water to the receiver temperatures. To raise the feed from 104° Fahr. to 207° takes 103 thermal units; from 207° to 265° takes 58 thermal units; from 265° to 327° takes 62 thermal units.

These amounts are measured by *ABda*, *adeb*, and *befc*.

If heat for this purpose is transferred from expanding steam, equivalent amounts must be taken from the receivers, as shown by rectangles on the right of the diagram. The work sacrificed by this transfer, for the low-pressure, second, and first intermediate cylinders, is 15.9, 12.8, and 17.6 thermal units respectively, shown by rectangles *RMLO*, *OKJP*, and *PIHS*.

The values obtained for these areas are:

Work lost in low-pressure cylinder $= \frac{103}{668} \times 103$ thermal units = 15.9 thermal units.

Work lost in second intermediate cylinder $= \frac{161}{726} \times 58$ thermal units = 12.8 thermal units.

Work lost in first intermediate cylinder $= \frac{223}{788} \times 62$ thermal units = 17.6 thermal units.

The maximum work of the cycle then equals the Rankine cycle, less the sum of the above; or is 333 − (15.9

$+ 12.8 + 17.6) = 286.7$ thermal units, represented by area *BCDHIJKLMR*. The heat applied will be that applied in the Rankine cycle, less the amount necessary to raise feed-water to temperature of first receiver, or $1096 - 223$ thermal units $= 837$ thermal units, and is represented by the area *cfCDE* = area *ABCDHIJKLMN*.

The efficiency will be $\dfrac{286.7}{873} = .3284$, which is greater than the Rankine efficiency.

The Carnot efficieney is the ratio between rectangles *TCDS*, or the equivalent area *BCDF*, and the rectangle *PCDE*, which is the same as the well-known relation $\dfrac{T - T_1}{T}$, or, in this case, $\dfrac{848 - 565}{848} = .3337$.

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